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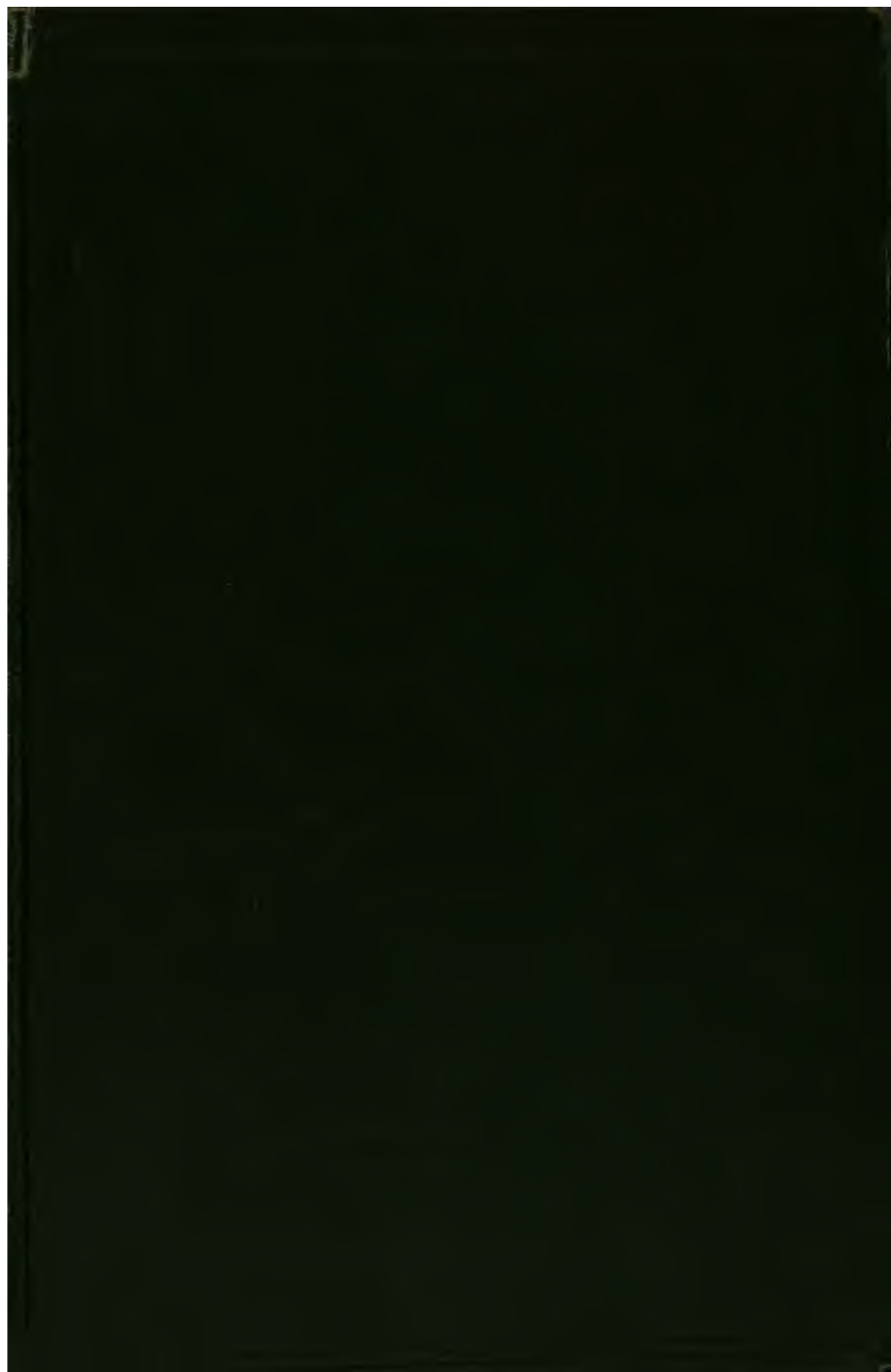
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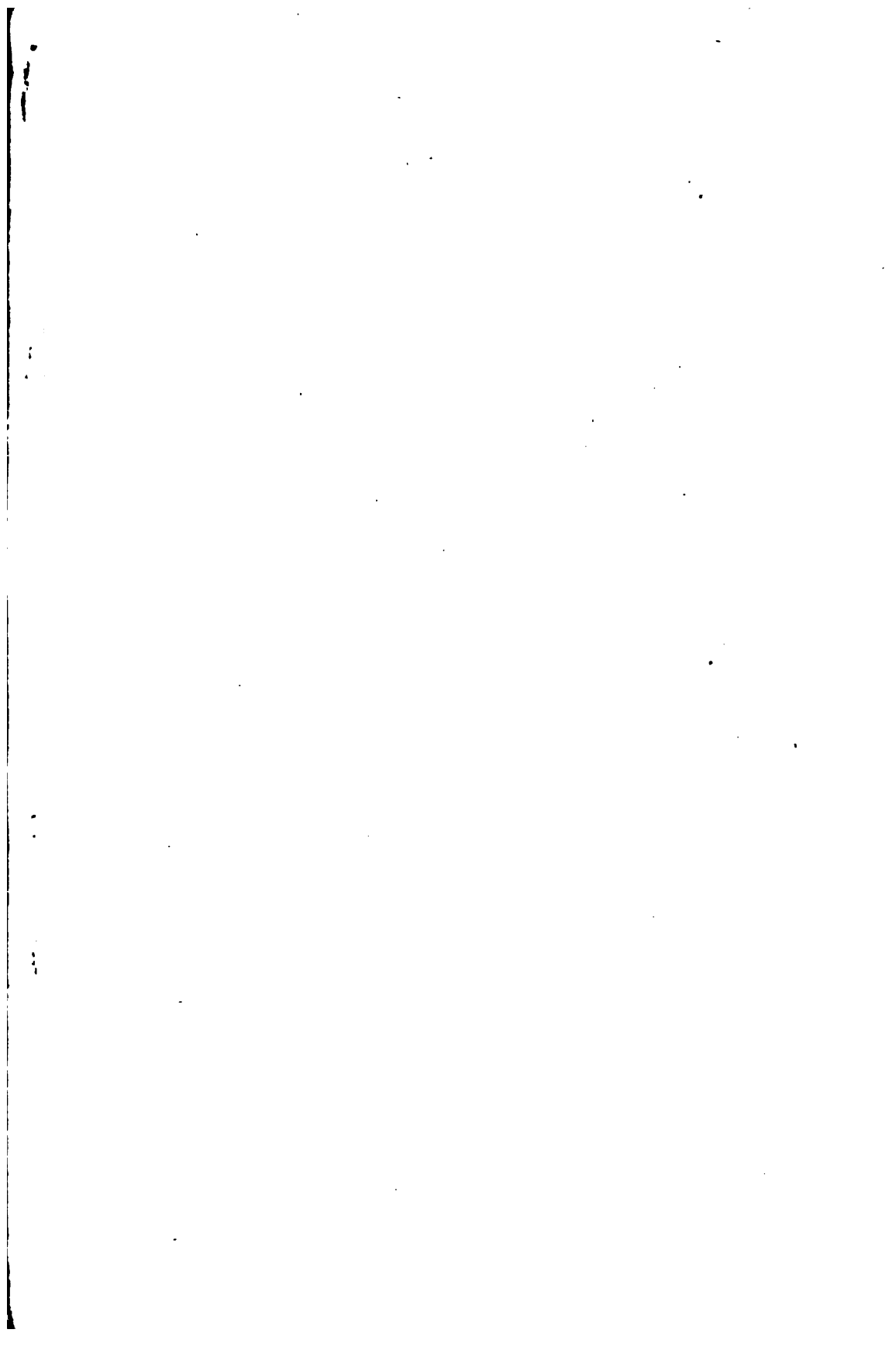
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TEXT BOOK
OF
ENGINEERING THERMODYNAMICS

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A TEXT BOOK
OF
ENGINEERING THERMODYNAMICS

BY
CHARLES EDWARD LUCKE, PH.D.,
Professor of Mechanical Engineering in Columbia University, New York City
AND
JOHN J. FLATHER, PH.B., M.M.E.,
Professor of Mechanical Engineering in the University of Minnesota, Minneapolis

AN ABRIDGMENT OF
ENGINEERING THERMODYNAMICS

BY
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PREFACE

THIS Textbook of Engineering Thermodynamics has been prepared to meet the requirements of technical schools desiring a briefer treatment of the subject than that contained in the original "Engineering Thermodynamics," by Charles E. Lucke, of which this book is an abridgment.

Since Rankine's time the science of thermodynamics has been highly developed and has become of great importance in the formulation of modern physical chemistry and its correlated branches in engineering. Thermodynamics, *per se*, is not concerned with any physical substance, it is rather a theory of energy in relation to matter.

Engineering thermodynamics, while making use of those principles of pure thermodynamics which may help to solve its problems, must rely on a great mass of facts or relations that have not attained the dignity of thermodynamic laws. Its field includes a portion of that of pure thermodynamics, but it extends far beyond the established provinces of that subject and reaches to the interpretation of all pertinent principles and facts for purely useful purposes. One of the most promising applications of engineering thermodynamics is to be found in the establishment of limits of possible performance of heat apparatus and machines. These limits show what might be expected of a steam engine, gas engine or refrigerating machine when its mechanism is quite perfect; thus they become standards of reference, and a measure of improvements yet possible. These methods and practices are also applicable to the analysis of the operating performance of complete plants to discover the amount of energy being lost, how the total amount is divided between the different elements of the apparatus, which of the losses can be prevented and how, and finally which are unavoidable. In this book the treatment has followed that of the larger work, based upon the application of the laws of pure thermodynamics, modified by conditions of practice, to guide computation on thermal problems which deal with physical substances under actual conditions of operation.

The subject is divided into three general parts: Part I deals with the conditions surrounding the doing of work without any consideration of heat changes; Part II, with heat gains and losses by substances without reference to work involved; and Part III, transformation of heat into work or work into heat in conjunction with changes in the condition of substances. The first part applies to the behavior of fluids in the cylinders of compressors and engines. The second part is concerned with the development of heat

by combustion, its transmission from place to place, and the effect on the physical condition of solids, liquids and gases with their mixtures, solutions and reactions. The third part is fundamental to the efficient production of power by gases in internal combustion gas engines or compressed-air engines, and by steam or other vapors in steam engines and turbines, and likewise to the production of mechanical refrigeration by ammonia, carbon dioxide and other vapors.

The nineteen chapters of the book treat these three Parts accordingly. The first six chapters deal with work without any particular reference to heat; the next six chapters, with heat, without any particular reference to work; while the last seven are concerned with the relation between heat and work.

After establishing in the first chapter the necessary units and basic principles governing work, the second chapter proceeds to the determination of the work done in compressor cylinders, which is followed by a discussion of the available work in engine cylinders in terms of all the different variables that may determine the work for given dimensions of cylinder or for given quantities of fluid. Chapters VII to IX, Part II, are devoted to the qualitative and quantitative heat content of substances and their physical-chemical state; the remaining portion of Part II relates to heating by combustion and discusses fuels, furnaces, gas-producers and steam boilers. In Part III the general relations between heat and work are presented, and the thermal efficiencies of steam, gas, and compressed-air engines are deduced. The flow of expansive fluids and the performance of refrigerating systems are discussed in the last two chapters.

Throughout the entire work there have been established a series of working formulas derived from a few simple principles having in view maximum clearness and utility, and the resulting equations have been presented in such form as to be readily available for numerical substitution, either directly or by the use of derived charts. This permits the solution of quite involved thermodynamic problems with very little labor or time, although it has necessarily required the expansion of the subject over a considerable number of pages, but it is confidently believed that the saving of time thus gained in facilitating numerical solutions more than justifies the presentation.

The book treats in the same consistent manner all the important related subjects which are frequently administered in a technical school under different courses. It is hardly expected that teachers of Thermodynamics will use the entire text; rather it is assumed that those portions will be selected which will best articulate with other related subjects, serving more or less as a basis for them. In this way the present text can be made to supplement courses on gas power, compressed air, steam engines and turbines, steam boilers and power plants, refrigeration, chemical engineering, laboratory practice and research.

The text includes many tables and diagrams which are essential to the proper presentation of the subject, but in order to save space most of the

tables and working charts which appear in the original work on Engineering Thermodynamics have been omitted from the present book. These with some additions are published in a separate volume as a Handbook of Thermodynamic Tables and Diagrams.

The authors desire to express their obligations to Mr. E. D. Thurston, Jr., whose invaluable help is gratefully acknowledged, and to Mr. T. M. Gunn for aid on part of the work.

C. E. LUCKE.
J. J. FLATHER.

December, 1914.

CONTENTS

PART I

WORK DETERMINATIONS INDEPENDENT OF THERMAL CONDITIONS

CHAPTER I. WORK AND POWER. GENERAL PRINCIPLES

SECTION	PAGE
1. Work defined.....	1
2. Power defined.....	2
3. Work in terms of pressure and volume.....	3
4. Work of acceleration and resultant velocity.....	5
5. Graphical representation of work.....	7
6. Work by pressure volume change.....	8
7. Work of expansion and compression.....	11
8. Values of exponent s defining special cases of expansion or compression.....	17
9. Work phases and cycles, positive, negative and net work.....	20
10. Work determination by mean effective pressure.....	27
11. Relation of pressure-volume diagrams to indicator cards.....	29
12. To find the clearance.....	31
13. Measurement of areas of PV diagrams and indicator cards.....	34
14. Indicated horse-power.....	36
15. Effective horse-power, brake horse-power, friction horse-power, mechanical efficiency, efficiency of transmission, thermal efficiency.....	37
16. Specific displacement, quantity of fluid per hour, or per minute per I.H.P.....	40
17. Velocity due to free expansion by PV method.....	42
18. Weight of flow through nozzles by PV method.....	44
19. Horse-power of nozzles and jets, by PV method.....	46

CHAPTER II. WORK OF COMPRESSORS. HORSE-POWER AND CAPACITY OF AIR, GAS AND VAPOR COMPRESSORS, BLOWING ENGINES AND DRY VACUUM PUMPS

20. General description of structures and processes.....	48
21. Standard reference diagrams or PV cycles for compressors and methods of analysis of compressor work and capacity.....	50
22. Single-stage compressor, no clearance, isothermal compression, Cycle I. Work, capacity, and work per cubic foot in terms of pressures and volumes.....	56
23. Single-stage compressor with clearance, isothermal compression, Cycle II. Work, capacity, and work per cubic foot in terms of pressures and volumes.....	59
24. Single-stage compressor, isothermal compression. Capacity, volumetric efficiency, work, mean effective pressure, horse-power and horse-power per cubic foot of substance, in terms of dimensions and cylinder clearance.....	61
25. Single-stage compressor, no clearance, exponential compression, Cycle III. Work, capacity and work per cubic foot, in terms of pressures and volumes.....	64
26. Single-stage compressor with clearance, exponential compression, Cycle IV. Work, capacity, and work per cubic foot in terms of pressures and volumes.....	68

SECTION	PAGE
27. Single-stage compressor, exponential compression. Relation between capacity, volumetric efficiency, work, mean effective pressure, horse-power and horse-power per cubic foot of substance and the dimensions of cylinder and clearance..	70
28. Two-stage compressor, no clearance, perfect intercooling, exponential compression, best-receiver pressure, equality of stages, Cycle V. Work and capacity in terms of pressures and volumes.....	74
29. Two-stage compressor, with clearance, perfect intercooling exponential compression, best-receiver pressure, equality of stages, Cycle VI. Work and capacity in terms of pressures and volumes.....	78
30. Two-stage compressor, <i>any receiver pressure</i> , exponential compression. Capacity, volumetric efficiency, work, mean effective pressure and horse-power, in terms of dimensions of cylinders and clearances.....	81
31. Two-stage compressor, <i>with best-receiver pressure</i> , exponential compression. Capacity, volumetric efficiency, work, mean effective pressure and horse-power in terms of dimensions of cylinders and clearances.....	87
32. Three-stage compressors, Cycles VII and VIII. Formulas for work and horse-power. Best two receiver pressures.....	91
 CHAPTER III. COMPRESSOR CHARACTERISTICS. ECONOMY. MAXIMUM WORK. WORK AT FRACTIONAL LOADS	
33. Comparative economy or efficiency of compressors.....	96
34. Conditions of maximum work of compressors.....	98
35. Compressor characteristics.....	100
36. Work at partial capacity in compressors of variable capacity.....	107
 CHAPTER IV. WORK OF SIMPLE PISTON ENGINES. STANDARD REFERENCE DIAGRAM FOR SINGLE-CYLINDER ENGINES. HORSE-POWER AND CONSUMPTION OF SIMPLE ENGINES	
37. Action of fluid in single cylinders. General description of structure and processes..	115
38. Standard reference cycles or <i>PV</i> diagrams for the work of expansive fluids in a single cylinder. Simple engines.....	120
39. Work of expansive fluid in single cylinder without clearance. Logarithmic expansion, Cycle I. Mean effective pressure, horse-power and consumption of simple engines.....	125
40. Work of expansive fluid in single cylinder without clearance. Exponential expansion, Cycle II. Mean effective pressure, horse-power and consumption of simple engines.....	130
41. Work of expansive fluid in single cylinder with clearance. Logarithmic expansion and compression, Cycle III. Mean effective pressure, horse-power, and consumption of simple engines.....	134
42. Work of expansive fluid in single cylinder with clearance; exponential expansion and compression, Cycle IV. Mean effective pressure, horse-power and consumption of simple engines.....	141
 CHAPTER V. WORK DISTRIBUTION IN MULTI-EXPANSION PISTON ENGINES	
43. Action of fluid in multiple-expansion cylinders. General description of structure and processes.....	148
44. Standard reference cycles or <i>PV</i> diagrams for the work of expansive fluids in two-cylinder compound engines.....	157
45. Compound engine with infinite receiver. Logarithmic law. No clearance, Cycle V. General relations between pressures, dimensions, and work.....	173

CONTENTS

xi

SECTION	PAGE
46. Compound engine with infinite receiver. Exponential law. No clearance, Cycle VI. General relations between pressures, dimensions and work.....	182
47. Compound engine with finite receiver. Logarithmic law. No clearance, Cycle VII. General relations between dimensions and work when H.P. exhaust and L.P. admission are not coincident.....	186
48. Compound engine with finite receiver. Exponential law, no clearance, Cycle VIII. General relations between pressures, dimensions, and work, when high pressure exhaust and low-pressure admission are independent.....	195
49. Compound engine without receiver. Logarithmic law. No clearance, Cycle IX. General relations between dimensions and work when high-pressure exhaust and low-pressure admission are coincident.....	199
50. Compound engine without receiver. Exponential law. No clearance, Cycle X. General relations between dimensions and work when high-pressure exhaust and low-pressure admission are coincident.....	205
51. Compound engine with infinite receiver. Logarithmic law, with clearance and compression, Cycle XI. General relations between pressures, dimensions and work.....	207
52. Compound engine with infinite receiver. Exponential law, with clearance and compression, Cycle XII. General relations between pressures, dimensions and work.....	218
53. Compound engine with finite receiver. Logarithmic law, with clearance and compression, Cycle XIII. General relations between pressures, dimensions, and work when H.P. exhaust and L.P. admission are independent.....	222
54. Compound engine with finite receiver. Exponential law, with clearance and compression, Cycle XIV. General relations between pressures, dimensions, and work when H.P. exhaust and L.P. admission are independent.....	229
55. Compound engine without receiver. Logarithmic law, with clearance and compression, Cycle XV. General relations between pressures, dimensions, and work when H.P. exhaust and L.P. admission are coincident.....	232
56. Compound engine without receiver. Exponential law, with clearance and compression, Cycle XVI. General relations between pressures, dimension, and work, when H.P. exhaust and L.P. admission are coincident.....	237
57. Triple-expansion engine with infinite receiver. Logarithmic law. No clearance, Cycle XVII. General relations between pressures, dimensions and work.....	240

CHAPTER VI. PISTON ENGINES. WORK, POWER AND CONSUMPTION. GRAPHICAL METHODS

58. Multiple-expansion engine. General case. Any relation between cylinders and receiver. Determination of pressure-volume diagram and work.....	246
59. Mean effective pressure, engine horse-power, and work distribution and their variation with valve movement and initial pressure. Diagram distortion and diagram factors. Mechanical efficiency.....	252
60. Consumption of steam engines and its variation with valve movement and initial pressure. Best cut-off as affected by condensation and leakage.....	259
61. Variation of steam consumption with engine load. The Willans line. Most economical load for more than one engine and best load division.....	267
62. Graphical solution of problems on engine horse-power and cylinder sizes.....	272

PART II

*EFFECT OF CHANGE IN HEAT CONTENT OF SUBSTANCES*CHAPTER VII. EFFECTS OF CHANGES IN HEAT CONTENT OF SUBSTANCES WITHOUT
CHANGE OF STATE. TEMPERATURE, PRESSURE, DENSITY

SECTION	PAGE
63. Substances and heat effects important in engineering.	274
64. Classification of heating processes. Heat addition and abstraction with or without temperature change, qualitative relations.	277
65. Thermometry based on temperature change, heat effects. Thermometer and absolute temperature scales.	283
66. Calorimetry based on proportionality of heat effects to heat quantity. Units of heat and mechanical equivalent.	288
67. Temperature change relation to amount of heat for solids, liquids, gases and vapors not changing state. Specific heats.	291
68. Volume or density variation with temperature of solids, liquids, gases and vapors not changing state. Coefficients of expansion. Coefficients of pressure change for gases and vapors.	299
69. Pressure, volume, and temperature relations for gases. Perfect and real gases. . . .	302
70. Gas density and specific volume and its relation to molecular weight and gas constant. .	307
71. Pressure and temperature relations for vapor of liquids or solids. Vaporization, sublimation and fusion curves. Boiling- and freezing-points for pure liquids and dilute solutions. Saturated and superheated vapors	311

CHAPTER VIII. CHANGES OF PHYSICAL STATE WITH HEAT CONTENT. LATENT
HEAT. GAS AND VAPOR MIXTURES. ABSORPTION OF GASES

72. Change of state with amount of heat at constant temperature. Latent heats of fusion and vaporization. Total heats of vapors. Relation of specific volume of liquid and of vapor to the latent heat.	319
73. Gas and vapor mixtures. Partial and total gas and vapor pressures. Volume, weight and gas constant relations. Saturated mixtures. Humidity.	327
74. Absorption of gases by liquids and by solids. Relative volumes and weights with pressure and temperature. Heats of absorption and of dilution. Properties of aqua ammonia.	333

CHAPTER IX. CHANGE OF CHEMICAL STATE WITH HEAT. COMBUSTION AND HEAT
TRANSMISSION

75. Combustion and related reactions. Relative weights and volumes of substances and elements before and after reaction.	341
76. Heats of reaction. Calorific power of combustible elements and of simple chemical compounds. B.T.U. per pound and per cubic foot.	348
77. Heat transmission processes. Factors of internal conduction, surface resistance, radiation and convection.	356
78. Heat transmission between separated fluids. Mean temperature differences, coefficients of transmission.	364
79. Variation in coefficient of heat transmission due to kind of substance, character of separating wall and conditions of flow	375

CHAPTER X. FUELS. NATURAL AND MANUFACTURED

SECTION	PAGE
80. Origin of heat and transformation to useful form. Complexity of fuels as sources of heat. General classification, solid, liquid, gaseous, natural and artificial.	385
81. Natural solid fuels, wood, peat, lignite, bituminous and anthracite coals. Chemical and physical properties. Classifications based on ultimate and proximate analysis and on behavior on heating.	390
82. Calorific power of coals and the combustibility of coals. Calculation of calorific power from ultimate and proximate analyses. Calorific power of the volatile.	398
83. Mineral oil and natural gas fuel. Chemical and physical properties. Calorific power direct and as calculated for oils from ultimate analysis or from density, and for gas from sum of constituent gases.	402
84. Charcoal, coke, coke oven and retort coal gas as products of heating wood and coal. Chemical, physical, and calorific properties per pound. Calorific power of gases per cubic foot in terms of constituent gases. Yield of gas and coke per pound of coal.	406
85. Distillate oils, kerosene and gasoline, residue oils, and oil gas as products of heating mineral oils. Chemical, physical, and calorific properties. Calorific power of fractionated oils in terms of, (a) carbon and hydrogen; (b) density per pound, and estimated value per cubic foot vapor. Calorific power of oil gas per pound and per cubic foot in terms of constituent gases. Yield of distillates and oil gas.	412

CHAPTER XI. GASIFICATION AND COMBUSTION OF FUEL

86. Gasification of fixed carbon and coke by air-blast reactions, producing air gas, and blast-furnace gas. Comparative yield per pound of coke and air. Sensible heat and heat of combustion of gas. Relation of constituents in gas. Efficiency of gasification.	418
87. Gasification of fixed carbon, coke, and coal previously heated, by steam-blast reactions, producing water gas. Composition and relation of constituents of water gas, yield per pound of steam and coal. Heat of combustion of gas and limitation of yield by negative heat of reaction.	428
88. Gasification of coals by steam and air blasts, resulting in producer gas. Composition and relation of constituents of producer gas, yield per pound of fixed carbon, air and steam. Modification of composition by addition of volatile of coal. Heat of combustion of gas, sensible heat, and efficiency of gasification. Horse-power of gas producers.	435
89. Combustion effects. Final temperature, volume and pressure for explosive and non-explosive combustion. Estimation of air weights and heat suppression due to CO in products from volumetric analysis.	449
90. Temperature of ignition and its variation with conditions. Limits of proportion air gas neutral, or detonating gas and neutral, for explosive combustion of mixtures. Limits of adiabatic compression for self-ignition of mixtures.	461
91. Rate of combustion of solid fuels with draft. Propagation rates, uniform and detonating for explosive gaseous mixtures.	467

CHAPTER XII. STEAM BOILER EVAPORATION. HORSE POWER AND EFFICIENCY

92. Steam-boiler evaporative capacity and horse-power. Horse-power units, equivalent rates of evaporation and of heat absorption. Factors of evaporation. Relation between absorption rates and rates of heat generation. Influence of heating and grate surface, calorific power of fuels, rates of combustion and furnace losses.	472
93. Steam-boiler efficiency, furnace and heating-surface efficiency. Heat balances and variation in heat distribution. Evaporation and losses per pound of fuel.	484

PART III

**TRANSFORMATION OF HEAT AND WORK BY CHANGES
OF SUBSTANCE CONDITION**

CHAPTER XIII. HEAT AND WORK. FUNDAMENTAL RELATIONS. THERMAL PHASES	
SECTION	PAGE
94. General heat and work relations. Thermal cycles. Work and efficiency determination by heat differences and ratios. Graphic method of temperature entropy heat diagram	496
95. General energy equation between heat change, intrinsic energy change, and work done. Derived relations between physical constants for gases and for changes of state, solid to liquid, and liquid to vapor	503
96. Quantitative relations for primary thermal phases, algebraic, and graphic to PV , and $T\Phi$ coordinates. Constancy of PV , and T for gases and vapors, wet, dry and superheated	510
97. Quantitative relations for secondary thermal phases. Adiabatics for gases and vapors. Constant quality, constant total heat, and logarithmic expansion lines for steam	519
CHAPTER XIV. THERMAL CYCLES FOR STEAM	
98. Thermal cycles representative of heat-engine processes. Cyclic efficiency. A reference standard for engines and fuel-burning power systems. Classification of steam cycles	533
99. The Rankine cycle. Work, mean effective pressure, jet velocity, water rate, heat consumption and efficiency of steam, Cycle I. Adiabatic expansion, constant pressure, heat addition and abstraction, no compression	541
100. Carnot steam cycle and derivatives. Work, mean effective pressure, water rate, heat consumption and efficiency of steam, Cycle II. Adiabatic expansion and compression, constant pressure heat addition and abstraction	554
CHAPTER XV. THERMAL CYCLES FOR GAS	
101. Gas cycles representative of ideal processes and standards of reference for gas engines	562
102. Brown, Lenoir, Otto and Langen non-compression gas-cycles. Work, mean effective pressure, efficiency, heat and gas consumption	569
103. Stirling and Ericsson gas cycles. Work, efficiency and derived quantities for isothermal compression with and without regenerators	571
104. Otto, Atkinson, Brayton, Diesel, and Carnot gas cycles. Work, efficiency and derived quantities for adiabatic compression gas cycles	575
CHAPTER XVI. EFFICIENCY OF TRANSFORMATION OF HEAT INTO WORK BY STEAM AND GAS CYCLIC AND ACTUAL	
105. Comparison of steam and gas cycles with the Rankine as standard for steam, and with the Otto and Diesel, as standard for gas. Relations of Otto and Diesel to Rankine cycle. Conditions for equal efficiency	588
106. Gas cycle performance as affected by variability of the specific heats of gases, applied to the Otto cycle	592

CONTENTS

XV

SECTION	PAGE
107. Actual performance of Otto and Diesel gas engines, and its relation to the cyclic. Diagram factors for mean effective pressure and thermal efficiency. Effect of load on efficiency. Heat balance of gas engines alone and with gas producers	596
108. Actual performance of piston steam engines and steam turbines at their <i>best load</i> and its relation to the cyclic. Effect on efficiency of initial pressure, vacuum, superheat, jacketing, and reheating. Heat balances of steam power plants . . .	607
CHAPTER XVII. FLOW OF EXPANSIVE FLUIDS THROUGH ORIFICES, NOZZLES, DUCTS, FLUES, CHIMNEYS AND PIPES	
109. Flow of hot water, steam and gases through orifices and nozzles. Velocity, weight per second, kinetic energy, and force of reaction of jets. Nozzle friction and reheating and coefficient of efflux. Relative proportions of series nozzles for turbines for proper division of work of expansion	620
110. Flow of expansive fluids under small pressure drops through orifices, valves, and Venturi tubes. Relation between loss of pressure and flow. Velocity heads and quantity of flow by Pitot tubes	627
111. Flow of gases and vapors in pipes, flues, ducts, and chimneys. Relation between quantity of flow and loss of pressure. Friction resistances. Draught and capacity of chimneys	634
CHAPTER XVIII. EFFICIENCY OF COMPRESSED-AIR ENGINES WITH AND WITHOUT PREHEAT	
112. Thermal efficiency of compressed-air engines alone and in combination with air compressors. Effect of preheating and reheating. Compressor suction heating, and volumetric efficiency. Wall action	646
CHAPTER XIX. MECHANICAL REFRIGERATION. QUANTITY OF FLUID. HORSE POWER AND HEAT ABSTRACTED PER TON	
113. Mechanical refrigeration, general description of processes and structures. Thermal cycles and refrigerating fluids. Limiting temperatures and pressures	656
114. Performance of mechanical refrigerating cycles and systems. Quantity of fluid circulated per minute per ton refrigeration, horse-power, and heat supplied per ton. Refrigeration per unit of work done and its relation to thermal efficiency of the system	666
INDEX	677

LIST OF TABLES

NUMBER	PAGE
I. Values of a and a^3 for use in determining size of receiver.....	193
II. Specific heat constants of gases.....	296
III. Conversion factors for weights in the complete combustion of hexane with air to water and carbon dioxide.....	346
IV. General classification of fuels.....	389
V. Classification of coals by composition.....	392
VI. Wilkesbarre anthracite sizes and average ash contents.....	396
VII. Density and calorific power of natural gas from constituents.....	405
VIII. Comparison of coke oven and retort coal gases.....	410
IX. Density and calorific power of coke oven and retort coal gas.....	411
X. American mineral oil products at 60° Fahrenheit.....	413
XI. U. S. gasolene and kerosene bearing crude oils.....	414
XII. Yield of retort oil gas.....	416
XIII. Density and calorific power of oil gas.....	416
XIV. Boudouard's equilibrium relations CO and CO ₂ with temperature.....	420
XV. Change of O ₂ in air to CO and CO ₂ at 1472° F. (Boudouard).....	421
XVI. Composition of hypothetical air gas. General.....	423
XVII. Composition of hypothetical air gas. No CO ₂ and no CO.....	424
XXVIII. Density and calorific power of blast furnace gas.....	427
XIX. Water gas characteristics with bed temperature (Bunte).....	428
XX. Comparison of hypothetical water gas. General.....	430
XXI. Comparison of hypothetical water gas. No CO ₂ and no CO.....	431
XXII. Density and calorific power of water gas.....	434
XXIII. Composition of hypothetical producer gas from the fixed carbon.....	441
XXIV. Density and calorific power of producer gas.....	447
XXV. Characteristics of explosive mixtures of Pintsch oil-gas and air.....	455
XXVI. Calculated ignition temperatures for producer gas.....	463
XXVII. Compressions commonly used in gas engines.....	465
XXVIII. Per cent detonating mixture in total air-gas mixture at explosive limits of proportion (Bunte).....	466
XXIX. Boiler efficiency summaries (Donkin).....	487
XXX. Three examples of heat balance for boilers.....	488
XXXI. Values of s for adiabatic expansion of steam.....	525
XXXII. Otto cycle thermal efficiencies for constant and variable specific heats. Equal limiting temperatures.....	595
XXXIII. Mechanical efficiencies of gas engines.....	601
XXXIV. Allowable compression of gas engines in standard atmospheres, absolute....	601
XXXV. Güldner's values of Otto engine, real volumetric efficiency with estimated mean suction resistances.....	603
XXXVI. Comparative heat balances of producer and engine plants.....	606
XXXVII. Steam plant balances.....	608
XXXVIII. Steam turbine efficiency with varying back pressure and steam approxi- mately at constant initial pressure, dry saturated.....	612
XXXIX. Coefficient of discharge C for air under various water heads and diameters of orifice (Durley).....	629

TABLE OF SYMBOLS

	USED IN PART
A = area in square feet.	
= constant, in formula for most economical load of a steam engine	I
= constant, in pipe flow formula.....	III
= excess air per pound of coal.....	II
= pounds of ammonia dissolved per pound of weak liquor	II
a = area in square inches.	
= coefficient of linear expansion.....	II
= constant in equation for the ratio of cylinder sizes for equal work distribution in compound engine	I
= constant in equation for change in intrinsic energy.....	III
= constant in equation for specific heat.....	II
= cubic feet of air per cubic foot of gas in explosive mixtures.....	II
= effective area of piston, square inches.....	I
B = constant in equation for the most economical load of the steam engine.....	I
= constant in equation for flow in pipes.....	III
Bé. = Baumé.	
B.H.P. = brake horse-power.....	I and III
= boiler horse-power.....	II
B.T.U. = British thermal unit.	
b = constant in equation for change in intrinsic energy.....	III
= constant in equation for specific heat.....	II
(bk.pr.) = back pressure in pounds per square inch.	
C = Centigrade.	
= coefficient for air flow.....	III
= circumference or perimeter of ducts in equations for flow	III
= constant.	
= heat suppression factor.....	II
= ratio of pressure after compression to that before compression in gas engine cycles	III
= specific heat	II
C_C = per cent of ammonia in weak liquor	III
C_p = specific heat at constant pressure.	
C_R = per cent of ammonia in rich liquor.....	III
C_s = specific heat of water	III
C_v = specific heat at constant volume.	
Cl = clearance expressed in cubic feet.	
c = clearance expressed as a fraction of the displacement.	
= constant.	
cu.ft. = cubic foot.	
cu.in. = cubic inch.	
D = constant in equations for pipe flow.....	III
= density.....	II
= diameter of pipe in feet.....	III
= displacement in cubic feet.	
D_s = specific displacement.....	I

	USED IN PART
d = constant in equation for change in intrinsic energy.....	III
= diameter of a cylinder in inches.....	I
= diameter of pipe in inches.....	III
= differential.	
(del.pr.) = delivery pressure in pounds per square inch.....	I
E = constant in equation for pipe flow.....	III
= external latent heat.....	II
= thermal efficiency.....	III
E_B = thermal efficiency referred to brake horse-power.....	I
E_b = boiler efficiency.....	II
E_f = furnace efficiency.....	II
E_I = thermal efficiency referred to indicated horse-power.....	I
E_m = mechanical efficiency.....	I
E_s = heating surface efficiency.....	II
E_V = volumetric efficiency (apparent).....	
E_V' = volumetric efficiency (true).....	
e = as a subscript to log to designate base e ($=2.71828+$).	
= constant in equation for change in intrinsic energy.....	III
e_1 = ratio of true volumetric efficiency to hypothetical.....	I
e_2 = ratio of true volumetric efficiency to apparent.....	I
e_3 = ratio of true indicated horse-power to hypothetical.....	I
F = constant in equation for pipe flow.....	III
= diagram factor for gas engine indicator cards.....	III
= Fahrenheit.	
= force in pounds.	
F_F = friction factor, $F_F \times$ velocity head = loss due to friction.....	III
F_R = resistance factor, $F_R \times$ velocity head = loss due to resistances.....	III
F_S = special resistances to flow in equations for chimney draft.....	III
f = constant in equation for changes in intrinsic energy.....	III
= function.	
ft. = foot.	
ft.-lb. = foot-pound.	
G = constant in equation for pipe flow.....	III
= weight of gas per hour in equation for chimney flow.....	III
G_m = maximum weight of gases in equation for chimney flow.....	III
G. S. = grate surface.	
g = acceleration due to gravity, 32.2 (approx.) feet per second, per second.	
H = as a subscript to denote high pressure cylinder.	
= heat per pound of dry saturated vapor above 32° F.	
= heat per cubic foot gas.	
= heat transmitted.....	II
= height of column of hot gases in feet.....	III
= pressure or head in feet of fluid.....	III
H_A = difference in pressure on two sides of an orifice in feet of air.....	III
H_G = equivalent head of hot gases.....	III
H_M = pressure in feet of mercury.....	III
H_W = pressure in feet of water.....	
H.P. = high pressure.	
= horse-power.....	
H.S. = heating surface.	
(H.P.cap.) = high pressure cylinder capacity.....	
h = heat of superheat.	
= height in inches.	

TABLE OF SYMBOLS

xxi

USED IN PART

h_M = difference in pressure on two sides of an orifice in inches of mercury	III
h_W = difference in pressure on two sides of an orifice in inches of water	III
I = as a subscript to denote intermediate cylinder	
I.H.P. = indicated horse-power.	
in. = inch.	
(in.pr.) = initial pressure in pounds per square inch.	
J = Joule's equivalent = 778 (approx.) foot-pounds per B.T.U.	
K = coefficient of thermal conductivity	II
= constant.	
= proportionality coefficient in equation for draft	III
K_e = engine constant = $\frac{Lan}{33000}$ in expression for horse-power, = $\frac{aS}{33000}$	I
L = as a subscript to denote low-pressure cylinder.	
= distance in feet.	
= latent heat	II and III
= length of stroke in feet	I
L = per cent of heat in fuel lost in furnace	II
L.P. = low pressure.	
(L.P.Cap.) = low-pressure capacity	
l = constant	I
= length	II
lb. = pound.	
log = logarithm to the base 10, = common log.	
\log_e = logarithm to the base e , = hyperbolic log.	
M = mass.	
(M.E.P.) = mean effective pressure, pounds per square foot.	
m = constant	I
= mean hydraulic radius = $\frac{\text{area}}{\text{perimeter}}$.	
= molecular weight	II
= ratio of initial pressure to that at end of expansion in Otto and Langen gas cycle	III
(m.b.p.) = mean back pressure in pounds per square inch.	
(m.e.p.) = mean effective pressure in pounds per square inch.	
(m.f.p.) = mean forward pressure in pounds per square inch.	
N = constant	I
= revolutions per minute, = R.P.M. = R.p.m.	
n = cycles per minute.	
= constant	I
= cubic feet of neutral per cubic foot of gaseous mixture	II
= number of degrees exposed on thermometer stem	II
= ratio of volume after expansion to volume before, in Atkinson gas cycle	II
= specific volume of dry saturated steam	III
O = volume of receiver of compound engine in cubic feet	
P = draft in pounds per square foot	III
= load in kilowatts	I
= pressure in pounds per square foot.	
P_f = static pressure in pounds per square foot lost in wall friction	
P_R = static pressure in pounds per square foot lost in changes of cross-section, etc.	III
PV = pressure volume.	
P_v = velocity head in pounds per square foot.	
p = pressure in pounds per square inch.	
p_e = mean exhaust pressure	III

	USED IN PART
p_s = mean suction pressure	III
p_v = partial pressure of water vapor in air	III
Q = quantity of heat or energy in B.T.U. gained or lost by a body passing from one state to another.	
Q_1' = heat added from fire in Stirling and Ericsson cycles	
Q_1'' = heat added from regenerator in Stirling and Ericsson cycles	
Q_2' = heat abstracted by water jacket in Stirling and Ericsson cycles	
Q_2'' = heat abstracted by regenerator in Stirling and Ericsson cycles	
q = quantity of heat per pound of liquid above 32° F.	
R = ratio of heating surface to grate surface	II
= gas constant.	
R_C = ratio of cylinder sizes in two-stage air compressor or compound engine	
R_H = ratio of expansion in high-pressure cylinder	
R_L = ratio of expansion in low-pressure cylinder	
R_P = ratio of initial to back pressure	
R_p = ratio of delivery to supply pressure	I
R_V = ratio of larger volume to smaller volume.	
r = rate of flame propagation in explosive mixtures	II
r_P = pressure differences (maximum—minimum) in gas cycles	III
r_V = volume differences (maximum—minimum) in gas cycles	III
(rec.pr.) = receiver pressure in pounds per square inch	
(rel.pr.) = release pressure in pounds per square inch	
S = per cent of ammonia in solution	II
= piston speed	
= pounds of steam per pound of air in producer blast	II
= specific heat	
= specific heat of superheated steam'	
(Sup.Vol.) = volume of steam supplied to the cylinder per stroke	
s = general exponent of V in expansion or compression of gases.	
sp.gr. = specific gravity.	
sp.ht. = specific heat.	
sq.ft. = square foot.	
sq.in. = square inch.	
(sup.pr.) = supply pressure, in pounds per square inch.	
T = temperature, degrees absolute.	
T_C = temperature of air	III
T_H = temperature of gases in chimney	
t = temperature in degrees scale.	
$T\Phi$ = temperature entropy.	
U = rate of heat transfer in B.T.U. per square foot per hour per degree difference in temperature	II
\bar{U} = intrinsic energy	III
u = velocity in feet per second.	
u_m = velocity in feet per minute	III
V = volume in cubic feet.	
V_A = cubic feet per pound air	III
V_G = cubic feet per pound, gas	III
V_L = volume of liquid in cubic feet per pound.	
V_S = volume of solid in cubic feet per pound.	
V_V = volume of vapor in cubic feet per pound.	
v = volume	II
W = work in foot-pounds.	
W.R. = water rate.	

TABLE OF SYMBOLS

xxiii

	USED IN PART
w = pounds of water per pound of ammonia in solution.....	II
= weight in pounds.	
w_R = pounds of rich liquor per pound of ammonia.....	III
wt. = weight.	
X = compression in the steam engine as a fraction of the stroke.....	I
$= 1 + \frac{\text{heat added}}{\text{temperature at beginning of addition} \times \text{specific heat at constant volume}}$	III
x = constant in the expression for missing water.....	I
= fraction of liquid made from solid or vapor made from liquid, = quality.....	III
= per cent of carbon burned to CO_2	II
= per cent of nozzle reheat.....	III
= per cent of steam remaining in high-pressure cylinder of compound engine at any point of the exhaust stroke.....	II
= quantity of heat added in generator of absorption system in addition to the amount of heat of absorption of 1 lb. of ammonia.....	III
= ratio of low-pressure admission volume to high-pressure admission volume.....	I
Y = total steam used per hour by an engine.....	I
$= 1 + \frac{\text{heat added}}{\text{temperature at beginning of addition} \times \text{specific heat at constant pressure}}$	III
y = per cent of vane reheat.....	III
= ratio of the volume of receiver to that of the high-pressure cylinder of the compound engine.....	I
Z = fraction of the stroke of the steam engine completed at cut-off.....	I
$= 1 + \frac{\text{heat added from regenerator}}{\text{temperature at beginning of addition} \times \text{specific heat at constant volume}}$	III
Z' = hypothetical best value of Z .	
$= 1 + \frac{\text{heat added from regenerator}}{\text{temperature at beginning of addition} \times \text{specific heat at constant pressure}}$	III
z = ratio of R.P.M. to cycles per minute.	
α = (alpha) = an angle.....	I
= coefficient of cubical expansion.....	III
= constant in the equation for latent heat.....	III
= α_s = constant in equation for variable specific heat at constant volume.....	III
$\alpha' = \alpha_p$ = constant in equation for variable specific heat at constant pressure.....	III
β = (beta) = constant in equation for latent heat.....	III
= fraction of fuel heat available for raising temperature.....	II
γ = (gamma) = constant in equation for latent heat.....	III
= ratio of cross-section to perimeter.....	II
= special value for s for adiabatic expansion or compression = $\frac{\text{sp.ht. at const. press.}}{\text{sp.ht. at const. vol.}}$	
γ' = ratio of specific heat at constant pressure to specific heat at constant volume when each is a variable.....	III
Δ = (delta) = increment.	
δ = (delta) = density in pounds per cubic foot.	
δ_C = density in cold gases in equations for chimney draft.....	
δ_H = density of hot gases in equations for chimney draft.....	
ζ = (zeta) = coefficient of friction.....	
μ = (mu) = material coefficient in heat transfer expression.....	II
ρ = (rho) = internal thermal resistance.....	II
Σ = (sigma) = summation.	

PART III

*TRANSFORMATION OF HEAT AND WORK BY CHANGES
OF SUBSTANCE CONDITION*

CHAPTER XIII. HEAT AND WORK. FUNDAMENTAL RELATIONS. THERMAL PHASES	
SECTION	PAGE
94. General heat and work relations. Thermal cycles. Work and efficiency determination by heat differences and ratios. Graphic method of temperature entropy heat diagram.....	496
95. General energy equation between heat change, intrinsic energy change, and work done. Derived relations between physical constants for gases and for changes of state, solid to liquid, and liquid to vapor.....	503
96. Quantitative relations for primary thermal phases, algebraic, and graphic to PV , and $T\Phi$ coordinates. Constancy of PV , and T for gases and vapors, wet, dry and superheated.....	510
97. Quantitative relations for secondary thermal phases. Adiabatics for gases and vapors. Constant quality, constant total heat, and logarithmic expansion lines for steam.....	519
CHAPTER XIV. THERMAL CYCLES FOR STEAM	
98. Thermal cycles representative of heat-engine processes. Cyclic efficiency. A reference standard for engines and fuel-burning power systems. Classification of steam cycles.....	533
99. The Rankine cycle. Work, mean effective pressure, jet velocity, water rate, heat consumption and efficiency of steam, Cycle I. Adiabatic expansion, constant pressure, heat addition and abstraction, no compression.....	541
100. Carnot steam cycle and derivatives. Work, mean effective pressure, water rate, heat consumption and efficiency of steam, Cycle II. Adiabatic expansion and compression, constant pressure heat addition and abstraction.....	554
CHAPTER XV. THERMAL CYCLES FOR GAS	
101. Gas cycles representative of ideal processes and standards of reference for gas engines.....	562
102. Brown, Lenoir, Otto and Langen non-compression gas-cycles. Work, mean effective pressure, efficiency, heat and gas consumption.....	569
103. Stirling and Ericsson gas cycles. Work, efficiency and derived quantities for isothermal compression with and without regenerators.....	571
104. Otto, Atkinson, Brayton, Diesel, and Carnot gas cycles. Work, efficiency and derived quantities for adiabatic compression gas cycles.....	575
CHAPTER XVI. EFFICIENCY OF TRANSFORMATION OF HEAT INTO WORK BY STEAM AND GAS CYCLIC AND ACTUAL	
105. Comparison of steam and gas cycles with the Rankine as standard for steam, and with the Otto and Diesel, as standard for gas. Relations of Otto and Diesel to Rankine cycle. Conditions for equal efficiency.....	588
106. Gas cycle performance as affected by variability of the specific heats of gases, applied to the Otto cycle.....	592

CONTENTS

XV

SECTION	PAGE
107. Actual performance of Otto and Diesel gas engines, and its relation to the cyclic. Diagram factors for mean effective pressure and thermal efficiency. Effect of load on efficiency. Heat balance of gas engines alone and with gas producers	596
108. Actual performance of piston steam engines and steam turbines at their <i>best load</i> and its relation to the cyclic. Effect on efficiency of initial pressure, vacuum, superheat, jacketing, and reheating. Heat balances of steam power plants....	607
 CHAPTER XVII. FLOW OF EXPANSIVE FLUIDS THROUGH ORIFICES, NOZZLES, DUCTS, FLUES, CHIMNEYS AND PIPES	
109. Flow of hot water, steam and gases through orifices and nozzles. Velocity, weight per second, kinetic energy, and force of reaction of jets. Nozzle friction and reheating and coefficient of efflux. Relative proportions of series nozzles for turbines for proper division of work of expansion.....	620
110. Flow of expansive fluids under small pressure drops through orifices, valves, and Venturi tubes. Relation between loss of pressure and flow. Velocity heads and quantity of flow by Pitot tubes.....	627
111. Flow of gases and vapors in pipes, flues, ducts, and chimneys. Relation between quantity of flow and loss of pressure. Friction resistances. Draught and capacity of chimneys.....	634
 CHAPTER XVIII. EFFICIENCY OF COMPRESSED-AIR ENGINES WITH AND WITHOUT PREHEAT	
112. Thermal efficiency of compressed-air engines alone and in combination with air compressors. Effect of preheating and reheating. Compressor suction heating, and volumetric efficiency. Wall action.....	646
 CHAPTER XIX. MECHANICAL REFRIGERATION. QUANTITY OF FLUID. HORSE POWER AND HEAT ABSTRACTED PER TON	
113. Mechanical refrigeration, general description of processes and structures. Thermal cycles and refrigerating fluids. Limiting temperatures and pressures.....	656
114. Performance of mechanical refrigerating cycles and systems. Quantity of fluid circulated per minute per ton refrigeration, horse-power, and heat supplied per ton. Refrigeration per unit of work done and its relation to thermal efficiency of the system.....	666
INDEX.....	677

acting at the water end. Then the work at the steam end of the pump may be considered to be positive and at the water end negative, so far as the movement of the rod is concerned; when, however, this same movement causes a movement of the water, work done at the water end (although negative with reference to the rod motion, since it opposes that motion) is positive with reference to the water, since it causes this motion. It may also be said that the steam does work on the steam end of the rod and the water end of the rod does work on the water, so that one end receives and the other delivers work, the rod acting as a transmitter; or that the work performed at the steam end is the input and that at the water end the output work.

Example. An elevator weighing 2000 lbs. is raised 80 ft. How much work is done in foot-pounds? Foot-pounds = force \times distance = $2000 \times 80 = 160,000$ ft.-lbs.

Prob. 1. A pump lifts 150 gallons of water to a height of 250 ft. How much work does it do?

Prob. 2. By means of a jack a piece of machinery weighing 10 tons is raised $\frac{3}{4}$ in. What is the work done?

Prob. 3. A rifle bullet weighing 2 oz. travels vertically upward $1\frac{1}{2}$ miles. What work was done in foot-pounds?

Prob. 4. A cubic foot of water falls 50 ft. in reaching a water-wheel. How much work can it do?

Prob. 5. A piston of an elevator is 12 ins. in diameter and has acting on it a pressure of 80 lbs. per square inch. What work is done per foot of travel?

Prob. 6. It has been found that a horse can exert 75 lbs. pull when going 7 miles per hour. How much work can be done per minute?

Prob. 7. How much work is done by an engine which raises a 10-ton casting 50 ft.?

Prob. 8. The pressure of the air on front of a train is 50 lbs. per square foot when the speed is 50 miles per hour. If the train presents an area of 50 sq.ft., what work is done in overcoming wind resistance?

Prob. 9. The pressure in a 10-inch gun during the time of firing is 2000 lbs. per square inch. How much work is done in ejecting the projectile if the gun is 33 ft. 4 ins. long?

2. Power Defined. Power is defined as the rate of working or the work done in a given time interval, thus introducing a third unit of mechanics, time, so that power will always be expressed as a quotient, the numerator being a product of force and distance, and the denominator time. This is in opposition to the popular use of the word, which is very hazy, but is most often applied to the capability of performing much work or the exertion of great force, thus, popularly, a powerful man is one who is strong, but in the technical sense a man would be powerful only when he could do much work continuously and rapidly. An engine has large power when it can perform against resistance many foot-pounds per minute. The unit of power in the English system is the horse-power, or the performance of 550 foot-pounds per second or 33,000 foot-pounds per minute, or 1,980,000 foot-pounds per hour. In the metric system the horse-power is termed cheval-vapeur, and is the performance of 75 kilogrammeters = $542\frac{1}{2}$ foot-pounds per second, or 4500 kilogrammeters = 32,549 foot-pounds per minute, or 270,000 kilogrammeters = 1,952,932 foot-pounds per hour.

Example. The piston of a steam engine travels 600 ft. per minute and the mean force of steam acting upon it is 65,000 lbs. What is the horse-power?

$$\text{H.P.} = \frac{\text{foot-pounds per minute}}{33,000} = \frac{\text{force} \times \frac{\text{distance}}{\text{time}}}{33,000} = \frac{65,000 \times 600}{33,000} = 1182 \text{ H.P.}$$

Prob. 1. The draw-bar pull of a locomotive is 3000 lbs. when the train is traveling 50 miles per hour. What horse-power is being developed?

Prob. 2. A mine cage weighing 2 tons is lifted up a 2000-ft. shaft in 40 seconds. What horse-power will be required if the weight of the cable is neglected?

Prob. 3. By direct pull on a cable it is found possible to lift 4 tons 20 ft. per second. With a differential pulley 40 tons may be lifted 3 ft. per second. What is the difference in power required?

Prob. 4. A horse exerts a pull of 100 lbs. on a load. How fast must the load be moved to develop one horse-power?

Prob. 5. The resistance offered to a ship at a speed of 12 knots was 39,700 lbs. What horse-power must be available to maintain this speed? (One knot is a speed of one nautical mile per hour.)

Prob. 6. It is estimated that 100,000 cu. ft. of water go over a fall 60 ft. high every second. What horse-power is available?

Prob. 7. The force acting on a piston of a pump is 80,000 lbs. If the piston speed is 150 ft. per minute, what is the horse-power?

Prob. 8. To draw a set of plows $2\frac{1}{2}$ miles per hour requires a draw-bar pull of 10,000 lbs. What must be the horse-power of a tractor to accomplish this?

Prob. 9. The horse-power to draw a car up a grade is the sum of the power necessary to pull it on a level and that necessary to lift it vertically the same number of feet as it rises on the grade. What will be the horse-power required to draw a car 20 miles per hour up a 12 per cent grade if the car weighs 2500 lbs. and the draw-bar pull on the level is 250 lbs.?

3. Work in Terms of Pressure and Volume. Another of the definitions of mechanics fixes pressure as *force per unit area* so that pressure is always a quotient, the numerator being force and the denominator area, or length to the second power. If, therefore, the pressure of a fluid be known, and according to hydromechanics it acts equally and normally over all surface in contact with it, then the force acting in a given direction against any surface will be the product of the pressure and the projected area of the surface, the projection being on a plane at right angles to the direction considered. In the case of pistons and plungers the line of direction is the axis of the cylinder, and the projected area is the area of the piston less the area of any rod passing completely through the fluid that may be so placed. When this plane area moves in a direction perpendicular to itself, the product of its area and the distance will be the volume swept through, and if a piston be involved the volume is technically the *displacement* of the piston. Accordingly, work may be expressed in three ways, as follows: Work = force \times distance; Work = pressure \times area \times distance; Work = pressure \times volume.

The product should always be in foot-pounds, but will be, only when appropriate units are chosen for the factors. These necessary factors are given as follows: Work in foot-pounds = force in lbs. \times distance in ft. = W .

$W = \text{pressure in lbs. per sq.ft.} \times \text{area in sq.ft.} \times \text{distance in ft.} = \text{pressure in lbs. per sq.in.} \times \text{area in sq.in.} \times \text{distance in ft.} = \text{pressure in lbs. per sq.ft.} \times \text{volume in cu.ft.} = \text{pressure in lbs. per sq.in.} \times 144 \times \text{volume in cu.ft.}$

In thermodynamic computations the pressure-volume product as an expression for work is most useful, as the substances used are always vapors and gases, which, as will be explained later in more detail, have the valuable property of changing volume indefinitely with or without change of pressure according to the mode of treatment. Every such increase of volume gives, as a consequence, some work, since the pressure never reaches zero, so that to derive work from vapors and gases they are treated in such a way as will allow them to change volume considerably with as much pressure acting as possible.

It should be noted that true pressures are always *absolute*, that is, measured above a perfect vacuum or counted from zero, while most pressure gages and other devices for measuring pressure, such as indicators, give results measured above or below atmospheric pressure, or as commonly stated, above or below atmosphere. In all problems involving work of gases and vapors, the absolute values of the pressures must be used; hence, if a gage or indicator measurement is being considered, the pressure of the atmosphere found by means of the barometer must be added to the pressure above atmosphere in order to obtain the absolute or true pressures. When the pressures are below atmosphere the combination with the barometric reading will depend on the record. If a record be taken by an indicator it will be in pounds per square inch below atmosphere and must be subtracted from the barometric equivalent in the same units to give the absolute pressure in pounds per square inch. When, however, a vacuum gage reads in inches of mercury below atmosphere, as such gages do, the difference between its reading and the barometric gives the absolute pressure in inches of mercury directly, which can be converted to the desired units by the proper factors.

While it is true that the barometer is continually fluctuating at every place, it frequently happens that standards for various altitudes enter into calculations, and to facilitate such work, values are given in the Handbook of Tables for the standard barometer at various altitudes with equivalent pressures in pounds per square inch.

Frequently in practice, pressures are given without a definite statement of what units are used. Such a custom frequently leads to ambiguity, but it is often possible to interpret them correctly from a knowledge of the nature of the problem in hand. Steam pressures are most commonly stated in pounds per square inch and should be designated as either gage or absolute. Pressures of compressed air are commonly expressed in the same units as steam, either gage or absolute, though sometimes in atmospheres. Steam pressures below atmosphere may be stated as a vacuum of so many inches of mercury, meaning that the pressure is less than atmosphere by that amount, or may be given as a pressure of so many inches of mercury absolute, or as so many pounds per square inch absolute. The pressures of gases stored in tanks under high pressure are frequently recorded in atmospheres, due to the convenience

of computation of quantities on this basis. Pressures of air obtained by blowers or fans are usually given by the manufacturers of such apparatus in ounces per square inch above (or below) atmosphere. Such pressures and also differences of pressure of air due to chimney draft, or forced draft, and the pressure of illuminating gas in city mains, are commonly stated in inches of water, each inch of water being equivalent to 5.196 lbs. per square foot. The pressure of water in city mains or other pressure pipes may be stated either in pounds per square inch or in feet of water head.

Example. A piston on which the mean pressure is 60 lbs. per square inch sweeps through a volume of 300 cu.ft. What is the work done?

$$W = P \times V, \text{ where } V = \text{cu.ft. and } P = \text{lbs. per sq.ft.}$$

$$\therefore W = 60 \times 144 \times 300 = 2,592,000 \text{ ft.-lbs.}$$

Prob. 1. The mean pressure acting per square inch when a mass of air changes in volume from 10 cu.ft. to 50 cu.ft. is 40 lbs. per square inch. How much work is done?

Prob. 2. An engine is required to develop 30 H.P. If the volume swept through per minute is 150 cu.ft., what must the mean pressure be?

Prob. 3. The mean effective pressure in compressing air from one to five atmospheres is 28.7 lbs. per square inch. How many horse-power are required to compress 1000 cu.ft. of free air per minute?

Prob. 4. At an altitude of 4100 ft. a pressure gage showed the pressure on one side of a piston to be 50 lbs. per square inch while the pressure on the opposite side is 3 lbs. per square inch absolute. What pressure was tending to move the piston?

Prob. 5. At an altitude of 1 mile the mean pressure in a gas engine cylinder during the suction stroke was found to be 12 lbs. per square inch absolute. What work was done by the engine to draw in a charge if the cylinder was 5 ins. in diameter and the stroke 6 ins.?

Prob. 6. After explosion the piston of the above engine was forced out so that the gas volume was five times that at the beginning of the stroke. What must the M.E.P. have been to get 20,000 ft.-lbs. of work?

Prob. 7. On entering a heating oven cold air expands to twice its volume. What work is done per cubic foot of air?

Prob. 8. A projectile is forced from a gun by a constant air pressure of 1000 lbs. per square inch. Before it begins to move there is $\frac{1}{2}$ cu.ft. of air in the barrel, and at the instant it leaves the barrel the volume is 10 cu.ft. What work was done on the projectile?

Prob. 9. Water is forced from a tank against a head of 75 ft. by filling the tank with compressed air. How much work is done in emptying a tank containing 1000 cu.ft.?

4. Work of Acceleration and Resultant Velocity. When a force acting on a mass is opposed by an equal resistance there may be no motion at all, or there may be motion of constant velocity. Any differences, however, between the two opposing forces will cause a change of velocity so long as the difference lasts, and this difference between the two forces may be itself considered as the only active force. Observations on unresisted falling bodies show that they increase in velocity 32.16 ft. per second for each second they are free to fall, and this quantity is universally denoted by g . If then, a body have any velocity, u_1 , and be acted on by a force equal to its own weight in the direction of its motion for a time, τ seconds, it will have a velocity u_2 after that time.

$$u_2 = u_1 + g\tau. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

It may be that the force acting is not equal to the weight of the body, in which case the acceleration will be different and so also the final velocity, due to the action of the force, but the force producing any acceleration will be to the weight of the body as the actual acceleration is to the gravitational acceleration.

So that $\frac{\text{Actual acceleration force}}{\text{Weight of body or gravitational force}} = \frac{\text{actual acceleration}}{\text{gravitational acceleration } (g)}$, or

Actual accelerating force = $\frac{\text{weight of body}}{\text{gravitational acceleration } (g)} \times \text{actual acceleration}$.

or Force = mass \times acceleration = mass $\times \frac{\text{change of velocity}}{\text{time of change}}$,

$$F = M \times \frac{u_2 - u_1}{\tau} \quad \dots \dots \dots (2)$$

The work performed in accelerating a body is the product of the resistance met into the distance covered, L , while the resistance, or the above-defined force, acts, or while the velocity is being increased. This distance is the product of the time of action and the mean velocity, or the distance in feet,

$$L = \frac{u_2 + u_1}{2} \tau \quad \dots \dots \dots (3)$$

The work is the product of Eqs. (2) and (3), or, work of acceleration is

$$\begin{aligned} W &= \frac{M(u_2 - u_1)}{\tau} \times \frac{(u_2 + u_1)\tau}{2} \\ &= \frac{1}{2} M(u_2^2 - u_1^2) = \frac{w}{2 \times 32.16} (u_2^2 - u_1^2) = \frac{w}{64.32} (u_2^2 - u_1^2), \quad \dots \quad (4) \end{aligned}$$

where w is the weight in pounds. Exactly the same result will be obtained by the calculus when the acceleration is variable, so that Eq. (4) is of universal application.

The work performed in accelerating a body depends on nothing but its mass and the initial and final velocities, and is in every case equal to the product of half the mass and the difference between the squares of the initial and final velocities, or the product of the weight divided by 64.32 and the difference between the squares of the initial and final velocities.

It frequently happens that the velocity due to the reception of work is desired, and this is the case with nozzle flow in injectors and turbines, where the steam performs work upon itself and so acquires a velocity. In all such cases the velocity due to the reception of the work energy is

$$u_2 = \sqrt{u_1^2 + \frac{64.32W}{w}}, \quad \dots \dots \dots (5)$$

where W is work in foot-pounds and w , as before, is weight in pounds. Or if the initial velocity be zero, as it frequently is,

$$u_2 = \sqrt{\frac{2gW}{w}} = \sqrt{64.32 \frac{W}{w}}. \quad \dots \dots \dots (6)$$

Example. A force of 100 lbs. acts for 5 seconds on a body weighing 10 lbs.; if the original velocity of the body was 5 ft. per second, what will be the final velocity, the distance traveled and the work done?

$$F = \frac{M(u_2 - u_1)}{\tau}; \text{ therefore } 100 = \frac{10}{32.2} \frac{(u_2 - 5)}{5}; \text{ and } u_2 = 1615 \text{ ft. per second};$$

$$L = \left(\frac{u_2 + u_1}{2} \right) \tau = 4050 \text{ ft. Hence } W = \frac{M(u_2^2 - u_1^2)}{2} = 405,000 \text{ ft.-lbs.}$$

Prob. 1. A stone weighing $\frac{1}{2}$ lb. is dropped from a height of 1 mile. With what velocity and in what length of time will it strike if the air resistance is zero?

Prob. 2. A car moving 20 miles per hour and weighing 25 tons is brought to rest in 500 ft. What is the negative acceleration, the time required to stop, and the work done?

Prob. 3. Steam escapes through an opening with a velocity of half a mile per second. How many foot-pounds of energy were imparted to each pound of it to accomplish this?

Prob. 4. A weight of 100 lbs. is projected upward with a constant force of 200 lbs. How much further will it have gone at the end of 10 seconds than if it had been merely falling under the influence of gravity for the same period of time?

Prob. 5. A projectile weighing 100 lbs. is dropped from an aeroplane at the height of mile. How soon will it strike, neglecting air resistance?

Prob. 6. A water-wheel is kept in motion by a jet of water impinging on flat vanes. The velocity of the vanes is one-half that of the jet. The jet discharges 1000 lbs. of water per minute with a velocity of 200 ft. per second. Assuming no losses, what is amount of the work done?

Prob. 7. With the wind blowing 30 miles per hour, how much work could a 12-ft. windmill perform if 25 per cent of the available work were utilized.

NOTE. The weight of a cubic foot of air may be taken as .075 lb.

Prob. 8. An engine has a piston speed of 600 ft. per minute and runs at 150 R.P.M. If the reciprocating parts weigh 500 lbs., how much work is done in accelerating the piston during each stroke?

Prob. 9. A flywheel with rim 10 ft. in diameter to center of section and weighing 5 tons, revolves at a rate of 150 R.P.M.; 100,000 ft.-lbs. of work are expended on it. How much will the speed change?

5. Graphical Representation of Work. As work is always a product of force and distance or pressure and volume, it may be graphically expressed by

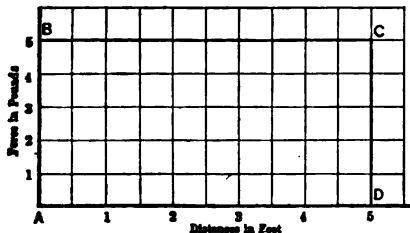


FIG. 1.—Constant Force, Work Diagram, Force-Distance Coordinates.

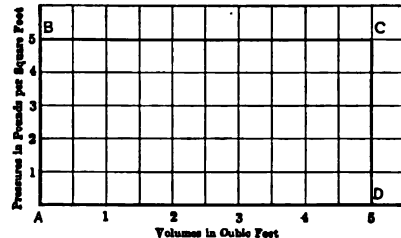


FIG. 2.—Constant Pressure Work Diagram, Pressure-Volume Coordinates.

an area on a diagram having as coordinates the factors of the product. It is customary in such representations to use the horizontal distances for volumes and the vertical for pressures, which, if laid off to appropriate scale and

in proper units, will give foot-pounds of work directly by the area enclosed. Thus in Fig. 1, if a force of 5 lbs. (AB) act through a distance of 5 ft. (BC) there will be performed 25 foot-pounds of work as indicated by the area of the rectangle $ABCD$, which encloses 25 unit rectangles, each representing one foot-pound of work.

If a steam cylinder piston suffers a displacement of 5 cu.ft. under the steam pressure (absolute) of 5 lbs. per square foot then the operation which results in the performance of 25 foot-pounds of work is represented by the diagram Fig. 2, $ABCD$.

Prob. 1. Following the method given for Fig. 1, draw a diagram for the example of Section 3.

Prob. 2. By means of a diagram, show that the work done by a pressure of 1000 lbs. per square foot traversing a distance of 10 ft. is 10,000 ft.-lbs.

Prob. 3. Draw a diagram for the case of a volume change from 1 to 10 cu.ft. while the pressure acting is 20 lbs. per square inch.

Prob. 4. Draw a pressure volume diagram for the case of forcing a piston out of a cylinder by a water pressure of 15,000 lbs. per square foot, the volume of the cylinder at the start is $\frac{1}{2}$ cu.ft. and at the end 6 cu.ft. Make a diagram to scale and report work per square inch of diagram.

Prob. 5. A pump draws in water at a constant suction pressure of 14 lbs. and discharges it at a constant delivery pressure of 150 lbs. per sq.in. Considering the pump barrel to be empty at beginning of suction and end of delivery and to contain 3 cu.ft. when full, draw the diagram for this case and find the foot-pounds of work done.

Prob. 6. In raising a weight a man pulls on a rope with a constant force of 80 lbs. If the weight is lifted 40 ft., find from a diagram the work done.

Prob. 7. In working a windlass a force of 100 lbs. is applied at the end of a 6-ft. lever, the drum of the windlass being 1 ft. in diameter. Draw a work diagram for work applied and for work done in lifting if there be no loss in the windlass.

Prob. 8. The steam and water pistons of a pump are on the same rod and the area of the former is twice that of the latter, the stroke being 3 ft. Show by a diagram that the work done in the two cylinders is the same if losses be neglected.

Prob. 9. An engine exerts a draw-bar pull of 8000 lbs. at speed of 25 miles an hour. A change in grade occurs and speed increases to 40 miles per hour and the pull decreases to 5000 lbs. Show by a diagram the change in horse-power.

6. Work by Pressure-Volume Change. Suppose that instead of being constant the pressure were irregular and, being measured at intervals of 1 cu.ft. displacement, found to be as in table:

Pressure. Lbs. per Sq.Ft.	Displacement Volume. Cu.Ft.
100	0
125	1
150	2
100	3
75	4
50	5

This condition might be plotted as in Fig. 3, A, B, C, D, E, F, G, H . The work done will be the area under the line joining the observation points. In the absence of exact data on the nature of the pressure variations between the two observation points A and B , a variety of assumptions might be made as to the precise evaluation of this area, as follows:

(a) The pressure may have remained constant at its original value for the first cubic foot of displacement, as shown dotted $A-B'$, and then suddenly have risen to B . In this case the work done for this step would be 100 foot-pounds.

(b) Immediately after the measurement at A the pressure may have risen to A' and remained constant during displacement A' to B , in which case the work done would be 125 foot-pounds.

(c) The pressure may have risen regularly along the solid line AB , in which case the work area is a trapezoid and has the value $\frac{100+125}{2} \times 1 = 112.5$ ft.lbs.

It thus appears that for the exact evaluation of work done by pressure-volume change, continuous data are necessary on the value of pressure with respect to the volume. If such continuous data, obtained by measurement or otherwise, be plotted, there will result a continuous line technically termed the pressure-volume curve for the process. Such a curve for a pressure-volume change starting at 1 cu.ft. and 45 lbs. per square foot, and ending at 7 cu.ft., and 30 lbs. per square foot, is represented by Fig. 4, A, B, C, D, E .

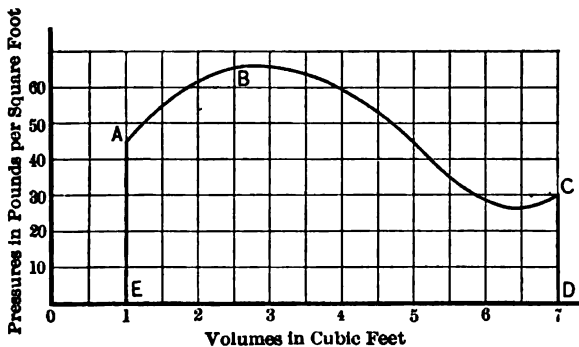


FIG. 4.—Work Diagram, Pressure-Volume Coordinates. Continuous Pressure-Volume Relations.

into small vertical rectangular strips, each so narrow that the pressure is sensibly constant, however much it may differ in different strips. The area of the rectangle is $P\Delta V$, each having the width ΔV and the height P , and the work area will be exactly evaluated if the strips are narrow enough to fulfill the conditions of sensibly constant pressure in any one. This condition is true only for infinitely narrow strips having the width dV and height P , so that each has the area PdV and the whole area or work done is

$$W = \int PdV. \quad \dots \dots \dots (7)$$

This is the general algebraic expression for work done by any sort of continuous

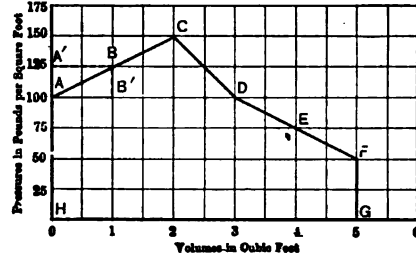


FIG. 3.—Work Diagram, Pressure-Volume Coordinates. Discontinuous Pressure-Volume Relations.

represented by Fig. 4, A, B, C, D, E .

The work done during this displacement under continuously varying pressure is likewise the area between the curve and the horizontal axis when pressures are laid off vertically, and will be in foot-pounds if the scale of pressures is pounds per square foot and volumes, cubic feet. Such an irregular area can be divided

pressure-volume change. It thus appears that whenever there are available sufficient data to plot a continuous curve representing a pressure volume change, the work can be found by evaluating the area lying under the curve and bounded by the curve coordinates and the axis of volumes. The work done may be found by actual measurement of the area or by algebraic solution of Eq. (7), which can be integrated only when there is a known algebraic relation between

the pressure and the corresponding volume of the expansive fluid, gas or vapor.

Pressure in Pounds per Sq.In.	Per Cent of Stroke.
100	0
100	10
100	30
100	50
83.3	60
71.5	70
62.5	80
55.5	90
50.0	100

Prob. 1. Draw the diagrams for the following cases: (a) The pressure in a cylinder 12 ins. in diameter was found to vary at different parts of an 18-in. stroke as shown in the table.

(b) On a gas engine diagram the following pressures were found for parts of stroke.

IN		OUT			
V	p	V	p	V	p
0.25 cu.ft.	14.7	0.1	45.2	0.13	146.2
0.20 "	19.5	0.102	79.7	0.15	116.7
0.14 "	29.7	0.104	123.2	0.17	95.7
0.10 "	45.2	0.106	157.7	0.19	80.7
		0.108	181.7	0.21	68.7
		0.11	188.2	0.23	58.7
		0.12	166.2		

Prob. 2. Steam at a pressure of 100 lbs. per square inch absolute is admitted to a cylinder containing .1 cu.ft. of steam at the same pressure, until the cylinder contains 1 cu.ft., when the supply valve closes and the volume increases so that the product of pressure and volume is constant until a pressure of 30 lbs. is reached. The exhaust valve is opened, the pressure drops to 10 lbs. and steam is forced out until the volume becomes 1 cu.ft., when the exhaust valve closes and the remaining steam decreases in volume so that product of pressure and volume is constant until the original point is reached. Draw the pressure-volume diagram for this case.

Volume in Cu.Ft.	Pressure in Lbs. Sq.In.
2.0	14.0
1.8	15.5
1.6	17.5
1.4	20.0
1.2	23.3
1.0	28.0
0.8	28.0
0.4	28.0
0.0	28.0

Prob. 3. During an air compressor stroke the pressures and volumes were as given in the accompanying table.

Draw the diagram to a suitable scale to give work area in foot-pounds directly.

Prob. 4. Draw the diagrams for last two problems of Section 3.

7. Work of Expansion and Compression. Any given quantity of gas or vapor confined and not subject to extraordinary thermal changes such as explosion, will suffer regular pressure changes for each unit of volume change, or conversely, suffer a regular volume change for each unit of pressure change, so that pressure change is dependent on volume change and vice versa. When the volume of a mass of gas or vapor, V_1 , is allowed to increase to V_2 by the movement of a piston in a cylinder, the pressure will regularly increase or decrease from P_1 to P_2 , and experience has shown that no matter what the gas or vapor or the thermal conditions, if steady, the volumes and pressures will have the relation for the same mass,

$$P_1 V_1^s = P_2 V_2^s = K, \quad (8)$$

or the product of the pressure and s power of the volume of a given mass will always be the same. The exponent s may have any value, but usually lies between 1 and 1.5 for conditions met in practice.

The precise value of s for any given case depends on

- (a) The substance.
- (b) The thermal conditions surrounding expansion or compression, s being different if the substance receives heat from, or loses heat to, external surroundings, or neither receives nor loses.
- (c) The condition of vapors as to moisture or superheat when vapors are under treatment.

Some commonly used values of s are given in the Handbook of Tables for various substances subjected to different thermal conditions during expansion or compression.

Not only does Eq. (8) express the general law of expansion, but it likewise expresses the law of compression for decreasing volumes in the cylinder with corresponding rise in pressure. Expansion in a cylinder fitted with a piston is called balanced expansion, because the pressure over the piston area is balanced by resistance to piston movement and the mass of gas or vapor is substantially at rest, the work of expansion being imparted to the piston and resisting mechanism attached to it. On the other hand, when the gas or vapor under pressure passes through a nozzle orifice to a region of lower pressure the falling pressure is accompanied by increasing volumes as before, but the work of expansion is imparted not to a piston, because there is none, but to the fluid itself, accelerating it until a velocity has been acquired as a resultant of the work energy received. Such expansion is termed free expansion and the law of Eq. (8) applies as well to free as to balanced expansion. This equation, then, is of very great value, as it is a convenient basis for computations of the work done in expansion or compression in cylinders and nozzles of all sorts involving every gas or vapor substance. Some expansion curves for different values of s are plotted to scale in Fig. 5, and the corresponding compression curves in Fig. 6, in which

Curve *A* has the exponent $s = 0$; Curve *B* has the exponent $s = .5$;
 Curve *C* " " $s = 1.0$; Curve *D* " " $s = 1.1$;

Curve *E* has the exponent $s = 1.2$; Curve *F* has the exponent $s = 1.3$;
 Curve *G* " " $s = 1.4$; Curve *H* " " $s = 1.5$.

The volume after expansion is given by $V_2 = V_1 \left(\frac{P_1}{P_2} \right)^{\frac{1}{s}}$, (9)
 so that the final volume depends on the original volume, on the ratio of the two

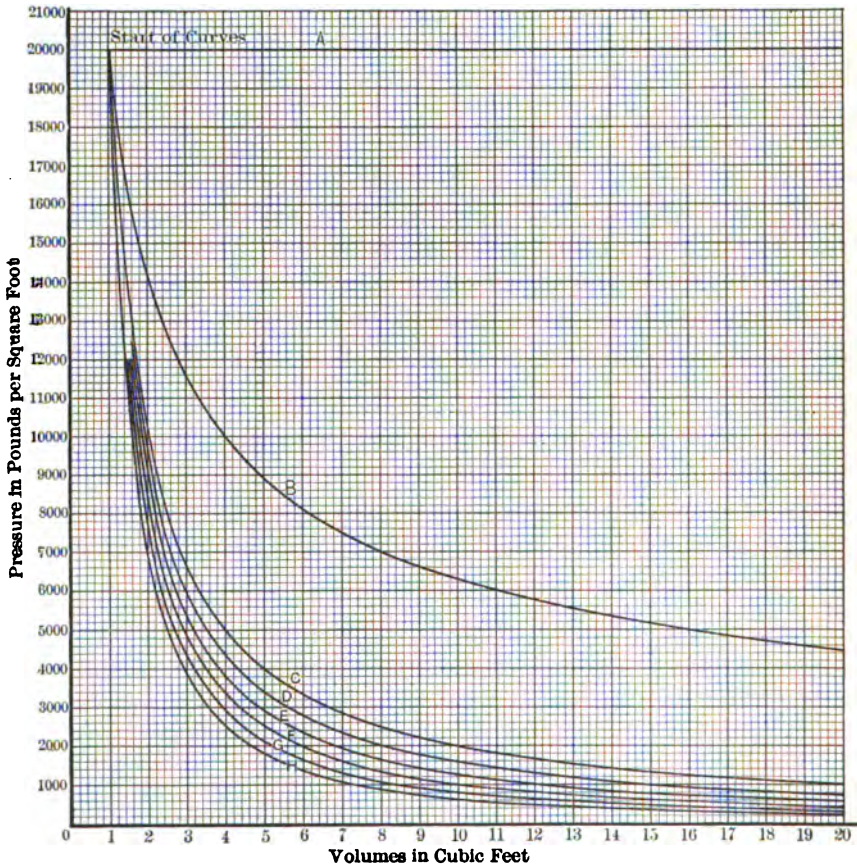


FIG. 5.—Comparison of Expansion Lines having Different Values of s .

pressures and on the value of the exponent. Similarly, the pressure after expansion

$$P_2 = P_1 \left(\frac{V_1}{V_2} \right)^s, \quad (10)$$

depends on the original pressure, on the ratio of the two volumes and on the exponent.

The general equation for the work of expansion or compression can now be integrated by means of the Eq. (8), which fixes the relation between pressures and volumes. From Eq. (8), $P = \frac{K}{V^s}$,

which, substituted in Eq.(7), gives $W = \int \frac{KdV}{V^s}$,

but as K is a constant, $W = K \int \frac{dV}{V^s}$ (11)

The integral of Eq. (11) will have two forms:

- (1) When s is equal to one, in which case $P_1 V_1 = P_2 V_2 = K$;
- (2) When s is not equal to one.

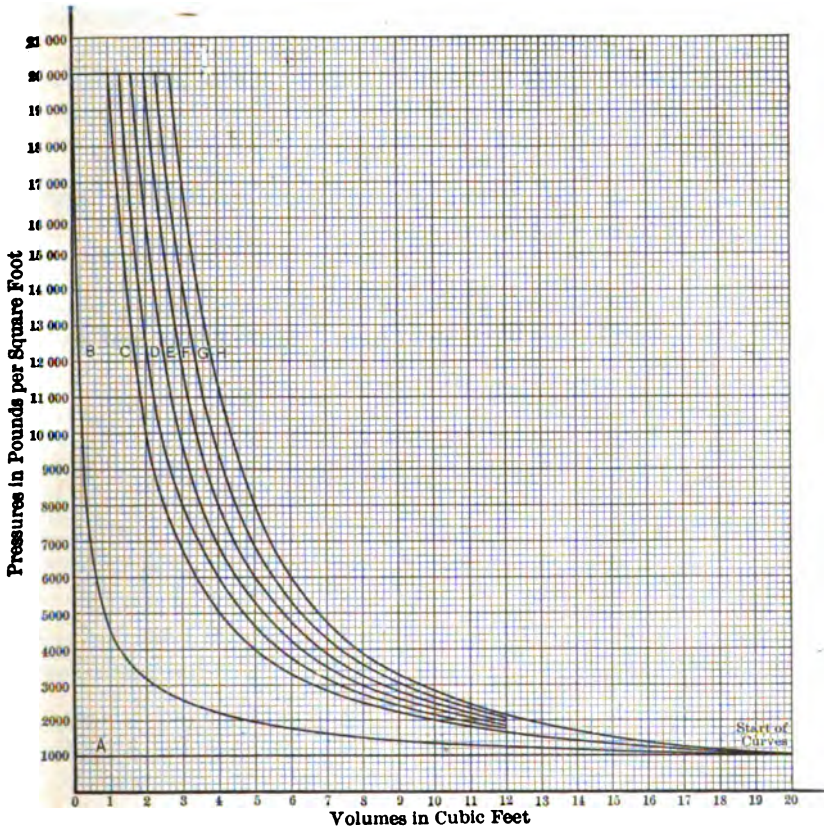


FIG. 6.—Comparison of Compression Curves having Different Values of s .

Taking first the case when s is equal to one, $W = K \int_{V_1}^{V_2} \frac{dV}{V}$. Whence

$$\begin{aligned} W &= K \log_e \frac{V_2}{V_1} \quad (a); & &= P_1 V_1 \log_e \frac{V_2}{V_1} \quad (b); & &= P_2 V_2 \log_e \frac{V_2}{V_1} \quad (c); \\ &= K \log_e \frac{P_1}{P_2} \quad (d); & &= P_1 V_1 \log_e \frac{P_1}{P_2} \quad (e); & &= P_2 V_2 \log_e \frac{P_1}{P_2} \quad (f). \end{aligned}$$

When $s=1$ (12)

Eqs. (12) are all equal and set down in different forms for convenience in computation; in them

V_2 = largest volume = initial vol. for compression = final vol. for expansion.

P_2 = smallest pressure = initial pres. for compression = final pres. for expansion.

V_1 = smallest volume = final vol. for compression = initial vol. for expansion.

P_1 = largest pressure = final pres. for compression = initial pres. for expansion.

These Eqs. (12) all indicate that the work of expansion and compression of this class is dependent only on the ratio of pressures or volumes at the beginning and end of the process, and the PV product at either beginning or end, this product being of constant value.

When the exponent s is not equal to one, the equation takes the form,

$$W = K \int_{V_1}^{V_2} \frac{dV}{V^s} = K \int_{V_1}^{V_2} V^{-s} dV = \frac{K}{1-s} \left[V_2^{1-s} - V_1^{1-s} \right].$$

As s is greater than one, the denominator and exponents will be negative, so, changing the form to secure positive values,

$$W = \frac{K}{s-1} \left(\frac{1}{V_1^{s-1}} - \frac{1}{V_2^{s-1}} \right).$$

This can be put in a still more convenient form. Multiplying and dividing by $\frac{1}{V_2^{s-1}}$ or $\frac{1}{V_1^{s-1}}$.

$$W = \frac{K}{s-1} \frac{1}{V_2^{s-1}} \left[\left(\frac{V_2}{V_1} \right)^{s-1} - 1 \right] = \frac{K}{s-1} \frac{1}{V_1^{s-1}} \left[1 - \left(\frac{V_1}{V_2} \right)^{s-1} \right].$$

Substituting the value of $K = P_2 V_2^s = P_1 V_1^s$,

$$W = \frac{1}{s-1} \frac{P_2 V_2^s}{V_2^{s-1}} \left[\left(\frac{V_2}{V_1} \right)^{s-1} - 1 \right] = \frac{1}{s-1} \frac{P_1 V_1^s}{V_1^{s-1}} \left[1 - \left(\frac{V_1}{V_2} \right)^{s-1} \right].$$

$$\begin{aligned} \text{Whence } W &= \frac{P_2 V_2}{s-1} \left[\left(\frac{V_2}{V_1} \right)^{s-1} - 1 \right] & (a) &= \frac{P_2 V_2}{s-1} \left[\left(\frac{P_1}{P_2} \right)^{\frac{s-1}{s}} - 1 \right] & (b) \\ &= \frac{P_1 V_1}{s-1} \left[1 - \left(\frac{V_1}{V_2} \right)^{s-1} \right] & (c) &= \frac{P_1 V_1}{s-1} \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{s-1}{s}} \right] & (d) \end{aligned}$$

When $s \neq 1$ (13)

Eqs. (13) gives the work for this class of expansion and compression in terms of pressure ratios and volume ratios as previously indicated.

V_2 = largest volume = initial vol. for compression = final vol. for expansion;

P_2 = smallest pressure = initial pres. for compression = final pres. for expansion;

V_1 = smallest volume = final vol. for compression = initial vol. for expansion;

P_1 = largest pressure = final pres. for compression = initial pres. for expansion.

The work of expansion or compression of this class is dependent according to Eqs. (13), upon the ratio of pressures or volumes at beginning and end of the process, the exponent, and on the pressure-volume product appropriately taken. It should be remembered that for the result to be in foot-pounds appro-

appropriate units should be used and all pressures taken absolute. Examination of Eqs. (12) and (13), for the work done by expansion or compression of both classes, shows that it is dependent on the initial and final values of pressures and volumes and on the exponent s , which defines the law of variation of pressure with volume between the initial and final states.

Example 1. Method of calculating Diagram, Figs. 5 and 6. Consider the curve for which $s=1.4$ as typical of the group.

Assumed Data: $V_1=1.0$ cu.ft. $P_1=20,000$ lbs. per square foot. $s=1.4$.

Then $P_1 V_1^s = K = 20,000 \times 1^{1.4} = 20,000$.

For any other value of P , V was found from the relation, $V = \left(\frac{K}{P}\right)^{\frac{1}{s}}$.

Let $P_z=6000$, then $V_z = \left(\frac{K}{P_z}\right)^{\frac{1}{s}} = \left[\frac{20,000}{6000}\right]^{.715} = [3.33]^{.715}$

$\log 3.33 = .5224$; hence $.715 \times .5224 = .373 = \log V_z$. $\therefore V_z = 2.36$.

A series of points, as shown below, were found, through which the curve was drawn.

P	$\frac{20,000}{P}$	$\log \frac{20,000}{P}$	$\frac{1}{s} \log \frac{20,000}{P}$	V
18000	1.111	0.0453	0.032	1.08
14000	1.430	0.1553	0.111	1.30
10000	2.000	0.3010	0.214	1.64
6000	3.330	0.5224	0.373	2.36
2000	10.000	1.0000	0.714	5.18
1000	20.000	1.3010	0.930	8.51

Curves for other values of s were similarly drawn. Starting at a common volume of 20 cu. ft. the compression curves of Fig. 6 were determined by the same methods.

Example 2. A pound of air at 32° F. and under atmospheric pressure is compressed to a pressure of five times the original. What will be the final volume and the work done if $s=1$ and if $s=1.4$? The volume of 1 lb. of air at 32° F. and one atmosphere is 12.4 cu.ft approx. The initial pressure in pounds per square foot is equal to $144 \times 14.7 = 2116$.

For $s=1$, $\frac{P_1}{P_2} = 5$, $V_2 = 12.4$ cu.ft.;

$$\frac{V_2}{V_1} = \frac{P_1}{P_2} = \frac{12.4}{V_1} = 5, \text{ whence } V_1 = 2.48 \text{ cu.ft.}$$

$$W = P_2 V_2 \log_e \frac{P_1}{P_2} = 2116 \times 12.4 \log_e 5; = 2116 \times 12.4 \times 1.61 = 42,300 \text{ ft.-lbs.}$$

$$\text{For } s=1.4, \quad \frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{1.4}} = \frac{12.4}{V_1} = (5)^{\frac{1}{1.4}} = (5)^{.71}.$$

5 may be raised to the .71 power by means of logarithms as follows: $(5)^{.71}$ is equal to the number whose logarithm is $.71 \log 5$.

$\log 5 = .699$, $.71 \times .699 = .4963$, and number of which this is the logarithm is 3.13, hence $V_1 = V_2 + 3.13$ or $V_1 = 3.96$;

$$W = \frac{P_2 V_2}{s-1} \left[\left(\frac{P_1}{P_2} \right)^{\frac{s-1}{s}} - 1 \right] = \frac{2116 \times 12.4}{.4} \left[(5)^{\frac{.4}{1.4}} - 1 \right].$$

$$= \frac{2116 \times 12.4}{.4} \times .583 = 38,200 \text{ ft.-lbs.}$$

The value of W can also be found by any other form of equation (13) such as

$$W = \frac{P_1 V_1}{s-1} \left[1 - \left(\frac{V_1}{V_2} \right)^{s-1} \right].$$

The value of V_1 being found as before, the work expression becomes after numerical substitution

$$W = \frac{10,580 \times 3.96}{.4} \left[1 - \left(\frac{3.96}{12.4} \right)^{.4} \right].$$

As the quantity to be raised to the .4 power is less than one, students may find it easier to use the reciprocal as follows:

$$\left(\frac{3.96}{12.4} \right)^{.4} = \frac{1}{\left(\frac{12.4}{3.96} \right)^{.4}} = \frac{1}{(3.13)^{.4}} = \frac{1}{1.58} = .632.$$

Hence

$$W = \frac{10,580 \times 3.96}{.4} (1 - .632) = 38,200 \text{ ft.-lbs.}$$

Prob. 1. Find V_1 and W for Example 2 if $s = 1.2$ and 1.3 .

Prob. 2. If a pound of air were compressed from a pressure of 1 lb. per square inch absolute to 15 lbs. per square inch absolute find V_1 and W when $s = 1$ and 1.4 . $V_2 = 180$ cu.ft. What would be the H.P. to compress 1 lb. of air per minute?

Prob. 3. Air expands so that $s = 1$. If $P_1 = 10,000$ lbs. per square foot, $V_1 = 10$ cu.ft. and $V_2 = 100$ cu.ft. and the expansion takes place in 20 seconds, what is the H.P. developed?

Prob. 4. 100 cu.ft. of air at atmospheric pressure are compressed in a cylinder to a pressure of 8 atmospheres and then expelled against this constant pressure. Find graphically and by calculation the foot-pounds of work done for the case where $s = 1$ and for the case where $s = 1.4$.

Prob. 5. At an altitude of 4000 ft., air is compressed to a pressure of 60 lbs. per sq.in. gage. Find the H.P. required to compress 1000 cu.ft. of free air per minute.

Prob. 6. From the algebraic equation show how much work is done for a volume change of 1 to 4, provided pressure is originally 1000 lbs. per square foot when

$$(a) PV^0 = K_1; \quad (b) PV = K_2; \quad (c) PV^2 = K_3.$$

Prob. 7. A vacuum pump compresses air from 1 lb. per square inch absolute to 15 lbs. per square inch absolute and discharges it. An air compressor compresses air from atmosphere to 15 atmospheres and discharges it. Compare the work done for equal initial volumes, $s = 1.4$.

Prob. 8. For steam expanding according to the saturation law, compare the work done by 1 lb. expanding from 150 lbs. per square inch absolute to 15 lbs. per square inch absolute with the work of the same quantity expanding from 15 lbs. to 1 lb. per square inch absolute.

NOTE. 1 lb. of steam occupies 3 cu.ft. at 150 lbs. per square inch absolute.

Prob. 9. Two air compressors of the same size compress air adiabatically from atmosphere to 100 lbs. gage and discharge it. One is at sea level, the other at 10,000 ft. elevation. Compare the work in the two cases.

8. Values of Exponent s Defining Special Cases of Expansion or Compression. There are three general methods of finding s for the definition of particular cases of expansion or compression to allow of the solution of numerical problems. The first is experimental, the second and third thermodynamic. If by measurement the pressures and volumes of a series of points on an expansion or compression curve, obtained by test with appropriate instruments, for example, the indicator, be set down in a table and they be compared in pairs, values of s can be found as follows: Calling the points A, B, C , etc., then,

$$P_a V_a^s = P_b V_b^s,$$

and $\log P_a + s \log V_a = \log P_b + s \log V_b,$

or $s(\log V_b - \log V_a) = \log P_a - \log P_b,$

hence $s = \frac{\log P_a - \log P_b}{\log V_b - \log V_a} \quad (a) \quad s = \frac{\log \left(\frac{P_a}{P_b} \right)}{\log \left(\frac{V_b}{V_a} \right)} \quad (b). \quad . \quad . \quad . \quad (14)$

According to Eq. (14a), if the difference between the logarithms of the pressures at B and A be divided by the difference between the logarithms of the volumes at A and B respectively, the quotient will be s . According to Eq. (14b), the logarithm of the ratio of pressures, B to A , divided by the logarithm of the ratio of volumes, A to B respectively will also give s . It is interesting to note that if the logarithms of the pressures be plotted vertically and logarithms of volumes horizontally as in Fig. 7, then the line AC equal to the intercept on the horizontal axis represents the difference between the logarithms of volumes or,

$$\overline{CA} = \log V_a - \log V_b,$$

and similarly $\overline{CB} = \log P_b - \log P_a$. Hence $s = \frac{\overline{CB}}{\overline{CA}} = \tan \alpha,$

or the slope of the line indicates the value of s . This is a particularly valuable method, as it indicates at a glance the constancy or variability of s , and there are many cases of practice where s does vary. Should s be constant the line will be straight; should it be variable the line will be curved, but can generally be divided into parts, each of which is substantially straight and each will have a different s . It is sometimes most convenient to take only the beginning

and end of the curve and to use the value of s corresponding to these points, neglecting intermediate values.

A second method for finding s for a given compression or expansion line by means of areas is indicated in a note in Section 17 of this Chapter that is omitted here because it depends on formulas not yet derived. It is by this sort of study of experimental data that most of the variable values of s have been obtained. There is, however, another method of finding a value for s by purely thermodynamic analysis based on certain fundamental hypotheses, and the value is as useful as the hypotheses are fair or true to the facts of a particular case.

One of the most common hypotheses of this sort is that the *gas* or *vapor* undergoing expansion or compression shall neither receive any heat from, nor give up any to bodies external to itself during the process, and such a process is given the name *adiabatic*. Whether adiabatic processes are possible in actual cylinders or nozzles does not affect the analysis with which pure thermody-

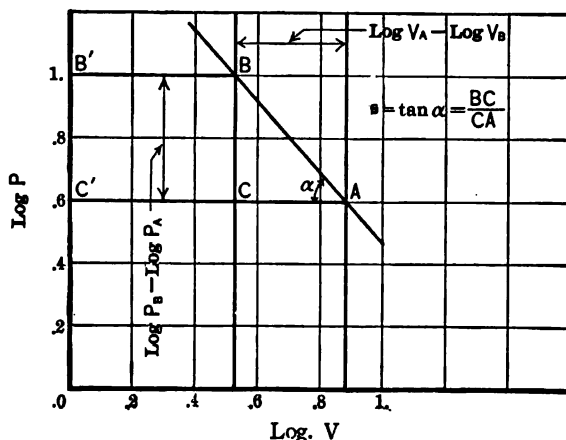


FIG. 7.—Graphic Method of Finding s , from Logarithms of Pressures and Volumes.

namics is concerned. By certain mathematical transformations, to be carried out later, and based on a fundamental thermodynamic proposition, the adiabatic hypothesis will lead to a value of s , the use of which gives results valuable as a basis of reference, and which when compared with an actual case will permit of a determination of how far the real case has departed from the adiabatic condition, and how much heat has been received or lost

at any part of the process. The particular value of s which exists in an adiabatic change is represented by the symbol γ .

Another common hypothesis on which another value of s can be derived, is that gases in expansion or compression shall remain at a constant temperature, thus giving rise to the name *isothermal*. This is generally confined to gases and superheated vapors, as it is difficult to conceive of a case of isothermal or constant temperature expansion or compression of wet vapors, as will be seen later.

In the study of vapors, which, it must be understood, may be dry or wet, that is, containing liquid, a common hypothesis is that during the expansion or compression they shall remain just barely dry or that they shall receive or lose just enough heat to keep any vapor from condensing, but no more than sufficient to keep any moisture that tends to form always evaporated. Expansion or compression according to this hypothesis is said to follow the saturation

law, and the substance to remain saturated. It will appear from this thermal analysis later that the value of s for the isothermal hypothesis is the same for all gases and equal to one, but for the adiabatic hypothesis $s=\gamma$ will have a different value for different substances, though several may have the same value, while for vapors γ will be found to be a variable for any one, its value depending not only on the substance, but on the temperatures, pressures and wetness.

When gases or vapors are allowed to expand in cylinders and nozzles or are compressed, it is often difficult and sometimes impossible or perhaps undesirable to avoid interference with the adiabatic conditions for vapors and gases, with the isothermal for gases or with the saturation law for vapors, yet the work to be done and the horse-power developed cannot be predicted without a known value of s , which for such cases must be found by experience. A frequent cause of interference with these predictions, which should be noted, is leakage in cylinders, which, of course, causes the mass under treatment to vary.

Mixtures of common gases such as constitute natural, producer, blast furnace or illuminating gas, alone or with air or products of combustion, such as used in internal combustion engines, have values of s that can be calculated from the elementary gases or measured under actual conditions.

All vapors, except those considerably overheated, have variable exponents for adiabatic expansion and compression. This fact makes the exact solution of problems of work for wet vapors, expanding or compressing, which form the bulk of the practical cases, impossible by such methods as have been described. This class of cases can be treated with precision only by strictly thermal methods to be described later.

Prob. 1. By plotting the values for the logarithms of the following pressures and volumes, see if the value for s is constant, and if not find the mean value in each case.

(a) GAS ENGINE COMPRESSION

V	p	V	p	V	p
10	45.2	13	32.2	18	21.0
11	39.7	14	29.7	20	19.5
12	35.7	16	24.7	25	14.7

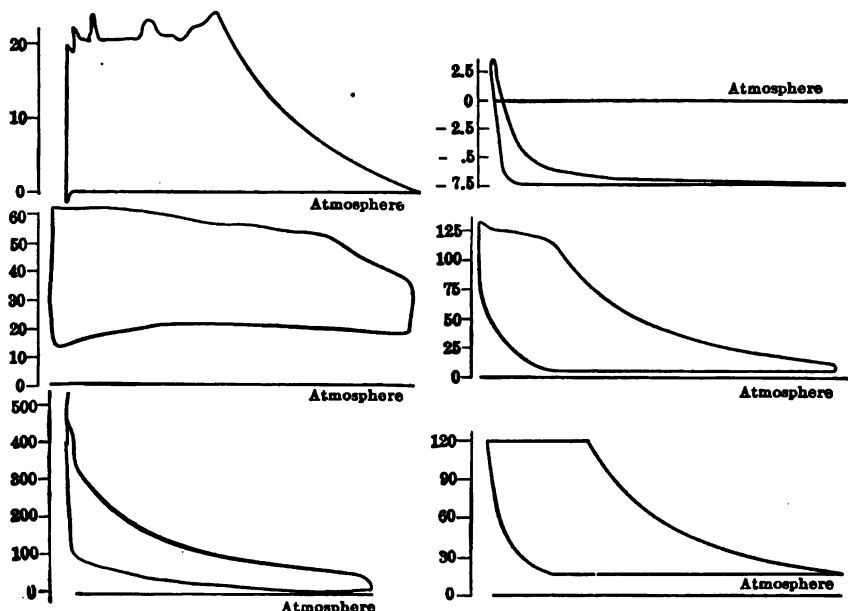
(b) GAS ENGINE EXPANSION

V	p	V	p	V	p
		13	146.2	19	80.7
11	188.2	15	116.7	21	68.7
12	166.2	17	95.7	23	58.7

(c) STEAM EXPANSION

V	p	V	p
2.242	203.3	7.338	52.5
2.994	145.8	12.44	28.8
4.556	89.9	22.68	14.7

Prob. 2. By plotting the values for the logarithms of the volumes and pressures on the expansion and compression curves of the following cards, find value for s .



Prob. 3. From the steam tables select the pressures and volumes for dry-saturated steam and find the value of s between

- (a) 150 lbs. per square inch and 1 lb. per square inch.
 (b) 15 " " "

Prob. 4. Find for superheated steam at 150 lbs. per square inch and with 100° of superheat expanding to 100 lbs. per square inch without losing any superheat, the corresponding value of s , using tabular data.

Prob. 5. From the ammonia table data for dry-saturated vapor find the value of s between

- (a) 150 lbs. per square inch and 1 lb. per square inch.
 (b) 15 " " "

9. Work Phases and Cycles, Positive and Negative and Net Work. According to the preceding it is easy to calculate or predict numerically the work of expansion or compression whenever the conditions are sufficiently definite to permit of the selection of the appropriate s . It very seldom happens, however, that the most important processes are single processes or that the work of expansion or compression is of interest by itself. For example, before expansion can begin in a steam cylinder steam must be first admitted, and in air compressors air must be drawn in before it can be compressed. Similarly, after expansion in a steam cylinder there must be an expulsion of used vapor before another admission and expansion can take place, while in the air compressor after compression the compressed air must be expelled before more can enter

for treatment. The whole series of operations is a matter of more concern than any one alone, and must be treated as a whole. The effect can be most easily found by the summation of the separate effects, and this method of summation will be found of universal application.

The whole series of processes taking place and involving pressure-volume changes is called a *cycle*, any one of them a *phase*. It is apparent that there can be only a limited number of phases so definite as to permit of the mathematical treatment necessary for prediction of work, but it is equally clear that there may be a far greater number of combinations of phases constituting cycles. Before proceeding to analyze the action of steam or gas in a cylinder it is necessary first to determine on structural, thermal or any other logical grounds, what series of separate processes will be involved, in what order, and the pressure-volume characteristics of each. Then and then only, can the cycle as a whole be treated. These phases or separate and characteristic processes affecting the work done, or involving pressure volume changes, are divisible into two classes so far as the causes producing them are concerned; the first thermal and the second mechanical. It requires no particular knowledge of thermodynamics to realize that if air be confined in a cylinder with a free piston and is heated, that the volume will increase while pressure remains constant, since the piston will move out with the slightest excess of pressure inside over what is outside. This is a pressure constant, volume increasing, phase, and is thermal since it is a heat effect. If an ample supply of steam be available from a boiler held at a constant pressure by the manipulation of dampers and fires by the fireman and the steam be admitted so a cylinder with a piston, the piston will move out, the pressure remaining constant and volume increasing. This is also a pressure constant, volume increasing phase, exactly as before, but is mechanical because it is due to a transportation of steam from the boiler to the cylinder, although in another sense it may be considered as thermal if the boiler, pipe and cylinder be considered as one part during the admission. A similar constant pressure phase will result when a compressor piston is forcibly drawn out, slightly reducing the pressure and permitting the outside atmosphere to push air in, to follow the piston; and after compression of air to a slight excess, the opening of valves to storage tanks or pipe lines having a constant pressure will allow the air to flow out or be pushed out of the cylinder at constant pressure. These two constant pressure phases are strictly mechanical, as both represent transmission of the mass. If a cylinder contain water and heat be applied without permitting any piston movement, there will be a rise of pressure at constant volume; a similar constant volume pressure rise phase will result from the heating of a contained mass of gas or vapor under the same circumstances, both of these being strictly thermal.

However much the causes of the various characteristic phases may differ, the work effects of similar ones is the same, and at present only work effects are under consideration. For example, all constant volume phases do no work as work cannot be done without change of volume.

The consideration of the strictly thermal phases is one of the principal

problems of thermodynamics, for by this means the relation between the work done to the heat necessary to produce the phase changes is established, and a basis laid for determining the ratio of work to heat, or efficiency. For the present it is sufficient to note that the work effects of any phase will depend only on the pressure-volume changes which characterize it.

Consider a cycle, Fig. 8, consisting of (*AB*), admission of 2 cu.ft. of steam at a constant pressure of 200,000 lbs. per square foot, to a cylinder originally containing nothing, followed by (*BC*), expansion with $s=1$, to a pressure of 20,000 lbs. per square foot; (*CF*), constant volume change of pressure, and (*FG*), constant pressure exhaust at 10,000 lbs. per square foot. These operations are plotted to scale. Starting at zero volume, because the cylinder

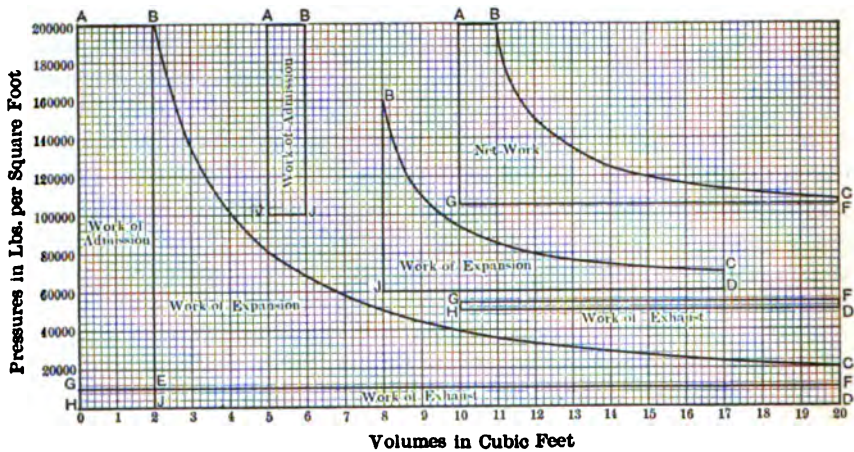


FIG. 8.—Analysis of Work Diagram for Admission Expansion and Exhaust of Engine without Clearance.

originally contains nothing, and at a pressure of 200,000 lbs. per square foot, the line *AB*, ending at volume 2 cu.ft., represents admission and the area under *AB* represents the 400,000 ft.-lbs. of work done during admission. At *B* the admission ceases by closure of a valve and the 2 cu.ft. of steam at the original pressure expands with lowering pressure according to the law $P_a V_a = P_b V_b = 200,000 \times 2 = 400,000$ ft.-lbs.,

so that when $V = 4$ cu.ft., $P = \frac{400,000}{4} = 100,000$ lbs. per sq.ft.;

$$V = 5 \text{ cu.ft.}, P = \frac{400,000}{5} = 80,000 \text{ lbs. per sq.ft.};$$

$$V = 10 \text{ cu.ft.}, P = \frac{400,000}{10} = 40,000 \text{ lbs. per sq.ft.}$$

This continues until $V = 20$ at point *C*, at which time $P = \frac{400,000}{20} = 20,000$ lbs. per square foot, and the work done during expansion is the area *JBCD*

under the expansion curve BC , the value of which can be found by measuring the diagram or by using the formula Eq. (12), which on substitution gives

$$\begin{aligned} W_{bc} &= 400,000 \log_e 10 = 400,000 \times 2.3; \\ &= 920,000 \text{ ft.-lbs.} \end{aligned}$$

This completes the stroke and the work for the stroke can be found by addition of the numerical values,

$$\begin{aligned} W_{ab} &= 400,000 \text{ ft.-lbs.}; & W_{bc} &= 920,000 \text{ ft.-lbs.}; \\ W_{ac} + W_{bc} &= 1,320,000 \text{ ft.-lbs.} \end{aligned}$$

It is often more convenient to find an algebraic expression for the whole, which for this case will be,

$$\begin{aligned} W_{ab} &= P_a V_a = P_b V_b; & W_{bc} &= P_b V_b \log_e \frac{V_c}{V_b}; \\ W_{ac} &= W_{ab} + W_{bc} = P_b V_b \left(1 + \log_e \frac{V_c}{V_b} \right), \\ &= 400,000(1 + \log_e 10) = 400,000 \times 3.3 = 1,320,000 \text{ ft.-lbs.} \end{aligned}$$

On the return of the piston it encounters a resistance due to a constant pressure of 10,000 lbs. per square inch, opposing its motion; it must, therefore, do work on the steam in expelling it. Before the return stroke begins, however, the pressure drops by the opening of the exhaust valve from the terminal pressure of the expansion curve to the exhaust or *back* pressure along the constant volume line, CF , of course, doing no work, after which the return stroke begins, the pressure volume line being FG and the work of the stroke being represented by the area $DFGH$, $W_{fd} = P_f V_f = 10,000 \times 20 = 200,000$ ft.-lbs. This is *negative* work, as it is done in opposition to the movement of the piston. The cycle is completed by admission of steam at constant zero volume, raising the pressure along GA . The net work is the difference between the positive and negative work, or algebraically

$$\begin{aligned} W &= P_b V_b \left(1 + \log_e \frac{V_c}{V_b} \right) - P_f V_f, \\ &= 1,320,000 - 200,000 = 1,120,000 \text{ ft.-lbs.} \end{aligned}$$

Consider now a cycle of an air compressor, Fig. 9. Admission or suction is represented by AB , compression by BC , delivery by CD and constant volume drop in pressure after delivery by DA . The work of admission is represented by the area $ABFE$ or algebraically by $W_{ab} = P_b V_b$, the work of compression by the area $FBCG$, or algebraically since $s = 1.4$ by

$$W_{bc} = \frac{P_b V_b}{s-1} \left[\left(\frac{P_c}{P_b} \right)^{\frac{s-1}{s}} - 1 \right],$$

the work of delivery by the area $CDEG$, or algebraically $W_{cd} = P_c V_c$.

The positive work is that assisting the motion of the piston during suction; the area $ABFE$ or algebraically $P_b V_b$. The negative work, that in opposition to the motion, is the sum of the compression and delivery work, the area $FBCDE$, or algebraically,

$$W_{bc} + W_{cd} = P_c V_c + \frac{P_b V_b}{s-1} \left[\left(\frac{P_c}{P_b} \right)^{\frac{s-1}{s}} - 1 \right].$$

The net work is the difference and is negative, as such a cycle is mainly resistant,

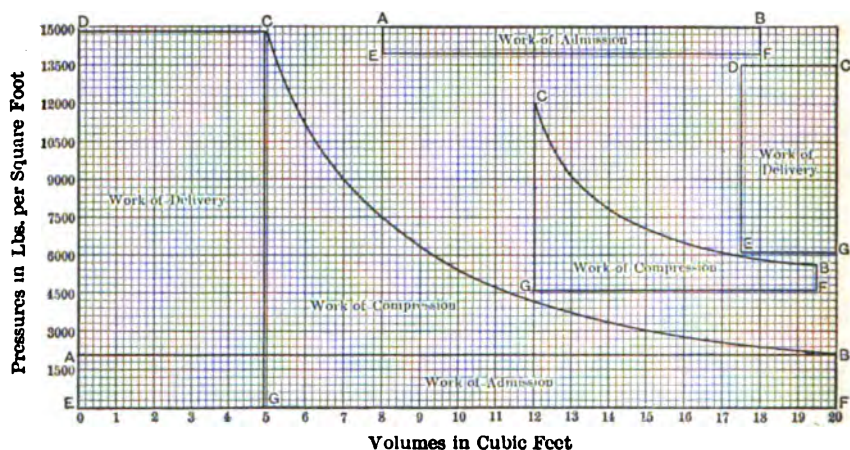


FIG. 9.—Analysis of Work Diagram for Admission Compression and Delivery of Compressor without Clearance.

and to execute it the piston must be driven with expenditure of work on the gas. The value of the net work is,

$$\begin{aligned} W &= W_{bc} + W_{cd} - W_{ab} \\ &= P_c V_c + \frac{P_b V_b}{s-1} \left[\left(\frac{P_c}{P_b} \right)^{\frac{s-1}{s}} - 1 \right] - P_b V_b, \end{aligned}$$

an expression which will be simplified in the chapter on compressors. This net work is represented by the area $ABCD$, which is the area enclosed by the cycle itself independent of the axes of coordinates.

It might seem from the two examples given as if net work could be obtained without the tedious problem of summation, and this is in a sense true if the cycle is plotted to scale or an algebraic expression be available, but these processes are practically equivalent to summation of phase results. It might also seem that the work area would always be that enclosed by the cycle, and this is true with a very important limitation, which enters when the cycle has loops. If, for example, as in Fig. 10, steam admitted A to B , expanded along BC to a pressure C , then on opening the exhaust the pressure instead of falling to the back pressure or exhaust line as in Fig. 8, would here

rise along CD , as the back pressure is higher than the terminal expansion pressure, after which exhaust will take place at constant back pressure along DE . The forward stroke work is that under AB and BC or $ABCEG$, the return stroke work is the area $DEGH$ and the net work is Area $ABCEG$ - Area $DEGH$. As the area $HGECH$ is common to both terms of the difference, the net work may be set down as equal to Area $ABXH - CDX$.

It may be said then, in general, for looped cycles that the net work area is the difference between that of the two loops. If, however, the method laid down for the treatment of any cycle be adhered to there need not be any distinction drawn between ordinary and looped cycles, that is, in finding the work

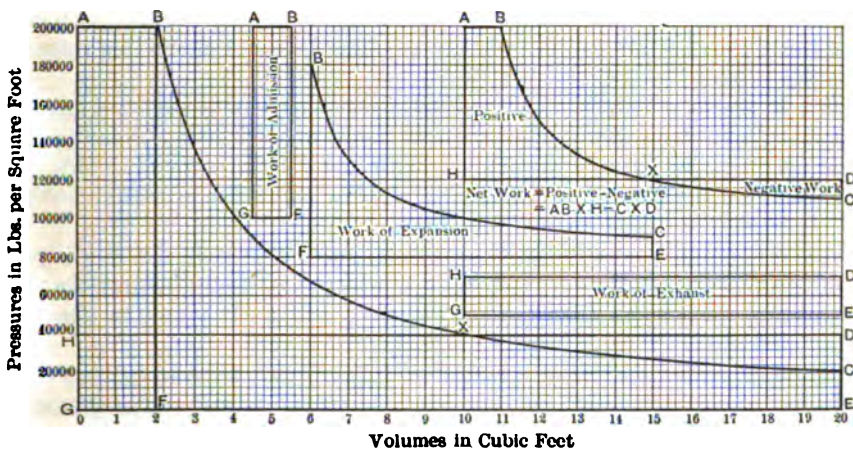


FIG. 10.—Analysis of Work Diagram for Engine with Over-expansion Negative Work Loop.

of a cycle divide it into characteristic phases and group them into positive and negative, find the work for each and take the algebraic sum.

Special cases of cycles and their characteristics for steam, compressor and gas engine cylinders, as well as nozzle expansion, will be taken up later in more detail.

Example 1. Method of calculating Diagram, Fig. 8.

$$\text{Assumed data } \left\{ \begin{array}{l} V_a = 0 \text{ cu.ft.} \\ V_b = 2 \text{ " } \\ V_d = V_c \text{ " } \\ V_e = 2 \text{ " } \\ s = 1 \end{array} \right\} \quad \left\{ \begin{array}{l} P_a = 200,000 \text{ lbs. per square foot.} \\ P_b = P_a \text{ " " } \\ P_c = 20,000 \text{ " " } \\ P_f = 10,000 \text{ " " } \\ P_e = P_f \text{ " " } \end{array} \right\}$$

To obtain point C, $P_c V_c = P_b V_b$ or $V_c = \frac{P_b V_b}{P_c} = \frac{200,000 \times 2}{20,000} = 20$.

Intermediate points B to C are obtained by assuming various pressures and finding the corresponding volumes as for V_c .

Example 2. Method of calculating Diagram, Fig. 9.

$$\text{Assumed data } \left\{ \begin{array}{l} V_a = 0 \text{ cu.ft.} \\ V_b = 20 \text{ " " } \\ V_c = 0 \text{ " " } \\ s = 1.4 \end{array} \right\} \quad \left\{ \begin{array}{l} P_a = 2116 \text{ lbs. per square foot.} \\ P_b = P_a \text{ " " " " } \\ P_c = 14,812 \text{ " " " " } \\ P_d = P_c \text{ " " " " } \end{array} \right\}$$

To obtain point C, $P_b V_b^{1.4} = P_c V_c^{1.4}$ or $V_c = V_b \div \left(\frac{P_c}{P_b} \right)^{\frac{1}{1.4}} = V_b \div \left(\frac{P_c}{P_b} \right)^{.715}$

$$\frac{P_c}{P_b} = 7, \log 7 = .845; \text{ and } .715 \times .845 = \log \left(\frac{P_c}{P_b} \right)^{\frac{1}{1.4}} = .6105,$$

Therefore,

$$V_c = 4.97, \text{ and } P_c = 14,812.$$

Intermediate values *B* to *C* may be found by assuming pressures and finding volumes corresponding as for *V_c*.

Prob. 1. Steam at 150 lbs. per square inch absolute pressure is admitted into a cylinder in which the volume is originally zero until the volume is 2 cu.ft., when the valve is closed and expansion begins and continues until the volume is 8 cu.ft., then exhaust valve opens and the pressure falls to 10 lbs. absolute and steam is entirely swept out. Draw the diagram and find the net work done.

Prob. 2. A piston moving forward in a cylinder draws in 10 cu.ft. of CO₂ at a pressure of .9 of an atmosphere at sea level and then compresses it adiabatically until the pressure rises to 9 atmospheres and discharges it at constant pressure. Draw the diagram and find net work done.

Prob. 3. A cylinder 18 ins. in diameter and 24 ins. piston stroke receives steam at 100 lbs. per square inch absolute pressure for $\frac{1}{4}$ of the stroke. It then expands to the end of the stroke and is exhausted at atmospheric pressure. Draw the diagram and find the H.P. if the engine makes 100 strokes per minute.

Prob. 4. Two compressors without clearance each with a cylinder displacement of 2 cu.ft. draw in air at 14 lbs. per square inch absolute and compress it to 80 lbs. per square inch absolute before delivery. Find the difference in H.P. per 1000 cu.ft. of free air per minute if one is compressing isothermally and the other adiabatically. Draw diagram for each case.

Prob. 5. A quantity of air 5 cu.ft. in volume and at atmospheric pressure is compressed in a cylinder by the movement of a piston until the pressure is 50 lbs. per square inch gage. If the air be heated, the pressure will rise, as in an explosion. In this case the piston remains stationary, while the air is heated until the pressure reaches 200 lbs. per square inch gage. It then expands adiabatically to the original volume when the pressure is reduced to atmosphere with no change in volume. Draw the diagram, and find the work done.

Prob. 6. The *Brayton cycle* is one in which gas is compressed adiabatically and then by the addition of heat, the gas is made to expand without change of pressure. Adiabatic expansion then follows to original pressure and the cycle ends by decrease in volume to original amount without change of pressure. Draw such a cycle starting with 5 cu.ft. of air at atmospheric pressure, compressing to 4 atmospheres, expanding at constant pressure to 5 cu.ft., expanding adiabatically to original pressure and finally ending at original point. Find also, work done.

Prob. 7. In the *Ericsson cycle* air is expanded at constant temperature, cooled at constant pressure, compressed at constant temperature and receives heat at constant

pressure. Draw a diagram for the case where 5 cu.ft. at atmospheric pressure are compressed to 1 cu.ft., heated until volume is 8 cu.ft., expanded to atmosphere and then cooled to original volume. Find the work.

Prob. 8. In the *Stirling cycle* constant volume heating and cooling replace that at constant pressure in the Ericsson. Draw diagram starting with 5 cu.ft. and atmospheric pressure compressing to 1 cu.ft. and then after allowing the pressure to double, expand to original volume and cool to atmosphere. Find the work.

Prob. 9. The *Joule cycle* consists of adiabatic compression and expansion and constant pressure heating and cooling. Assuming data as in last problem draw the diagram and find the work.

Prob. 10. The *Carnot cycle* consists of isothermal expansion, adiabatic expansion, isothermal compression and adiabatic compression. Draw the diagram for this cycle and find the work.

10. Work Determination by Mean Effective Pressure. While the methods already described are useful for finding the work done in foot-pounds for a defined cycle with *known pressure and volume limits*, they are not, as a rule, convenient for the calculation of the work done in a cylinder of *given dimensions*. As work done can always be represented by an area, this area divided by its length will give its mean height. If the area be in foot-pounds with coordinates pounds per square foot, and cubic feet, then the division of area in foot-pounds by length in cubic feet will give the mean height or the mean pressure in pounds per square foot. Again, dividing the work of the cycle into forward-stroke work and back-stroke work, or the respective foot-pound areas divided by the length of the diagram in cubic feet, will give the *mean forward pressure* and the *mean back pressure*. The difference between mean forward pressure and mean back pressure will give the *mean effective pressure*, or that average pressure which if maintained for one stroke would do the same work as the cycle no matter how many strokes the cycle itself may have required for its execution, which is very convenient considering the fact that most gas engines require four strokes to complete one cycle. The mean effective pressure may also be found directly from the enclosed cycle area, taking proper account of loops, as representative of net work by dividing this net work area by the length of the diagram in appropriate units. This method is especially convenient when the diagram is drawn to odd scales so that areas do not give foot-pounds directly, for no matter what the scale the mean height of the diagram, when multiplied by the pressure scale factor, represents the mean effective pressure. This mean height can always be found in inches for any scale of diagram by finding the area of the diagram in square inches and dividing it by the length in inches; and this mean height in inches multiplied by the scale of pressures in whatever units may be used will give the mean effective pressure in the same units.

Mean pressures, forward, back or effective, are found and used in two general ways; first, algebraically, and second graphically; generally in this case from test records. By the first method, formulas, based on some assumed laws for the phases, can be found, and the mean effective pressure and its value

predicted. This permits of the prediction of work that may be done by a given quantity of gas or vapor, or the work per cycle in a cylinder, or finally the horsepower of a machine, of which the cylinder is a part, operating at a given speed and all without any diagram measurement whatever. By the second method, a diagram of pressures in the cylinder at each point of the stroke can be obtained by the indicator, yielding information on the scale of pressures. The net work area measured in square inches, when divided by the length in inches, gives the mean height in inches, which, multiplied by the pressure scale per inch of height, gives the mean effective pressure in the same units, which are usually pounds per square inch in practice.

As an example of the algebraic method of prediction, consider the cycle represented by Fig. 8. The forward work is determined from

$$\text{Forward work} = P_b V_b \left(1 + \log_e \frac{V_c}{V_e} \right),$$

the length of the diagram representing the volume swept through in the performance of this work is V_c , hence

$$\text{Mean forward pressure} = \frac{P_b V_b}{V_c} \left(1 + \log_e \frac{V_c}{V_e} \right).$$

But $P_b V_b = P_c V_c$ by the law of this particular expansion curve, hence

$$\text{Mean forward pressure} = P_c \left(1 + \log_e \frac{V_c}{V_e} \right).$$

As the back pressure is constant its mean value is this constant value, hence

$$\text{Constant (mean) back pressure} = P_f. \quad \text{By subtraction}$$

$$\begin{aligned} \text{Mean effective pressure} &= P_c \left(1 + \log_e \frac{V_c}{V_e} \right) - P_f \\ &= 3.3 P_c - P_f = 3.3 \times 20,000 - 10,000; \\ &= 66,000 - 10,000 = 56,000 \text{ lbs. per sq.ft.} \end{aligned}$$

The work done in foot-pounds is the mean effective pressure in pounds per square foot, multiplied by the displacement in cubic feet.

$$W = 56,000 \times 20 = 1,120,000 \text{ ft.-lbs. as before.}$$

As an example of the determination of mean effective pressure from a test or indicator diagram of unknown scale except for pressures, and without axes of coordinates, consider Fig. 11, which represents a gas engine cycle in four strokes, the precise significance of the lines being immaterial now. The pressure scale is 120 lbs. per square inch, per inch of height.

By measurement of the areas in square inches it is found that

Large loop area $CDEXC$	= 2.6 sq.in.
Small loop area $ABXA$	= 0.5 sq.in.
Net cycle area	= 2.1 sq.in.
Length of diagram	= 3.5 in.
Mean height of net work cycle	= 0.6 in.
Mean effective pressure	= $120 \times 0.6 = 72$ lbs. per square inch.

It is quite immaterial whether this diagram were obtained from a large or a small cylinder; no matter what the size, the same diagram might be secured and truly represent the pressure-volume changes therein. If this particular cylinder happened to have a diameter of 10 ins. and a stroke of 12 ins. the work per stroke can be found. The area of the cylinder will be 78.54 sq.ins., hence the average force on the piston is 72 lbs. per square inch $\times 78.54$ sq.ins. = 5654.88 lbs., and the stroke being 1 ft. the work per stroke is 5654.88 ft.-lbs. Both of these methods are used in practical work and that one is adopted in any particular case which will yield results by the least labor.

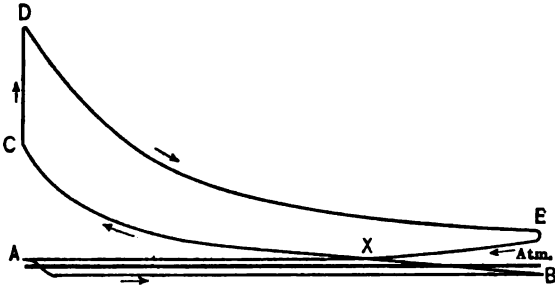


FIG. 11.—Gas-Engine Indicator Card. For Determination of Mean Effective Pressure without Volume Scale.

Prob. 1. An indicator card from an air compressor is found to have an area of 3.11 sq. ins., while the length is $2\frac{1}{2}$ ins. and scale of spring is given as 80 lbs. per square inch per inch height. What is m.e.p. and what would be the horse-power if the compressor ran with a piston speed of 250 ft. per minute and had a piston 9 ins. in diameter?

Prob. 2. For the same machine another card was taken with a 60-lb. spring and had an area of 4.12 sq.ins. How does this compare with first card, the two having the same length?

Prob. 3. A steam engine having a cylinder 18 ins. in diameter and a stroke of 24 ins., takes in $\frac{1}{2}$ cu.ft. of steam at 100 lbs. absolute, allows it to expand and exhausts at atmospheric pressure. An indicator card taken from the same engine showed a length of 3 ins., an area of .91 sq.in. when an 80-lb. spring is used. How does the actual m.e.p. compare with the computed?

Prob. 4. Find m.e.p. by the algebraic method of prediction for the following cycles: (a) Brayton; (b) Carnot; (c) Stirling; (d) Ericsson; (e) Joule.

(See problems following Section 9.)

11. Relation of Pressure-Volume Diagrams to Indicator Cards. The Indicator. When a work cycle or diagram of pressure-volume changes is drawn to scale with pressures and volumes as coordinates, it is termed a pressure volume or *PV* diagram, and may be obtained by plotting point by point from the algebraic expression for the law of each phase or by modifying the indicator card. The indicator card is that diagram of pressures and stroke obtained by applying a steam engine-indicator to a cylinder in operation.

By means of this instrument the pressures within the cylinder corresponding to the movement of the engine piston may be recorded autographically; but it must be clearly understood that such indicator diagrams or cards do not give the true or absolute pressures nor the true volumes of steam or gas in the cylinder, but only the pressures above or below atmosphere and the changes of volume of the fluid corresponding to the piston movement. Of

course, if there is no gas or steam in the cylinder at the beginning of the stroke, the true volume of the fluid will be always equal to the displacement, but no such cylinder can be made.

While the indicator card is sufficient for the determination of mean effective pressure and work per stroke, its lack of axes of coordinates of pressure and volume prevents any study of the laws of its curves. That such study is important must be clear, for without it no data nor constants, such as the exponent s , can be obtained for prediction of results in other similar cases; neither can the presence of leaks be detected, nor the gain or loss of heat be determined during the various processes studied. In short, the most valuable analysis of the operations is impossible.

To convert the indicator card into a pressure-volume diagram, there must first be found (a) the relation of true or absolute pressures to gage pressures,

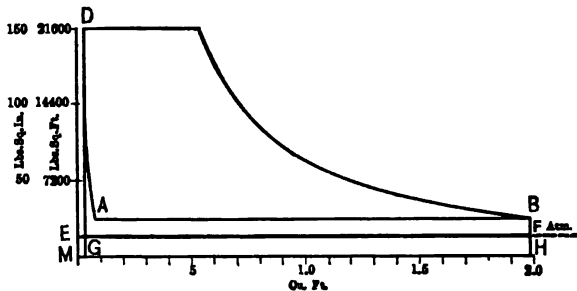


FIG. 12.—Ammonia Compressor Indicator Card with Coordinates of Pressures and Volumes Added to Convert it into a Pressure-Volume Diagram.

which involves the pressure equivalent of the barometer, and (b) the relation of displacement volumes to true volumes of vapor or gas present, which involves the clearance or inactive volume of the cylinder. The conversion of gage to absolute pressures by the barometer reading has already been explained, Section 3; the conversion of displacement

volumes to true fluid volumes is made by adding to the displacement volume the constant value, in the same units, of the clearance, which is usually the result of irregularity of form at the cylinder ends dictated by structural necessities of valves, and of linear clearance or free distance between the piston at the end of its stroke and the heads of the cylinder to avoid any possibility of touching, due to wear or looseness of the bearings.

Let $ABCD$, Fig. 12, represent an indicator card from an ammonia compressor on which EF is the atmospheric line. The cylinder bore is 14 ins., stroke 22 ins., and the scale of the indicator spring 100, barometer 28 ins., and measured clearance 32 cu.in. According to the Tables, 28 ins. of mercury corresponds to 13.753 lbs. per square inch, and as 100 lbs. per square inch, according to the spring scale, corresponds to 1 in. of height on the diagram, 1 lb. per square inch corresponds to 0.01 in. of height, or 13.75 lbs. per square inch atmospheric pressure to .137 in. of height. The zero of pressures then on the diagram must lie .137 in. below the line EF . Lay off then a line MH , this distance below EF . This will be the position of the axis of volume coordinates.

Actual measurement of the space in the cylinder with the piston at the end of its stroke gave the clearance volume of 32 cu.ins. As the bore is 14 ins. the piston area is 153.94 sq.ins. which in connection with the stroke

of 22 ins. gives a displacement volume of $22 \times 153.94 = 3386.68$ cu.ins. Compared with this the clearance volume is $\frac{32}{3386.68} = .94$ per cent of the displacement. It should be noted here that clearance is generally expressed in per cent of displacement volume. Just touching the diagram at the ends drop two lines at right angles to the atmospheric line intersecting the axis of volumes previously found at G and H . The intercept GH then represents the displacement, or 3386.68 cu.in. or 1.96 cu.ft. Lay off to the left of G , .0094, or in round numbers $1/100$ of GH , fixing the point M , MG representing the clearance to scale, and a vertical through M the axis of pressures. The axes of coordinates are now placed to scale with the diagram but no scale marked thereon. The pressure scale can be laid off by starting at M and marking off inch points each representing 100 lbs. per square inch. Pounds per square foot can also be marked by a separate scale 144 times as large. As the length of the diagram is 2.94 ins. and displacement 1.96 cu.ft., 1 in. of horizontal distance corresponds to .667 cu.ft. or 1 cu.ft. to 1.50 ins. of distance. Lay off then from M distances of 1.50 ins. and mark the first 1 cu.ft. and the second 2 cu.ft., dividing the intervals into fractions. A similar scale of volumes in cubic inches may also be obtained.

By this process any indicator card may be converted into a pressure volume diagram for study and analysis, but there will always be required the two factors of *true atmospheric pressure* to find one axis of coordinates and the *clearance volume* to find the other.

Prob. 1. If in cards Nos. 1 and 2, Section 8, the clearances are 5 per cent and 3 per cent respectively of the displacement, convert the cards to PV diagrams on the same base to scales of 4 ins. to 1 cu.ft. and 1 in. to 1000 lbs. per square foot, for cylinders $9\frac{1}{4}$ ins. and $14\frac{1}{4}$ ins. respectively in diameter and stroke 12 ins.

Prob. 2. Do the same for cards Nos. 3 and 4, if clearances are 7 per cent and 4 per cent respectively, for cylinders 10 ins. and 17 ins. respectively in diameter and stroke 12 ins.

12. To Find the Clearance. There are two general methods for the finding of clearance: the first, a direct volumetric measurement of the space itself which is filled with measured liquid; and the second, a determination by algebraic or graphic means from the location of two points on the expansion or compression curves of the indicator card, based on an assumed law for the curves.

The first method of direct measurement is the only one that offers even a promise of accuracy, but even this is difficult to carry out because of the tendency of the measuring liquid to leak past piston or valves, which makes the result too large if the liquid be measured before the filling of the clearance space and too small if the liquid be measured after filling and drawing off. There is also a tendency in the latter case for some of the liquid to remain inside the space, besides the possibility in all cases of the failure to fill the space completely due to air pockets at high places.

By the second general method any two points, A and B , on an expansion or

compression curve, Fig. 13, may be selected and horizontals drawn to the vertical line indicating the beginning of the stroke. The points A' and B' are distant from the unlocated axis an amount $A'A''=B'B''$, representing the clearance.

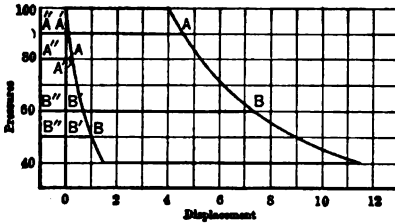


FIG. 13.—Diagram Illustrating Location of Clearance Line from Expansion or Compression Lines of Known Laws.

Let the clearance volume $= Cl$;
the displacement up to $A = D_a$;
the displacement up to $B = D_b$;
the whole displacement $= D$;
 s be the exponent in $PV^s = \text{constant}$, which defines the law of the curve.

Then in general, $P_a V_a^s = P_b V_b^s$.

But

$$V_a = D_a + Cl, \quad \text{and} \quad V_b = D_b + Cl,$$

hence $P_a^{\frac{1}{s}} (D_a + Cl) = P_b^{\frac{1}{s}} (D_b + Cl)$, or $Cl (P_a^{\frac{1}{s}} - P_b^{\frac{1}{s}}) = P_b^{\frac{1}{s}} D_b - P_a^{\frac{1}{s}} D_a$,

whence the clearance in whatever units the displacement may be measured will be

$$Cl = \frac{P_b^{\frac{1}{s}} D_b - P_a^{\frac{1}{s}} D_a}{P_a^{\frac{1}{s}} - P_b^{\frac{1}{s}}}, \quad \text{or} \quad Cl = \frac{P_b^{\frac{1}{s}} \left[D_b - \left(\frac{P_a}{P_b} \right)^{\frac{1}{s}} D_a \right]}{P_b^{\frac{1}{s}} \left[\left(\frac{P_a}{P_b} \right)^{\frac{1}{s}} - 1 \right]} = \frac{D_b - \left(\frac{P_a}{P_b} \right)^{\frac{1}{s}} D_a}{\left(\frac{P_a}{P_b} \right)^{\frac{1}{s}} - 1}.$$

$$\text{Clearance as a fraction of displacement} = c = \frac{\frac{D_b}{D} - \left(\frac{P_a}{P_b} \right)^{\frac{1}{s}} \frac{D_a}{D}}{\left(\frac{P_a}{P_b} \right)^{\frac{1}{s}} - 1}.$$

When $s=1$ this takes the form

$$\text{Clearance in fraction of displacement} = c = \frac{\frac{D_b}{D} - \left(\frac{P_a}{P_b} \right) \frac{D_a}{D}}{\left(\frac{P_a}{P_b} \right) - 1} \quad \dots (15)$$

To use such an expression it is only necessary to measure off the atmospheric pressure below the atmospheric line, draw verticals at ends of the diagram and use the length of the horizontals and verticals to the points in the formula, each horizontal representing one D and each vertical a P .

Graphic methods for the location of the axis of pressures, and hence the clearance, depend on the properties of the curves as derived from analytics. For example, when $s=1$, $P_a V_a = P_b V_b$, which is the equation of the equilateral hyperbola, a fact that gives a common name to the law, i.e., hyperbolic expansion or hyperbolic compression. Two common characteristics of this curve may be used either separately or together, the proof of which need not

be given here, first that the diagonal of the rectangle having two opposite corners on the curve when drawn through the other two corners will pass through the origin of coordinates; and second, that the other diagonal drawn through two points of the curve and extended to intersect the axes of coordinates will have equal intercepts between each point and the nearest axis cut.

According to the first principle, lay off, Fig. 14, the vacuum line or axis of volume XY and selecting any two points A and B , construct the rectangle $ACBD$. Draw the diagonal CDE and erect at E the axis of pressures EZ , then will EZ and EY be the axes of coordinates. According to the second principle, proceed as before to locate the axes of volumes XY and select two points, A and B , Fig. 15. Draw a straight line through these points, which represents the other diagonal of the rectangle $ACBD$, producing it to inter-

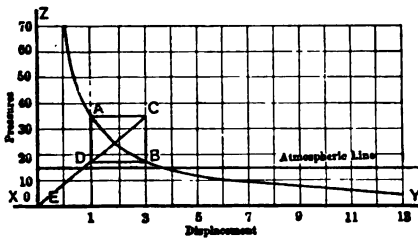


FIG. 14.

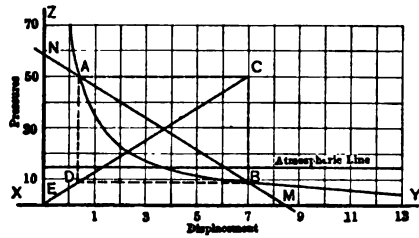


FIG. 15.

Graphic Methods of Locating the Clearance Line for Logarithmic or Hyperbolic Expansion and Compression Curves.

sect XY at M and lay off $\overline{AN} = \overline{BM}$. Then will the vertical NE be the axis of pressures. It should be noted that these two graphic methods apply only when $s=1$; other methods must be used when s is not equal to 1.

A method of finding the axis of zero volume is based upon the slope of the exponential curve, $PV^s = c$.

Differentiation with respect to V gives

$$PsV^{s-1} + V^s \frac{dP}{dV} = 0, \quad \text{or} \quad \frac{Ps}{V} + \frac{dP}{dV} = 0,$$

whence

$$V = \frac{Ps}{\left(-\frac{dP}{dV}\right)} = sP \left(-\frac{dV}{dP}\right). \quad \dots \dots (16)$$

In other words, the true volume at any given point on the known curve may be found by dividing the product of P and s by the tangent or the slope of the line at the given point, with the sign changed. This method gives results dependent for their accuracy upon the determination of the tangent to the curve, which is sometimes difficult.

The following graphical solution is dependent upon the principle just given, and while not mathematically exact, gives results so nearly correct that the error is not easily measured. The curve ACB , Fig. 16, is first known experimentally or otherwise and therefore the value of s , and the axis FV from which pressures are measured is located. Assume that the axis of zero volume,

KP , is not known but must be found. Selecting any two convenient points, A and B , on the curve, complete the rectangle $AHBG$ with sides parallel and perpendicular to FV . The diagonal HG cuts the curve at C and the horizontal axis at E . From C drop the perpendicular CD . If now the distance DE be multiplied by the exponent s , and laid off DK , and the vertical KP erected, this may be taken as the zero volume axis, from Eq. (16).

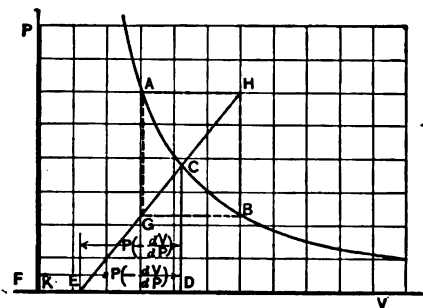


FIG. 16.—Graphic Method of Locating the Clearance Line for Exponential Expansion and Compression Curves.

It cannot be too strongly stated that methods for the finding of clearance or the location of the axes of pressures from the indicator card, much as they have been used in practice, are inaccurate and practically useless unless it is positively known beforehand just what value s has, since the assumed value of s enters into the work, and s for the actual diagram, as already explained, is affected by the substance, leakage, moisture or wetness of vapor and by all heat interchange or exchange between the gas of vapor and its container.

Prob. 1. If in card No. 6, Section 8, compression follows the law $PV^s = K$, where $s = 1.4$ and the barometer reads 29.9 ins. of mercury, locate the axes algebraically and graphically.

Prob. 2. If in card No. 3, Section 8, expansion follows the law $PV^s = K$, where $s = 1.37$ and the barometer reads 29.8 ins. of mercury, locate the axes algebraically and graphically.

Prob. 3. If in card No. 5, Section 8, expansion follows the law $PV^s = K$, where $s = 1$ and the barometer reads 27.5 ins. of mercury, locate axes algebraically and graphically.

13. Measurement of Areas of PV Diagrams and Indicator Cards. Areas of pressure volume diagrams or indicator cards must be evaluated for the determination of work or mean effective pressure, except when calculation by formula and hypothesis is possible. There are two general methods applicable to both the indicator card and PV diagram, that of average heights, and the planimeter measure, besides a third approximate but very useful method, especially applicable to plotted curves on cross-section paper.

The third method assumes that the diagram may be divided into strips of equal width as in Fig. 17, which is very easily done if the diagram is plotted on cross-section paper. At the end of each strip, a line is drawn perpendicular to the axis of the strip, such that the area intercepted inside the figure is apparently equal to that outside the figure. If this line is correctly located, the area of the rectangular strip will equal the area of the strip bounded by the irregular lines. If the entire figure has irregular ends it may be necessary to subdivide one or both ends into strips in the other direction, as shown at the left-hand side of Fig. 17. The area of the entire figure will be equal to the summation of lengths

of all such strips, multiplied by the common width. This total length may be obtained by marking on the edge of a strip of paper the successive lengths in such a way that the total length of the strip of paper when measured will be the total length of the strips.

The mean height will be the total length of such strips divided by the number of strip-widths in the length of the diagram. By a little practice the proper location of the ends of the strips can be made with reasonable accuracy, and consequently the results of this method will be very nearly correct if care be exercised.

By dividing the diagram into equal parts, usually ten, and finding the length of the middle of each strip, an approximation to the mean height of each strip will be obtained; these added together and divided by the number will give the mean height in inches from which the mean effective pressure may be found

by multiplying by the scale as above, or the area in square inches by multiplying by the length in inches, which can be converted into work by multiplying by the foot-pounds per square inch of area as fixed by the scales. As the pressures usually vary most, near the ends of the diagram a closer approximation can be made by subdividing the end strips, as shown in Fig. 18, which represents two steam engine indicator cards taken from opposite ends of the same cylinder and superimposed.

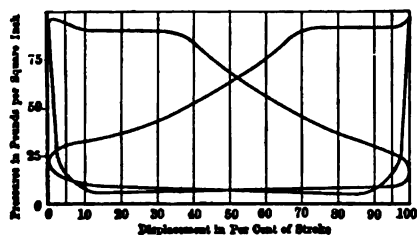


FIG. 18.—Simpson's Method for Finding Mean Effective Pressure of Indicator Cards.

The two diagrams are divided into ten equal spaces and then each end space is subdivided. The mean heights of the subdivisions are measured and averaged to get the mean height of the whole end division, or average pressure in this case for the division. The average heights of divisions may be set down in columns on the left and right respectively; the sum of each column divided by ten and multiplied by the spring scale gives the whole m.e.p. The heights of the one in inches marked off continuously on a slip of paper measured a total of 11.16 ins. and of the other, 10.79 ins., each of these quantities divided by 10 (number of strips) and multiplied by the spring scale, 50, gives the m.e.p., as before. This method is often designated as Simpson's rule.

The best and most commonly used method whether for work or m.e.p. determination, is to evaluate the area by means of the planimeter, a well-known instrument specially designed for direct measurements of area.

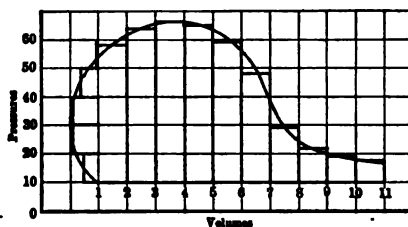


FIG. 17.—Approximate Method of Evaluating Areas and Mean Effective Pressures of Indicator Cards and P.V. Diagrams.

14. Indicated Horse-power. Work done by the fluid in a cylinder, because it is most often determined by indicator card measurements, measures the indicated horse-power, but the term is also applicable to work that would be done by the execution of a certain cycle of pressure volume changes carried out at a specified rate. The mean effective pressure in pounds per square inch, whether of an indicator card or *PV* cycle, when multiplied by piston area in square inches, gives the average force acting on the piston for one stroke, whether the cycle required one, two or z strokes for its execution, and this mean force multiplied by the stroke in feet gives the foot-pounds of work done by the cycle. Therefore,

Let m.e.p. = mean effective pressure in pounds per square inch for the cycle referred to one stroke;

a = effective area of piston in square inches;

L = length of stroke in feet;

n = number of equal cycles completed per minute;

N = number of revolutions per minute;

S = mean piston speed = $2LN$ feet per minute;

z = number of revolutions to complete one cycle = $\frac{N}{n}$. Then will the

indicated horse-power be given by,

$$\text{I.H.P.} = \frac{(\text{m.e.p.})Lan}{33000}; (a) \quad = \frac{(\text{m.e.p.})LaN}{33000z}; (b) \quad = \frac{(\text{m.e.p.})aS}{33000 \times 2z}. (c) \quad (17)$$

When there are many working chambers, whether in opposite ends of the same cylinder or in separate cylinders, the indicated horse-power of each should be found and the sum taken for that of the machine. This is important not only because the effective areas are often unequal, as, for example, in opposite ends of a double-acting cylinder with a piston rod passing through one side only or with two piston rods or one piston rod and one tail rod of unequal diameters, but also because unequal valve settings which are most common will cause different pressure-volume changes in the various chambers.

It is frequently useful to find the *horse-power per pound mean effective pressure*, which may be symbolized by K_e , and its value given by

$$K_e = \frac{Lan}{33000} = \frac{LaN}{33000z}$$

Using this constant, which may be tabulated for various values of n , stroke and bore, the indicated horse-power is given by two factors, one involving cylinder dimensions and cyclic speed or machine characteristics, and the other the resultant *PV* characteristic, of the fluid, symbolically,

$$\text{I.H.P.} = K_e(\text{m.e.p.}).$$

These tables of horse-power per pound m.e.p. are usually based on piston speed rather than rate of completion of cycles and are, therefore, directly applicable

when $z = \frac{1}{2}$ or $n = 2N$, which means that the two cycles are completed in one revolution, in which case,

$$S = 2LN = Ln, \quad \text{and} \quad K_e = \frac{aS}{33000},$$

whence
$$\text{I.H.P.} = K_e (\text{m.e.p.}) = \frac{(\text{m.e.p.})aS}{33000} \dots \dots \dots (18)$$

Tables are frequently given for what is called the engine constant, which is variously defined as either

- (a) $\frac{La}{33000}$, which must be multiplied by m.e.p. $\times n$ to obtain H.P., or
- (b) $\frac{a}{33000}$, which must be multiplied by m.e.p. $\times L \times n$ to obtain H.P.

For an engine which completes two cycles per revolution, this is the same as multiplying by m.e.p. $\times S$. Before using such a table of engine constants it must be known whether it is computed as in (a) or in (b).

Example. A 9 in. \times 12 in. double-acting steam engine runs at 250 R.P.M. and the mean effective pressure is 30 lbs. What is H.P. per pound m.e.p. and the I.H.P.?

$$K_e = \frac{Lan}{33000} = \frac{1 \times 63.6 \times 500}{33000}; \quad \text{I.H.P.} = .9636 \times 30 = 28.908.$$

Prob. 1. A pump has a piston speed of 250 ft. per minute; piston diameter is 24 ins. What is the H.P. per pound mean effective pressure?

Prob. 2. A simple single-acting 2-cylinder engine has pistons 10 ins. in diameter with 2-in. rods and a stroke of 15 ins. It runs with a mean effective pressure of 45 lbs. per square inch at a speed of 220 R.P.M. What is the H.P.?

Prob. 3. A gas engine has one working stroke in every four. If the speed is 300 R.P.M. what must be the m.e.p. to give 6 H.P. when the cylinder has a diameter of 6 ins. and a stroke of 12 ins.?

Prob. 4. An air compressor is found to have a mean effective pressure of 50 lbs. If the cylinder is double acting and is 12 ins. diameter and 16 ins. stroke, what H.P. will be needed to drive it at 80 R.P.M.?

Prob. 5. A gasoline engine has an engine constant (a) of .3. What must be the m.e.p. to give 25 H.P. for 150 cycles per minute?

Prob. 6. A blowing engine has a m.e.p. of 10 lbs. Its horse-power is 500. What is the H.P. per pound m.e.p.?

Prob. 7. Two engines of the same size and speed are so run that one gives twice the power of the other. How will the engine constants and m.e.p. vary?

Prob. 8. From the diagrams following Section 9 what must have been the H.P. per pound m.e.p. to give 300 H.P. in each case?

Prob. 9. How will the H.P. per pound m.e.p. vary in two engines if the speed of one is twice that of the other, if the stroke is twice, if the diameter of piston is twice?

15. Effective Horse-power, Brake Horse-power, Friction Horse-power, Mechanical Efficiency, Efficiency of Transmission, Thermal Efficiency. Work is done and power developed primarily in the power cylinder of engines, and is transmitted through the mechanism with friction loss to some point at which

it is utilized. There is frequently a whole train of transmission which may involve transformation of the energy into other forms, but always with some losses, including the mechanical friction. For example, a steam cylinder may drive the engine mechanism which in turn drives a dynamo; this transforms mechanical into electrical energy which is transmitted to a distance over wires and used in motors to hoist a cage in a mine or to drive electric cars. There is mechanical work done at the end of the system and at a certain rate, so that there will be a certain *useful* or *effective* horse-power output for the system, which may be compared to the horse-power primarily developed in the power cylinders. A similar comparison may be made between the primary power or input and the power left after deducting losses to any intermediate point in the system. For example, the electrical energy per minute delivered to the motor, or *motor input*, is, of course, the output of the transmission line. Again, the electrical energy delivered to the line, or electrical transmission *input*, is the same as dynamo output, and mechanical energy delivered to the dynamo from a direct-connected engine is identical with engine output. The comparison of these measurements of power usually takes one of two forms, and frequently both; first, a comparison by differences, and second, a comparison by ratios. The ratio of any horse-power measurement in the system to the I.H.P. of the power cylinder is the *efficiency* of the power system up to that point, the difference between the two is the horse-power loss up to that point. It should be noted that, as both the dynamo and motor transform energy from mechanical to electrical or vice versa, the engine mechanism transmits mechanical energy and the wires electrical energy; the system then, is made up of parts which have the function of (a) transmission without change of form, and (b) transformation of form. The ratio of output to input is always an *efficiency*, so that the *efficiency of the power system* is the *product* of all the *efficiencies of transformation* and of *transfer or transmission*, and the power loss of the system is the sum of transformation and transmission losses. Some of these efficiencies and losses have received names which are generally accepted and the meaning of which is generally understood, but it is equally important to note that others have no names, simply because there are not names enough to go around. In dealing with efficiencies and power losses that have accepted names these names may with reason be used, but in other cases where names are differently understood or where there is no name, accurate description must be relied on. As a matter of fact controversy should be avoided by definition of the quantity considered, whether descriptive names be used or not.

Effective horse-power is a general term applied to the output of a machine, or power system, determined by the form of energy output. Thus, for an engine it is the power that might be absorbed by a friction brake applied to the shaft, and in this case is universally called Brake Horse-power. The difference between brake and indicated horse-power of engines is the *friction horse-power* of the engine, and the ratio of brake to indicated horse-power is the *mechanical efficiency* of the engine. For an engine, then, the effective horse-power or useful horse-

power is the brake horse-power. When the power cylinders drive in one machine a pump or an air compressor, the friction horse-power of the machine is the difference between the indicated horse-power of the power cylinders and that for the pump or compressor cylinders, and the mechanical efficiency is the ratio of pump or air cylinder indicated horse-power to indicated horse-power of the power cylinders. Whether the indicated horse-power of the air or pump cylinders can be considered a measure of useful output or not is a matter of difference of opinion. From one point of view the machine may be considered as built for doing work on water or on air, in which case these horse-powers may properly be considered as useful output. On the other hand, the power pump is more often considered as a machine for moving water, in which case the useful work is the product of the weight of water moved into its head in feet, and includes all friction through ports, passages and perhaps even in pipes or conduits, which the indicated horse-power of the pump cylinder does not include, especially when leakage or other causes combine to make the pump cylinder displacement differ from the volume of water actually moved. With compressors the situation is still more complicated, as the air compressor may be considered useful only when its discharged compressed air has performed work in a rock drill, hoist or other form of an engine, in which case various measures of useful output of the compressor may be devised, even, for example, the purely hypothetically possible work derivable from the subsequent admission and complete expansion of the compressed air in a separate air engine cylinder.

Too great care in definition of *output* and *input energy* in machines and power systems is not possible for avoidance of misunderstanding, which may affect questions both of power losses and efficiency of transmission and transformation whether in a power system or single machine. It is interesting to note here that not only is the *indicated work* of the power cylinder always considered the measure of power input for the system or machine, but, as in the other cases, it is itself *an output* or result of the action of heat on the vapor or gas and of the cycle of operations carried out. The ratio of the indicated power or cylinder work, to the heat energy both in foot-pound units, that was expended on the fluid is the *thermal efficiency* of the engine referred to indicated horse-power or the *efficiency of heat transformation* into work, the analysis of which forms the bulk of the subject matter of Part III. Similarly, the ratio of any power measurement in the system to the equivalent of the heat supplied, is the thermal efficiency of as much of the system that is included.

Example. It has been found that when the indicated horse-power of an engine is 250, a generator is giving out 700 amperes at 220 volts. At end of a transmission line is a motor using the output of the generator. This motor on test gave out 180 brake horse-power. Assuming no losses in the transmission line, what was the efficiency of the motor, of the generator, of the engine, and of the system?

$$\text{Motor efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{180}{\frac{220 \times 700}{746}} = 87.2\%.$$

NOTE: Volts \times amperes = watts, and watts \div 746 = H.P.

$$\text{Engine and generator efficiency} = \frac{220 \times 700}{\frac{746}{250}} = 82.4\%.$$

$$\text{Efficiency of system} = \frac{180}{250} = 72\% \quad \text{or} \quad 82.4 \times 87.2 = 72\%.$$

Prob. 1. An engine is belted to a pump; the I.H.P. of the engine is 50, of the pump 40, and the pump delivers 1200 gallons water per minute against 100-ft. head. What is the efficiency of each part and of the entire system?

Prob. 2. An engine is geared to air compressor. Upon test, the efficiency of the engine alone, gearing alone and compressor alone were each 80 per cent. When the compressor H.P. was 100 what was that of the engine?

Prob. 3. A water-wheel is run by the discharge from a pump. The B.H.P. of wheel is found to be 20 when the pump is delivering 45 gallons of water per minute at a head of 1000 lbs. per square inch. The water I.H.P. of the pump is 30 and the steam I.H.P. is 40. What are the efficiencies of each part of the system and the over-all efficiency?

Prob. 4. Perry gives a rule for the brake horse-power of steam engines as being equal to .95 I.H.P. $- 10$. On this basis find the mechanical efficiency of a 500-H.P. engine from 200 to 500 H.P. Show results by a curve with B.H.P. and per cent efficiency as coordinates.

Prob. 5. Perry gives a rule for the efficiency of an hydraulic line as $H = .7I - 25$ where H is the useful power of the pump and I is the indicated. Find I for values of H from 100 to 300 and plot a curve of results.

Prob. 6. An engine gives one I.H.P. for every 3 lbs. of coal per hour. One pound of coal contains 9,500,000 ft.-lbs. of energy. What is the thermal efficiency?

Prob. 7. A gas engine has a mechanical efficiency of 70 per cent when delivering power to a generator which in turn has an efficiency of 90 per cent. If the engine uses 15 cu.ft. of gas per indicated horse-power hour and the gas contains 700,000 ft.-lbs. per cubic foot, what is the net thermal efficiency of the system?

16. Specific Displacement, Quantity of Fluid per Hour or per Minute per I.H.P. It has been shown that the work done in cylinders by pressure-volume changes of the vapor or gas depends on the mean effective pressure and on the displacement, or that there is a relation between I.H.P. and displacement. The quantity of fluid used also depends on the displacement and may be expressed in cubic feet per minute at either the low pressure or high pressure condition when the work is done between two definite pressure limits; or in terms of pounds per minute or hour, which involves the application of fluid densities to volumes and eliminates the double expression for the two conditions of pressure. The displacement per hour per horse-power, termed the *specific displacement*, is the basis of computations on the steam consumption of steam engines; the horse-power per cubic foot of free air per minute for air compressors; the horse-power per ton refrigeration for refrigerating machines, and the consumption of fuel per hour per horse-power for gas and oil engines. It is, therefore, a quantity of great importance in view of these applications. Apply-

ing the symbols already defined to displacement in one direction of one side of a piston

$$\text{Displacement in cu.ft. per stroke} = L \times \frac{a}{144};$$

$$\text{Displacement in cu.ft. per minute} = L \times \frac{a}{144} \times N;$$

$$\text{Displacement in cu.ft. per hour} = 60L \times \frac{a}{144} \times N.$$

$$\text{Indicated horse-power} = \frac{(\text{m.e.p.})LaN}{33000} = \frac{(\text{m.e.p.})LaN}{33000z}.$$

Whence expressing displacement per hour per I.H.P. or specific displacement in one direction for one side of a piston by D_s ,

$$D_s = \frac{60L \times \frac{a}{144} \times N}{\frac{(\text{m.e.p.})LaN}{33000}} = \frac{60 \times 33000z}{144(\text{m.e.p.})} = \frac{13750z}{(\text{m.e.p.})} \quad (19)$$

From Eq. (19) it appears that the specific displacement is equal to $z \times 13,750$ divided by the mean effective pressure in pounds per square inch.

If two points, A and B , be so located on the indicator card, Fig. 19, as to

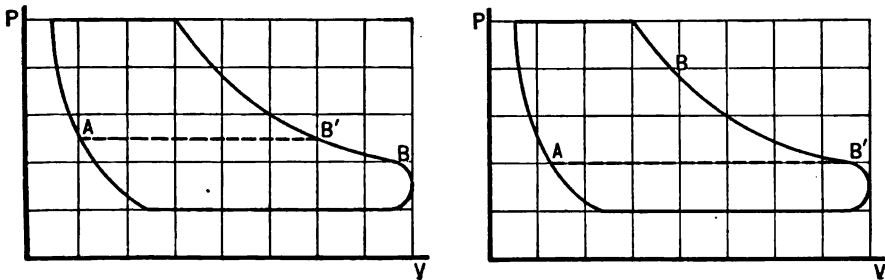


FIG. 19.—Determination of Consumption of Fluid per Hour per Indicated Horse-power from the Indicator Card.

have included between them a fluid transfer phase, either admission to, or expulsion from the cylinder, then calling δ_a =pounds per cubic foot or density at point A , and δ_b =pounds per cubic foot or density at point B , the weight of fluid present at A is $(D_a + Cl)\delta_a$ lbs., and weight of fluid present at B is $(D_b + Cl)\delta_b$ lbs., whence the weight that has changed places or passed in and out per stroke is $(D_b + Cl)\delta_b - (D_a + Cl)\delta_a$ lbs. per stroke. If both A and B lie on the same horizontal as A and B' , $\delta_a = \delta_b = \delta$, the density of fluid at the pressure of measurement, whence the weight of fluid used per stroke, will be $(D_b' - D_a)\delta$, and the volume per stroke used at density δ is $D_b' - D_a$ cu.ft., which compared to the displacement is $\frac{D_b' - D_a}{D}$. This is the fraction of the displace-

ment representing the volume of fluid passing through the machine at the selected pressure. Multiplying the specific displacement by this, there results,

$$\text{Cu.ft. of fluid per hr. at density } (\delta) \text{ per I.H.P.} = \frac{13750}{(\text{m.e.p.})} \frac{D_b' - D_a}{D},$$

$$\text{and Lbs. of fluid per hr. per I.H.P.} = \frac{13750}{(\text{m.e.p.})} \left(\frac{D_b' - D_a}{D} \right) \delta. \quad (20)$$

More generally, that is, when A and B are not taken at the same pressures

$$\text{Lbs. fluid per hr. per I.H.P.} = \frac{13750}{(\text{m.e.p.})D} \left[(D_b + Cl)\delta_b - (D_a + Cl)\delta_a \right] \quad (21)$$

The particular forms which this may take when applied to special cases will be examined in the succeeding chapters.

Example. An air compressor whose cylinder is 18×24 ins. (18 ins. in diameter and stroke 2 ft.) runs at 60 R.P.M. and is double acting. The mean effective air pressure is 50 lbs. per square inch. What is the specific displacement?

$$\text{Cu.ft. per hour} = \frac{60 \text{ Lan}}{144} = 60 \times 2 \times \frac{254.5}{144} \times 120 = 25,600.$$

$$\text{I.H.P.} = \frac{\text{m.e.p. Lan}}{33000} = \frac{50 \times 2 \times 254.5 \times 120}{33000} = 92.3;$$

$$\frac{\text{Cu.ft. per hour}}{\text{I.H.P.}} = \frac{25,600}{92.3} = 277, \quad \text{or by the formula directly,} \quad \frac{13750}{\text{m.e.p.}} = \frac{13750}{50} = 275.$$

Prob. 1. What will be the cubic feet of free air per hour per horse-power delivered by a 56×72 -in. blowing engine with 4 per cent clearance and mean effective pressure of 10 lbs. per square inch?

Prob. 2. An 18×22 -in. ammonia compressor works with a mean effective pressure of 45 lbs. per square inch. What is the weight of NH_3 per I.H.P. hour if the speed is 50 R.P.M. and compressor is double acting having a volumetric efficiency of 90 per cent? Use tabular NH_3 densities.

Prob. 3. A steam engine whose cylinder is 9×12 ins. runs at a speed of 300 R.P.M. and is double acting. If the m.e.p. is 60 lbs. and the density of steam at end of the stroke is .03, how many pounds of steam are used per hour per indicated horse-power?

17. Velocity Due to Free Expansion by PV Method. All the cases examined for the work done by PV cycles have been so far applied only to their execution in cylinders, but the work may be developed in nozzles accelerating the gas or vapor in free expansion, giving, as a consequence, a high velocity to the fluid. It was noted that for cylinders many combinations of phases might be found worthy of consideration as typical of possible actual conditions of practice, but this is not true of proper nozzle expansion, which has but one cycle, that of Fig. 20. That this is the cycle in question is seen from the following considerations. Consider a definite quantity of the gas or vapor approaching the nozzle from a source of supply which is capable of maintaining the pressure. It pushes forward that in front of it and work will be done, $ABCD$, equal to the admission of the same substance to a cylinder, so that its approach AB may be considered as a constant pressure, volume increasing phase for which the energy comes from the source of supply. This same substance expanding to the lower pres-

sure will do the work *CBEF*; but there will be negative work equivalent to the pushing away or displacing of an equivalent quantity of fluid at the low pressure, or *FEGD*, making the work cycle *ABEG*, in which *AG* is the excess of initial over back pressure or the effective working pressure, remaining constant during approach and lessening regularly during expansion to zero excess at *E*. The work done will be from Eq. (13),

$$W = P_b V_b + \frac{P_b V_b}{s-1} \left[1 - \left(\frac{P_e}{P_b} \right)^{\frac{s-1}{s}} \right] - P_e V_e.$$

But

$$P_e V_e^s = P_b V_b V_e^{s-1} = P_b V_b^s = P_b V_b V_b^{s-1};$$

$$\therefore P_e V_e = P_b V_b \left(\frac{V_b}{V_e} \right)^{s-1} = P_b V_b \left(\frac{P_e}{P_b} \right)^{\frac{s-1}{s}};$$

Whence

$$\begin{aligned} W &= P_b V_b + \frac{P_b V_b}{s-1} \left[1 - \left(\frac{P_e}{P_b} \right)^{\frac{s-1}{s}} \right] - P_b V_b \left(\frac{P_e}{P_b} \right)^{\frac{s-1}{s}} \\ &= \frac{s}{s-1} P_b V_b \left[1 - \left(\frac{P_e}{P_b} \right)^{\frac{s-1}{s}} \right]. \end{aligned} \quad (22)$$

Assuming the initial velocity to be zero, and the work of Eq. (22) to be done on 1 lb., the final or resultant velocity will be according to Eq. (6),

$$u = \sqrt{2gW} = \sqrt{2g \frac{s}{s-1} P_b V_b \left[1 - \left(\frac{P_e}{P_b} \right)^{\frac{s-1}{s}} \right]}, \quad (23)$$

or

$$u = 8.02 \sqrt{\frac{s}{s-1} P_b V_b \left[1 - \left(\frac{P_e}{P_b} \right)^{\frac{s-1}{s}} \right]}. \quad (24)$$

This velocity is in feet per second when pressures are in pounds per square foot and volumes in cubic feet of 1 lb. of substance, and is known as Zeuner's equation for the velocity of a gas or vapor expanding in a nozzle. It is generally assumed that such expansion, involving as it does very rapid motion of the fluid past the nozzle, is of the adiabatic sort, as there seems to be no time for heat exchange between fluid and walls. As already noted, the value of *s* for adiabatic expansion of vapors is not constant, making the correct solution of problems on vapor flow through orifices practically impossible by this method of pressure-volume analysis, but as will be seen later the thermal method of solution is exact and comparatively easy.

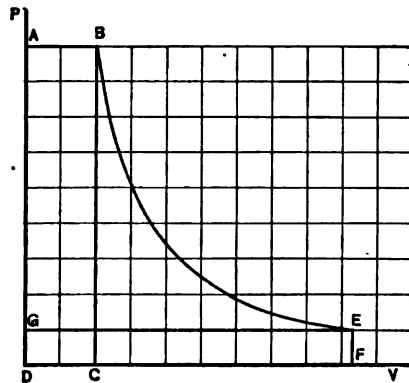


FIG. 20.—Pressure-Volume Diagram for Nozzle Expansion Measuring the Acceleration Velocity and Horse-power of Jets.

NOTE. A comparison of Eqs. (22) and (13) and the figures corresponding

will show that the area under the process curve, which is the same as the work done during the compression or expansion, if multiplied by s will equal the area to the left of the process curve, which in turn represents, as in Fig. 19, for engines, the algebraic sum of admission, complete expansion, and exhaust work areas, or as in Fig. 9 for compressors, the algebraic sum of suction, compression and delivery work areas. This statement must not be thought to refer to the work area of such a cycle as that of Fig. 8, where expansion is incomplete, nor a case of over-expansion, Fig. 10.

Example. In Fig. 20 assume the initial pressure at 100 lbs. per square inch absolute, back pressure at atmosphere, and expansion as being adiabatic. What will be the work per pound of steam and the velocity of the jet, if V_b is 4.36 cu.ft. and $s=1.3$ for superheated steam?

$$\begin{aligned}
 W &= \frac{s}{s-1} P_b V_b \left[1 - \left(\frac{P_c}{P_b} \right)^{\frac{s-1}{s}} \right] \\
 &= \frac{1.3}{.3} \times 144 \times 100 \times 4.36 \left[1 - \left(\frac{14.7}{100} \right)^{\frac{.3}{1.3}} \right] = 272,060 \times .608 = 165,410 \text{ ft.-lbs.}; \\
 u &= \sqrt{2gW} = 8.02 \sqrt{165,410}; \quad = 3262 \text{ ft. per second.}
 \end{aligned}$$

Prob. 1. Taking the same pressure range as above, find W and u for adiabatic expansion of air, also for isothermal expansion.

Prob. 2. How large must the effective opening of the suction valve be, in an air compressor 18×24 ins. to allow the cylinder to properly fill if the mean pressure-drop through the valve is 1 lb. per square inch and the compressor runs at 80 R.P.M.?

Prob. 3. What must be the diameter of the inlet valve in a gas engine to fill a cylinder of $\frac{1}{2}$ cu.ft. capacity if the lift of the valve is $\frac{3}{8}$ ins., allowing a pressure-drop of 1 lb. per square inch? Engine makes 150 working strokes per minute.

Prob. 4. It has been found from experiment that the velocity of air issuing from a hole in plate orifice is 72 per cent of what would be expected from calculation as above when the absolute pressure ratio is 2 to 1, and 65 per cent when the absolute pressure ratio is $1\frac{1}{2}$ to 1. What will be the actual velocity for air flowing from a tank to atmosphere for these pressure ratios?

Prob. 5. CO₂ stored in a tank is allowed to escape through an orifice into the air. What will be the maximum velocity of the jet if the pressure on the tank be 100 lbs. per square inch gage?

NOTE: 1 lb. CO₂ at pressure of 100 lbs. per square inch gage occupies 1.15 cu.ft.

Prob. 6. If ammonia gas and hydrogen were allowed to expand from the same pressure, how would their maximum velocities compare? Vol. of 1 lb. of NH₃ at 50 lbs. per square inch gage is 4.5 cu.ft. Vol. of 1 lb. of H at same pressure is 77.5 cu.ft.

18. Weight of Flow through Nozzles. Applying an area factor to the velocity equation will give an expression for cubic flow per second which becomes weight per second by introducing the factor, density.

Let the area of an orifice at the point of maximum velocity, u , be A sq.ft., then will the cubic feet per second efflux be Au . Assume the point of maximum velocity, having area A , to be that part of the nozzle where the pressure has fallen to P , Fig. 20, and the gas or vapor to have the density δ , pounds

per cubic foot. Then will the nozzle flow in pounds per second be $w = uA \delta_e$. But the weight per cubic foot is the reciprocal of the cubic feet per pound, V_e , which it has already been assumed, is the final volume, of one pound of the fluid. Hence,

$$w = \frac{uA}{V_e}.$$

This may be put in terms of initial gas or vapor conditions for, $V_e = V_b \left(\frac{P_b}{P_e} \right)^{\frac{1}{s}}$.

Whence,

$$w = \frac{uA}{V_b \left(\frac{P_b}{P_e} \right)^{\frac{1}{s}}} = \frac{uA}{V_b} \left(\frac{P_e}{P_b} \right)^{\frac{1}{s}}.$$

Substituting in this the value of u from Eq. (24),

$$\begin{aligned} w &= \frac{A}{V_b} \left(\frac{P_e}{P_b} \right)^{\frac{1}{s}} \sqrt{2g \frac{s}{s-1} P_b V_b \left[1 - \left(\frac{P_e}{P_b} \right)^{\frac{s-1}{s}} \right]} \\ &= 8.02 \frac{A}{V_b} \left(\frac{P_e}{P_b} \right)^{\frac{1}{s}} \left\{ \frac{s}{s-1} P_b V_b \left[1 - \left(\frac{P_e}{P_b} \right)^{\frac{s-1}{s}} \right] \right\}^{\frac{1}{2}} \text{ lbs. per sec.} \end{aligned} \quad (25)$$

This weight will be a maximum for a certain value of the pressure ratio, depending on the value of s only, and this value can be found by placing the first differential coefficient of w with respect to $\left(\frac{P_e}{P_b} \right)$ equal to zero: $\frac{dw}{d\left(\frac{P_e}{P_b} \right)} = 0$.

To accomplish this, rearrange Eq. (25) as follows:

$$w = A \left\{ 2g \frac{s}{s-1} \frac{P_b}{V_b} \left[\left(\frac{P_e}{P_b} \right)^{\frac{2}{s}} - \left(\frac{P_e}{P_b} \right)^{\frac{s+1}{s}} \right] \right\}^{\frac{1}{2}}.$$

But as the other factors do not enter to affect the result so long as P_b does not vary, w is a maximum when the bracket

$$\left[\left(\frac{P_e}{P_b} \right)^{\frac{2}{s}} - \left(\frac{P_e}{P_b} \right)^{\frac{s+1}{s}} \right], \text{ is a maximum or when } \frac{2}{s} \left(\frac{P_e}{P_b} \right)^{\frac{2}{s}-1} - \left(\frac{s+1}{s} \right) \left(\frac{P_e}{P_b} \right)^{\frac{s+1}{s}-1} = 0;$$

or

$$2 \left(\frac{P_e}{P_b} \right)^{\frac{1}{s}} \left(\frac{P_e}{P_b} \right)^{\frac{1}{s}-1} - (s+1) \left(\frac{P_e}{P_b} \right)^{\frac{1}{s}} = 0;$$

or

$$\left(\frac{P_e}{P_b} \right)^{\frac{1}{s}} \left[2 \left(\frac{P_e}{P_b} \right)^{\frac{1}{s}-1} - (s+1) \right] = 0.$$

But since $\left(\frac{P_e}{P_b} \right)^{\frac{1}{s}}$ cannot be equal to zero in practice, it follows that

$$2 \left(\frac{P_e}{P_b} \right)^{\frac{1}{s}-1} - (s+1) = 0,$$

which gives the condition that w is a maximum when $\left(\frac{P_e}{P_b}\right)^{\frac{1-s}{s}} = \frac{s+1}{2}$,
or maximum flow for given initial pressure occurs when

$$\left(\frac{P_e}{P_b}\right) = \left(\frac{2}{s+1}\right)^{\frac{s}{s-1}} \quad \dots \quad (26)$$

For air expanding adiabatically $s=1.407$. Maximum flow occurs when $\frac{P_e}{P_b}=.528$ and for most common values of s it will be between .50 and .60.

This result is quite remarkable and is verified by experiment reasonably closely. It shows that, contrary to expectation, the weight of efflux from nozzles will not continuously and regularly increase with increasing differences in pressure, but for a given initial pressure the weight discharged per second will have reached its limit when the final pressure has been diminished to a certain fraction of the initial, and any further decrease of the discharge pressure will not increase the flow through an orifice of a given area.

The subject of flow in nozzles will be treated more completely in Part III.

Prob. 1. For the following substances under adiabatic expansion determine the pressure ratio for maximum flow and find the rate of flow per square inch of orifice under this condition when flow is into a vacuum of 10 ins. of mercury with standard barometer:

(a) Carbon dioxide; (b) Nitrogen; (c) Hydrogen; (d) Ammonia; (e) Dry steam according to saturation law.

19. Horse-power of Nozzles and Jets. Although, strictly speaking, nozzles can have no horse-power, the term is applied to the nozzle containing the orifice through which flow occurs and in which a certain amount of work is done per minute in giving to a jet of gas or vapor initially at rest a certain final velocity, and amount of kinetic energy. The foot-pounds of work per pound of fluid multiplied by the pounds flowing per second will give the foot-pounds of work developed per second within the nozzle, and this divided by 550 will give the horse-power developed by the jet, or the nozzle horse-power. Accordingly,

$$\left. \begin{aligned} \text{H.P. of jet} &= \frac{W \times w}{550} = \frac{W}{550} \times \frac{A}{V_b} \left(\frac{P_e}{P_b}\right)^{\frac{1}{s}} \times u \quad \dots \quad (a) \\ &= \frac{W}{550} \frac{A}{V_b} \left(\frac{P_e}{P_b}\right)^{\frac{1}{s}} \sqrt{2gW} \quad \dots \quad (b) \\ &= .01458 \frac{A}{V_b} \left(\frac{P_e}{P_b}\right)^{\frac{1}{s}} \left\{ \frac{s}{s-1} P_b V_b \left[1 - \left(\frac{P_e}{P_b}\right)^{\frac{s-1}{s}} \right] \right\}^{\frac{3}{2}}, (c) \end{aligned} \right\} \quad (27)$$

where the expression in the bracket is the work done per pound of substance. The pressures are expressed in pounds per square foot, areas in square feet and volumes in cubic feet.

Example. A steam turbine operates on wet steam at 100 lbs. per square inch absolute pressure which is expanded adiabatically to atmospheric pressure. What must be the area of the nozzles if the turbine is to develop 50 H.P. ideally?

NOTE: 1 cu.ft. of steam at 100 lbs. = .23 lb.

By Eq. (26), maximum flow occurs when the pressure ratio is $\left(\frac{2}{s+1}\right)^{\frac{s}{s-1}}$, or, for

this case when the pressure is $100 \div \left(\frac{2}{2.11}\right)^{\frac{1.11}{.11}} = 58$ lbs. per square inch absolute. As the back pressure is one atmosphere, the flow will not be greater than for the above critical pressure. Substituting it in Eq. (25) will give the flow weight w , and using the actual back pressure in Eq. (22) will give the work W .

$$\text{By Eq. (22), } W = \left(\frac{1.11}{.11}\right) \times \left(\frac{14400}{.23}\right) \left[1 - \left(\frac{2116}{14400}\right)^{\frac{.11}{1.11}}\right] \\ = 110000 \text{ ft.-lbs. per pound of steam.}$$

$$\text{By Eq. (25), } w = 8.02A \times .23 \times (.58)^{\frac{1}{1.11}} \left\{ \frac{1.11}{.11} \times \frac{14400}{.23} \left[1 - (.58)^{\frac{.11}{1.11}}\right] \right\}^{\frac{1}{2}} \\ = 198A \text{ lbs. of steam per second.}$$

$$\text{By Eq. (27a), H.P.} = \frac{W \times w}{550} = 50 = \frac{110000 \times 198A}{550}$$

$$\text{Whence } A = \frac{50}{39600} \text{ square feet.}$$

Prob. 1. What will be the horse-power per square inch of nozzle for a turbine using hot gases if expansion follows law $PV^s = k$, when $s = 1.37$, the gases being at a pressure of 200 lbs. per square inch absolute and expanding to atmosphere?

Let the volume per pound at the high pressure be 2 cu.ft.

Prob. 2. What will be the horse-power per square inch of nozzle for the problems of Section 17?

Prob. 3. Suppose steam to expand according to law $PV^s = k$, where $s = 1.111$, from atmosphere to a pressure of 2 lbs. per sq. inch absolute. How will the area of the orifice compare with that of the example to give the same horse-power?

NOTE: $V_b = 26.4$.

Prob. 4. Suppose steam to be superheated in the case of the example and of the last problem, how will this affect the area of nozzle?

NOTE: Let $V_b = 5$ and 32 respectively.

Prob. 5. How much work is done per inch of orifice if initial pressure is 100 lbs. absolute on one side and final 10 lbs. absolute on other side of a valve through which air is escaping?

CHAPTER II

WORK OF COMPRESSORS, HORSE-POWER AND CAPACITY OF AIR, GAS AND VAPOR COMPRESSORS, BLOWING ENGINES AND DRY VACUUM PUMPS.

20. General Description of Structure and Processes. There is quite a large class of machines designed to receive a cylinder full of some gas at one constant pressure, and, after doing work on the gas through decreasing volumes and rising pressures, discharge the lesser volume of gas against a constant higher pressure. These machines are in practice grouped into sub-classes, each having some specific distinguishing characteristic. For example, blowing engines take in air at atmospheric pressure or as nearly so as the valve and port resistance will permit, and after compression deliver the air at a pressure of about three atmospheres absolute for use in blast furnaces. These blowing engines are usually very large, work at low and variable speeds, but always deliver against comparatively low pressures; therefore, they have the characteristics of large and variable capacity and low pressures. A great variety of valves and driving gears are used, generally mechanically moved suction and automatic spring closed discharge valves, but all valves may be automatic. The compressor cylinder is often termed the blowing tub and the compressed or blast air frequently is spoken of as wind by furnace men. They are all direct-connected machines, an engine forming with the compressor one machine. The engine formerly was always of the steam type, but now a change is being effected to permit the direct internal combustion of the blast furnace waste gases in the cylinders of gas engines. These gas-driven blowing engines, showing approximately twice the economy of steam-driven machines, will in time probably entirely displace steam in steel plants, and this change will take place in proportion to the successful reduction of cost of repairs, increase of reliability and life of the gas-driven blowing engines to equal the steam-driven. Some low-pressure blowers are built on the rotary plan without reciprocating pistons, some form of rotating piston being substituted, and these, by reason of greater leakage possibilities, are adapted only to such low pressures as 5 lbs. per square inch above atmosphere or thereabouts. These blowers are coming into favor for blasting gas producers, in which air is forced through thick coal beds either by driving the air or by drawing on the gas produced beyond the bed. They are also used for forcing illuminating gas in cities through pipes otherwise too small, especially when the distances are long. In general, very low pressures and large capacities are the characteristics of the service whether the work be that of blowing or exhausting or both. For still lower pressures, measurable by water or mercury columns, fans are used of the disk or propeller or centrifugal type. These fans are most used for ventilation of buildings and mines, but a modification, based on the principles of the steam turbine reversed, and termed turbo-compressors, is being rapidly adapted to such higher pressures as have heretofore required piston compressors.

When high-pressure air is required for driving rock drills in mines and for hoisting engines, tools, as metal drills, riveters, chipping chisels, and for car air brakes, the compressors used to provide the air are termed simply air compressors. These compressors usually take in atmospheric air and compress it to the desired pressure, the capacity required being usually adjustable; they have valves of the automatic type throughout commonly, but in large sizes frequently are fitted with mechanically operated suction valves to decrease the resistance to entrance of air and so increase economy, a complication not warranted in small machines. When the pressures of delivery are quite high the compression is done in stages in successive cylinders, the discharge from the first or low-pressure cylinder being delivered through a water cooler or *intercooler* to the second cylinder and occasionally to a third in turn. This staging with intermediate or intercooling results in better economy, as will be seen later in detail, and permits the attainment of the desired quantity of cool compressed air for subsequent use with the expenditure of less work; the extra complication and cost being warranted only when machines are large and final pressures high.

In the operation of large steam condensers, non-condensable gases will collect and spoil the vacuum, which can be maintained only by the continuous removal of these gases, consisting of air, carbon dioxide and gases of animal and vegetable decomposition originally present in the water. When these gases are separately removed the machine used is a special form of compressor termed a dry vacuum pump which, therefore, receives a charge at the absolute pressure corresponding to the vacuum, or as nearly so as the entrance resistance permits, and after compression discharges into the atmosphere at a pressure in the cylinder above atmosphere, equivalent to discharge resistance. Natural-gas wells near exhaustion can sometimes be made to flow freely by the application of a compressor capable of drawing a charge at a pressure below atmosphere; but whether the charge be received below atmospheric pressure or above as in normal wells, the compressor will permit the delivery of the gas to distant cities or points of consumption, even 250 miles away, through smaller pipes than would otherwise be possible. Natural-gas compressors, some steam- and some gas-engine driven, are in use for both these purposes, compressing natural gas from whatever pressure may exist at the well to that desired at the beginning of the pipe line.

In the preparation of liquid ammonia or carbonic acid gas for the market, or in the operation of refrigerating machinery, wet or dry vapor is compressed into a condenser to permit liquefaction by the combined effect of high pressure and cooling. One form of refrigerating machine merely compresses air, subsequently expanding it after preliminary cooling by water, so that after expansion is complete it will become extremely cold.

All these compressing machines have, as a primary purpose, either the removal of a quantity of low-pressure gases from a given place, or the delivery of a quantity of higher-pressure gas to another place or both, but all include compression as an intermediate step between constant-pressure admission and

constant-pressure discharge, as nearly as structure may permit. They will all involve the same sort of physical operations and can be analyzed by the same principles except the wet-vapor or wet-gas compressors, in which condensation or evaporation may complicate the process and introduce elements that can be treated only by thermal analysis later. Safe compressors cannot be built with zero cylinder clearance, hence at the end of delivery there will remain in the clearance space a volume of high-pressure gases equal to the volume of the clearance space. On the return stroke this clearance volume will expand until the pressure is low enough to permit suction, so that the new charge cannot enter the cylinder until some portion of the stroke has been covered to permit this *re-expansion* of clearance gases.

It is quite impossible to study here all the effects or influences of structure as indicated by the compressor indicator cards, but a quite satisfactory treatment can be given by establishing reference diagrams as standards of comparison, and noting the nature of the differences between the actual cases and the standard reference diagram. These standard reference diagrams will really be pressure-volume diagrams, the phases of which correspond to certain hypotheses capable of mathematical expression, such as constant pressure, constant volume, expansion, and compression, according to some law, or with some definite value of s fixing either the heat-exchange character of the process or the substance, as already explained.

21. Standard Reference Diagrams or PV Cycles for Compressors and Methods of Analysis of Compressor Work and Capacity. All the standard reference diagrams will include constant-pressure lines corresponding to delivery and supply at pressures assumed equal to whatever exists outside the cylinder on either delivery or suction side, that is, assuming no loss of pressure on delivery or suction. The compression may be single or multi-stage with various amounts of cooling in the intercooler, but in multi-stage compression the standard reference diagram will be assumed to involve intermediate cooling of the gases to their original temperature, so that the gases entering all cylinders will be assumed to have the same temperature and to maintain it constant during admission. Another difference entering into the classification of standard reference diagrams is that shown by the law of compression as defined by the exponent s . Integration of the differential work expression will take a logarithmic form for $s=1$, and an exponential form for all other values, thus giving two possible reference compression curves and two sets of work equations.

(a) The *isothermal*, for which $s=1$, no matter what the gas; this is the consequence of assuming that all the heat liberated by compression is continuously carried away as fast as set free, so that the temperature cannot rise at all.

(b) The *exponential*, for which s has a value greater than one, generally different for every gas, vapor or gas-vapor mixture, but constant for any one gas, and also for dry vapors that remain dry for the whole process. Wet vapors having variable values of s cannot be treated by the simple pressure-volume analysis that suffices for the gases, but must be analyzed thermally. The adiabatic value of s is a consequence of assuming no heat exchange at all

between the gas and anything else and is a special case of the general exponential class.

Just why these two assumptions of thermal condition should result in the specified values of s will be taken up later under the thermal analysis part of this work.

As a consequence of these phase possibilities there may be established various standard reference diagrams or pressure-volume cycles defined by their phases; thus in Fig. 21, four cycles are shown for single-stage compression. Similarly for multi-stage compression there are two additional cycles for two-stage compression: cycle 5—two-stage exponential compression without clearance, perfect intercooling at best-receiver pressure; and cycle 6—two-stage exponential compression with clearance and perfect intercooling at best-receiver pressure. These are shown in the second series of Fig. 21 in which the first diagram represents cycle 5 and the other two represent cycle 6; the difference in the latter being due to the relative clearances. The lower set of diagrams in Fig. 21 indicates three-stage compression. The first of these is typical of cycle 7 for three-stage exponential compression, without clearance, perfect intercooling at best two receiver-pressures. The others represent cycle 8, three-stage compression with clearance and perfect intercooling at best two receiver-pressures. Multi-stage compression could be extended to four-stage by adding two more cycles, and so on.

Since three-stage and four-stage compressors are less frequently used in practice the analysis of the various cycles will not be carried beyond that required for two-stage compression, although the phase relations and the more important formulas for three-stage compressions will be presented.

SINGLE-STAGE COMPRESSION REFERENCE CYCLES OR PV DIAGRAMS

Cycle 1. Single-stage Isothermal Compression without Clearance.

- Phase (a) Constant pressure supply.
- " (b) Isothermal compression.
- " (c) Constant pressure delivery.
- " (d) Constant zero-volume pressure drop.

Cycle 2. Single-stage Isothermal Compression with Clearance.

- Phase (a) Constant pressure supply.
- " (b) Isothermal compression.
- " (c) Constant pressure delivery.
- " (d) Isothermal re-expansion.

Cycle 3. Single-stage Exponential Compression without Clearance.

- Phase (a) Constant pressure supply.
- " (b) Exponential compression.
- " (c) Constant pressure delivery.
- " (d) Constant zero-volume pressure drop.

Cycle 4. Single-stage Exponential Compression with Clearance.

- Phase (a) Constant pressure supply.
- " (b) Exponential compression.
- " (c) Constant pressure delivery.
- " (d) Exponential re-expansion.

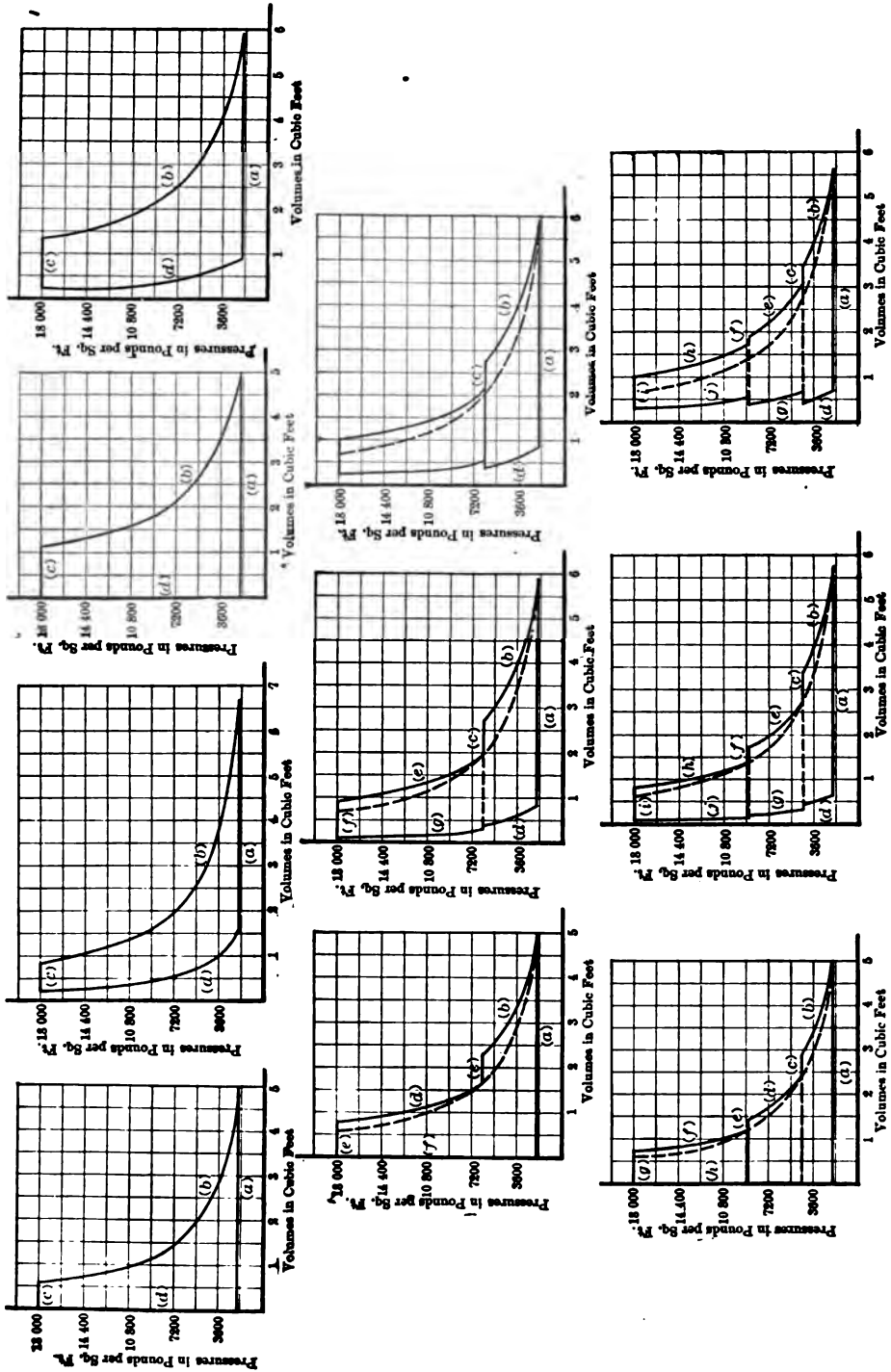


FIG. 21.—Standard Reference Cycles for Compressors, One-, Two-, and Three-Stage, with and without Clearance, Logarithmic and Exponential Laws.

MULTI-STAGE COMPRESSION

The phases making up multi-stage compression cycles may be considered in two ways; first, as referred to each cylinder and intercooler separately, or second, as referred to the pressure-volume changes of the gases themselves regardless of whether the changes take place in cylinders or intercoolers.

For example, if 10 cu.ft. of hot compressed air be delivered from the first cylinder of 50 cu.ft. displacement, the phase referred to this cylinder is a constant-pressure decreasing volume delivery line, whose length is $\frac{1}{5}$ of the whole diagram, exactly as in single-stage compression. If this 10 cu.ft. of air delivered to an intercooler became 8 ft. at the same constant pressure as the first cylinder delivery, the phase would be indicated by a constant-pressure volume reduction line 2 cu.ft. long to scale, or referred to the original volume of air admitted to the first cylinder, a line $\frac{2}{5}$ of its length. Finally, admitting this 8 cu.ft. of cool air to the second cylinder and compressing it to $\frac{1}{5}$ of its volume would result in a final delivery line at constant pressure of a length of $\frac{1}{5}$ of the length of the second cylinder diagram, but as this represents only 8 cu.ft., the final delivery will represent only $\frac{1}{5} \times 8 = 1.6$ cu.ft. This 1.6 cu.ft. will, when referred to the original 50 cu.ft. admitted to the first cylinder, be represented by a constant-pressure line, $\frac{1.6}{50} = .032$, of the whole diagram length,

which in volume is equivalent to $\frac{1}{5}$ of the length of the second cylinder diagram. It should be noted also that three volume change operations take place at the intermediate pressure; first, first cylinder delivery; second, volume decrease due to intercooling; third, second cylinder admission: the net effect of which referred to actual gas volumes, regardless of place where the changes happen, is represented by the volume decrease due to intercooling only. A diagram of volumes and pressures representing the resultant of all the gas processes is called in practice the *combined PV diagram* for the two cylinders, or when plotted from actual indicator cards with due regard for the different clearances of each cylinder the *combined indicator diagrams*. It is proper in the study of the whole process of compression to consider the cycle as consisting of phases referred to true gas volumes rather than phases referred to separate cylinder processes, which is equivalent to imagining the whole cycle carried out in one cylinder.

Intercooling effects measured by the amount of decrease of volume at constant pressure will, of course, depend on the amount of cooling or reduction of temperature, but in establishing a standard reference diagram some definite amount capable of algebraic description must be assumed as an intercooling hypothesis.

It has already been shown, Fig. 6, Chapter I, that from any original state of pressure and volume the exponential and isothermal could be drawn, diverging an amount depending on the defining exponent, s . If, after reaching a given state on the exponential curve, the gas be cooled at constant pressure to its original temperature, the point indicating its condition will lie, by

definition, on the other curve or isothermal, and the cooling process will be represented by a horizontal joining the two curves. Such intercooling as this will be defined as *perfect intercooling*, for want of a better name, and its pressure-volume effects can be treated by the curve intersections. It is now possible to set down the phases for the standard reference diagrams of multi-stage compression, if in addition to the above it be admitted, that there is a best or most economical receiver pressure.

TWO-STAGE COMPRESSION REFERENCE CYCLES OR PV DIAGRAMS

Cycle 5. Two-stage Exponential Compression without Clearance, Perfect Intercooling at Best Receiver Pressure.

- Phase (a) Constant pressure supply.
- “ (b) Exponential compression to best receiver pressure.
- “ (c) Constant pressure perfect intercooling of delivered gas.
- “ (d) Exponential compression from best receiver pressure.
- “ (e) Constant pressure delivery.
- “ (f) Constant zero-volume pressure drop.

Cycle 6. Two-stage Exponential Compression with Clearance, Perfect Intercooling at Best Receiver Pressure.

- Phase (a) Constant pressure supply.
- “ (b) Exponential compression to best receiver pressure.
- “ (c) Constant pressure perfect intercooling of delivered gas.
- “ (d) Exponential re-expansion of first stage clearance.
- “ (e) Exponential compression from best receiver pressure.
- “ (f) Constant pressure delivery.
- “ (g) Exponential re-expansion of second stage clearance.

THREE-STAGE COMPRESSION REFERENCE CYCLES OR PV DIAGRAMS

Cycle 7. Three-stage Exponential Compression, without Clearance, Perfect Intercooling at Best Two Receiver Pressures.

- Phase (a) Constant pressure supply.
- “ (b) Exponential compression to first receiver pressure.
- “ (c) Perfect intercooling at best first receiver pressure.
- “ (d) Exponential compression from best first to best second receiver pressure.
- “ (e) Perfect intercooling at best second receiver pressure.
- “ (f) Exponential compression from best second receiver pressure.
- “ (g) Constant pressure delivery.
- “ (h) Constant zero-volume pressure drop.

Cycle 8. Three-stage Adiabatic Compression with Clearance, Perfect Intercooling at Best Two Receiver Pressures.

- Phase (a) Constant pressure admission.
- “ (b) Exponential compression to best first receiver pressure.
- “ (c) Perfect cooling of delivered gas at best first receiver pressure.
- “ (d) Exponential re-expansion of first stage clearance.

- Phase (e) Exponential compression from best first to best second receiver pressure.
- “ (f) Perfect intercooling of delivered gas at best second receiver pressure.
- “ (g) Exponential re-expansion of second stage clearance.
- “ (h) Exponential compression from best second receiver pressure.
- “ (i) Constant pressure delivery.
- “ (j) Exponential re-expansion of third stage clearance.

It should be noted that cycles 6 and 8 may be sub-divided into any number of cases, of which some of the most characteristic are shown: (a) where the clearance volume in each cylinder bears the same ratio to the displacement of that cylinder, and commonly called equal clearances; (b) where the clearances are such that the volume after re-expansion in the higher-pressure cylinder is equal to the volume of clearance in the next lower-pressure cylinder, causing the combined diagram to have a continuous re-expansion line, a case which may be called proportionate clearance; and (c) the general case in which there is no particular relation between clearances in the several cylinders.

By means of these definitions or their mathematical equivalents in symbols it will be possible to calculate work as a function of pressures and volumes, and by various transformations of a general expression for work of a reference cycle, to calculate the horse-power corresponding to the removal of a given volume of gas per minute from the low-pressure supply, or to the delivery of another volume per minute to the high-pressure receiver, or per unit weight, per minute. It will also be possible to calculate the necessary cylinder size or displacement per unit of gas handled, and the horse-power necessary to drive the compressing piston at a specified rate, and further to calculate the work and horse-power of cylinders of given size and speed. In order that these calculations of a numerical sort may be quickly made, which is quite necessary if they are to be useful, the formulas must be definite and of proper form, the form being considered proper when little or no algebraic transformation is necessary before numerical work is possible. While special expressions for each case are necessary to facilitate numerical work, it is equally important, if not more so, to make clear the broad general principles or methods of attack, because it is quite impossible to set down every case or even to conceive at the time of writing of all different cases that must in future arise. The treatment, then, must be a combination of general and special; the general methods being applied successively, to make them clear and as a matter of drill, not to every possible case, but only to certain characteristic or type forms of cases, such as are here set down as standard reference diagrams. Individual cases may be judged by comparison with these and certain factors of relation established which, being ratios, may be and are called efficiencies. Thus, if a single-stage compressor should require two horse-power per cubic foot of free air compressed per minute, and Cycle I should for the same pressure limits require only one horse-power for its execution, then the efficiency

of the real compression would be 50 per cent referred to Cycle I, and similar factors or efficiencies for other compressors similarly obtained; a comparison of the factors will yield information for a judgment of the two compressors.

In what follows on the work and gas capacity of compressors two methods of attack will be used.

1. General pressure-volume analysis *in terms of gas pressures and volumes* resulting in the evaluation of work per cubic foot of low- or high-pressure gaseous substance.

2. Transformation of results of (1) to yield volumetric efficiencies, mean effective pressures, work, horse-power, and capacity *in terms of dimensions* of cylinders and clearances.

22. Single-stage Compressor, No Clearance, Logarithmic Compression (Cycle 1), Work, Capacity, and Work per Cubic Foot in Terms of Pressures and Volumes. The standard reference diagram is represented by Fig. 22, on which the process (*A to B*) represents admission or supply at constant pressure; (*B to C*) compression at constant temperature; (*C to D*) delivery at constant pressure; and (*D to A*) zero-volume.

Let V_b = The number of cubic feet of low pressure gas in the cylinder after admission, represented to scale on the diagram by \overline{AB} and equal to the volume at *B*;

V_c = volume in cubic feet of the gas in cylinder when discharge begins, represented by \overline{DC} , which is the volume at *C*;

P_b = absolute pressure in pounds per square foot, at which supply enters cylinder = (Sup.Pr.) = pressure at *B*;

$p_b = P_b \div 144$ = absolute supply pressure in pounds per square inch = (sup.pr.);

P_c = absolute pressure in pounds per square foot, at which delivery occurs = (Del. Pr.) = pressure at *C*;

$p_c = P_c \div 144$ = absolute delivery pressure in pounds per square inch = (del.pr.);

$R_p = \frac{P_c}{P_b}$ = ratio of delivery pressure to supply pressure;

W = foot-pounds work done for the cycle;

(H. P. Cap.) = volume of gas delivered in cubic feet per cycle, at temperature same as that of supply;

(L. P. Cap.) = volume of gas drawn into cylinder, cubic feet per cycle.

For this no-clearance case (L. P. Cap.) = V_b .

Referring to Fig. 22, the work for the cycle is the sum of compression and delivery work, less admission work, or by areas

Net work \overline{ABCD} = compression work \overline{EBCG} + delivery work $\overline{GCD\overline{F}}$
 - admission work $\overline{EBA\overline{F}}$,

Algebraically this is equivalent to $W = P_b V_b \log \frac{P_c}{P_b} + P_c V_c - P_b V_b$.

The work per cubic foot of low pressure gas, foot-pounds, will be the above expression divided by (L. P. Cap.), or

$$\frac{W}{(\text{L. P. Cap.})} = 144 (\text{sup.pr.}) \log_e R_p. \quad (31)$$

The work per cubic foot of high-pressure gas delivered will be

$$\frac{W}{(\text{H. P. Cap.})} = 144 (\text{sup.pr.}) R_p \log_e R_p, \quad (32)$$

since $P_b V_b = P_c V_c$; then $V_b = V_c \frac{P_c}{P_b}$,

which expressed symbolically is

$$(\text{L.P. Cap.}) = (\text{H. P. Cap.}) \times R_p. \quad (33)$$

Expressions (31) and (32) for the isothermal compressor are especially useful as standards of comparison for the economy of the compressors using methods other than isothermal. It will be found that the work per cubic foot of either low-pressure or cooled high-pressure gas is less by the isothermal process than by any other process discussed later, and that it is the limiting case for the economy of multi-stage compressors with a great number of stages. The fact that this process of isothermal compression is seldom if ever approached in practice does not make it any the less a suitable basis for comparison.

Example 1. Method of calculating Diagram Fig. 22.

Assumed Data. $P_a = P_b = 2116$ lbs. per square foot. $V_a = V_d = 0$.

$P_c = P_d = 18,000$ lbs. per square foot. Capacity = 5 cu.ft. $s = 1$.

To obtain point C, $P_b V_b = P_c V_c$ or $V_c = \frac{P_b}{P_c} V_b = \frac{5 \times 2116}{18,000} = .59$.

Intermediate points B to C are obtained by assuming various pressures and finding corresponding volumes as for V_c .

Example 2. To compress and deliver 5 cu.ft. of air from atmospheric pressure (2116 lbs. per square foot) to 8.5 atmospheres (18,000 lbs. per square foot) isothermally without clearance, how much work is necessary?

$$P_b = 2116 \quad P_c = 18,000 \quad V_b = 5 \quad \frac{V_b}{V_c} = \frac{P_c}{P_b} = 8.5 \quad \therefore V_c = .588$$

Work of admission = $P_b V_b = 2116 \times 5 = 10,585$ ft.-lbs.

Work of compression = $P_b V_b \log_e \frac{P_c}{P_b} = 10,585 \times \log_e 8.5 = 22,600$ ft.-lbs.

Work of delivery = $P_c V_c = 10,585$ ft.-lbs.

Total work = $10,585 + 22,600 - 10,585 = 22,600$ ft.-lbs.

Or by the general formula,

$$W = 144 (\text{sup.pr.}) (\text{L.P. Cap.}) \log_e R_p = 2116 \times 5 \times \log_e 8.5 = 2116 \times 5 \times 2.14 = 22,652 \text{ ft.-lbs.}$$

Prob. 1. How many cubic feet of free air may be compressed and delivered per minute from 14 lbs. absolute to 80 lbs. per square inch absolute, per horse-power in a compressor with zero clearance if compression is isothermal?

Prob. 2. Gas is being forced through mains at the rate of 10,000 cu.ft. per minute under a pressure of 5 lbs. per square inch above atmosphere. The gas is taken into the compressor at atmospheric pressure and compression is isothermal. What horse-power will be needed at sea level, and at an elevation of 5000 feet?

Prob. 3. Natural gas is drawn from a well, compressed isothermally and forced through a main at the rate of 200,000 cu.ft. per hour measured at the pressure on the suction side. What horse-power will be required to operate the compressor if the mechanical efficiency be 80 per cent? Suction pressure is 8 lbs. per square inch absolute, delivery pressure 60 lbs. per square inch absolute.

Prob. 4. A vacuum cleaning pump is required to maintain a pressure of 14 lbs. per square inch absolute, move 500 cu.ft. of free air per minute and discharge it against an atmospheric pressure of 15 lbs. per square inch absolute. What horse-power will be required (isothermal)?

Prob. 5. A blower furnishes 45 cu.ft. of air a minute at a pressure of 5 ins. of mercury above atmosphere. Assuming compression to be isothermal and supply pressure to be atmospheric, what horse-power will be needed?

Prob. 6. A compressor has a piston displacement of 3 cu.ft. At what speed can it be run if air be compressed isothermally from 1 to 10 atmospheres and the horse-power supplied is 100?

Prob. 7. A tank of 1000 cu.ft. capacity contains air at atmospheric pressure. A compressor taking air from atmosphere compresses it isothermally and discharges it into the tank until the pressure reaches 100 lbs. per square inch gage. What horse-power will be required to fill tank at this pressure in ten minutes?

Prob. 8. A compressor receives air at atmosphere and compresses it isothermally to five atmospheres above atmosphere. It takes in 1000 cu.ft. of free air per minute. How much would the capacity increase if the discharge pressure dropped to 3 atmospheres and the horse-power remained the same?

Prob. 9. Suppose that the pressure in the preceding problem were raised to 8 atmospheres. How much would the capacity decrease if the horse-power remained the same and how much more power would be required to keep the capacity the same?

Prob. 10. By means of suitable apparatus, the water from the side of a waterfall is diverted to a vertical shaft, and in falling 126 ft. compresses air from atmospheric pressure to a value equal to 90 per cent of the head of the water. To deliver 1000 cu.ft. of compressed air per hour, how much water is required if the work of falling water is 80 per cent useful in compressing the air?

23. Single-stage Compressor with Clearance, Isothermal Compression, (Cycle 2). Work, Capacity, and Work per Cubic Foot in Terms of Pressures and Volumes.

Referring to Fig. 23, the work of the cycle is, by areas.

$$\text{Net work area} = \overline{EBCG} + \overline{GCDF} - \overline{HADF} - \overline{EBAH} = \text{Area } \overline{ABCD}.$$

It is easily seen that this area is also equal to $(\overline{JBCL}) - (\overline{JADL})$, both of which are areas of the form evaluated in the preceding section. Accordingly

$$\text{Net work area} = \overline{JBCL} - \overline{JADL},$$

Algebraically,

$$W = P_b V_b \log_e \frac{P_c}{P_b} - P_a V_a \log_e \frac{P_d}{P_a} = P_b (V_b - V_a) \log_e \frac{P_c}{P_b} \quad \dots \quad (34)$$

which is the general expression for the work of the cycle in foot-pounds when pressures are in pounds per square foot, and volumes in cubic feet. Substituting the symbolic equivalents and using pressures in pounds per square inch, there results, since $(V_b - V_a) = (\text{L. P. Cap.})$,

$$\text{Work} = 144 (\text{sup.pr.})(\text{L. P. Cap.}) \log_e R_p, \quad (35)$$

which is identical with Eq. (30), showing that for a given low-pressure capacity the work of isothermal compressors is independent of clearance. The value of

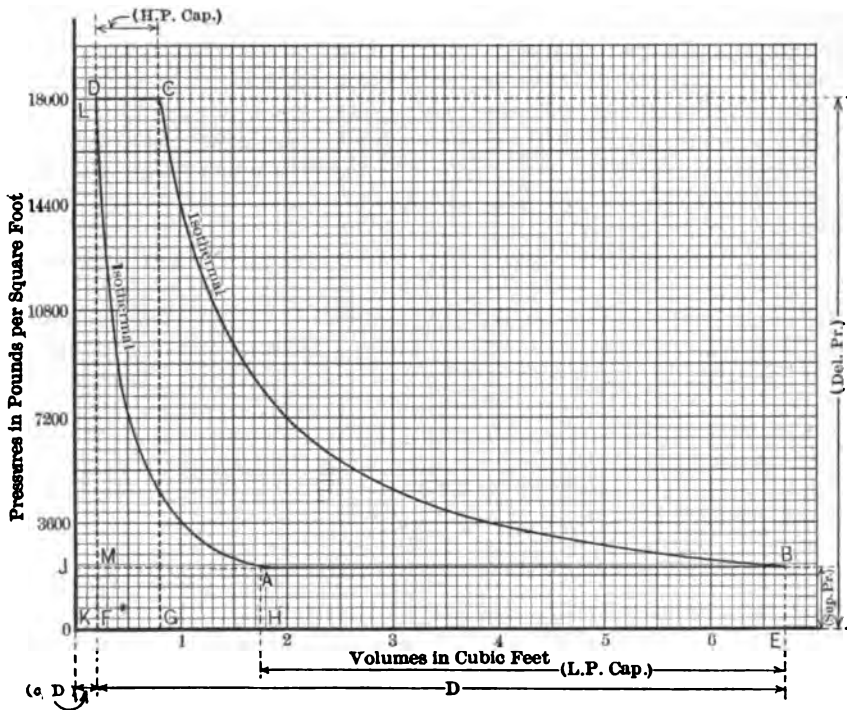


FIG. 23.—One-stage Compressor Cycle 2, Clearance, Isothermal.

the low-pressure capacity $(V_b - V_a)$ may not be known directly, but may be found if the volume before compression, V_b , the clearance volume before re-expansion, V_a , and the ratio of delivery to supply pressure, R_p , are known, thus $V_a = V_b R_p$, from which

$$(\text{L. P. Cap.}) = (V_b - V_a R_p). \quad (36)$$

Since Eq. (35) is the same as Eq. (30) it follows that the work per cubic foot of low-pressure gas, in foot-pounds, may be obtained from Eq. (31); and the work per cubic foot of high-pressure gas delivered, ft.-lbs. will be that obtained from Eq. (32).

That is *clearance has no effect on the work done for a given volume of gas admitted, however much it may affect the work of the cycle between given volume limits or work per unit of displacement.*

It is interesting to note that the work areas of Figs. 22 and 23 are equal when plotted on equal admission lines \overline{AB} or delivery lines \overline{CD} and any horizontal intercept xy will be equal in length on both if drawn at the same pressure.

In what precedes, it has been assumed that \overline{AB} represents admission volume and \overline{CD} represents delivery volume which is true for these established cycles of reference, but it is well to repeat that for real compressors these are only apparent admission and delivery lines, as both neglect heating and cooling effects on the gas during its passage into and out of the cylinder. Also that in real compressors the pressure of the admission line cannot ever be as high as the pressure from which the charge is drawn, and the delivery pressure must be necessarily higher than that which receives the discharge, in which cases the volume of gas admitted, as represented by \overline{AB} , even if the temperature did not change, would not equal the volume taken from the external supply, because it would exist in the cylinder at a lower pressure than it originally had. A similar statement would be true for delivered gas.

Problems. Repeat all the problems of the last section, assuming any numerical value for the clearance up to 10 per cent of the displacement.

24. Single-stage Compressor Isothermal Compression. Capacity, Volumetric Efficiency, Work, Mean Effective Pressure, Horse-power and Horse-power per Cubic Foot of Substance, in Terms of Dimensions of Cylinder and Clearance.

Consider first the case where clearance is not zero. Then Fig. 23 is the reference diagram.

Let D = displacement = volume, in cubic feet, displaced by piston in one stroke = area of piston in sq.ft. \times stroke in ft. = $(V_b - V_a)$.

(H. P. Cap.) = high pressure capacity = vol. cu.ft. of gas delivered per cycle at temperature equal to that of supply = $(V_c - V_d)$;

(L. P. Cap.) = low pressure capacity = vol. in cu.ft. of gas entering cylinder per cycle = $(V_b - V_a)$;

$$E_v = \text{volumetric efficiency} = \frac{\text{L.P. Cap.}}{D} = \frac{V_b - V_a}{V_b - V_d};$$

Cl = volume of clearance, cubic feet = V_d

c = clearance volume expressed as a fraction of the displacement;

$$= \frac{Cl}{D} = \frac{V_d}{V_b - V_a} \text{ whence } Cl = cD;$$

$$\text{M.E.P.} = \text{mean effective pressure, lbs. per square foot} = \frac{W}{D};$$

$$\text{m.e.p.} = \text{mean effective pressure, lbs. per square inch} = \frac{W}{144D};$$

N = number of revolutions per minute;

n = number of cycles per minute;

$$z = \text{number of revolutions per cycle} = \frac{N}{n};$$

I.H.P. = indicated horse-power of compressor;

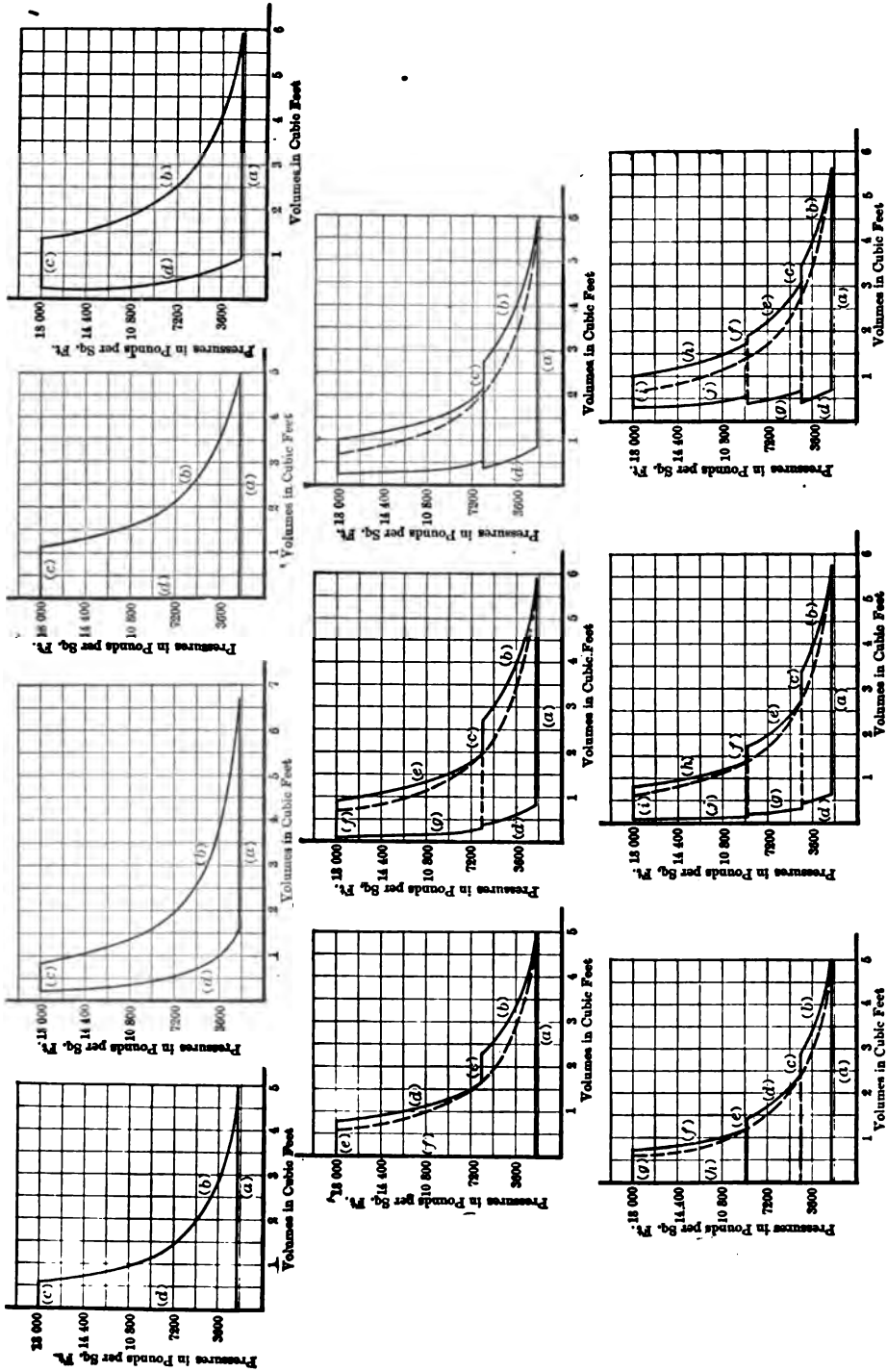


FIG. 21.—Standard Reference Cycles or P.V. Diagrams for Compressors, One-, Two-, and Three-Stage, with and without Clearance, Logarithmic and Exponential Laws.

MULTI-STAGE COMPRESSION

The phases making up multi-stage compression cycles may be considered in two ways; first, as referred to each cylinder and intercooler separately, or second, as referred to the pressure-volume changes of the gases themselves regardless of whether the changes take place in cylinders or intercoolers.

For example, if 10 cu.ft. of hot compressed air be delivered from the first cylinder of 50 cu.ft. displacement, the phase referred to this cylinder is a constant-pressure decreasing volume delivery line, whose length is $\frac{1}{5}$ of the whole diagram, exactly as in single-stage compression. If this 10 cu.ft. of air delivered to an intercooler became 8 ft. at the same constant pressure as the first cylinder delivery, the phase would be indicated by a constant-pressure volume reduction line 2 cu.ft. long to scale, or referred to the original volume of air admitted to the first cylinder, a line $\frac{1}{25}$ of its length. Finally, admitting this 8 cu.ft. of cool air to the second cylinder and compressing it to $\frac{1}{5}$ of its volume would result in a final delivery line at constant pressure of a length of $\frac{1}{5}$ of the length of the second cylinder diagram, but as this represents only 8 cu.ft., the final delivery will represent only $\frac{1}{5} \times 8 = 1.6$ cu.ft. This 1.6 cu.ft. will, when referred to the original 50 cu.ft. admitted to the first cylinder, be represented by a constant-pressure line, $\frac{1.6}{50} = .032$, of the whole diagram length,

which in volume is equivalent to $\frac{1}{5}$ of the length of the second cylinder diagram. It should be noted also that three volume change operations take place at the intermediate pressure; first, first cylinder delivery; second, volume decrease due to intercooling; third, second cylinder admission: the net effect of which referred to actual gas volumes, regardless of place where the changes happen, is represented by the volume decrease due to intercooling only. A diagram of volumes and pressures representing the resultant of all the gas processes is called in practice the *combined PV diagram* for the two cylinders, or when plotted from actual indicator cards with due regard for the different clearances of each cylinder the *combined indicator diagrams*. It is proper in the study of the whole process of compression to consider the cycle as consisting of phases referred to true gas volumes rather than phases referred to separate cylinder processes, which is equivalent to imagining the whole cycle carried out in one cylinder.

Intercooling effects measured by the amount of decrease of volume at constant pressure will, of course, depend on the amount of cooling or reduction of temperature, but in establishing a standard reference diagram some definite amount capable of algebraic description must be assumed as an intercooling hypothesis.

It has already been shown, Fig. 6, Chapter I, that from any original state of pressure and volume the exponential and isothermal could be drawn, diverging an amount depending on the defining exponent, s . If, after reaching a given state on the exponential curve, the gas be cooled at constant pressure to its original temperature, the point indicating its condition will lie, by

definition, on the other curve or isothermal, and the cooling process will be represented by a horizontal joining the two curves. Such intercooling as this will be defined as *perfect intercooling*, for want of a better name, and its pressure-volume effects can be treated by the curve intersections. It is now possible to set down the phases for the standard reference diagrams of multi-stage compression, if in addition to the above it be admitted, that there is a best or most economical receiver pressure.

TWO-STAGE COMPRESSION REFERENCE CYCLES OR PV DIAGRAMS

Cycle 5. Two-stage Exponential Compression without Clearance, Perfect Intercooling at Best Receiver Pressure.

- Phase (a) Constant pressure supply.
- “ (b) Exponential compression to best receiver pressure.
- “ (c) Constant pressure perfect intercooling of delivered gas.
- “ (d) Exponential compression from best receiver pressure.
- “ (e) Constant pressure delivery.
- “ (f) Constant zero-volume pressure drop.

Cycle 6. Two-stage Exponential Compression with Clearance, Perfect Intercooling at Best Receiver Pressure.

- Phase (a) Constant pressure supply.
- “ (b) Exponential compression to best receiver pressure.
- “ (c) Constant pressure perfect intercooling of delivered gas.
- “ (d) Exponential re-expansion of first stage clearance.
- “ (e) Exponential compression from best receiver pressure.
- “ (f) Constant pressure delivery.
- “ (g) Exponential re-expansion of second stage clearance.

THREE-STAGE COMPRESSION REFERENCE CYCLES OR PV DIAGRAMS

Cycle 7. Three-stage Exponential Compression, without Clearance, Perfect Intercooling at Best Two Receiver Pressures.

- Phase (a) Constant pressure supply.
- “ (b) Exponential compression to first receiver pressure.
- “ (c) Perfect intercooling at best first receiver pressure.
- “ (d) Exponential compression from best first to best second receiver pressure.
- “ (e) Perfect intercooling at best second receiver pressure.
- “ (f) Exponential compression from best second receiver pressure.
- “ (g) Constant pressure delivery.
- “ (h) Constant zero-volume pressure drop.

Cycle 8. Three-stage Adiabatic Compression with Clearance, Perfect Intercooling at Best Two Receiver Pressures.

- Phase (a) Constant pressure admission.
- “ (b) Exponential compression to best first receiver pressure.
- “ (c) Perfect cooling of delivered gas at best first receiver pressure.
- “ (d) Exponential re-expansion of first stage clearance.

Phase (e) Exponential compression from best first to best second receiver pressure.

- “ (f) Perfect intercooling of delivered gas at best second receiver pressure.
- “ (g) Exponential re-expansion of second stage clearance.
- “ (h) Exponential compression from best second receiver pressure.
- “ (i) Constant pressure delivery.
- “ (j) Exponential re-expansion of third stage clearance.

It should be noted that cycles 6 and 8 may be sub-divided into any number of cases, of which some of the most characteristic are shown: (a) where the clearance volume in each cylinder bears the same ratio to the displacement of that cylinder, and commonly called equal clearances; (b) where the clearances are such that the volume after re-expansion in the higher-pressure cylinder is equal to the volume of clearance in the next lower-pressure cylinder, causing the combined diagram to have a continuous re-expansion line, a case which may be called proportionate clearance; and (c) the general case in which there is no particular relation between clearances in the several cylinders.

By means of these definitions or their mathematical equivalents in symbols it will be possible to calculate work as a function of pressures and volumes, and by various transformations of a general expression for work of a reference cycle, to calculate the horse-power corresponding to the removal of a given volume of gas per minute from the low-pressure supply, or to the delivery of another volume per minute to the high-pressure receiver, or per unit weight, per minute. It will also be possible to calculate the necessary cylinder size or displacement per unit of gas handled, and the horse-power necessary to drive the compressing piston at a specified rate, and further to calculate the work and horse-power of cylinders of given size and speed. In order that these calculations of a numerical sort may be quickly made, which is quite necessary if they are to be useful, the formulas must be definite and of proper form, the form being considered proper when little or no algebraic transformation is necessary before numerical work is possible. While special expressions for each case are necessary to facilitate numerical work, it is equally important, if not more so, to make clear the broad general principles or methods of attack, because it is quite impossible to set down every case or even to conceive at the time of writing of all different cases that must in future arise. The treatment, then, must be a combination of general and special; the general methods being applied successively, to make them clear and as a matter of drill, not to every possible case, but only to certain characteristic or type forms of cases, such as are here set down as standard reference diagrams. Individual cases may be judged by comparison with these and certain factors of relation established which, being ratios, may be and are called efficiencies. Thus, if a single-stage compressor should require two horse-power per cubic foot of free air compressed per minute, and Cycle I should for the same pressure limits require only one horse-power for its execution, then the efficiency

(L. P. Cap.), p_b is the supply pressure (sup.pr.) pounds per square inch absolute, and $\frac{p_c}{p_b}$ is the ratio of delivery to supply pressure, R_p .

Accordingly, the work of an exponential, single-stage compressor with no clearance is

$$W = 144 \frac{s}{s-1} (\text{sup.pr.}) (\text{L. P. Cap.}) \left(R_p^{\frac{s-1}{s}} - 1 \right). \quad (48)$$

The work per cubic foot of low pressure gas, foot-pounds is

$$\frac{W}{(\text{L. P. Cap.})} = 144 \frac{s}{s-1} (\text{sup.pr.}) \left(R_p^{\frac{s-1}{s}} - 1 \right). \quad (49)$$

Before obtaining the work per cubic foot of high-pressure gas, it is necessary to describe two conditions that may exist. Since the exponential compression is not isothermal, it may be concluded that a change in temperature will take place during compression. This change is a rise in temperature and its law of variation will be presented in another chapter.

1. If the compressed air is to be used immediately, before cooling takes place, the high-pressure capacity, or capacity of delivery, will be equal to the volume at C , V_c and may be represented by (H. P. Cap. hot).

2. It more commonly occurs that the gas passes to a constant-pressure holder or reservoir, in which it stands long enough to cool approximately to the original temperature before compression, and the volume available after this cooling takes place is less than the actual volume discharged from the cylinder in the heated condition. Let this volume of discharge when reduced to the initial temperature be represented by (H. P. Cap. cold) which is represented by V_k , Fig. 24.

Since B and C in Fig. 24 lie on the exponential compression line, $P_b V_b^s = P_c V_c^s$,

$$V_b = V_c \left(\frac{P_c}{P_b} \right)^{\frac{1}{s}},$$

or $(\text{L. P. Cap.}) = (\text{H. P. Cap. hot}) (R_p)^{\frac{1}{s}}. \quad (50)$

Hence, the work in foot-pounds per cubic foot of hot gas delivered from compressor is

$$\frac{W}{(\text{H. P. Cap. hot})} = 144 \frac{s}{s-1} (\text{sup.pr.}) R_p^{\frac{1}{s}} \left(R_p^{\frac{s-1}{s}} - 1 \right). \quad (51)$$

On the other hand, B and K lie on an isothermal and $P_b V_b = P_k V_k$, or since $P_k = P_c$,

$$V_b = V_k \frac{P_c}{P_b},$$

whence $(\text{L. P. Cap.}) = (\text{H. P. Cap. cold}) R_p. \quad (52)$

The work foot-pounds per cubic foot of gas cooled to its original temperature is, therefore,

$$\frac{W}{(\text{H. P. Cap. cold})} = 144 \frac{s}{s-1} (\text{sup.pr.}) R_p \left(R_p^{\frac{s-1}{s}} - 1 \right), \quad (53)$$

or $\frac{W}{(\text{H. P. Cap. cold})} = 144 \frac{s}{s-1} (\text{del.pr.}) \left(R_p^{\frac{s-1}{s}} - 1 \right), \quad (54)$

This last equation is useful in determining the work required for the storing or supplying of a given amount of cool compressed air or gas, under conditions quite comparable with those of common practice.

Example 1. Method of calculating Diagram Fig. 24.

Assumed data:

$$P_a = P_b = 2116 \text{ lbs. per square foot; } P_c = P_d = 18,000 \text{ lbs. per square foot.}$$

$$C_l = 0; \quad V_a = V_d = 0; \quad \text{L. P. Capacity} = 5 \text{ cu.ft.}; \quad s = 1.4 \text{ (adiabatic value of } s).$$

$$\text{To obtain point } C: \quad P_c V_c^{1.4} = P_b V_b^{1.4} \text{ or } V_c = V_b \div \left(\frac{P_c}{P_b} \right)^{\frac{1}{1.4}}$$

$$P_c/P_b = 8.5; \log 8.5 = .929, \text{ and } .71 \log 8.5 = .665; (P_c/P_b)^{\frac{1}{1.4}} = 4.6,$$

$$\text{hence} \quad V_c = 5 \div 4.6 = 1.09 \text{ cu.ft.} \quad P_c = 18,000 \text{ lbs. per sq.ft.}$$

Intermediate points *B* to *C* are obtained by assuming various pressures and finding the corresponding volumes as for *V_c*.

Example 2. To compress 5 cu.ft. of air from atmospheric pressure (2116 lbs. per square foot) to 8.5 atmospheres (18,000 lbs. per square foot) adiabatically, with no clearance, requires how many foot-pounds of work?

$$V_b = 5 \text{ cu.ft., } P_b = 2116 \text{ lbs. sq.ft., } P_c = 18,000 \text{ lbs. sq.ft.,}$$

$$V_c = \frac{V_b}{\left(\frac{P_c}{P_b} \right)^{\frac{1}{1.4}}} = 5 \div 4.57 = 1.092 \text{ cu.ft.}$$

$$\text{Work of admission is } P_b V_b = 2116 \times 5 = 10,585 \text{ ft.-lbs.}$$

Work of compression, using γ to represent the adiabatic value of s is,

$$\frac{P_b V_b}{\gamma - 1} \left[\left(\frac{P_c}{P_b} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] = \frac{10585}{.41} [(8.5)^{.29} - 1] = \frac{10585}{.41} \times .860 = 22,350 \text{ ft.-lbs.}$$

$$\text{Work of delivery is } P_c V_c = 18,000 \times 1.092 = 19,650 \text{ ft.-lbs.}$$

$$\text{Total work} = 19,650 + 22,350 - 10,585 = 31,425 \text{ ft.-lbs.,}$$

$$\begin{aligned} \text{or by the formula Eq. (48) directly } W &= 144 \frac{\gamma}{\gamma - 1} (\text{sup.pr.}) (\text{L. P. Cap.}) \left(R_p^{\frac{\gamma - 1}{\gamma}} - 1 \right) \\ &= 3.46 \times 2116 \times 5 \times [(8.5)^{.29} - 1]; \\ &= 3.46 \times 2116 \times 5 \times .86 = 31,450 \text{ ft.-lbs.} \end{aligned}$$

Prob. 1. A single-stage zero clearance compressor compresses air adiabatically from 1 to 6 atmospheres. How many cubic feet of free air per minute can be handled if the compressor is supplied with 25 H. P. net?

Prob. 2. The same compressor is used for superheated ammonia under the same pressure conditions. For the same horse-power will the capacity be greater or less and how much?

Prob. 3. A dry-vacuum pump receives air at 28 ins. of mercury vacuum and delivers it against atmospheric pressure. What will be the work per cubic foot of low-pressure air and per cubic foot of high-pressure air hot? Barometer reads 29.9 ins.

Prob. 4. The manufacturer gives for a $10\frac{1}{2} \times 12$ in. double acting compressor running at 160 R.P.M., a capacity of 177 cu.ft. of free air per minute and a horse-power of 25 to 35 when delivering against pressures from 50 to 100 lbs. Check these figures.

Prob. 5. A set of drills, hoists, etc., are operated on compressed air. For their operation 3000 cu.ft. of air at 70 lbs. gage pressure are required per minute. What must be the pitson displacement and horse-power of a compressor plant to supply this air if compression is adiabatic and there is assumed to be no clearance?

Prob. 6. Air is compressed from atmosphere to 60 lbs. per square inch gage by a compressor running at 100 R.P.M. having a 12×18 in. cylinder. Find its capacity and horse-power at sea level and loss in capacity and horse-power if operated at an altitude of 10,000 ft. for zero clearance.

Prob. 7. 10,000 cu.ft. of free air per minute are compressed and delivered by a blowing engine at a pressure of 15 lbs. above atmosphere. Find the horse-power required to do this and find how much free air could be delivered by same horse-power if the pressure were tripled.

Prob. 8. In a gas engine the mixture of air and gas is compressed in the cylinder before ignition. If the original pressure is 14 lbs. per square inch absolute, final pressure 85 lbs. absolute and compression is adiabatic, what will be the work of compression only, per pound of mixture?

NOTE: Weight per cubic foot may be taken as .07 and γ as 1.38.

Prob. 9. A vacuum pump is maintaining a 25-in. vacuum and discharging the air removed against atmospheric pressure. Compare the work per cubic foot of low pressure air with that of a compressor compressing from atmosphere to 110 lbs. above atmosphere.

26. Single-stage Compressor with Clearance, Exponential Compression, (Cycle 4). Work, Capacity, and Work per Cubic Foot in Terms of Pressures and Volumes. When clearance exists in the cylinder, it is evident that a volume equal to the clearance, V_a , will not be expelled during the delivery of compressed gas, and this volume will expand with fall in pressure as the piston returns, causing pressure-volume changes represented by the line DA on the diagram, Fig. 25. Until the pressure has fallen to that of supply, the admission valve will not open, so that while the total volume in the cylinder at end of admission is V_b , the volume V_a was already present by reason of the clearance, and the volume taken in is $(V_b - V_a)$ which is the low-pressure capacity (L. P. Cap.).

The work area of the diagram is $ABCD$, which may be expressed as

$$\text{Work area} = \overline{JBCL} - \overline{JADL},$$

which areas are of the form evaluated in Section 25. Hence, the above expression in algebraic terms is

$$\begin{aligned} W &= \frac{s}{s-1} P_b V_b \left[\left(\frac{P_c}{P_b} \right)^{\frac{s-1}{s}} - 1 \right] - \frac{s}{s-1} P_a V_a \left[\left(\frac{P_d}{P_a} \right)^{\frac{s-1}{s}} - 1 \right] \\ &= \frac{s}{s-1} P_b (V_b - V_a) \left[\left(\frac{P_c}{P_b} \right)^{\frac{s-1}{s}} - 1 \right] \dots \dots \dots (55) \end{aligned}$$

This is the general expression for the work of the cycle, in foot-pounds, when the pressures are expressed in pounds per square foot, and volumes in cubic feet. Using symbolic equivalents

$$W = 144 \frac{s}{s-1} (\text{sup.pr.}) (\text{L. P. Cap.}) \left[(R_p)^{\frac{s-1}{s}} - 1 \right]. \quad (56)$$

Eq. (56) is identical with Eq. (48), showing that for adiabatic as for isothermal compressors, the work done for a given low-pressure capacity is inde-

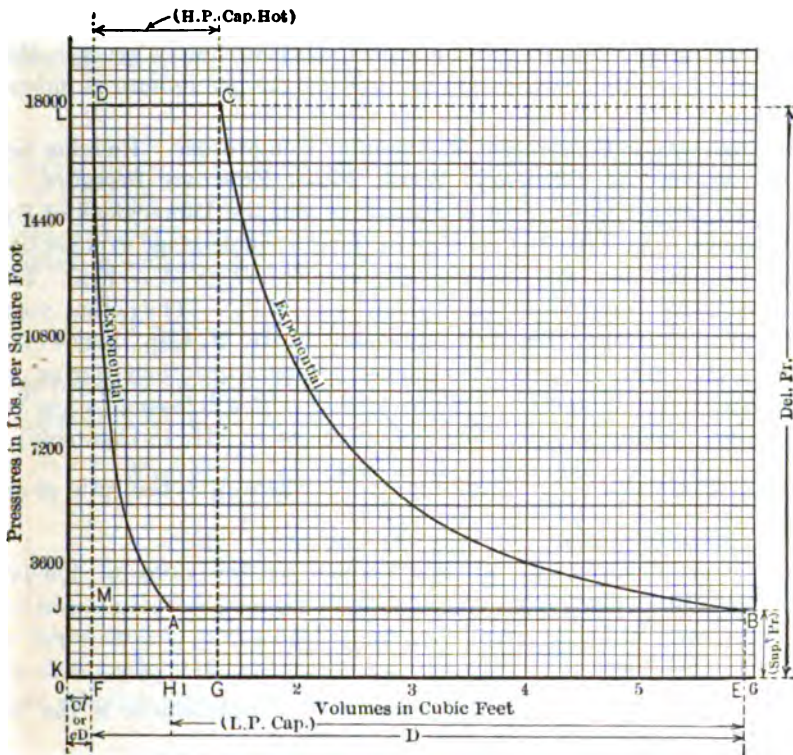


FIG. 25.—One-stage Compressor Cycle 4, Clearance, Exponential.

pendent of clearance. Owing to this fact, the expressions derived for the exponential compressor without clearance will hold for those with clearance, hence the work, in foot-pounds per cubic foot of low-pressure gas is that given in Eq. (49); and the work, in foot-pounds per cubic foot of hot gas delivered is that given in Eq. (51).

The work, in foot-pounds per cubic foot of gas cooled to its original temperature may be obtained from Eq. (54).

The relation of high-pressure capacity either hot or cold to the low-pressure capacity is the same as that given for the case of no clearance, as will be shown.

In Fig. 25, the high-pressure capacity, hot, is $\overline{DC} = V_c - V_d$. The low-pressure capacity is $\overline{AB} = V_b - V_a$, but $V_c P_c^{\frac{1}{\gamma}} = V_b P_b^{\frac{1}{\gamma}}$ and $V_d P_d^{\frac{1}{\gamma}} = V_a P_a^{\frac{1}{\gamma}}$, or

$$V_b = V_c R_p^{\frac{1}{\gamma}} \quad \text{and} \quad V_a = V_d R_p^{\frac{1}{\gamma}}$$

Hence $(\text{L. P. Cap.}) = (\text{H. P. Cap. hot}) R_p^{\frac{1}{\gamma}} \dots \dots \dots (57)$

which is identical with Eq. (50). Therefore if the delivered gas be cooled to its original temperature, then the volume after delivery and cooling will be that obtained from Eq. (52).

From the work relations given above, it is seen that in general, the work per unit of gas, or the horse-power per unit of gas per minute is independent of clearance.

27. Single-stage Compressor Exponential Compressor. Relation between Capacity, Volumetric Efficiency, Work, Mean Effective Pressure, Horse-power and H.P. per Cubic Foot of Substance and the Dimensions of Cylinder and Clearance. As indicated on Fig. 25, for the single-stage exponential compressor with clearance, the cylinder displacement D , is $(V_b - V_a)$. The low-pressure capacity per cycle is $(\text{L. P. Cap.}) = (V_b - V_a)$. The actual volume of gas or vapor delivered by the compressor is $(\text{H. P. Cap. hot}) = (V_c - V_d)$. This is, in the case of a gas at a higher temperature than during supply, but if cooled to the temperature which existed at B will become a less volume. This delivered volume after cooling is symbolized by (H. P. Cap. cold) and is equal to $(\text{L. P. Cap.}) \times \frac{(\text{sup.pr.})}{(\text{del.pr.})}$ or $\frac{(\text{L. P. Cap.})}{R_p}$ where R_p is the ratio of delivery pressure to supply pressure.

Volumetric efficiency, E_v , already defined as the ratio of low-pressure capacity to displacement is

$$E_v = \frac{V_b - V_a}{V_b - V_d} = \frac{(\text{L. P. Cap.})}{D}$$

Clearance, c , expressed as a fraction of the displacement is the ratio of clearance volume, Cl , to displacement, D , and is,

$$c = \frac{Cl}{D} = \frac{V_d}{V_b - V_a}$$

Mean effective pressure, pounds per square foot (M.E.P.), is the mean height of the diagram or the work area, W , divided by displacement D . If expressed in pounds per square inch the mean effective pressure will be indicated by

$$(\text{m.e.p.}) = \frac{W}{144D}$$

Let (I.H.P.) be indicated horse-power of the compressor;

N the number of revolutions per minute;

n the number of cycles per minute and

z the number of revolutions per cycle, whence $n \times z = N$.

Then, the low-pressure capacity is (L. P. Cap.) = $(V_b - V_a)$.

But
$$V_a = V_d \times \left(\frac{P_d}{P_a}\right)^{\frac{1}{s}},$$

since the re-expansion DA is exponential and similar to compression as to value of s , whence (L. P. Cap.) = $(V_b - V_a) = V_b - V_d R_p^{\frac{1}{s}};$

$$= D + Cl - V_d R_p^{\frac{1}{s}}; = D + cD - cD R_p^{\frac{1}{s}};$$

or
$$(\text{L. P. Cap.}) = D \left(1 + c - cR_p^{\frac{1}{s}}\right) \dots \dots \dots (58)$$

From this, by definition, the volumetric efficiency is

$$E_v = \frac{(\text{L.P.Cap.})}{D} = 1 + c - cR_p^{\frac{1}{s}} \dots \dots \dots (59)$$

Referring to Eq. (55), in which may be substituted the value Eq. (58) for $(V_b - V_a)$, the work of the single-stage exponential compressor in terms of displacement, clearance (as a fraction of displacement), and pressures of supply and delivery in pounds per square foot, is,

$$W = \frac{s}{s-1} P_b D \left[1 + c - c\left(\frac{P_c}{P_b}\right)^{\frac{1}{s}}\right] \left[\left(\frac{P_c}{P_b}\right)^{\frac{s-1}{s}} - 1\right], \dots \dots \dots (60)$$

or using pressures, pounds per square inch, and inserting the symbols, this may be stated in either of the following forms:

$$W = 144 \frac{s}{s-1} (\text{sup.pr.}) D \left(1 + c - cR_p^{\frac{1}{s}}\right) \left[R_p^{\frac{s-1}{s}} - 1\right] \dots \dots \dots (61)$$

$$= 144 \frac{s}{s-1} (\text{sup.pr.}) D E_v \left[R_p^{\frac{s-1}{s}} - 1\right] \dots \dots \dots (62)$$

The mean effective pressure in pounds per square foot is this work divided by the displacement, in cubic feet, and may be converted to pounds per square inch by dividing by 144, whence

Mean effective pressure, pounds per square inch,

$$(\text{m.e.p.}) = \frac{s}{s-1} (\text{sup.pr.}) \left(1 + c - cR_p^{\frac{1}{s}}\right) \left[R_p^{\frac{s-1}{s}} - 1\right], \dots \dots (63)$$

$$= \frac{s}{s-1} (\text{sup.pr.}) E_v \left[R_p^{\frac{s-1}{s}} - 1\right] \dots \dots \dots (64)$$

The indicated horse-power of the single-stage exponential compressor from (61) is,

$$\text{I.H.P.} = \frac{Wn}{33000} = \frac{s}{s-1} \frac{(\text{sup.pr.})nDE_v}{229.2} \left[R_p^{\frac{s-1}{s}} - 1\right] \dots \dots \dots (65)$$

Where n is the number of cycles per minute, or in terms of piston speed S and effective area of piston, square inches, and z the number of revolutions per cycle,

$$\text{I.H.P.} = \frac{s}{s-1} \frac{(\text{sup.pr.})aSE_v}{66000z} \left[R_p^{\frac{s-1}{s}} - 1\right] \dots \dots \dots (66)$$

Since it was found in Section 26, that the work per unit volume of gas is the same with clearance as without clearance, the horse-power per cubic foot per minute will also be independent of clearance. (See Eqs. (49), (51) and (54).)

Horse-power per cubic foot of gas supplied per minute

$$\frac{\text{I.H.P.}}{n(\text{L. P. Cap.})} = \frac{s}{s-1} \frac{(\text{sup.pr.})}{229.2} \left[R_p^{\frac{s-1}{s}} - 1 \right]. \quad (67)$$

The horse-power per cubic foot of hot gas delivered per minute is

$$\frac{\text{I.H.P.}}{n(\text{H.P. Cap. hot})} = \frac{s}{s-1} \frac{(\text{sup.pr.})}{229.2} R_p^{\frac{1}{s}} \left[R_p^{\frac{s-1}{s}} - 1 \right]. \quad (68)$$

Horse-power per cubic foot of gas delivered and cooled is

$$\frac{\text{I.H.P.}}{n(\text{H. P. Cap. cold})} = \frac{s}{s-1} \frac{(\text{sup.pr.})}{229.2} R_p \left[R_p^{\frac{s-1}{s}} - 1 \right], \quad (69)$$

$$= \frac{s}{s-1} \frac{(\text{del.pr.})}{229.2} \left[R_p^{\frac{s-1}{s}} - 1 \right]. \quad (70)$$

In the above formulæ (del.pr.) and (sup.pr) indicate delivery pressure and supply pressure, in pounds per square inch.

Example 1. Method of calculating Diagram, Fig. 25.

Assumed data: $P_a = P_b = 2116$ lbs. per square foot.
 $P_c = P_d = 18,000$ lbs. per square foot.

$Cl = 3.5$ per cent. L. P. Capacity = 5 cu.ft. $s = 1.4$.

To obtain point D : L. P. Cap. = $D \left(1 + c - cR_p^{\frac{1}{s}} \right)$ or $5 = D \left(1 + .035 - .035(8.5)^{.715} \right)$;
Hence $D = 5 \div (1 + .035 - .035 \times 4.6) = 5.72$ cu.ft. and $Cl = .035 \times 5.72 = .2$ cu.ft. = V_d .

To obtain point A : $V_a = \left(\frac{P_d}{P_a} \right)^{\frac{1}{1.4}} V_d = 4.6 \times .2 = .92$.

Intermediate points D to A are obtained by assuming various pressures and finding the corresponding volumes as for V_a .

To obtain point B : $V_b = V_a + \text{L. P. Cap.} = .92 + 5 = 5.92$ cu.ft.

To obtain point C : $V_c = V_b + \left(\frac{P_c}{P_b} \right)^{\frac{1}{1.4}} = 5.92 + 4.6 = 1.29$ cu.ft.

Example 2. It is required to compress 1000 cu.ft. of air per minute from 1 to 8.5 atmospheres absolute so that $s = 1.4$, in a compressor having 4 per cent clearance. What must be the displacement of the compressor, work per 100 cu.ft. of supplied and delivered air, hot and cold, and horse-power of machine? Speed is 150 R.P.M., compressor is double acting and stroke = 1.5 diameters.

$$D = \text{L. P. Cap.} + E_s, \text{ and } E_s = \left(1 + c - cR_p^{\frac{1}{s}} \right).$$

$$\therefore E_s = \left(1 + .04 - .04 \times (8.5)^{.71} \right) = .86;$$

$$\therefore D = 1000 + .86 = 1162 \text{ cu.ft. per min.}$$

$$\text{Work per cubic foot of supplied air} = 144 \frac{s}{s-1} (\text{sup.pr.}) [R_p^{\frac{s-1}{s}} - 1],$$

$$= 144 \times 3.46 \times 14.7 \times .86 = 6300 \text{ ft.-lbs.}$$

$$\therefore \text{Work per 100 cu.ft.} = 6,300,000 \text{ ft.-lbs.}$$

Work per cubic foot of delivered air cold is R_p times work per cubic foot of supplied air, hence work per 100 cu.ft. of delivered cooled air is 5,350,000 ft.-lbs.

Work per cubic foot of delivered air hot is $R_p^{\frac{1}{s}}$ times work per cubic foot of supplied air, hence work per 100 cu.ft. of hot delivered air is 2,800,000 ft.-lbs.

$$\text{I.H.P.} = \frac{(\text{m.e.p.})aS}{66000z}; \quad z = \frac{1}{2}; \quad (\text{m.e.p.}) = \frac{s}{s-1} (\text{sup.pr.}) E_s \left[R_p^{\frac{s-1}{s}} - 1 \right],$$

$$\text{or } (\text{m.e.p.}) = 3.46 \times 14.7 \times .86 \times .86 = 37.7 \text{ lbs. per square inch.}$$

$$a = \frac{\pi d^2}{4}; \quad S = 150 \times 2 \times \frac{1.5d}{12}; \quad \frac{aS}{288z} = D = 1162;$$

$$\therefore d^2 = 5690 \quad \text{or} \quad d = 17.85.$$

$$a = 250 \text{ square inches.} \quad S = 670 \text{ ft. per min.} \quad \therefore \text{I.H.P.} = 191.$$

Prob. 1. A dense-air ice machine requires that 4000 cu.ft. of air at 50 lbs. per square inch absolute be compressed each minute to 150 lbs. per square inch absolute. The compression being such that $s = 1.4$, clearance being 6 per cent, find the work required. What would be the work if clearance were twice as great? One-half?

Prob. 2. The compressor for an ammonia machine compresses from one atmosphere to 8 atmospheres absolute. With adiabatic compression and 4 per cent clearance, what will be work per cubic foot of vapor at the low pressure and at the high? Assume vapor to be superheated.

Prob. 3. On a locomotive an air-brake pump compresses air adiabatically from atmosphere to 80 lbs. per square inch gage. It is required to compress 50 cu.ft. of free air per minute; clearance is 5 per cent. What horse-power must be supplied to it?

Prob. 4. In a manufacturing process a tank must be maintained with a vacuum of 29 ins. when barometer reads 30 ins. To do this 100 cu.ft. of carbon dioxide must be removed from it per minute and returned under atmospheric pressure. Compression is adiabatic and clearance 7 per cent. How much power must be supplied to compressor and what should be its displacement?

Prob. 5. Two compressors each 12×18 in., double acting, running at 150 R.P.M. with 8 per cent clearance, compress in the one case air, in the other carbon disulphide. The compression being adiabatic in each case, what (a), is the difference in power required, (b), in low-pressure capacities? Take pressures as 2 and 15 atmospheres of 26 inches mercury.

Prob. 6. A compressor is supplied with 40 horse-power. If it draws in air from atmosphere to what pressure can 500 cu.ft. per minute be compressed, when $s = 1.38$ and clearance 10 per cent?

Prob. 7. For forcing gas through a main, a pressure of 50 lbs. per square inch gage is required. What is the work done per cubic foot of high-pressure gas, if a compressor having 6 per cent clearance is used, and s for the gas is 1.36? What should be its displacement?

Prob. 8. A gas compressor 20×22 ins. has a volumetric efficiency of 90 per cent, supply pressure = 4 lbs. per square inch and delivery 110 lbs. per square inch gage.

What are its L. P., H. P., hot and cold capacities, and its I.H.P. if single acting at 70 R.P.M. when $s=1.35$?

28. Two-Stage Compressor, no Clearance, Perfect Intercooling, Exponential Compression, Best Receiver Pressure, Equality of Stages (Cycle V). Work and Capacity in Terms of Pressures and Volumes. The common assumption in considering the multi-stage compressor is that in passing from one cylinder to the next, the gas is cooled to the temperature it had before entering the compressor, which has already (Section 21) been defined as "perfect intercooling."

This condition may be stated in other words by saying that the product of pressure and volume must be the same for gas entering each cylinder. If then the volume and pressure of gas entering the first stage be determined, fixing the volume entering the second stage will determine the pressure of the gas entering the second stage, or fixing the pressure of the gas entering the second stage will determine the volume that must be taken in.

Using subscripts referring to Fig. 26, for the no-clearance case,

$$P_b V_b = P_d V_d \quad \dots \quad (71)$$

The net work of the compressor, area $\overline{ABCDE F}$ = area \overline{ABCH} first stage + area \overline{HDEF} second stage. Using the general expression, Eq. (46) for these work areas with appropriate changes in subscripts.

$$W = \frac{s}{s-1} P_b V_b \left[\left(\frac{P_c}{P_b} \right)^{\frac{s-1}{s}} - 1 \right] \quad \dots \quad (\text{first stage})$$

$$+ \frac{s}{s-1} P_d V_d \left[\left(\frac{P_e}{P_d} \right)^{\frac{s-1}{s}} - 1 \right] \quad \dots \quad (\text{second stage})$$

But from the above, and since $P_c = P_d$,

$$W = \frac{s}{s-1} P_b V_b \left[\left(\frac{P_c}{P_b} \right)^{\frac{s-1}{s}} + \left(\frac{P_e}{P_c} \right)^{\frac{s-1}{s}} - 2 \right], \quad \dots \quad (72)$$

which is the general expression for work of a two-stage compressor without clearance, perfect intercooling, and may be restated with the usual symbols as follows:

$$W = 144 \frac{s}{s-1} (\text{sup.pr.}) (\text{L. P. Cap.}) \left[(R_{p1})^{\frac{s-1}{s}} + (R_{p2})^{\frac{s-1}{s}} - 2 \right], \quad \dots \quad (73)$$

in which (R_{p1}) and (R_{p2}) are the ratios of delivery to supply pressures for the first stage and for the second stage respectively. From Eq. (73), work per cubic foot of gas supplied is,

$$\frac{W}{(\text{L. P. Cap.})} = 144 \frac{s}{s-1} (\text{sup.pr.}) \left[R_{p1}^{\frac{s-1}{s}} + R_{p2}^{\frac{s-1}{s}} - 2 \right], \quad \dots \quad (74)$$

Work per cubic foot of gas discharged and cooled to its original temperature is

$$\frac{W}{(\text{H. P. Cap. cold})} = 144 \frac{s}{s-1} (\text{sup.pr.}) R_p \left[R_{p1}^{\frac{s-1}{s}} + R_{p2}^{\frac{s-1}{s}} - 2 \right],$$

$$= 144 \frac{s}{s-1} (\text{del.pr.}) \left[R_{p1}^{\frac{s-1}{s}} + R_{p2}^{\frac{s-1}{s}} - 2 \right]. \quad \dots \quad (75)$$

The low-pressure capacity stated in terms of high-pressure capacity hot, as actually discharged is

$$(L. P. Cap.) = (H. P. Cap. hot) R_{p2}^{\frac{1}{s}} R_{p1}, \dots \dots \dots (76)$$

whence, work per cubic foot of hot gas discharged

$$\frac{W}{(H. P. Cap. hot)} = 144 \frac{s}{s-1} (\text{sup.pr.}) R_{p2}^{\frac{1}{s}} R_{p1} \left[R_{p1}^{\frac{s-1}{s}} + R_{p2}^{\frac{s-1}{s}} - 2 \right]. \quad (77)$$

Examination of Fig. 26 will show without analysis that there must be some

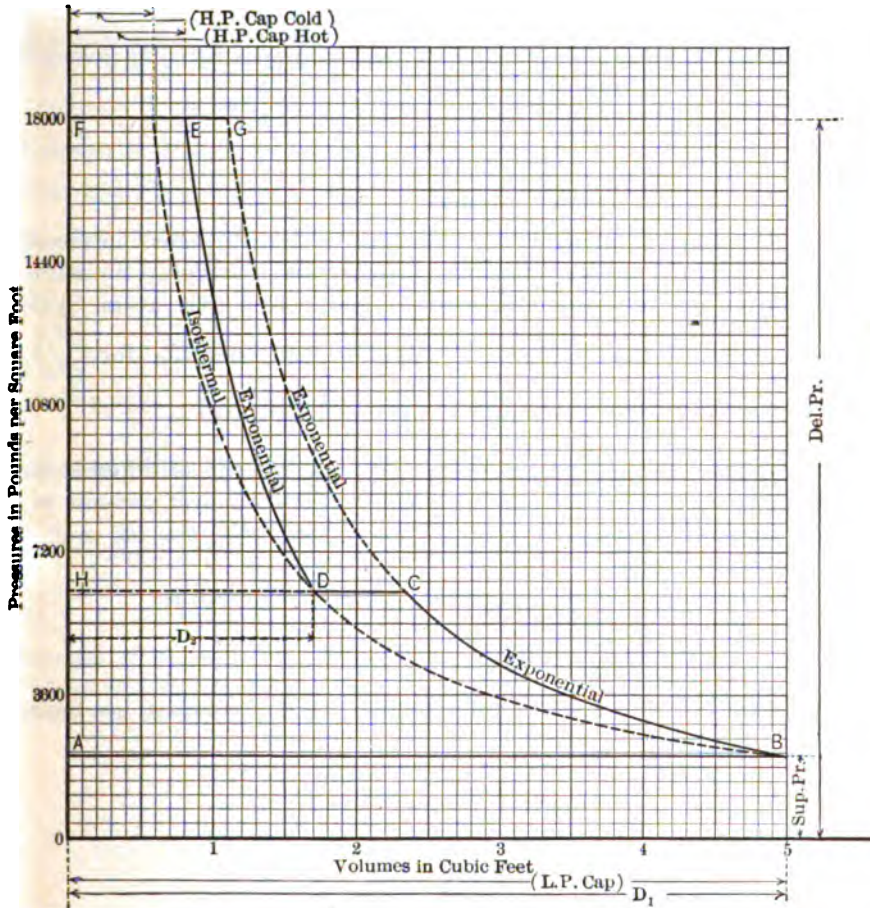


FIG. 26.—Two-stage Compressor Cycle 5, No Clearance, Exponential. Perfect Intercooling, Best Receiver Pressure.

best-receiver pressure at which least work will be required. For if the receiver pressure approached P_s then the compression would approach single stage and the compression line approach BCG . The same would be true as the receiver pressure approached $P_s = P_c$, whereas at any intermediate point C , intercooling causes the process to follow $BCDE$ with a saving of work over single-stage operation represented by the area \overline{DCGE} . This area being zero when C is at

either B or G , it must have a maximum value somewhere between, and the pressure at which this least-compressor work will be attained is the *best-receiver pressure*.

By definition the best-receiver pressure is that for which W is a minimum, or that corresponding to $\frac{dW}{dP_c} = 0$.

Performing this differentiation upon Eq. (72), equating the result to zero, and solving for P_c ,

$$(\text{Best rec.pr.}) = (P_b P_c)^{1/2} = [(\text{sup.pr.}) (\text{del.pr.})]^{1/2} \quad (78)$$

Substituting this value in the general expression for work Eq. (72), noting that

$$\frac{P_c}{P_b} = \frac{(P_b P_c)^{1/2}}{P_b} = \left(\frac{P_c}{P_b}\right)^{1/2} \quad \text{and} \quad \frac{P_c}{P_c} = \left(\frac{P_c}{P_b}\right)^{1/2}$$

$$W = 2 \frac{s}{s-1} P_b V_b \left[\left(\frac{P_c}{P_b}\right)^{\frac{s-1}{2s}} - 1 \right] \quad (79)$$

Eq. (79) is the general expression for two-stage work with perfect intercooling at best-receiver pressure in terms of pressures and volumes. Substituting the symbols for the pressures and volumes and noting that as in Cycle 1,

$$V_b = (\text{L. P. Cap.}) \quad \text{and} \quad V_c = (\text{H. P. Cap. hot}) \quad \text{and using } (R_p) \text{ for } \left(\frac{P_c}{P_b}\right),$$

$$W = 288 \frac{s}{s-1} (\text{sup.pr.}) (\text{L. P. Cap.}) \left(R_p^{\frac{s-1}{2s}} - 1 \right) \quad (80)$$

This equation gives the same value as Eq. (79), but in terms of different units.

It should be noted here that the substitution of best-receiver pressure in the expressions for the two stages preceding Eq. (76), will show that the work done in the two cylinders is equal.

The work per cubic foot of low-pressure gas, from Eq. (80) is,

$$\frac{W}{(\text{L. P. Cap.})} = 288 \frac{s}{s-1} (\text{sup.pr.}) \left[R_p^{\frac{s-1}{2s}} - 1 \right] \quad (81)$$

To transform Eq. (79) into a form involving delivery volumes, use the relation from the diagram,

$$V_c = V_d \left(\frac{P_d}{P_c}\right)^{\frac{1}{s}} = V_b \left(\frac{P_b}{P_c}\right) \left(\frac{P_d}{P_c}\right)^{\frac{1}{s}}$$

Whence

$$V_b = V_c \left(\frac{P_c}{P_b}\right) \left(\frac{P_c}{P_d}\right)^{\frac{1}{s}},$$

which for the best-receiver pressure becomes $V_b = V_c R_p^{\frac{s+1}{2s}}$.

$$\text{Substituting in Eq. (79), } W = 2 \frac{s}{s-1} P_b V_c (R_p)^{\frac{s+1}{2s}} \left[R_p^{\frac{s-1}{2s}} - 1 \right] \quad (82)$$

Introducing the symbols,

$$W = 288 \frac{s}{s-1} (\text{sup.pr.}) (\text{H. P. Cap. hot}) R_p^{\frac{s+1}{2s}} \left[R_p^{\frac{s-1}{2s}} - 1 \right] \quad (83)$$

$$\text{and } \frac{W}{(\text{H. P. Cap. hot})} = 288 \frac{s}{s-1} (\text{sup.pr.}) R_p^{\frac{s+1}{2s}} \left[R_p^{\frac{s-1}{2s}} - 1 \right] \quad (84)$$

The volume of gas discharged at the higher pressure when reduced to its original temperature will become such that

$$\frac{(\text{L. P. Cap.})}{(\text{H. P. Cap. cold})} = \frac{P_c}{P_b} = R_p,$$

or $(\text{sup.pr.}) (\text{L. P. Cap.}) = (\text{del.pr.}) (\text{H. P. Cap. cold}), \dots (85)$

which may be substituted in Eq. (80),

$$W = 288 \frac{s}{s-1} (\text{del.pr.}) (\text{H. P. Cap. cold}) \left[R_p^{\frac{s-1}{2s}} - 1 \right], \dots (86)$$

from which the work per cubic foot of gas delivered and cooled is,

$$\frac{W}{(\text{H. P. Cap. cold})} = 288 \frac{s}{s-1} (\text{del.pr.}) \left[R_p^{\frac{s-1}{2s}} - 1 \right]. \dots (87)$$

Example 1. Method of calculating diagram, Fig. 26.

Assumed data: $V_a = 0$ cu.ft. $P_a = 2116$ lbs. per square foot.

$V_f = 0$ cu.ft. $P_c = P_d = \sqrt{P_a P_e} = 6172$ lbs. sq.ft.

$V_b = 5$ cu.ft. $P_f = P_e = P_g = 18,000$ lbs. sq.ft.

$$V_d = V_b \frac{P_b}{P_d} \quad s = 1.4.$$

To obtain point C: $V_c = V_b \div \left(\frac{P_c}{P_b} \right)^{\frac{1}{1.4}} = 2.36$ cu.ft.

To obtain point D: $V_d = V_b \times \frac{P_b}{P_c} = 5 \times \frac{2116}{6172} = 1.71$ cu.ft.

To obtain point E: $V_e = V_d \div \left(\frac{P_e}{P_d} \right)^{\frac{1}{1.4}}$, but by definition $\left(\frac{P_e}{P_d} \right)^{\frac{1}{1.4}} = \left(\frac{P_d}{P_b} \right)^{\frac{1}{1.4}} = 2.14$,

hence, $V_e = 1.71 \div 2.14 = .8$ cu.ft. $P_e = 18000$ lbs. sq.ft.

Example 2. To compress 5 cu.ft. of air from one atmosphere (2116 lbs. per square foot) to 8.5 atmospheres (18,050 lbs. per square foot) in two stages with best-receiver pressure and perfect intercooling requires how much work?

$$W = 288 \frac{s}{s-1} (\text{sup.pr.}) (\text{L. P. Cap.}) \left(R_p^{\frac{s-1}{2s}} - 1 \right),$$

$$(\text{sup.pr.}) = 14.7. \quad (\text{L. P. Cap.}) = 5. \quad R_p = 8.5.$$

$$\therefore W = 288 \times 3.463 \times 14.7 \times 5 \times \left(8.5^{\frac{s-1}{2s}} - 1 \right) = 26,800 \text{ ft.-lbs.}$$

Prob. 1. Air at 14 lbs. per square inch absolute is compressed to 150 lbs. per square inch absolute by a two-stage compressor. What will be the work per cubic foot of air delivered? What will be the work per cubic foot if the air be allowed to cool to the original temperature, and how will this compare with the work per cubic foot of supplied air? Best receiver-pressure and perfect intercooling are assumed for the above compressor, $s = 1.4$.

Prob. 2. A compressor receives air at atmosphere and compresses it to half its volume, whereupon the air is discharged to the cooler and its temperature reduced to the original point. It then enters a second cylinder and is compressed to 80 lbs. absolute.

What will be the work per cubic foot of supplied air in each cylinder? How will the work of compressing a cubic foot to the delivery pressure compare with the work done if compression were single stage, compression being adiabatic?

Prob. 3. Air is to be compressed from 15 lbs. per square inch absolute to 10 times this pressure. What would be the best-receiver pressure for a two-stage compressor? How many more cubic feet may be compressed per minute in two stage than one stage by the same horse-power?

Prob. 4. A manufacturer sells a compressor to run at best-receiver pressure when (sup.pr.) is 14 lbs. per square inch absolute and (del.pr.) 100 lbs. per square inch absolute. What will be the work done in each cylinder per cubic foot of supply-pressure air? Another compressor is so designed that the receiver pressure for same supply pressure and delivery pressure is 30 lbs. per square inch absolute, while a third is so designed that receiver pressure is 50 lbs. per square inch absolute. How will the work done in each cylinder of these machines compare with that of first machine?

Prob. 5. For an ice machine a compressor works between 50 and 150 lbs. per square inch absolute. It is single stage. What would be the saving by making compression two stage at best-receiver pressure?

Prob. 6. A compressor has been designed to compress 1000 cu.ft. of carbon dioxide per minute from 15 to 150 lbs. per square inch absolute. What horse-power will be required at best-receiver pressure? Should delivery pressure change to 200 lbs., what power would be required? To 100 lbs. what power?

Prob. 7. A gas-compressing company operates a compressor which has to draw CO₂ gas from a spring and compress it to 150 lbs. per square inch gage. In the morning pressure on the spring is 10 lbs. gage, while by evening it has dropped to 5 lbs. absolute. If the compressor was designed for the first condition, how will the high-pressure capacity and horse-power per cubic foot of high-pressure gas at night compare with corresponding values in morning? Assume a barometric reading.

Prob. 8. On a mining operation a compressor is supplying a number of drills and hoists with air at 150 lbs. per square inch absolute, the supply pressure being 14 lbs. What will be the difference in horse-power per cubic foot of delivered air at compressor and per cubic foot received at drills if air is a long time in reaching drills?

Prob. 9. With a best-receiver pressure of 40 lbs. per square inch absolute and a supply pressure of 14 lbs. per square inch absolute, what horse-power will be required to compress and deliver 1000 cu.ft. of high-pressure air per minute at the delivery pressure for which compressor is designed and what is that delivery pressure?

29. Two-stage Compressor, with Clearance, Perfect Intercooling Exponential Compression, Best-receiver Pressure, Equality of Stages, (Cycle 6). Work and Capacity in Terms of Pressures and Volumes. The two-stage exponential compressor with clearance and perfect intercooling is represented by the PV diagrams Figs. 27, 28, 29, which are clearly made up of two single-stage compression processes, each with clearance.

Applying Eq. (55) to the two stages and supplying proper subscripts, referring to Fig. 27,

$$W = \frac{s}{s-1} P_b (V_b - V_a) \left[\left(\frac{P_c}{P_b} \right)^{\frac{s-1}{s}} - 1 \right] \quad \text{. . (first stage)}$$

$$+ \frac{s}{s-1} P_d (V_d - V_h) \left[\left(\frac{P_e}{P_d} \right)^{\frac{s-1}{s}} - 1 \right] \quad \text{. . (second stage)}$$

If the condition of perfect intercooling be imposed, it is plain that since the weight of gas entering the second stage must equal that entering the first stage, and the temperature in each case is the same,

$$(V_d - V_h)P_d = (V_b - V_a)P_b,$$

$$P_c = P_d$$

and noting also that

$$W = \frac{s}{s-1} P_b (V_b - V_a) \left[\left(\frac{P_c}{P_b} \right)^{\frac{s-1}{s}} + \left(\frac{P_c}{P_c} \right)^{\frac{s-1}{s}} - 2 \right], \quad \dots (88)$$

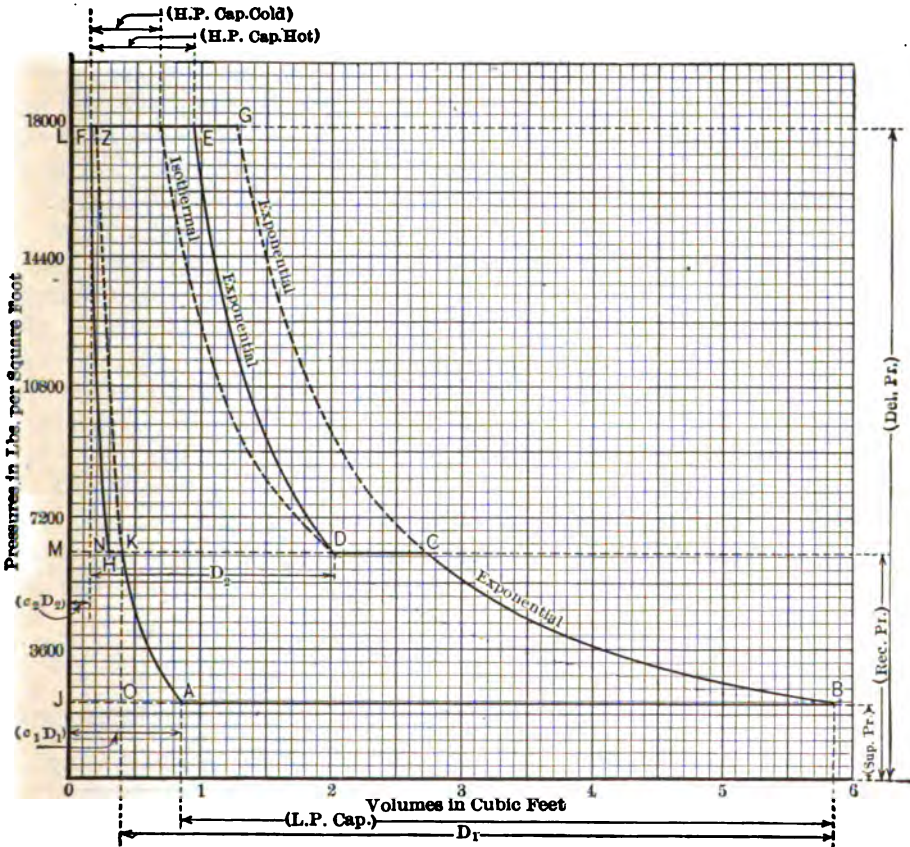


FIG. 27.—Two-stage Compressor, Cycle 6, Clearance, Exponential. Perfect Intercooling, Best-receiver Pressure. (H.P. Re-expansion Volume Less than L.P. Clearance.)

Eq. (88) is the general expression for the work of two-stage exponential compressor with perfect intercooling, P_c being the receiver pressure.

As in Section 28, let (R_{p1}) be the pressure ratio $\frac{P_c}{P_b}$ for the first stage and (R_{p2}) the pressure ratio $\frac{P_c}{P_c}$ for the second stage and using instead of P_b its equivalent 144 (sup.pr.) lbs. per square inch.

$$W = 144 \frac{s}{s-1} (\text{sup.pr.}) (\text{L.P. Cap.}) \left[(R_{p1})^{\frac{s-1}{s}} + (R_{p2})^{\frac{s-1}{s}} - 2 \right], \quad \dots (89)$$

which is identical with (73), showing that for two-stage compressors with perfect intercooling, (as for single stage Section 26), the work for a given low-pressure capacity is independent of clearance.

The work per cubic foot of gas supplied is given by Eq. (80); per cubic foot of cold gas delivered, by Eq. (75) and per cubic foot hot gas delivered, by Eq. (77).

The reasoning regarding best-receiver pressure followed out in Section 28, will

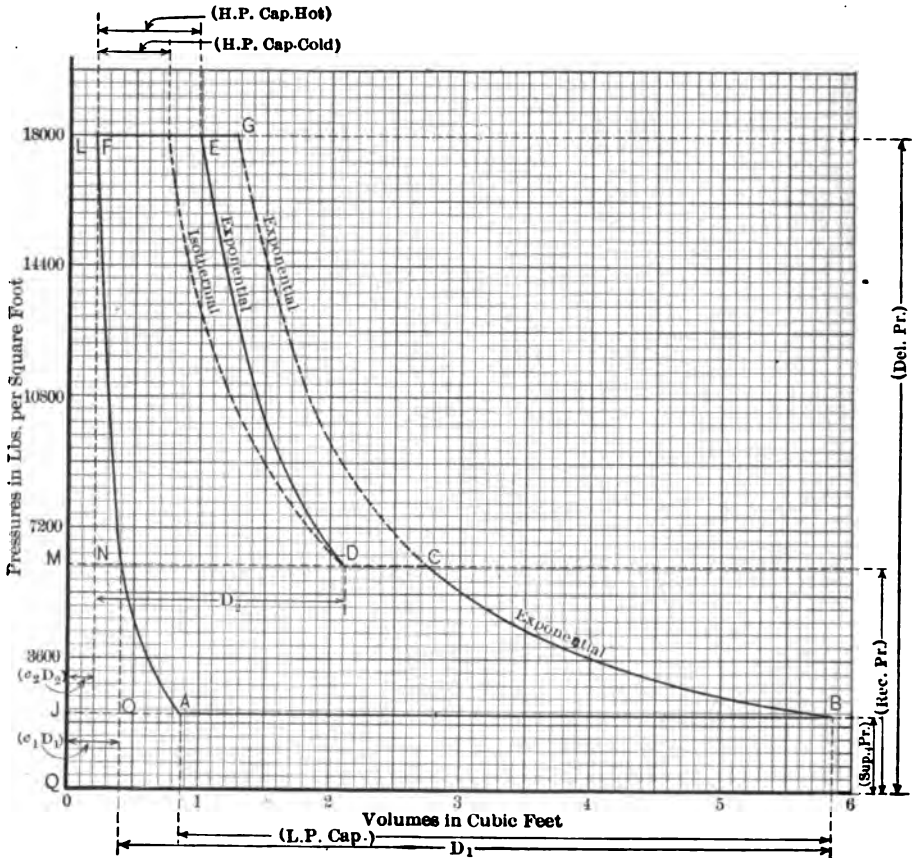


FIG. 28.—Two-stage Compressor, Cycle 6, Clearance, Exponential. Perfect Intercooling. Best-receiver Pressure. (H.P. Re-expansion Volume Equal to L.P. Clearance.)

hold again in this case, and by putting $\frac{dW}{dP_c} = 0$ in Eq. (88), and solving for P_c it will again be found that best-receiver pressure is that given in Eq. (78).

Substitution of this value for P_c in Eq. (88), gives the following expression for work of the two-stage exponential compressor with best-receiver pressure,

$$W = 2 \frac{s}{s-1} P_b (V_b - V_a) \left[\left(\frac{P_a}{P_b} \right)^{\frac{s-1}{2s}} - 1 \right], \quad \dots \quad (90)$$

which may be expressed in terms of supply pressure, pounds per square inch, low-pressure capacity, cubic feet, and ratio of compression,

$$W = 288 \frac{s}{s-1} (\text{sup.pr.})(\text{L. P. Cap.}) \left[R_p^{\frac{s-1}{2s}} - 1 \right], \quad . . . \quad (91)$$

which is the same as Eq. (80).

Substitution of the value of best-receiver pressure in the expression for the work of the two stages separately will show the equality of work done in the respective stages for this case with clearance.

Owing to the fact that clearance has no effect upon the work per cubic foot of substance, as previously noted, the equation for work per cubic foot of gas supplied to compressor will be identical with Eq. (81). In the same way the work per cubic foot of high-pressure gas hot is given by Eq. (84) and the work per cubic foot of air delivered and cooled to its original temperature is given by Eq. (87).

30. Two-stage Compressor, any Receiver Pressure, Exponential Compression. Capacity, Volumetric Efficiency, Work, Mean Effective Pressure and Horse-power, in Terms of Dimensions of Cylinders and Clearances. Referring to Fig. 27, let D_1 be the displacement of the first stage cylinder in cubic feet = $(V_b - V_a)$, D_2 the displacement of the cylinder of the second stage in cubic feet = $(V_d - V_c)$, c_1 the clearance of the first stage, stated as a fraction of the displacement of that cylinder, so that the clearance of the first stage cubic feet = $c_1 D_1$, and that of the second stage = $c_2 D_2$.

The low-pressure capacity of the first stage (L.P. Cap.) in cubic feet is $(V_b - V_a)$, and, as for the single-stage compressor, is expressed in terms of displacement, clearance, and ratio of compression of the first stage as follows see Eq. (58) and (59).

$$(\text{L.P. Cap.}_1) = D_1 \left(1 + c_1 - c_1 R_{p1}^{\frac{1}{s}} \right) = D_1 E_{v1}. \quad . . . \quad (92)$$

For the second stage, the low-pressure capacity (L.P. Cap.₂) is $(V_d - V_c)$,

$$\text{and is equal to } (\text{L.P. Cap.}_2) = D_2 \left(1 + c_2 - c_2 R_{p2}^{\frac{1}{s}} \right) = D_2 E_{v2}. \quad . . . \quad (93)$$

$$\text{Volumetric efficiency of the first stage is given by } E_{v1} = 1 + c_1 - c_1 R_{p1}^{\frac{1}{s}}, \quad . \quad (94)$$

$$\text{Volumetric efficiency for second stage } E_{v2} = 1 + c_2 - c_2 R_{p2}^{\frac{1}{s}}. \quad . . . \quad (95)$$

It may be required to find the receiver pressure (incidental to the finding of work or horse-power) for a compressor with given cylinder sizes and delivery pressure. The condition assumed of perfect intercooling stipulates that

$$(\text{L.P. Cap.}_1)(\text{sup.pr.}) = (\text{L.P. Cap.}_2)(\text{rec.pr.}),$$

$$\begin{aligned} \text{whence } (\text{rec.pr.}) &= (\text{sup.pr.}) \frac{(\text{L.P. Cap.}_1)}{(\text{L.P. Cap.}_2)} \\ &= (\text{sup.pr.}) \frac{D_1 E_{v1}}{D_2 E_{v2}}. \quad . . . \quad (96) \end{aligned}$$

If the volumetric efficiencies are known or can be sufficiently well approximated this can be solved directly. If, however, E_{v1} and E_{v2} are not known,

but the clearances are known, since these are both dependent upon the receiver, pressure sought, the substitution of the values of these two quantities will give

$$\text{(rec.pr.)} = \text{(sup.pr.)} \frac{D_1 \left[1 + c_1 - c_1 \left(\frac{\text{rec.pr.}}{\text{sup.pr.}} \right)^{\frac{1}{s}} \right]}{D_2 \left[1 + c_2 - c_2 \left(\frac{\text{del.pr.}}{\text{rec.pr.}} \right)^{\frac{1}{s}} \right]}, \dots \dots (97)$$

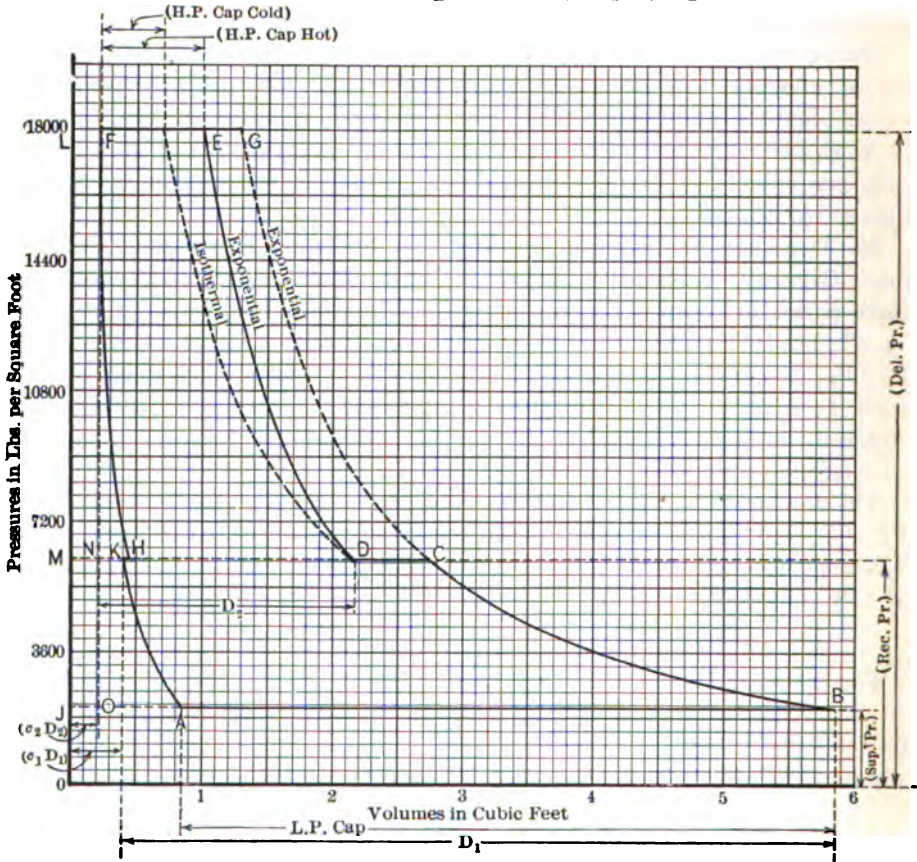


FIG. 29.—Two-stage Compressor, Cycle 6, Clearance, Exponential. Perfect Intercooling, Best-receiver Pressure. (H.P. Re-expansion Volume Greater than L.P. Clearance.)

an expression which contains the receiver pressure on both sides of the equation. This can be rearranged with respect to (rec.pr.), but results in a very complex expression which is difficult to solve and not of sufficient value ordinarily to warrant the expenditure of much labor in the solution. Therefore, the relations are left in the form (97). It may be solved by a series of approximations, the first of which is

$$\text{(rec.pr.)} = \text{(sup.pr.)} \frac{D_1}{D_2} \text{ approx.} \dots \dots (98)$$

With this value for the receiver pressure, substitution may be made in the second member of the Eq. (97), giving a result which will be very nearly correct. If desirable, a third approximation could be made.

To find the work of a two-stage exponential compressor in terms of displacement of cylinders, supply pressure, receiver pressure and delivery pressure, pounds per square inch, and volumetric efficiency of the first stage, E_{v1} , from (73) or (88)

$$W = 144 \frac{s}{s-1} (\text{sup.pr.}) D_1 E_{v1} \left[R_{p1}^{\frac{s-1}{s}} + R_{p2}^{\frac{s-1}{s}} - 2 \right], \quad . \quad . \quad . \quad (99)$$

in which $R_{p1} = \frac{(\text{rec.pr.})}{(\text{sup.pr.})}$ and $R_{p2} = \frac{(\text{del.pr.})}{(\text{rec.pr.})}$.

To solve this, the receiver pressure must be found as previously explained and the volumetric efficiency must be computed by Eq. (94) or otherwise be known.

It is impracticable to state work for this general case in terms of displacement and clearances directly, due to the difficulty of solving for (rec. pr.) in Eq. (97). It may, however, be stated purely in terms of supply and delivery pressures, in pounds per square inch, displacement, in cubic feet, and volumetric efficiencies as follows:

From Eq. (96), $R_{p1} = \frac{D_1 E_{v1}}{D_2 E_{v2}}$,

and $R_{p2} = \frac{\text{del.pr.}}{\text{sup.pr.}} \times \frac{D_2 E_{v2}}{D_1 E_{v1}} = R_p \frac{D_2 E_{v2}}{D_1 E_{v1}}$,

hence $W = 144 \frac{s}{s-1} (\text{sup.pr.}) D_1 E_{v1} \left[\left(\frac{D_1 E_{v1}}{D_2 E_{v2}} \right)^{\frac{s-1}{s}} + \left(R_p \frac{D_2 E_{v2}}{D_1 E_{v1}} \right)^{\frac{s-1}{s}} - 2 \right]. \quad . \quad (100)$

The mean effective pressure of the two-stage compressor referred to the low-pressure cylinder is found by dividing the work of the entire cycle Eq. (100), by the displacement of the first-stage cylinder, and by 144, to give pounds per square inch.

m.e.p. referred to first-stage cylinder, pounds per square inch is,

$$\frac{W}{144 D_1} = \frac{s}{s-1} (\text{sup.pr.}) E_{v1} \left[\left(\frac{D_1 E_{v1}}{D_2 E_{v2}} \right)^{\frac{s-1}{s}} + \left(R_p \frac{D_2 E_{v2}}{D_1 E_{v1}} \right)^{\frac{s-1}{s}} - 2 \right]. \quad . \quad (101)$$

It is well to note that this may also be found by multiplying (work done per cubic foot of gas supplied) by (volumetric efficiency of the first stage, E_{v1}), and dividing the product by 144.

In terms of the same quantities, an expression for indicated horse-power may be given as follows:

$$\text{I.H.P.} = \frac{s}{s-1} \frac{(\text{sup.pr.})}{229.2} n D_1 E_{v1} \left[\left(\frac{D_1 E_{v1}}{D_2 E_{v2}} \right)^{\frac{s-1}{s}} + \left(R_p \frac{D_2 E_{v2}}{D_1 E_{v1}} \right)^{\frac{s-1}{s}} - 2 \right], \quad (102)$$

where n is the number of cycles completed per minute by the compressor.

For n may be substituted the number of revolutions per minute, divided by the revolutions per cycle, $n = \frac{N}{z}$.

The horse-power per cubic foot of gas supplied per minute is

$$\frac{\text{I.H.P.}}{n(\text{L.P. Cap.})} = \frac{s}{s-1} \frac{(\text{sup.pr.})}{229.2} \left[\left(\frac{D_1 E_{s1}}{D_2 E_{s2}} \right)^{\frac{s-1}{s}} + \left(R_p \frac{D_2 E_{s2}}{D_1 E_{s1}} \right)^{\frac{s-1}{s}} - 2 \right] \quad (103)$$

Horse-power per cubic foot of gas delivered and cooled per minute.

$$\frac{\text{I.H.P.}}{n(\text{H.P. Cap. cold})} = \frac{s}{s-1} \frac{(\text{del.pr.})}{229.2} \left[\left(\frac{D_1 E_{s1}}{D_2 E_{s2}} \right)^{\frac{s-1}{s}} + \left(R_p \frac{D_2 E_{s2}}{D_1 E_{s1}} \right)^{\frac{s-1}{s}} - 2 \right] \quad (104)$$

Horse-power per cubic foot of hot gas delivered per minute

$$\frac{\text{I.H.P.}}{n(\text{H.P. Cap. hot})} = \frac{s}{s-1} \frac{\text{sup.pr.}}{229.2} \left(\frac{D_1 E_{s1}}{D_2 E_{s2}} \right)^{\frac{s-1}{s}} R_p^{\frac{1}{s}} \left[\left(\frac{D_1 E_{s1}}{D_2 E_{s2}} \right)^{\frac{s-1}{s}} + \left(R_p \frac{D_2 E_{s2}}{D_1 E_{s1}} \right)^{\frac{s-1}{s}} - 2 \right] \quad (105)$$

For the case where clearance is zero or negligible, these expressions may be simplified by putting E_{s2} and E_{s1} equal to unity.

$$\text{I.H.P.} = \frac{s}{s-1} \frac{(\text{sup.pr.})}{229.2} n D_1 \left[\left(\frac{D_1}{D_2} \right)^{\frac{s-1}{s}} + \left(R_p \frac{D_2}{D_1} \right)^{\frac{s-1}{s}} - 2 \right] \quad (106)$$

I.H.P. per cubic foot gas supplied per minute

$$\frac{\text{I.H.P.}}{n(\text{L.P. Cap.})} = \frac{s}{s-1} \frac{(\text{sup.pr.})}{229.2} \left[\left(\frac{D_1}{D_2} \right)^{\frac{s-1}{s}} + \left(R_p \frac{D_2}{D_1} \right)^{\frac{s-1}{s}} - 2 \right] \quad (107)$$

I.H.P. per cubic foot gas delivered and cooled per minute

$$\frac{\text{I.H.P.}}{n(\text{H.P. Cap. cold})} = \frac{s}{s-1} \frac{(\text{del.pr.})}{229.2} \left[\left(\frac{D_1}{D_2} \right)^{\frac{s-1}{s}} + \left(R_p \frac{D_2}{D_1} \right)^{\frac{s-1}{s}} - 2 \right] \quad (108)$$

I.H.P. per cubic foot hot gas delivered per minute

$$\frac{\text{I.H.P.}}{n(\text{H.P. Cap. hot})} = \frac{s}{s-1} \frac{\text{sup.pr.}}{229.2} \left(\frac{D_1}{D_2} \right)^{\frac{s-1}{s}} R_p^{\frac{1}{s}} \left[\left(\frac{D_1}{D_2} \right)^{\frac{s-1}{s}} + \left(R_p \frac{D_2}{D_1} \right)^{\frac{s-1}{s}} - 2 \right] \quad (109)$$

Example 1. Method of calculating diagram, Figs. 27, 28, 29. *Assumed data:*

$$P_a = P_b = 2116 \text{ lbs. per sq.ft.}; P_c = P_d = P_h = P_k = 6172 \text{ lbs. per sq.ft.};$$

$$P_g = P_e = P_f = 18,000 \text{ lbs. per sq.ft.}$$

$$Cl(\text{H.P.}) = 7.5 \text{ per cent}; Cl(\text{L.P.}) = 7.5 \text{ per cent}; s = 1.4; \text{L.P. Capacity} = 5 \text{ cu.ft.}$$

To obtain point K. From formula Eq. (58), $\text{L.P. Cap.} = D_1 (1 + c_1 - c_1 R_p^{\frac{1}{s}})$

$$5 = D_1 (1 + .075 - .075 \times 2.14), \text{ hence } D_1 = 5.45 \text{ cu.ft.}$$

$$Cl_1 = V_k = 5.45 \times .075 = .41 \text{ cu.ft.}$$

To obtain point A: $V_a = V_k \left(\frac{P_k}{P_a} \right)^{\frac{1}{1.4}} = .4 \times 2.14 = .856 \text{ cu.ft.}$

To obtain point B: $V_b = V_a + 5 = .85 + 5 = 5.85$ cu.ft.; $P_b = 2116$ lbs. sq.ft.

To obtain point C: $V_c = V_b + \left(\frac{P_c}{P_b}\right)^{\frac{1}{1.4}} = 5.85 + 2.14 = 2.73$.

To obtain point D:

Volume at D is the displacement plus clearance of H.P. cylinder. This cannot be found until the capacity is known. The capacity is the amount of gas which must be taken in each stroke and which is also the amount actually delivered by L.P. cylinder cooled to original temperature. The amount of cool gas taken in by the second cylinder

is $(\text{L.P. Cap.}_2) = \left(\frac{\text{sup.pr.}}{\text{rec.pr.}}\right) (\text{L.P. Cap.}_1) = \frac{2116}{6172} \times 5 = 1.7$ cu.ft.

But $D_2 = \frac{(\text{L.P. Cap.}_2)}{\left(1 + c_2 - c_2 R_{p_2}^{\frac{1}{s}}\right)} = \frac{1.7}{1 + .075 - .075 \times 2.14} = 1.88$ cu.ft.

$$Cl_2 = V_f = c_2 D_2 = .075 \times 1.88 = .14 \text{ cu.ft.}$$

$$V_d = Cl_2 + D_2 = 1.88 + .14 = 2.02 \text{ cu.ft.}$$

Other points are easily determined by obvious relations.

Example 2. What will be the capacity, volumetric efficiency and horse-power per 1000 cu.ft. of free air and per 1000 cu.ft. of hot compressed air per minute for the following compressor: Two-stage, double-acting cylinders, $22\frac{1}{2}$ and $34\frac{1}{2} \times 24$ in., running at 100 R.P.M. High-pressure clearance 6 per cent, low-pressure 4 per cent. Supply pressure 14 lbs. per square inch absolute. Delivery pressure 115 lbs. per square inch absolute.

The capacity will be the cylinder displacement times the volumetric efficiency.

D_1 = displacement of a $34\frac{1}{2} \times 24$ in. cylinder, or 12.8 cu.ft. and D_2 = displacement of a $22\frac{1}{2} \times 24$ in. cylinder or 5.4 cu.ft. To obtain the volumetric efficiency, make use of approximation of formula Eq. (98).

$$(\text{rec.pr.}) = (\text{sup.pr.}) \frac{D_1}{D_2} = 14 \times \frac{12.8}{5.4} = 33.2 \text{ lbs. sq.in.,}$$

and then by Eq. (97) checking,

$$(\text{rec.pr.}) = 14 \times \frac{12.8}{5.4} \left[\frac{1 + .04 - .04 \left(\frac{33.2}{14}\right)^{.714}}{1 + .06 - .06 \left(\frac{115}{33.2}\right)^{.714}} \right] = 35.1 \text{ lbs. sq. in.}$$

$$E_{v1} = 1 + c_1 - c_1 (R_{p1})^{\frac{1}{s}} \text{ from Eq. (94), } = 1 + .04 - .04 \times (2.5)^{\frac{1}{s}} = 96.8 \text{ per cent.}$$

Therefore the capacity will be, $200 \times 12.8 \times .968 = 2480$ cu.ft. per minute;

$$E_{v2} = 1 + c_2 - c_2 (R_{p2})^{\frac{1}{s}} \text{ from Eq. (95), } = 1 + .06 - .06 \times (3.28)^{.714} = 92 \text{ per cent.}$$

From Eq. (113), I.H.P. per cu.ft. (sup.pr.) air per minute is,

$$\begin{aligned} &= \frac{s}{s-1} \frac{\text{sup.pr.}}{229.2} \left[\left(\frac{D_1 E_{v1}}{D_2 E_{v2}}\right)^{\frac{s-1}{s}} + \left(R_p \frac{D_2 E_{v2}}{D_1 E_{v1}}\right)^{\frac{s-1}{s}} - 2 \right] \\ &= \frac{1.4}{.4} \times \frac{14}{229.2} \left[\left(\frac{12.8 \times .968}{5.4 \times .92}\right)^{.286} + \left(8.22 \frac{5.4 \times .92}{12.8 \times .968}\right)^{.286} - 2 \right] = .15. \end{aligned}$$

Whence horse-power per 1000 cu.ft. of free air per minute is 150.

From Eq. (105) horse-power per cubic foot (del.pr.) air, hot = that of (sup.pr.) air $\times R_p^{\frac{1}{s}} \left(\frac{D_1 E_1}{D_2 E_2} \right)^{\frac{s-1}{s}}$ or 5.85 times that of (sup.pr.) air.

\therefore Horse-power per 1000 cu.ft. of hot (del.pr. air) = $150 \times 5.85 = 877$.

Prob. 1. A two-stage double-acting compressor has volumetric efficiencies as shown by cards of 98 per cent and 90 per cent for the high- and low-pressure cylinders respectively. It compresses from atmosphere to 80 lbs. per square inch gage. If the cylinders are $15\frac{1}{4} \times 25\frac{1}{4} \times 18$ ins., and speed is 120 R.P.M., what horse-power is being used and how many cubic feet of free and compressed air (hot and cold) are being delivered per minute, when s equals 1.41?

Prob. 2. What horse-power will be needed to drive a two-stage compressor $10\frac{1}{4}$ ins. and $16\frac{1}{4} \times 12$ ins., double acting, with 5 per cent clearance in each cylinder at 160 R.P.M. when the supply pressure is atmosphere, delivery pressure 100 lbs. per square inch gage, when s equals 1.35?

Prob. 3. A thousand cubic feet of free air per minute must be compressed by a two-stage compressor to 80 lbs. per square inch gage from a supply pressure of 10 lbs. per square inch absolute. The volumetric efficiencies for the high- and low-pressure cylinders are 85 per cent and 95 per cent respectively, and the receiver pressure is 25 lbs. per square inch absolute. What will be the displacement of each cylinder and the horse-power per cubic foot of (sup.pr.) air?

Prob. 4. How many cubic feet of free air can be compressed in two-stage compressor $18\frac{1}{4} \times 30\frac{1}{4} \times 24$ ins. with 5 per cent clearance in high-pressure cylinder and 3 per cent in low if (sup.pr.) is atmosphere and (del.pr.) 80 lbs. per square inch gage? How would the answer be affected if clearance were taken as zero? Take $s = 1.41$.

Prob. 5. The volumetric efficiency of the low-pressure cylinder is known to be 95 per cent, and of the high-pressure cylinder 85 per cent. The cylinder sizes are $15\frac{1}{4} \times 25\frac{1}{4} \times 18$ ins. and speed is 120 R.P.M. What horse-power must be supplied to the machine if the mechanical efficiency is 80 per cent and the pressure ratio 10 with a (sup.pr.) of one atmosphere?

Prob. 6. A compressor runs at 120 R.P.M. and is double acting. It is compressing air from 14 lbs. per square inch absolute to pressures ranging from 70 lbs. per square inch gage to 100 lbs. per square inch gage. The cylinders are $20\frac{1}{4} \times 32\frac{1}{4} \times 24$ ins., and clearances 8 per cent and 4 per cent. Find the approximate receiver pressure, capacity and horse-power for the range of discharge pressure, for $s = 1.3$.

Prob. 7. The volumetric efficiency of the low-pressure cylinder of a two-stage compressor is known to be 95 per cent, the receiver pressure as shown by gage is 40 lbs., delivery pressure 100 lbs., and supply pressure one atmosphere. What will be the horse-power if the machine runs at 120 R.P.M. and the low-pressure cylinder is 18×12 in.?
 $s = 1.4$.

Prob. 8. An air compressor appears to require more power to run it than should be necessary. It is a double-acting $18 \times 30 \times 24$ in. machine running at 100 R.P.M. The volumetric efficiencies are 85 per cent and 90 per cent respectively and supply and delivery pressures 14 lbs. and 110 lbs. per square inch, both absolute. What should be the minimum work per cubic foot of (sup.pr.) air, per cubic foot of (del.pr.) air, hot and cold, for adiabatic compression?

Prob. 9. The efficiency of the driving gear on an electric-driven compressor is 75 per cent. Power is being supplied at the rate of 150 H.P. How much air should

be compressed per minute from 4 lbs. per square inch absolute to 100 lbs. per square inch gage, if the receiver pressure is 35 lbs. per square inch absolute and the low-pressure volumetric efficiency is 90 per cent, s being 1.4?

31. Two-stage Compressor with Best-receiver Pressure Exponential Compression. Capacity, Volumetric Efficiency, Work, Mean Effective Pressures and Horse-power in Terms of Dimensions of Cylinders and Clearances. For the two-stage exponential compressor with or without clearance, and perfect intercooling, the best-receiver pressure was found to be (Eq. 78),

$$(\text{best-rec.pr.}) = [(\text{sup.pr.})(\text{del.pr.})]^{\frac{1}{2}} \quad \dots \quad (110)$$

This expression Eq. (110) for best-receiver pressure makes it possible to evaluate R_{p1} and R_{p2} as follows:

$$R_{p1} \text{ for (best-rec.pr.)} = \frac{\text{best-rec.pr.}}{\text{sup.pr.}} = \frac{[(\text{sup.pr.})(\text{del.pr.})]^{\frac{1}{2}}}{\text{sup.pr.}} = \left[\frac{\text{del.pr.}}{\text{sup.pr.}} \right]^{\frac{1}{2}} = R_p^{\frac{1}{2}}, \quad (111)$$

$$\begin{aligned} \text{and } R_{p2} \text{ for (best-rec.pr.)} &= \frac{(\text{del.pr.})}{(\text{best-rec.pr.})} = \frac{(\text{del.pr.})}{[(\text{sup.pr.})(\text{del.pr.})]^{\frac{1}{2}}} \\ &= \left[\left(\frac{\text{del.pr.}}{\text{sup.pr.}} \right) \right]^{\frac{1}{2}} = R_p^{\frac{1}{2}} \quad \dots \quad (112) \end{aligned}$$

The use of these values for R_{p1} and R_{p2} in the expressions previously given for volumetric efficiency for the general case, Eqs. (94) and (95) results in

$$\text{Volumetric efficiency, first stage } E_{v1} = (1 + c_1 - c_1 R_p^{\frac{1}{2}}), \quad \dots \quad (113)$$

$$\text{and volumetric efficiency, second stage } E_{v2} = (1 + c_2 - c_2 R_p^{\frac{1}{2}}) \quad \dots \quad (114)$$

The work was found to be represented by Eq. (91), which may be stated in terms of displacement and volumetric efficiency of the first stage, as follows:

$$W = 288 \frac{s}{s-1} (\text{sup.pr.}) D_1 E_{v1} \left[R_p^{\frac{s-1}{2s}} - 1 \right], \quad \dots \quad (115)$$

where $R_p = \frac{(\text{del.pr.})}{(\text{sup.pr.})}$ and where (sup.pr.) is in pounds per square inch.

If the clearance is known for the first stage this becomes by the use of Eq. (94),

$$W = 288 \frac{s}{s-1} (\text{sup.pr.}) D_1 \left(1 + c_1 - c_1 R_p^{\frac{1}{2}} \right) \left[R_p^{\frac{s-1}{2s}} - 1 \right], \quad \dots \quad (116)$$

which is a direct statement of the work of a two-stage adiabatic compressor with perfect intercooling in terms of supply pressure and delivery pressure, pounds per square inch, displacement, cubic feet and clearance as a fraction of displacement, *provided the cylinder sizes and clearances are known to be such as to give best-receiver pressures.*

The mean effective pressure reduced to first-stage displacement, in pounds per square inch, may be derived from either Eq. (115) or (116) by dividing the

work by the displacement of the first-stage cylinder, and again dividing by 144.

$$\left. \begin{aligned} \text{m.e.p.} &= \frac{W}{144D_1} = \frac{2s}{s-1} (\text{sup.pr.}) E_{s1} \left[R_p^{\frac{s-1}{2s}} - 1 \right] \\ &= \frac{2s}{s-1} (\text{sup.pr.}) \left(1 + c_1 - c_1 R_p^{\frac{1}{2s}} \right) \left[R_p^{\frac{s-1}{2s}} - 1 \right] \end{aligned} \right\} \dots \dots (117)$$

Since the work done is equally divided between the two cylinders when best-receiver pressure is maintained, the mean effective pressure, in pounds per square foot, for each cylinder will be one-half the total work divided by the displacement of the cylinder in question,

$$\text{m.e.p., first stage} = \frac{W}{288D_1} = \frac{s}{s-1} (\text{sup.pr.}) E_{s1} \left[R_p^{\frac{s-1}{2s}} - 1 \right] \dots (118)$$

Note that this is one-half as great as the m.e.p. of the compressor reduced to first stage (117),

$$\text{m.e.p., second stage} = \frac{W}{288D_2} = \frac{s}{s-1} (\text{sup.pr.}) \frac{D_1}{D_2} E_{s1} \left[R_p^{\frac{s-1}{2s}} - 1 \right] \dots (119)$$

But $(\text{sup.pr.}) \frac{D_1 E_{s1}}{D_2 E_{s2}} = (\text{rec.pr.}) = [(\text{sup.pr.})(\text{del.pr.})]^{\frac{1}{s}}$, whence,

$$\text{m.e.p., second stage} = \frac{s}{s-1} [(\text{sup.pr.})(\text{del.pr.})]^{\frac{1}{s}} E_{s2} \left[R_p^{\frac{s-1}{2s}} - 1 \right] \dots (120)$$

It is next necessary to investigate what conditions must be fulfilled to obtain the best-receiver pressure, the value of which is stated, Eq. (110). The condition of perfect intercooling provides that the temperature of the gas entering the second stage is the same as that entering the first stage, and hence that the product (volume entering second stage) \times (pressure when entering second stage) must be equal to the product (volume entering first stage) \times (pressure of supply to first stage), or

$$(\text{L.P. Cap.}_2)(\text{rec. pr.}) = (\text{L.P. Cap.}_1)(\text{sup. pr.}) \dots (121)$$

Combining with Eq. (110)

$$\begin{aligned} \frac{(\text{L.P. Cap.}_1)}{(\text{L.P. Cap.}_2)} &= \frac{[(\text{sup.pr.})(\text{del.pr.})]^{\frac{1}{s}}}{(\text{sup.pr.})} = \left[\frac{(\text{del.pr.})}{(\text{sup.pr.})} \right]^{\frac{1}{s}} = R_p^{\frac{1}{s}}, \\ &\quad (1) \qquad (2) \qquad (3) \qquad 1 \\ \text{or } R_p^{\frac{1}{s}} &= \frac{(\text{L.P. Cap.}_1)}{(\text{L.P. Cap.}_2)} = \frac{D_1 E_{s1}}{D_2 E_{s2}} = \frac{D_1 \left[1 + c_1 - c_1 R_p^{\frac{1}{2s}} \right]}{D_2 \left[1 + c_2 - c_2 R_p^{\frac{1}{2s}} \right]} \dots (122) \end{aligned}$$

From this three-part equation proper values may be found to fulfill requirements of best-receiver pressure for:

1. The ratio of capacities for a given ratio of pressures, or conversely, the ratio of pressures when capacities are known.
2. The ratio of cylinder displacements for known volumetric efficiencies
3. The ratio of cylinder displacements when the clearances and ratio of compression are known, or conversely, with known displacements and clearances

the ratio of pressures which will cause best-receiver pressure to exist. This last case in general is subject to solution most easily by a series of approximations.

There is, however, a special case which is more or less likely to occur in practice, and which lends itself to solution—that of *equal clearance percentages*. If $c_1 = c_2$ the parenthesis in the numerator of part (3) of Eq. (122) becomes equal to the parenthesis in the denominator, and evidently the volumetric efficiency of the two cylinders is equal, hence for equal clearance percentages in the two stages,

$$\frac{D_1}{D_2} = R_p^{\frac{1}{2s}}. \quad (123)$$

A case which leads to the same expression, Eq. (123), is that of zero clearance, a condition that is often assumed in machines where the clearance is quite small.

The work per cycle, Eq. (116), when multiplied by the number of cycles performed per minute, n , and divided by 33,000, gives

$$\text{I.H.P.} = \frac{s}{s-1} \frac{(\text{sup.pr.})}{114.6} n D_1 \left(1 + c_1 - c_1 R_p^{\frac{1}{2s}} \right) \left(R_p^{\frac{s-1}{2s}} - 1 \right), \quad . . . (124)$$

from which are obtained the following:

I.H.P. per cubic foot supplied per minute

$$\frac{\text{I.H.P.}}{n(\text{L.P. Cap.})} = \frac{s}{s-1} \frac{(\text{sup.pr.})}{114.6} \left(R_p^{\frac{s-1}{2s}} - 1 \right), \quad . . . (125)$$

I.H.P. per cubic foot delivered and cooled per minute

$$\frac{\text{I.H.P.}}{n(\text{H.P. Cap. cold})} = \frac{s}{s-1} \frac{(\text{del.pr.})}{114.6} \left(R_p^{\frac{s-1}{2s}} - 1 \right), \quad . . . (126)$$

and I.H.P. per cubic foot delivered hot per minute

$$\frac{\text{I.H.P.}}{n(\text{H.P. Cap. hot})} = \frac{s}{s-1} \frac{(\text{sup.pr.})}{114.6} R_p^{\frac{s+1}{2s}} \left(R_p^{\frac{s-1}{2s}} - 1 \right). \quad (127)$$

These expressions, Eqs. (125), (126) and (127) are all independent of clearance.

Example. What will be the capacity, volumetric efficiency and horse-power per 1000 cu.ft. of free air and per 1000 cu.ft. of hot compressed air per minute for the following compressor for $s = 1.4$? Two-stage, double-acting, cylinders $22\frac{1}{4} \times 34\frac{1}{4} \times 24$ ins., running at 100 R.P.M. Low-pressure clearance 5 per cent, high-pressure clearance such as to give best-receiver pressure. Supply pressure 15 lbs. per square inch absolute, delivery pressure 105 lbs. per square inch absolute.

Capacity will be cylinder displacement times low pressure volumetric efficiency, or, $200 D_1 \times E_{v1}$.

$D_1 = 12.8$ cu.ft. E_{v1} from Eq. (113) $= \left(1 + c_1 - c_1 R_p^{\frac{1}{2s}} \right) = 1 + .05 - .05 \times 7^{.357} = 95$ per cent.

Therefore low pressure capacity $= 200 \times 12.8 \times .95 = 2430$ cu.ft. per minute.

Horse-power per cubic foot of (sup.pr.) air per minute is from Eq. (125)

$$\frac{s}{s-1} \frac{\text{sup.pr.}}{114.6} \left(R_p^{\frac{s-1}{2s}} - 1 \right), \quad = \frac{1.4}{.4} \times \frac{15}{114.6} \left(72.8 - 1 \right) = 0.147$$

Therefore, horse-power per 1000 cu.ft. of sup.pr. air = 147.

Horse-power per cubic foot of (del.pr.) air, hot, is from Eq. (127)

$$R_p^{\frac{1+s}{2s}} \text{ times power per cu.ft. of (sup.pr.) air, hence,}$$

$$147 \times 5.3 = 780 = \text{horse-power per 1000 cu.ft. of (del.pr.) air, hot, per minute.}$$

Problem Note. In the following problems, cylinders are assumed to be proportioned with reference to pressures so as to give best-receiver pressure. Where data conflict, the conflict must be found and eliminated.

Prob. 1. Air is compressed adiabatically from 14 lbs. per square inch absolute to 80 lbs. per square inch gage, in a $20\frac{1}{2} \times 32\frac{1}{2} \times 24$ in. compressor, running at 100 R.P.M., the low-pressure cylinder has 3 per cent clearance. What will be horse-power required to run compressor and what will be the capacity in cubic feet of low pressure and in cubic feet of (del.pr.) air?

Prob. 2. What must be the cylinder displacement of a two-stage compressor with 5 per cent clearance in each cylinder to compress 500 cu.ft. of free air per minute from 14 lbs. per square inch absolute to 85 lbs. per square inch gage, so that s equals 1.4? What will be the horse-power per cubic foot of (del.pr.) air hot and cold?

Prob. 3. A two-stage compressor is compressing gas with a value of $s = 1.25$ from 10 lbs. per square inch gage to 100 lbs. per square inch gage. The cylinders are $18\frac{1}{2} \times 30\frac{1}{2} \times 24$ ins., and speed is 100 R.P.M. If the clearance is 5 per cent in the low-pressure cylinder and 10 per cent in the high, what will be the cubic feet of (sup.pr.) gas handled per minute and what will be the horse-power at best-receiver pressure?

Prob. 4. A manufacturer states that his $20\frac{1}{2} \times 32\frac{1}{2} \times 24$ in. double-acting compressor when running at 100 R.P.M. at sea level will have a capacity of 2390 cu.ft. of free air per minute, pressure range being from atmosphere to 80 lbs. per square inch gage. At best-receiver pressure what clearance must the compressor have, compression being adiabatic?

Prob. 5. The cylinder sizes of a two-stage compressor are given as $10\frac{1}{2} \times 16\frac{1}{2} \times 12$ ins., and clearance in each is 5 per cent. What will be the best-receiver pressures when operating between atmosphere and following discharge pressures, 60, 70, 80, 90, 100 and 110 lbs. per square inch gage, for s equal 1.4?

Prob. 6. 1500 cu.ft. of air at 150 lbs. per square inch gage pressure are needed per minute for drills, hoists, etc. The air is supplied from 3 compressors of the same size and speed, 120 R.P.M. Each has 4 per cent clearance in each cylinder. What will be sizes of cylinders and the horse-power of the plant for best-receiver pressure, when $s = 1.41$?

Prob. 7. The cards taken from a compressor show volumetric efficiencies of 95 per cent and 80 per cent in low- and high-pressure cylinder respectively. What will be (del.pr.) for best-receiver pressure if compressor is $15\frac{1}{2} \times 25\frac{1}{2} \times 18$ ins., and (sup.pr.) 15 lbs. per square inch absolute to 10 lbs. absolute, and what will be the work in each case, s being 1.35?

Prob. 8. A manufacturer gives a range of working pressure of his $10\frac{1}{2} \times 16\frac{1}{2} \times 12$ in. compressor from 80–100 lbs. per square inch gage. If clearances are, low 4 per cent, high 8 per cent, and (sup.pr.) is atmosphere, find by trial which end of the range comes

nearer to giving best-receiver pressure? If clearances were equal which would give best-receiver pressure?

Prob. 9. A $16\frac{1}{4} \times 25\frac{1}{4} \times 16$ in. compressor is rated at 1205 cu.ft. free air per minute at 135 R.P.M. at sea level. What would be the clearance if compressor were compressing air from atmosphere to 100 lbs. gage at sea level? With same clearance what would be the size of a low-pressure cylinder to give the same capacity at altitude of 10,000 ft. with the same clearance and the same (del.pr.), best-receiver pressure always being maintained?

32. Three-Stage Compressors, Cycles 7 and 8. Formulas for Work and Horse-power. Best Two Receiver Pressures. The work and capacity of three-stage compressors may be determined in exactly the same manner as indicated for two-stage compressors, provided the pressures and volumes in each be known. The three-stage compressor may have one of two reference cycles, namely, that obtained with no clearance (cycle 7); or, that obtained with clearance (cycle 8). See page 54, also Fig. 21.

Cycle 8 may be sub-divided into any number of cases depending upon the relation of clearance in the several cylinders. The discussion of these cycles offers an interesting phase of the subject of multi-stage compression, but such a discussion is outside the scope of the present text-book. For a complete presentation of the three-stage compressor phases, see Dr. Lucke's larger work on Engineering Thermodynamics.

It is sufficient to state here that the three-stage exponential compression cycle with no clearance, perfect intercooling (cycle 7) is evaluated from a net work-area made up of three areas computed individually by the formula for single-stage Eq. (46), provided the requisite pressures and volumes are known. Under these conditions the work done may be obtained from

$$W = 144 \frac{s}{s-1} (\text{sup.pr.}) (\text{L.P. Cap.}) \left[(R_{p1})^{\frac{s-1}{s}} + (R_{p2})^{\frac{s-1}{s}} + (R_{p3})^{\frac{s-1}{s}} - 3 \right], \quad (128)$$

in which the supply pressure is given in pounds per square inch, low pressure capacity in cubic feet, and ratios of pressures—first stage R_{p1} , second stage R_{p2} and third stage R_{p3} . It is evident that if either receiver pressure be fixed and the other varied, the work necessary to compress a given initial volume of gas will be varied, and will have a minimum value for some particular value of the varying receiver pressure. By a variation of both receiver pressures a minimum may be found for the work when both receiver pressures have some specific relation to supply and delivery pressures. The work of the compressor with best-receiver pressure, no clearance, is equally divided between the three stages and is equal to

$$W = 432 \frac{s}{s-1} (\text{sup.pr.}) (\text{L.P. Cap.}) \left(R_p^{\frac{s-1}{3s}} - 1 \right) \quad (129)$$

In this equation R_p is the ratio of final to initial pressure (absolute).

As previously shown in connection with two-stage compression, clearance has no effect upon the work for a given capacity, therefore, Eqs. (128) and

(129) will give the work done in three-stage compression *with clearance* (cycle 8) under the conditions of perfect intercooling, exponential compression, best-receiver pressure and equality of stages.

The expression for work in three-stage compressors with *any receiver pressure*, exponential compression, with clearance and perfect intercooling is somewhat more involved inasmuch as the displacement and volumetric efficiencies must be considered. The resulting equation will be identical in form with Eq. (99) for two-stage compressors working under similar conditions. That is, the work of the three-stage compressor stated in terms of supply pressure, pounds per square inch, displacement in cubic feet, volumetric efficiencies, and overall ratios of compression R_p , is:

$$W = 144 \frac{s}{s-1} (\text{sup.pr.}) D_1 E_{s1} \left[\left(\frac{D_1 E_{s1}}{D_2 E_{s2}} \right)^{\frac{s-1}{s}} + \left(\frac{D_2 E_{s2}}{D_3 E_{s3}} \right)^{\frac{s-1}{s}} + \left(R_p \frac{D_3 E_{s3}}{D_1 E_{s1}} \right)^{\frac{s-1}{s}} - 3 \right] \quad (130)$$

The value of the volumetric efficiency, E_s , in each stage, may readily be obtained by substitution in Eq. (59). The method of obtaining the mean-effective pressure will be obvious from a consideration of the discussion of two-stage compression, see Eq. (101), from which the m.e.p. is obtained by dividing the work of the entire cycle, Eq. (130) by the displacement, D_1 , of the first stage and by 144 to reduce to pounds per square inch.

In the same way the expression for the indicated horse-power of a three-stage compressor may be obtained by analogy from that for two-stage compression under similar conditions. See Eq. (102). The I.H.P., then, of a three-stage compressor working under the stated conditions will be equal to the work of the entire cycle multiplied by n and divided by 33,000; in this case n is the number of cycles per minute. The foregoing formulas Eq. (130) and derivatives apply to compressors with *any receiver pressure*; if it be desired to obtain the work of compression in a three-stage compressor with best-receiver pressure the Eq. (129), when expressed in terms of displacement and volumetric efficiency, becomes

$$W = 432 \frac{s}{s-1} (\text{sup.pr.}) D_1 E_{s1} \left(R_p^{\frac{s-1}{3s}} - 1 \right) \quad (131)$$

in which $R_p = \frac{(\text{del.pr.})}{(\text{sup.pr.})}$. If the clearance is known E_{s1} may readily be determined from Eq. (59). The mean effective pressure, and indicator horse-power formulas may be determined from Eq. (131) by obvious methods.

It is evident that the use of formulas involving best-receiver pressures apply only when all the conditions necessary to the existence of the best-receiver pressures are fulfilled.

By referring to Eq. (78) it will be seen that the best-receiver pressure for two-stage compressors equals $[(\text{sup.pr.})(\text{del.pr.})]^{\frac{1}{2}}$; in the same way it

can be shown that the best-receiver pressure for three-stage adiabatic compression with perfect intercooling may be found from the following:

$$(\text{best 1 rec.pr.})=[(\text{sup.pr.})^2(\text{del.pr.})]^{1/4} \dots \dots (132)$$

$$(\text{best 2 rec.pr.})=[(\text{sup.pr.})(\text{del.pr.})^2]^{1/4} \dots \dots (133)$$

In order, then, that best-receiver pressure may be obtained, there must be a certain relation between the given ratio of compression and dimensions of cylinders and clearances. Since these dimensions are fixed in an actual compressor, a given multi-stage compressor can be made to give best-receiver pressures only when compressing through a given range, i.e., when R_p in each cylinder has the same value.

When clearance percentages are equal in all three cylinders, the volumetric efficiencies are all equal, then, when best-receiver pressures exist,

$$R_p = \frac{D_1}{D_2} = \frac{D_2}{D_3}.$$

Evidently this same expression holds if clearances are all zero or negligible. What constitutes negligible clearance is somewhat indeterminate since it is dependent upon the ratio of compression and the percentage of error allowable.

It is useful to note that the expressions for horse-power which may be derived from Eq. (131) are all independent of clearance; this is to be expected, since the multi-stage compressor may be regarded as a series of single-stage compressors, and in single-stage such an independence was found for work and horse-power per unit of capacity.

EXAMPLES AND PROBLEMS ON THREE-STAGE COMPRESSION

Example 1. What will be the horse-power required to compress 100 cu.ft. of free air per minute from 15 lbs. per square inch absolute to 90 lbs. per square inch gage in a no-clearance, three-stage compressor if compression be adiabatic?

$$\text{From Eq. (129)} \quad W = 432 \frac{s}{s-1} (\text{sup.pr.})(\text{L.P. Cap.}) \left(R_p^{\frac{s-1}{3s}} - 1 \right),$$

therefore the work per cubic foot of (sup.pr.) air is, $432 \frac{s}{s-1} (\text{sup.pr.}) \left(R_p^{\frac{s-1}{3s}} - 1 \right),$

$$= 432 \times \frac{1.4}{.4} \times 15 \times (7^{.0952} - 1) = 4500 \text{ ft.-lbs., hence,}$$

$$\text{H.P. for 100 cu.ft. per minute} = \frac{4500 \times 100}{33,000} = 13.6.$$

Prob. 1. What work will be required to supply 2000 cu.ft. of air at 200 lbs. per inch gage pressure if compressing is done adiabatically by three-stage compressors, taking air at atmosphere, neglecting the clearances?

Prob. 2. A table in "Power" gives the steam used per hour in compressing air to various pressures single stage. A value for air compressed to 100 lbs. is 9.9 lbs. steam per hour per 100 cu.ft. of free air. Using the same ratio of work to steam, find the value for the steam if compression had been three-stage, zero clearances to be assumed.

Prob. 3. A 5 in. drill requires 200 cu.ft. of free air per minute at 100 lbs. per square

inch gage pressure. What work will be required to compress air for 20 such drills if three-stage compressors are used, compared to single-stage for no clearance?

Prob. 4. What would be the steam horse-power of a compressor delivering 150 cu.ft. of air per minute at 500 lbs. per square inch pressure if compression is three-stage, adiabatic, clearance zero, and mechanical efficiency of compressor 80 per cent?

Example 2. A three-stage compressor is compressing air from atmosphere to 140 lbs. per square inch absolute. The low-pressure cylinder is 32×24 ins. and is known to have a clearance of 5 per cent. From gages on the machine it is noted that the first-receiver pressure is 15 lbs. per square inch gage and the second-receiver pressure is 55 lbs. per square inch gage. What horse-power is being developed if the speed is 100 R.P.M. and $s=1.4$? From the formula Eq. (130),

$$W = 144 \frac{s}{s-1} (\text{sup.pr.}) D_1 E_{v1} \left[\left(\frac{D_1 E_{v1}}{D_2 E_{v2}} \right)^{\frac{s-1}{s}} + \left(\frac{D_2 E_{v2}}{D_3 E_{v3}} \right)^{\frac{s-1}{s}} + \left(R_p \frac{D_3 E_{v3}}{D_1 E_{v1}} \right)^{\frac{s-1}{s}} - 3 \right].$$

From Eq. (96), $\frac{D_1 E_{v1}}{D_2 E_{v2}} = R_{p1}$; in the same way $\frac{D_2 E_{v2}}{D_3 E_{v3}} = R_{p2}$.

Since by definition, $(\text{del.pr.}) = R_p (\text{sup.pr.})$, $\therefore R_{p2} = R_p \frac{D_3 E_{v3}}{D_1 E_{v1}}$.

Hence, $W = 144 \frac{s}{s-1} (\text{sup.pr.}) D_1 E_{v1} \left[R_{p1}^{\frac{s-1}{s}} + R_{p2}^{\frac{s-1}{s}} + R_{p2}^{\frac{s-1}{s}} - 3 \right]$.

From gage readings $R_{p1} = \frac{30}{15} = 2$. $R_{p2} = \frac{70}{30} = 2.33$; $R_{p2} = \frac{140}{70} = 2$.

$$E_{v1} = (1 + c_1 - c_1 R_{p1}^{\frac{1}{s}}) \text{ from Eq. (59),}$$

or,

$$E_{v1} = (1 + .05 - .05 \times 1.65) = 67.5 \text{ per cent.}$$

Hence, $W = 144 \times \frac{1.4}{.4} \times 15 \times 11.2 \times .675 (1.22 + 1.28 + 1.22 - 3);$

$$= 59,200 \text{ ft.-lbs. per stroke or } 200 \times 59,200 \text{ ft.-lbs. per minute; } = 358 \text{ I.H.P.}$$

Example 3. Another compressor has cylinders 12×20×32×24 in. and it is known that the volumetric efficiencies of the high, intermediate and low-pressure cylinders are respectively 70 per cent, 85 per cent and 98 per cent. The (del.pr.) is 150 lbs. per square inch absolute. What is the horse-power in this case if the speed is 100 R.P.M.?

From the formula Eq. (130).

$$W = 144 \times \frac{14}{4} \times 15 \times 11.2 \times .98 \left[\left(\frac{11.2 \times .98}{4.36 \times .85} \right)^{.286} + \left(\frac{4.36 \times .85}{1.59 \times .70} \right)^{.286} + \left(10 \frac{1.59 \times .7}{11.2 \times .98} \right)^{.286} - 3 \right]$$

$$= (1.309 + 1.495 + 1 - 3) = 66,400 \text{ ft.-lbs. per stroke,}$$

whence $\text{I.H.P.} = \frac{200 \times 66,400}{33,000} = 402.$

Prob. 5. What will be the horse-power required to drive a 12×22×34×30 in. three-stage compressor with volumetric efficiencies of 75, 85, and 95 per cent in the high, intermediate and low-pressure cylinders, at 100 R.P.M. when compressing natural gas from 25 lbs. per square inch gage to 300 lbs. per square inch gage, adiabatically?

Prob. 6. Find the work done on a gas, the value for s of which is 1.3, in compressing it from atmosphere to 7 atmospheres absolute, adiabatically in three stages, the compressor having a low-pressure cylinder displacement of 60 cu.ft. per minute and a volumetric efficiency of 95 per cent, first receiver pressure being 2 atmospheres absolute,

and second-receiver pressure 4 atmospheres absolute. If air were being compressed instead of the above gas, how would the work vary?

Prob. 7. For special reasons it is planned to keep the first-receiver pressure of a three-stage compressor at 30 lbs. per square inch absolute, the second-receiver pressure at 60 lbs. per square inch absolute, and the line pressure at 120 lbs. per square inch absolute. The (sup.pr.) is 14 lbs. per square inch absolute. If the clearances are 4 per cent in the low and 8 per cent in the intermediate and high-pressure cylinders, what must be the cylinder sizes to handle 500 cu.ft. of free air per minute at 120 R.P.M. and what power must be supplied to the compressor on a basis of 80 per cent mechanical efficiency, for a value of s equal to 1.39?

Prob. 8. Should the above pressures (Prob. 6) be gage pressures instead of absolute, how would the quantities to be found be affected?

Prob. 9. The receiver pressures on a CO_2 gas compressor are 50 lbs. per square inch absolute, and 200 lbs. per square inch absolute, the (del.pr.) being 1000 lbs. per square inch absolute. The machine has a low-pressure cylinder 8×10 ins. with 3 per cent clearance. What horse-power will be required to run it at 100 R.P.M. and what would be the resultant horse-power and capacity if each pressure were halved? (Sup.pr.) = 14.7 lbs. per square inch.

Example 4. If the following three-stage compressor be run at best-receiver pressures what will be the horse-power and the best-receiver pressures? Compressor has low-pressure cylinder 32×24 ins. with 5 per cent clearance, is compressing air from atmosphere to 140 lbs. per square inch absolute, so that s equals 1.4 and it runs at 100 R.P.M.

$$\text{From the formula Eq. (131)} \quad W = \frac{432s}{s-1} (\text{sup.pr.}) D_1 E_n \left(R_p^{\frac{s-1}{s}} - 1 \right).$$

$$\text{From the formula Eq. (132) (best 1 rec.pr.)} = [(\text{sup.pr.})^s (\text{del.pr.})]^{1/s} = [(15)^s \times 140]^{1/s} = 31.6,$$

$$\text{From Eq. (133) (best 2 rec.pr.)} = [(\text{sup.pr.}) (\text{del.pr.})^{s-1}]^{1/s} = [15 \times (140)^{s-1}]^{1/s} = 66.5.$$

The volumetric efficiency, E_n , is obtained from a modification of Eqs. (59) and (113), from which

$$E_n = \left(1 + c_1 - c_1 R_p^{\frac{1}{s}} \right) = 1 + .05 - .05 \times \left(\frac{140}{15} \right)^{1/s} = 96.5;$$

$$\text{hence,} \quad W = 432 \times \frac{1.4}{.4} \times 15 \times 11.2 \times 96.5 \times (9.35^{.95} - 1) = 59,000 \text{ ft.-lbs.,}$$

$$\text{or,} \quad \text{I.H.P.} = \frac{59,000 \times 200}{33,000} = 358.$$

Prob. 10. There is available for running a compressor 175 H.P. How many cu.ft. of free air per minute can be compressed from atmosphere to 150 lbs. per square inch gage by a three-stage adiabatic compressor with best-receiver pressures?

Prob. 11. The low-pressure cylinder of a three-stage compressor has a capacity of $4\frac{1}{2}$ cu.ft. per stroke. If the stroke of all three cylinders is 18 ins., what must be the diameters of the intermediate and high to insure best-receiver pressures, if clearance be neglected, and (sup.pr.) be 1 lb. per square inch absolute and (del.pr.) 15 lbs. per square inch absolute, s being 1.4.

Prob. 12. Will a $15 \times 22 \times 34 \times 24$ in. compressor with clearances of 3, 5 and 8 per cent in low, intermediate and high-pressure cylinders respectively be working at best-receiver pressures when (sup.pr.) is 15 lbs. per square inch absolute and (del.pr.) 150 lbs. per square inch absolute? If not, find by trial, the approximate (del.pr.), for which this machine is best, with s equal to 1.4?

CHAPTER III

COMPRESSOR CHARACTERISTICS. ECONOMY. MAXIMUM WORK. WORK AT FRACTIONAL LOADS.

33. Comparative Economy or Efficiency of Compressors. As the prime duty of compressors of all sorts is to move gas or vapor from a region of low to a region of high pressure, and as this process always requires the expenditure of work, the compressor process which is most economical is the one that accomplishes the desired transference with the least work. In this sense, then, economy of compression means something different than efficiency, as ordinarily considered. Ordinarily, efficiency is the ratio of the energy at one point in a train of transmission or transformation, to the energy at another point, whereas with compressors, economy of compression is understood to mean the ratio of the work required to compress and deliver a unit of gas, moving it from a low- to a high-pressure place, to the work that would have been required by some other process or hypothesis, referred to as a standard. This economy of compression must not be confused with efficiency of compressors as machines, —it is merely a comparison of the work in the compressor cylinder for an actual case or hypothesis to that for some other hypothesis taken as a standard. The standard of comparison may be any one of several possible, and unfortunately there is no accepted practice with regard to this standard. It will, therefore, be necessary to specify the standard of reference whenever economy of compression is under consideration. The following standards have been used with some propriety and each is useful, since it supplies the sort of information really desired.

First Standard. The work per cubic foot of supply gas necessary to compress *isothermally* (Cycle 1), from the supply pressure to the delivery pressure of the existing compressor, and to deliver at the high pressure, is less than that of any commercial process of compression, and may be taken as a standard for comparison. Since, however, actual compressors never depart greatly from the adiabatic law, their economy compared with the isothermal standard will always be low, making their performance seem poor, whereas they may be as nearly perfect as is possible, so that some other standard would be a better indication of their excellence.

Second Standard. The work per cubic foot of gas supplied when compressed *adiabatically in a single stage* (Cycle 3), if taken as a standard, will indicate a high economy, nearly unity for single-stage compressors, and an economy above unity for most multi-stage compressors. For the purpose of comparison it will be equally as good as the first standard, and the excess of the economy over unity will be a measure of the saving over single-stage adiabatic compression.

Since, however, single-stage adiabatic compression is not the most economical obtainable in practice for many cases, this standard may give an incorrect idea of the perfection of the compressor.

Third Standard. Owing to the facts noted above, it may be a better indication of the degree of perfection of the compressor to compare the work per cubic foot of gas supplied with that computed for the standard *adiabatic cycle most nearly approaching that* of the compressor. This standard is, however, open to the objection that a multi-stage compressor is not referred to the same cycle as a single-stage compressor, and a multi-stage compressor with other than best-receiver pressure is not referred to the same cycle as another operating with best-receiver pressure. This is, therefore, not a desirable standard for comparing compressors of different types with one another, although it ~~does~~ show to what extent the compressor approaches the hypothetical best condition for its own type and size.

Other standards might be chosen for special reasons, each having a value in proportion as it supplies the information that is sought.

It is seen from the discussion of the second standard that its only advantage over the first is in that it affords a measure of the saving or loss as compared with the single-stage adiabatic compressor cycle.

If the first standard, that of the isothermal compressor cycle, be adopted for the purpose of comparison, it at once gives a measure of comparison with the isothermal, which is more nearly approached though never quite reached, as the number of stages is increased, or as the gas is more effectively cooled during compression. It may be regarded as the limiting case of multi-stage compression with perfect intercooling, or the limiting case of continuous cooling.

In order to ascertain how nearly the actual compressor approaches the adiabatic cycle most nearly representing its working conditions, the economy of the various reference cycles heretofore discussed may be tabulated or charted, and the economy of the cycle as compared with that of the actual performance of the compressor will give the required information. The process of computation by which this information is obtained will depend upon the nature of information sought. The economy of actual compressor compared with the isothermal may be stated in any of the following ways:

$$\left. \begin{array}{l}
 \frac{\text{Computed work per cubic foot supplied, isothermal}}{\text{Indicated work per cu.ft. actual gas supplied to compressor}} \quad (a) \\
 \frac{\text{I.H.P. per cubic foot per minute supplied, isothermal}}{\text{I.H.P. per cubic foot per minute actual supplied}} \quad (b) \\
 \text{Single stage} \\
 \frac{(\text{m.e.p.}) \text{ isothermal, pounds per square inch, no clearance}}{(\text{m.e.p.}) \text{ actual} \div \text{true volumetric efficiency}} \quad (c) \\
 \text{Multi-stage} \\
 \frac{(\text{m.e.p.}) \text{ isothermal, no clearance}}{(\text{m.e.p.}) \text{ reduced to first stage} \div \text{first stage vol. eff.}} \quad (d)
 \end{array} \right\} . \quad (134)$$

In this connection it is useful to note that for the case of the no-clearance cycles, the work per cubic foot of supply is equal to the mean effective pressure (M.E.P.) in pounds per square foot, and when divided by 144 gives (m.e.p.) in pounds per square inch. Also, that in cases with clearance, or even actual compressors with negligible clearance, but in which, due to leakage and other causes, the true volumetric efficiency is not equal to unity,

$$\text{Work per cubic foot gas supplied} \times E_v = 144(\text{m.e.p.}) \quad (135)$$

The information that is ordinarily available to determine the economy of the compressor will be in the form of indicator cards from which the (m.e.p.) for the individual cylinders may be obtained with ordinary accuracy. The volumetric efficiency may be approximated from the indicator cards also, but with certain errors due to leakage and heating, that will be discussed later. If by this, or other more accurate means the true volumetric efficiency is found, the information required for the use of Eq. (134) (c) or (d) is available. Evaluation of the numerator may be had by Eq. (31), repeated here for reference:

$$(\text{m.e.p.}) \text{ isothermal} = (\text{sup.pr.}) \log_e R_p \quad (136)$$

The evaluation of the numerator may also be determined by means of the curve sheets given in the Handbook of Tables (Chart No. 7).

In addition to the mean-effective pressure this diagram also gives the economy of adiabatic cycles of single stage, also two and three stages with best-receiver pressures. The value of s will depend upon the substance compressed and its condition. The curve sheet is arranged to give the choice of the proper value of s applying to the specific problem.

If it is required to find the economy of an actual compressor referred to the third standard, i.e., that hypothetical adiabatic cycle which most nearly approaches the actual, then

Economy by third standard is

$$\frac{\text{Econ. actual referred to isothermal}}{\text{Econ. hypothetical referred to isothermal}} \quad (137)$$

It is important to notice that for a vapor an isothermal process is not one following the law $P \times V = \text{constant}$. What has, in this section, been called an isothermal is correctly so called only so long as the substance is a gas. Since, however, the pressure-volume analysis is not adequate for the treatment of vapors, and as they will be discussed under the subject of Heat and Work, Part III, it is best to regard this section as referring only to the treatment of gases, or superheated vapors which act very nearly as gases. However, it must be understood that whenever the curve follows the law $P \times V = \text{constant}$, the isothermal equations for work apply, even though the substance be a vapor and the process is not isothermal.

34. Conditions of Maximum Work of Compressors. Certain types of compressors are intended to operate with a delivery pressure approximately con-

stant, but may have a varying supply pressure. Such a case is found in pumps or compressors intended to create or maintain a vacuum and to pump natural gas from wells to pipe lines. The former deliver to atmosphere, thus having a substantially constant delivery pressure. The supply pressure, however, is variable, depending upon the vacuum maintained. In order that such a compressor may have supplied to it a sufficient amount of power to keep it running under all conditions, it is desirable to learn in what way this power required will vary; what its maximum value will be; and under what conditions this value will be obtained.

The work per cycle of a single-stage adiabatic compressor with clearance will vary directly as the mean effective pressure. As shown in Eq. (63),

$$(m.e.p.) = \frac{s}{s-1} (\text{sup.pr.}) \left[1 + c - c \left(\frac{\text{del.pr.}}{\text{sup.pr.}} \right)^{\frac{1}{s}} \right] \left[\left(\frac{\text{del.pr.}}{\text{sup.pr.}} \right)^{\frac{s-1}{s}} - 1 \right]. \quad (138)$$

This will have a maximum value when $\frac{d(m.e.p.)}{d(\text{sup.pr.})} = 0$.

$$\text{or when} \quad \left(\frac{\text{del.pr.}}{\text{sup.pr.}} \right)^{\frac{s-1}{s}} - \frac{s}{1+c} \left[1 + c - c \frac{s-1}{s} \left(\frac{\text{del.pr.}}{\text{sup.pr.}} \right)^{\frac{1}{s}} \right] = 0. \quad (139)$$

Solving this for the value of supply pressure will give that supply pressure at which the work will be a maximum, in terms of a given delivery pressure, clearance and the exponent s .

The assumption most commonly used is that clearance is zero. If this is true or the assumption permissible, the above equation becomes simplified.

$$\frac{(\text{del.pr.})}{(\text{sup.pr.})} = s^{s-1}. \quad (140)$$

The value of s for air, for instance, is 1.406, and hence the ratio of compression for maximum work for the hypothetical air compressor is

$$(1.406)^{3.46} = 3.26. \quad (141)$$

It may be noted that when $s=1$ in the above expression, the value of the ratio of compression becomes indeterminate. To find the supply pressure for maximum work in this case, take the expression for mean effective pressure for the isothermal compressor ($s=1$), Eq. (41), differentiate with respect to (sup.pr.) and place the differential coefficient equal to zero. This process results in the expression

$$\log_e \left(\frac{\text{del.pr.}}{\text{sup.pr.}} \right) + \frac{c}{1+c} \left(\frac{\text{del.pr.}}{\text{sup.pr.}} \right) - 1 = 0. \quad (142)$$

$$\text{When } c=0, \text{ this becomes, } \log_e \left(\frac{\text{del.pr.}}{\text{sup.pr.}} \right) = 1 \text{ or } \left(\frac{\text{del.pr.}}{\text{sup.pr.}} \right) = 2.72. \quad (143)$$

The expressions Eqs. (140) and (143) are easily solved, but Eqs. (139) and (142) are not. To facilitate computations requiring their solution the results of the computation are given graphically in chart No. 5, in the Handbook of Tables.

This chart also gives the mean effective pressures in the form of curves calculated from Eqs. (138) and (41). These curves, it will be noted, give the mean effective pressure expressed as a fraction of the delivery pressure.

The discussion so far applies only to single-stage compressors. The problem of *maximum work for multi-stage compressors* is somewhat different, and its solution is not so frequently required. Moreover, if the assumption of perfect intercooling is made, the results are not of great value, as a still greater amount of power might be required, owing to the failure for a period of time of the supply of cooling water.

If *intercooling be discontinued* in a multi-stage compressor, the volume entering the second stage will equal that delivered from the first, and similarly for the third and second stages. The entire work done in all stages will be the same as if it had all been done in a single stage. It might be questioned as to whether this would hold, when the ratio of compression is much less than designed. The first stage will compress until the volume has become as small as the low-pressure capacity of the second stage. If the delivery pressure is reached before this volume is reached, there is no work left to be done in the second or any subsequent stages, and, owing to the pressure of the gas, the valves, if automatic, will be lifted in the second and higher stages, and the gas will be blown through, with only friction work. It appears then that under the condition of no intercooling the multi-stage compressor acts the same as a single stage, and the conditions of maximum work will be the same.

If *intercooling is maintained perfect* there will still be a range of pressures on which all the work of compression is done in the first stage, merely blowing the discharge through receivers, valves and cylinders in the upper stages. If this range is such that this continues beyond a ratio of pressures, which gives a maximum (m.e.p.) for the single stage, then the maximum will have been reached while the compressor is operating single stage, and the *single-stage* formulæ and curves may be applied to this case also.

That this condition frequently exists with multi-stage compressors of ordinary design is shown by the fact that the ratio of compression in each stage is seldom less than 3, and more frequently 3.5, 4 or even more. The ratio of compression giving maximum work for single stage, has values from 2.5 to 3.26, dependent on clearance and the value of s for the gas compressed, and is, therefore, less for the majority of cases.

If the ratio of cylinders is such that the single-stage maximum is not reached before the second stage begins to operate, the maximum work for a given delivery pressure will occur when the ratio of supply and delivery pressures is such as to make first-receiver pressure equal to delivery pressure.

35. Actual Compressor Characteristics. Air or gas compressors are very commonly made double acting, so that for a single cylinder, two cycles will be performed during one revolution, one in each end of the cylinder. If a rod extends through one of these spaces and not through the other, the displacement of that end of the cylinder will be less than the other by a volume equal to the area of rod multiplied by the stroke. To avoid mechanical shock

at the end of either stroke, it is necessary to leave some space between the piston and cylinder head. Passages must also be provided, communicating with inlet and discharge valves. The total volume remaining in this space and in the passages when the piston is at the nearer end of its stroke constitutes the clearance. The amount of this clearance volume varies from .5 or .6 of one per cent in some very large compressors to as much as 4 or 5 per cent of the volume of displacement in good small cylinders.

In order to study the performance of an actual compressor and to compare it with the hypothetical cycle, it is necessary to obtain an indicator card, and knowing the clearance and barometric pressure to convert the indicator card into a pressure-volume diagram, by methods explained in Chapter I. Fig. 30

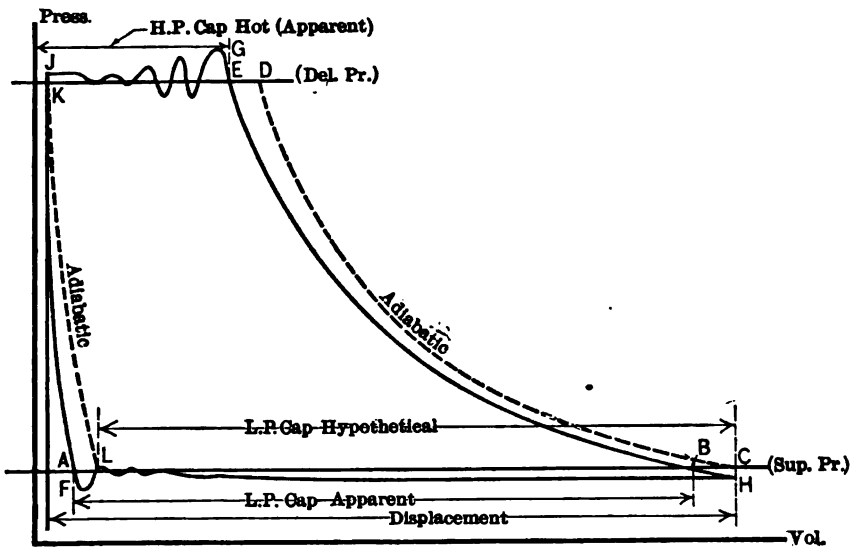


FIG. 30.—Compressor Indicator Card Illustrating Departure from Reference Cycle.

is such a diagram for a single-stage compressor. In the pipe leading to the intake valve the pressure is determined and a horizontal line \overline{AB} is drawn on the diagram at a height to represent the supply pressure. Similarly, discharge pressure is determined and drawn on the diagram, \overline{KE} . Consider the four phases of this diagram in succession.

1. *Intake Line.* At a point somewhat below *A* the intake valve opens say at the point *F*. This remains open till a point *H* is reached at or near the end of the stroke. The line connecting these two points indicates variations of pressures and volumes throughout the supply stroke. In general this line will lie below the supply-pressure line *AB* due to first, the pressure necessary to lift the inlet valve from its seat against its spring and inertia, if automatic, and support it, and second, friction in the passages leading to the cylinder from the point where the supply pressure was measured. While the former is nearly constant the latter varies, depending upon the velocity of gases in the passages.

The piston attains its highest velocity near the middle of the stroke, thus causing the intake line to drop below the supply pressure more at this part of stroke. These considerations do not, however, account completely for the form of the intake line. Frequently the first portion of the line lies lower than the last portion, even at points where the piston velocities are equal. This is more prominent on a compressor having a long supply pipe, and is due to the forces required to accelerate the air in the supply pipe while piston velocity is increasing, and to retard it while piston velocity is decreasing. In compressors where the inlet valve is mechanically operated and the supply pipe long, it is possible to obtain a pressure at the end of the intake, line H , even in excess of the supply pressure. The effect of this upon volumetric efficiency will be noted later.

The apparent fluctuations in pressure during the first part of the intake line may be attributed, first, to inertia vibrations of the indicator arm, in which case the fluctuations may not indicate real variations of pressure; second, the indicator card may show true variations of pressure due to inertia of the gases in the supply pipe, since a moment before the valve opened at F the gases were stationary in the supply pipe. When F is reached the piston is already in motion and a very considerable velocity is required in the supply pipe to supply the demand. This sudden acceleration can be caused only by a difference in pressure, which is seen to exist below and to the right of F on the diagram. The suddenness of this acceleration may start a surging action which will cause rise and fall of pressure to a decreasing extent immediately after. A third cause is possible, that is, a vibration of the inlet valve due to its sudden opening when it is of the common form; mechanical valves change the conditions. It is closed by weight or a spring and opened by the pressure difference. Between these forces the valve disk may vibrate, so affecting the pressure.

2. *Compression Line.* From the time the inlet valve closes at the point H until the discharge valve opens at the point G , the gases within the cylinder are being compressed. The compression is very nearly adiabatic in ordinary practice, but owing to the exchange of heat between the cylinder walls, at first from walls and later from gas to walls, which are cooled by water jacket to prevent the metal from overheating, there is a slight departure from the adiabatic law almost too small to measure.

A second factor which influences the form of this curve to a greater extent is leakage. This may occur around the piston, permitting gas to escape from one end of the cylinder to the other. During the compression process there is first an excess of pressure in the other end of the cylinder due to re-expansion, tending to increase pressures on the first part of compression. Later, the pressure rises and that on the other side of the piston falls to the supply pressure. During this period leakage past piston tends to decrease successive pressures or lowers the compression line. Leakage also occurs through either discharge or inlet valves. The former will raise the compression line, while excessive leakage of the inlet valve will lower it.

It is then evident that unless the nature of the leakage is known, it is

impossible to predict the way in which it will change this line. It is, however, more frequently the case that the piston and inlet leakage are large as compared with the discharge valves, in which case the actual compression line has a tendency to fall lower than the adiabatic as the volumes are decreased. Compression lines lowered by leakage are often mistaken for proofs of effective cooling, and cases have been known where isothermal compression of air was claimed on what proved to be evidence only of bad leakage.

3. *Delivery Line.* After the delivery pressure of the compressor has been exceeded sufficiently, the discharge valve is opened and the gas is delivered to the discharge pipe or receiver till the end of the stroke is reached and at the point *J* the valve closes. The same group of factors which act upon the intake line also influence the form of the delivery line; spring resistance of discharge valve; friction in discharge passages varying with piston velocity; inertia of gases in delivery pipe; sudden acceleration of gases in delivery pipe when discharge valve opens; inertia of indicator arm; and in addition a strong tendency for the valve to chatter or jump open and shut alternately. Leakage also occurs through intake valve and past the piston during this process, with the result that less gas passes through the discharge valve than is shown on the indicator card.

4. *Re-expansion Line.* From the time the discharge valve closes, at *J*, till the intake valve opens, at *F*, the gas which remained in the clearance space after delivery expands, due to the advancing of the piston, till the pressure has fallen to such an amount that the intake valve will open. The same factors influence this line as the compression line. Heat is exchanged with the jacketed cylinder walls, at first cooling and later heating the gas as the pressure falls. This, for any given volume, changes the pressure. Leakage occurs inward through the discharge valve and outward through intake valve and past the piston. If these last two are in excess, the pressures will fall more rapidly than if the expansion were that of a constant quantity of gas.

Work due to gas friction and inertia, it should be noted, is fully represented on the indicator card, and may be regarded as being equal to that extra area below the supply-pressure line and above the delivery-pressure line. In the combined card of a two-stage compressor there would be an overlapping of the diagrams due to this frictional loss.

Low-pressure Capacity. Referring to the adiabatic compression and expansion lines, *CD* and *KL*, Fig. 30, it is seen that the *low-pressure capacity of the hypothetical cycle* is the volume, *LC*.

The *apparent low-pressure capacity of the actual compressor*, measured at the supply pressure is *AB*. This is not, however, the true volume of gas at supply pressure and temperature that is taken in, compressed, and finally delivered per cycle. First, the valves, passages and walls are not at the same temperature as the entering gas, due to the heat left from the compression of the previous charge. This causes the temperature of the gas within the cylinder to be something higher and less dense than the supply gas outside; hence an equivalent weight of gas at supply temperature and pressure would

occupy a volume somewhat less than AB . Second, the gas which occupies the volume AB has not all entered the cylinder through the intake valve. After re-expansion is completed the intake valve opens and gas enters the end of the cylinder under consideration. At the same time compression is taking place in the other end, and later, delivery. During these processes whatever gas leaks past the piston tends to fill the end of the cylinder in which intake is going on. Leakage past the discharge valve also tends to fill the cylinder with leakage gas. Both of these tendencies decrease the quantity of gas entering through this intake valve, and its true amount when reduced to external supply pressure and temperature is, therefore, less than the volume AB .

The *true low-pressure capacity* of the compressor is the true volume of gas, under external supply conditions, that enters the cylinder for each cycle. This cannot be determined from the indicator card except by making certain assumptions which involve some error at best. It can, however, be ascertained by means of additional apparatus, such as meters or calibrated nozzles or receivers, by means of which the true amount of gas compressed per unit of time is made known. This reduced to the volume per cycle under supply pressure and temperature will give the true low-pressure capacity.

Volumetric efficiency is defined as being the ratio of low-pressure capacity to displacement. On the diagram, Fig. 30, the displacement is represented to the volume scale by the horizontal distance between verticals through the extreme ends of the diagram, K and H . Since there are three ways in which the low-pressure capacity may be approximated or determined, there is a corresponding number of expressions for volumetric efficiency.

1. The *volumetric efficiency* of the *hypothetical* cycle is

$$E_v(\text{hypothetical}) = \frac{(\text{hypothetical L.P. Cap.})}{(\text{displacement})}, \quad \dots \quad (144)$$

and this is evaluated and used in computations in the foregoing sections of this chapter.

2. The *apparent volumetric efficiency* is

$$E_v(\text{apparent}) = \frac{(\text{apparent L.P. Cap.})}{(\text{displacement})}, \quad \dots \quad (145)$$

which would be very nearly equal to the true volumetric efficiency were it not for leakage, valve resistance and heating during suction, but owing to these causes may be very different.

3. The *true volumetric efficiency* is $E_v(\text{true}) = \frac{(\text{true L.P. Cap.})}{(\text{displacement})} \dots \quad (146)$

In *problems of design or prediction* it is necessary either to find dimensions, speeds and power necessary to give certain actual results, or with given dimensions and speeds to ascertain the probable power and capacity or other characteristics of actual performance. Since it is impossible to obtain actual performance identical with the hypothetical, and since the former cannot be computed, the most satisfactory method of estimate is to perform the computa-

tions on the hypothetical cycle, as explained in previous sections of this chapter, and then to apply to these results factors which have been found by comparing actual with hypothetical performance on existing machines as nearly like that under discussion as can be obtained. This necessitates access to data on tests performed on compressors in which not only indicator cards are taken and speed recorded, but also some reliable measurement of gas compressed.

The following factors or ratios will be found of much use, and should be evaluated whenever such data are to be had:

$$e_1 = \frac{E_s(\text{true})}{E_s(\text{hypothetical})} = \frac{(\text{true L.P. Cap.})}{(\text{hypothetical L.P. Cap.})} \quad \dots (147)$$

$$e_2 = \frac{E_s(\text{true})}{E_s(\text{apparent})} = \frac{(\text{true L.P. Cap.})}{(\text{apparent L.P. Cap.})} \quad \dots (148)$$

$$e_3 = \frac{\text{true I.H.P.}}{\text{hypothetical I.H.P.}} = \frac{\text{true m.e.p.}}{\text{hypothetical m.e.p.}} \quad \dots (149)$$

$$\begin{aligned} \text{Then } & \frac{\text{true work per cu.ft. gas, supplied}}{\text{hypothetical work per cu.ft. gas, supplied}} \\ &= \frac{\text{true I.H.P. per cu.ft. gas supplied}}{\text{hypothetical I.H.P. per cu.ft. supplied}} = \frac{\text{true } \frac{\text{I.H.P.}}{(\text{L.P. Cap.})}}{\text{hypothetical } \frac{\text{I.H.P.}}{(\text{L.P. Cap.})}} \quad \dots (150) \\ &= \frac{\text{true m.e.p.}}{\text{hypothetical m.e.p.}} \div \frac{\text{true L.P. Cap.}}{\text{hypothetical L.P. Cap.}} = \frac{e_3}{e_1} \end{aligned}$$

This ratio can be used to convert from hypothetical work per cubic foot of gas supplied to probable true work per cubic foot.

Multi-stage Compressors are subject in each stage to all of the characteristics described for single stage to a greater or less extent. Valve resistance, friction and inertia affect the intake and discharge lines; heat transfer and leakage influence the form of compression and re-expansion lines, and the true capacity of the cylinder is made different from the apparent due to leakage, pressure and temperature changes.

In addition to these points it is useful to note one special way in which the multi-stage compressor differs from the single stage. The discharge of the first stage is not delivered to a reservoir in which the pressure is constant, but to a receiver of limited capacity. The *average* rate at which gas is delivered to the receiver must equal the average rate at which it passes to the next cylinder. The momentary rate of supply and removal is not constantly the same, however. This causes a rise or fall of pressure. It is evident that this pressure fluctuation is greatest for a small receiver. Very small receivers are not, however, used on gas compressors owing to the necessity of cooling the gas as it passes from one stage to the next. To accomplish this a large amount of cooling surface must be exposed, requiring a large chamber in which it can be done. Thus, it is seen that the hypothetical cycles assumed for multi-stage compressors do not truly represent the actual cycle, but the difference can never be very great, on account of the large size of receiver which must always be used.

Another way in which the performance of this multi-stage compressor commonly differs from assumptions made in the foregoing discussions is in regard to intercooling. It seldom occurs that the gases enter all stages at the same temperature. In the several stages the temperature of the gases will depend on the amount of compression, on the cooling surface and on the amount and temperature of cooling water. The effect of variations in temperature upon the work and receiver pressures will be taken up later. It may be noted now, however, that if all cooling water is shut off, the gas passes from one cylinder to the next *without cooling*, and there is no decrease in volume in the receiver. For simplicity take the case of zero clearance, two-stage (Fig. 31).

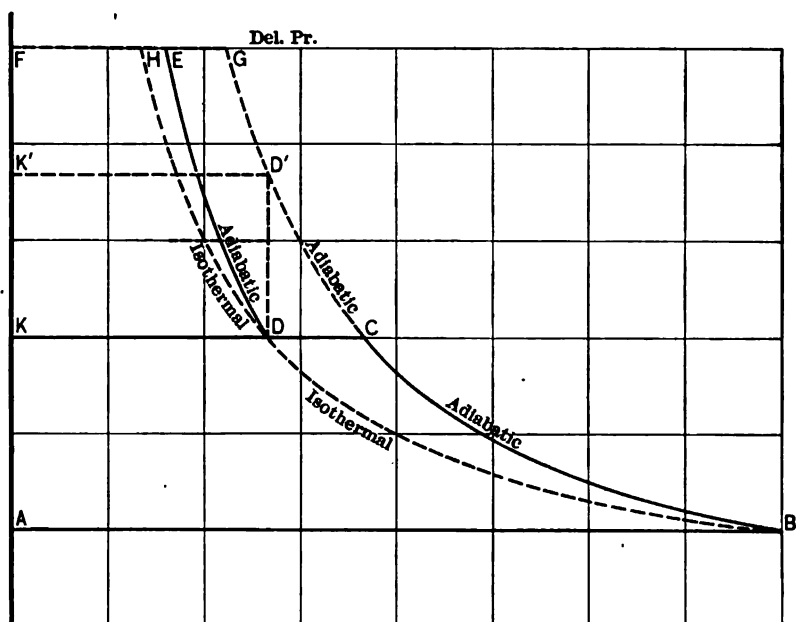


FIG. 31.—Effect of Loss of Intercooling in Two-stage Compressors on Receiver-pressure and Work Distribution in the Two Cylinders.

Let \overline{ABCDEF} be the cycle for perfect intercooling. AB and KD are the low-pressure capacities of the first and second stages respectively. If now, intercooling ceases, the gas will no longer change volume in the receiver. The receiver gas, in order to be made sufficiently dense to occupy the same volume (KD) as it did before, must be subjected to a greater pressure in the first stage. The new receiver line will be $K'D'$. The work of the first stage will therefore be $\overline{ABD'K'}$; of the second stage $\overline{K'D'GF}$, and the total work in the new condition is greater than when intercooling was perfect by an amount represented by the area \overline{DCGE} .

In the case where clearance is considered, the effect is the same, except that the increasing receiver pressure, increasing the ratio of compression of the first stage, causes the volumetric efficiency of the first stage to become less, and

hence lessens the capacity of the compressor. The effect on work per unit of capacity is the same as without clearance.

The determination of the number of stages that should be used for a given compressor is dependent largely upon the ratio of compression and involves the following considerations: first, is the question of economy; this can readily be understood from the preceding sections; second, the mechanical features must be considered in order to obviate high pressures in large cylinders; and third, the thermal effects must be studied to prevent excessive temperatures which make difficult the lubrication of the cylinders and produce other serious troubles including explosions.

Practice varies very widely as to the limiting pressures for single, two, three or four-stage compressors. Air compressors of a single stage are commonly used for ratios of compression as high as six or seven (75 to 90 lbs. gage). For ratios greater than these, two-stage compressors are used, especially for larger sizes, up to ratios of 34 to 51 (500 to 750 lbs. gage). Some three-stage compressors are used for ratios as low as 11 or 14 (150 or 200 lbs. gage), although installations of this nature are rare; they are warranted only when power is costly and the installation permanent and continuously used. As a minimum ratio for three stages, 11 (150 lbs. gage) is used for large units, while a few small units compress as high as 135 or even 170 atmospheres (2000 or 2500 lbs. gage). A notable use for the four-stage compressor is in the charging of the air flask of automobile torpedoes used by the various navies, which use pressures from 1600 to 3000 lbs. per square inch (110 to 220 atmospheres). These require special design of valves, cylinders and packings to withstand the extremely high pressures, small clearances, and special precautions against leakage, because of the great loss of volumetric efficiency and economy that would otherwise result.

36. Work at Partial Capacity in Compressors of Variable Capacity. It is seldom that a gas compressor is run continuously at its full capacity. If the duty of the compressor is to charge storage tanks, it may be made to run at its full capacity until the process is completed and then may be stopped entirely, by hand. Even where the compressed gas is being used continuously it is common practice to have a storage reservoir into which the compressor may deliver. This enables the compressor to deliver a little faster or slower than the demand for a short period without a great fluctuation pressure in the reservoir. For many purposes hand regulation is not sufficient or is too expensive, hence the demand for automatic systems of capacity regulation. These systems may be classified in a general way in accordance with the method of driving. Some methods of power application permit of speed variations while others require constant speed. The former provides in itself a means of regulating capacity within certain limits, while, if the compressor must run at constant speed, some additional means of gas capacity control must be provided.

Compressors driven by an independent steam engine, or steam cylinders constituting part of the same machine, may be made to run at any speed required within a very wide range and still be kept low enough for safety. If driven

by gear, belt, rope, chain or direct drive from a source of power whose speed is constant, the speed of the compressor cannot be varied. Electric motor, gas-engine, oil-engine or water-power drives are subject only to limited speed alteration and may, therefore, be placed in the constant speed class.

Regulation of Capacity by Means of Speed Change. If the speed of the compressor is decreased below normal:

1. Displacement of piston is decreased in proportion to the speed.
2. Mean effective pressure, from hypothetical considerations, is the same, but owing to the decrease of velocities in gas passages, the frictional fall of pressure during inlet and delivery is not so great, and hence the mean effective pressure is not quite so great. If the compressor is multi-stage, it is probable that the intercooling is more nearly perfect, since a smaller quantity of gas is passing through the intercooler, thus decreasing the mean effective pressures in the succeeding stages.
3. The volumetric efficiency is changed, first, by reason of the fact that leakage is about the same in total amount per minute as at full speed, but the total quantity of gas being less, leakage is a larger percentage of the total; second, the inertia of gases in the supply pipe, as well as their friction, has been decreased. The former tends to decrease volumetric efficiency, while the latter may tend to increase or decrease it. It may be expected that the true volumetric efficiency will be somewhat greater at fractional speed than at full speed.

For any compressor there is a speed of maximum economy above and below which the economy is less, though it may be that this most economical speed is greater than any speed of actual operation.

It is not desirable at this point to discuss the effect of speed variation upon the economy of the engine or other motor supplying the power. The reasoning above applies to the term economy as related to the compression effect obtained per unit of power delivered to the compression cylinder. It might be noted here, however, that the decrease of speed has little effect upon the mechanical efficiency of the compressor as a machine, since frictional resistance between solid parts remains nearly constant, and, therefore, power expended in friction will vary as the speed, as does approximately the power to drive the compressor. The ratio of frictional power to total may then be expected to remain nearly constant.

Regulation of Capacity at Constant Speed may be accomplished in a number of ways:

1. Intermittent running;
2. Throttling the supply to compressor;
3. Periodically holding open or shut the intake valve;
4. Closing intake valve before end of intake stroke, or holding intake valve open until compression stroke has been partially completed;
5. Large clearance;
6. Variable clearance.

The *first* necessitates some means for stopping and starting the compressor,

which is easily arranged with electric drive, and may be accomplished in other cases by means of a detaching clutch or other mechanical device. The pressure in the reservoir is made to control this stopping and starting device by means of a regulator. This arrangement keeps the pressure in the reservoir between certain fixed limits, but does not maintain a constant pressure. The economy of compression under these conditions is evidently the same as at full speed continuous running, provided there is no loss in the driving system due to starting and stopping, which may not be the case. This method of regulation is used mainly for small compressors in which inertia is not great, such as supply the air brakes on trolley cars. The sudden change of load on the driving machinery would be too great if large compressors were arranged in this way.

If the compressor whose capacity is regulated by intermittent running is *multi-stage*, the constant supply of water to the intercoolers while the compressor is stopped will lower the temperature of the cooling surface, causing more nearly perfect intercooling when the compressor is started. Leakage, on the other hand, will permit the loss of pressure to a greater or less extent in the receivers while the compressor is stationary, which must be replaced after starting before effective delivery is obtained.

Throttling the gas supply to the compressor causes work to be done in overcoming useless frictional resistances, so that the economy is seriously reduced. Such compressors may use almost as much power at partial loads as at full capacity.

It is easily seen that this method of regulation would be undesirable, its only advantage being simplicity.

The effect of throttling upon a multi-stage compressor, as for instance, a two-stage without clearance, causes a reduction in the receiver pressure; hence less work is done in the first stage and far more than half the work of compression is done in the second stage.

If best-receiver pressure should exist at normal capacity, it would not exist in the throttled condition.

The intake valve may be held wide open or completely closed during one or more revolutions, thereby avoiding the delivery of any gas during that period. If the intake valve is held wide open, the indicator card would be as shown in Fig. 32 A in full lines, *ABCD*, the dotted lines showing the cycle performed when normal operation is permitted. With the inlet valve open in this way there is a loss of power due to friction of the gas in passage during both strokes, measured by the area within the loop.

Closing the inlet valve and holding it shut will give an indicator card of the form *EFG*, Fig. 32 B, which will be a single line retraced in both directions except for probable leakage effects. If leakage is small, there will be but little area enclosed between the lines. At a high speed this might be expected to incur less lost power than by holding the valve wide open.

Certain types of compressors are made with an intake valve controlled by a drop cut-off, much like the steam valve of the Corliss engine. The effect

of this is to cut off the supply of gas before the end of the stroke, after which time the gas must expand hypothetically according to the adiabatic law. The return stroke causes it to compress along the same line continued up to the delivery pressure, as indicated by the line FEG , Fig. 32C. There is little work lost in the process, none, if the line is superimposed as in the figure, and hence the process is the same as if only the cycle $AEGD$ were performed.

The same quantity of gas might have been entrapped in the cylinder by holding the intake valve open until the end of the stroke and on the return till the point E , Fig. 32D, was reached, then closing it. The same compression

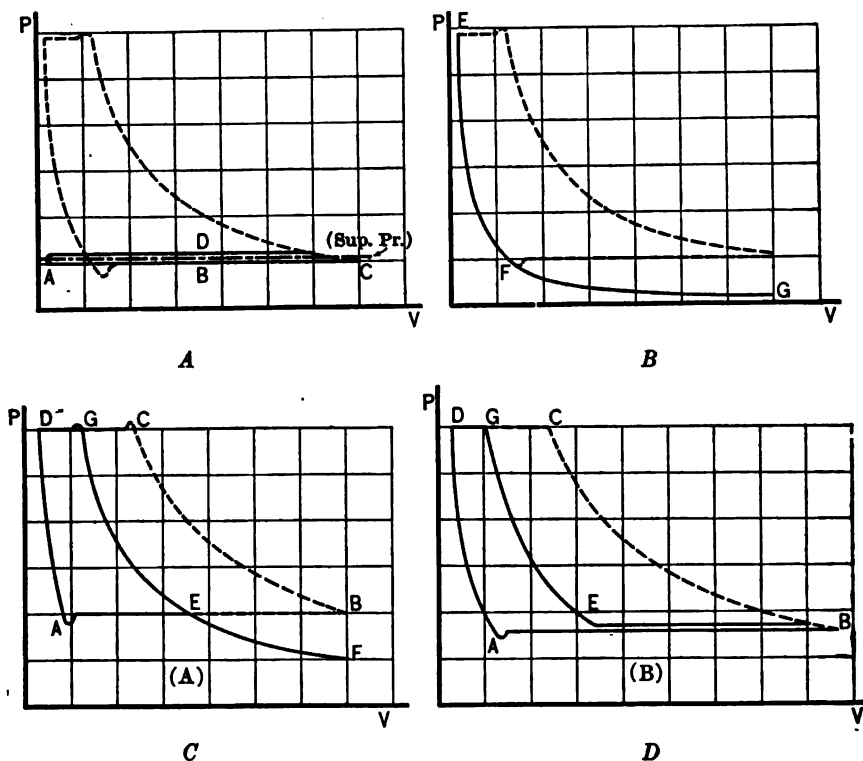


FIG. 32.—Control of Compressor Capacity by A. Open Inlet Valve; B. Closed Inlet Valve; C. Suction Cut-off; D. Delayed Suction Closure.

line EG will be produced. The line AB will not coincide with BE , due to friction of the gas in passages, and hence will enclose between them a small area representing lost work, which may be no larger than that lost in the process EFE , Fig. 32C.

If such an automatic cut-off were applied only to the first stage of a *multi-stage* compressor, the effect would be to lower receiver pressures as in the throttling process. To avoid this, the best practice is to have a similar cut-off to act on the supply to all of the stages. If this is properly adjusted, the receiver pressures can be maintained the same as at full load. An additional advantage

of this system is that even if the compressor is to be used for a delivery pressure for which it was not originally designed, the relative cut-offs may be so adjusted as to give and maintain best-receiver pressure.

Since the low-pressure capacity per cycle of a compressor involves clearance and ratio of compression as two of its variables, it is possible to change capacity by changing either the clearance or the ratio of compression. From Eq. (59),

$$(\text{L.P. Cap.}) = DE_e = D \left(1 + c - cR_p^{\frac{1}{s}} \right). \quad \dots \quad (151)$$

Assuming that clearance is a fixed amount and not zero, it is evident that an increase in the ratio of compression decreases the capacity, and when it has reached a certain quantity will make the capacity zero. If the clearance is large, making the coefficient of $R_p^{\frac{1}{s}}$ large in the equation the effect of a change in that factor is increased.

Fig. 33 indicates the hypothetical performance of a compressor with large clearance. When the pressure of delivery is low (say P_c) the capacity is large, AB . The cycle is then $ABCD$. An increase of the delivery pressure to P_c' changes the cycle to $A'BC'D'$ and the low pressure capacity is $A'B$. If the compressor is delivering to a receiver from which no gas is being drawn, the delivery pressure will continue to rise and the capacity to decrease, approaching zero as the delivery pressure approaches the pressure P , as a limit.

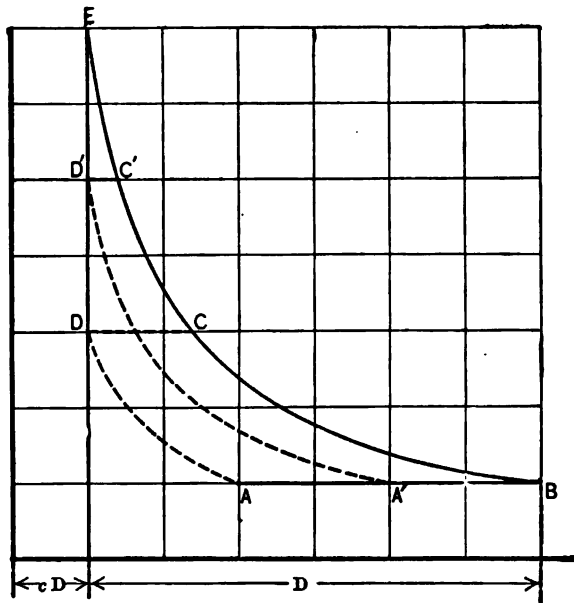


FIG. 33.—Variation of Compressor Capacity with Rise of Delivery Pressure, Fixed Clearance, Pressure for Zero Delivery.

$$(\text{limiting del.pr.}) = (\text{sup.pr.}) \left(\frac{1+c}{c} \right)^s \quad \dots \quad (152)$$

When the limiting condition has been reached and the capacity has become zero, the compression and re-expansion lines coincide and enclose zero area between them; hence, the mean effective pressure and the indicated horsepower are zero, for the hypothetical case. Leakage will prevent a perfect coincidence of the lines and cause some power to be required in addition to that of friction.

Such a simple method of regulation as this is used for small compressors driven constantly from some source of power used primarily for other purposes. When it is not necessary to have a constant delivery pressure, but only to keep it between certain limits, this may be made use of, especially if the limits of pressure are quite wide.

The expression for low-pressure capacity Eq. (151), suggests the possibility of decreasing capacity by the increase of clearance. The effect of this is shown in Fig. 34. The original compression cycle (full capacity) is shown by $ABCD$, with a clearance volume of cD , so that the axis of zero volume is OP . Increasing the clearance to $c'D$ causes a smaller volume $C'D$ to be delivered and owing to the more sloping re-expansion line DA' , a smaller volume of gas $A'B$ is taken in.

It has been shown in previous sections that clearance has no effect upon the

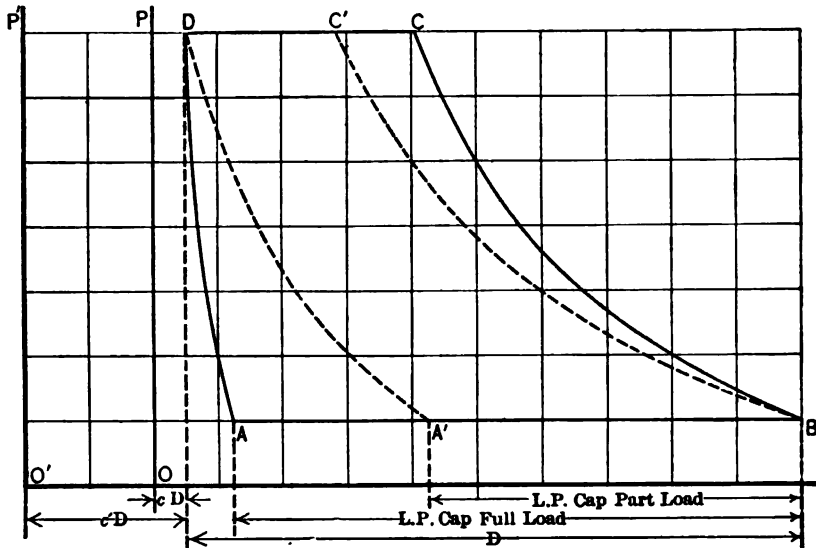


FIG. 34.—Variation of Compressor Capacity by Changing Clearance.

economy of a compressor so far as hypothetical considerations are regarded. In practice it is found that a slight loss of economy is suffered at light load, as might be expected, due to greater leakage per unit of capacity. The additional clearance is provided in the form of two or more chambers connected to the clearance space of the compressor by a passage in which is a valve automatically controlled by the receiver pressure.

In the *multi-stage* compressor, decreasing the capacity of the first stage by an increase of its clearance would evidently permit a decrease of receiver pressures unless the capacity of each of the various stages is decreased in the same proportion. Eq. (122) gives the condition which must be fulfilled to give best receiver pressure for a two-stage compressor.

Since D_1 , D_2 , and R_p remain fixed, for any chosen value of clearance of the

first stage, c_1 , the clearance of the second stage, c_2 , to give best-receiver pressure can be found,

$$c_2 = \frac{[1 - c_1(R_p^{\frac{1}{2s}} - 1)]D_1}{(R_p^{\frac{1}{2s}} - 1)R_p^{\frac{1}{2s}}D_2} \quad \dots \quad (153)$$

For every value of first-stage clearance there is a corresponding clearance of second stage that will give best-receiver pressure, found by this equation. Similar reasoning can be applied to three- or four-stage compressors.

It is often convenient to solve problems relating to this work, by means of graphic charts or diagrams, which obviate the necessity of working out the formulas in each case. Series of curves which constitute the respective charts have been calculated from the formulas discussed in this chapter and are given in the Handbook of Tables. It will be found that many problems may be solved directly from the diagrams; in other cases their use will permit the work to be shortened by the omission of certain steps. A description of the curves, their derivation and method of use accompany each chart.

Following are some of the problems which may be satisfactorily and conveniently solved by means of these charts.

Example 2, page (72). Solved by means of chart No. 1. This is to find the work to compress 1000 cu.ft. of free air from 1 to $8\frac{1}{2}$ atmospheres adiabatically. On the curve project upward from $R_p = 8.5$ to curve of $s = 1.406$, then over to 14.7 (sup.pr.) curve and down to read work = 6,300,000 as found, for example, by use of formulas in Section (27).

Example 2, page (77). Solved by means of chart No. 2. This problem calls for the work to compress 5 cu.ft. of free air from 1 to $8\frac{1}{2}$ atmospheres adiabatically in two stages. Project upward from $R_p = 8.5$ to curve $s = 1.406$, then over to 14.7 curve and down to read 5320 ft.-lbs. per cubic foot, which is the same as found from the formula in Section (28).

Example 1, page (93). Solved by means of chart No. 3. This calls for the horse-power to compress 100 cu.ft. free air per minute adiabatically in three stages from 15 lbs. per square inch abs. to 90 lbs. per square inch gage. From $R_p = 7$, project to curve of $s = 1.4$ then over to (sup.pr.) = 15 and down, and the horse-power will be found to be 13.6 as before by use of formulas.

Example 4, page (95). Solved by means of chart No. 4. In this example the pressure ratio is 9.35, therefore projecting upward from the pressure ratio of 9.35 to the line of $s = 1.4$ and then over to (sup.pr.) = 15 in diagram 4, since compression is three stage, and from 15 lbs. per square inch to 140 lbs. per square inch, work per cubic foot or (m.e.p.), is found for no clearance to be 37.8 abs. per square inch. Since best-receiver pressure assumed is 31.6, this gives a ratio of 2.1 for the low-pressure cylinder. From diagram 3, by projecting upward from $R_p = 2.1$ and over to the 5 per cent clearance line the volumetric efficiency is 96.5. The product gives (m.e.p.) reduced to low-pressure cylinder and is 36.5. From the $\frac{(\text{m.e.p.})L_{an}}{33,000}$ formula, horse-power is found to be 358 as before.

As an example of the use of chart No. 5, let it be required to find the (sup.pr.) for the case of maximum work for 9×12 in. double-acting compressor running 200

R.P.M., having 5 per cent clearance and delivering against 45 lbs. per square inch gage. Also the horse-power. Compression such that $s=1.3$.

Projecting from the value 1.3 for s on the left-hand diagram to the line of 5 per cent clearance find R_p to be 2.8, hence $(\text{sup.pr.}) = \frac{60}{2.8} = 21.4$ lbs. per square inch absolute = 6.4 lbs. per square inch gage. Again, projecting from value 1.3 for s on right-hand diagram to line of 5 per cent clearance find that

$$\frac{(\text{m.e.p.})}{(\text{del.pr.})} = 0.383, \text{ hence } (\text{m.e.p.}) = 23 \text{ and } \text{I.H.P.} = \frac{23 \times 1 \times 64 \times 400}{33,000} = 17.8.$$

To illustrate the use of chart No. 6, find the per cent of work needed to compress a cubic foot of air adiabatically from 1 to $8\frac{1}{2}$ atmospheres in two stages compared to doing it in one stage. From examples under charts Nos. 1 and 2 it was found that work was 6300 ft.-lbs. and 5320 ft.-lbs. respectively, for one- and two-stage compression, or that two stage was 84.5 per cent of one stage. From R_p , $8\frac{1}{2}$ project up on the chart to $s=1.406$ for two stage and over to read 84.6 per cent. which is practically the same.

As an example of the use of chart No. 8, let it be required to find the low-pressure cylinder size for a compressor to handle 1500 cu.ft. of free air per minute. Receiver pressure to be 45 lbs. per square inch gage and (sup.pr.) to be atmosphere. Piston speed limited to 500 ft. per minute. Compression to be such that $s=1.4$ and clearance = 4 per cent. Projecting upward from $R_p=4$ to $s=1.4$, across to $c=4$ per cent, and down to piston speed = 500, find the diameter of a cylinder for 100 cu.ft. per minute is 6.3. For 1500 cu.ft. diameter will be as $\sqrt{15 \times 6.3} = 3.9 \times 6.3 = 24$ ins.

CHAPTER IV

WORK OF SIMPLE PISTON ENGINES. STANDARD REFERENCE DIAGRAMS FOR SINGLE-CYLINDER ENGINES. HORSE-POWER, AND CONSUMPTION OF SIMPLE ENGINES.

37. Action of Fluid in Single Cylinders. General Description of Structure and Processes. The most commonly used class of engines is that in which the operation is dependent on the pushing action of high-pressure fluids on pistons in cylinders, and this includes all piston steam engines of the reciprocating or straight-line piston path group as well as the less common rotary group, having pistons moving in curved and generally circular paths. In these same engines there may be used compressed air as well as steam, and equally as well the vapors of other substances or any other gases, without change of structure, except perhaps as to proportions, providing only that the substance to be used be drawn from a source of supply under high pressure, be admitted to the cylinder, there used and from it discharged or exhausted to a place of lower pressure. This place of lower pressure may be the open air or a closed chamber; the used fluid may be thrown away and wasted or used again for various purposes without in any way affecting the essential process of obtaining work at the expense of high-pressure gases or vapors. It is evident that, regarding a piston as a movable wall of a cylinder, whenever a fluid acts on one side with greater pressure than on the other, the piston will move toward the lower pressure end of the cylinder, and in so moving can exert a definite force or overcome a definite resistance, measured by the difference in pressure existing on the areas exposed to the pressure. It is not so evident, but just as true, that the piston may be made to move from one end of the cylinder to the other when the average pressure on one side is greater than the average pressure on the other, and also do work even if the excess of pressure should reverse in direction during the stroke, provided only that some energy storage device is added. In the common steam or compressed-air engine this is a flywheel with the usual connecting rod and crank mechanism, uniting the reciprocating piston movement with the continuous rotary movement of the flywheel mass. In certain forms of pumps the energy is stored in extra cylinders at times of excess and given out at times of deficiency in the path of the piston, so that its motion from end to end of cylinder may not be interrupted even if the pressure on the driving side should fall below that on the resisting side, assuming, of course, the average pressure for the whole stroke to be greater on the driving side than on the resisting side.

It appears, therefore, that piston movement in engines of the common form and structure, and the doing of work by that movement is not a question of maintaining a continuously greater pressure on one side than on the other. On the contrary, the process is to be studied by examination of the average pressure on the driving side and that on the resisting side, or by comparing the whole work done on one side with the whole work done on the other side by the fluid. The work done by the fluid on one side of a piston may be positive or negative, positive when the pressures are assisting motion, negative when they are resisting it. It is most convenient to study the action of fluids in cylinders by considering the whole action on one side from the beginning of movement at one end to the end of movement at the same point, after the completion of one complete forward and one complete return stroke. All the work done by the pressure of the fluid on the forward stroke on the side of the piston that is apparently moving away from the fluid is positive work, all the work done by the pressure of the fluid on the same side of the piston during the return stroke is negative, and for this stroke the side of the piston under consideration is apparently moving toward the fluid or pushing it.

For the complete cycle of piston movement covering the two strokes the work done on one side is the algebraic sum of the forward stroke work, considered positive, and the back stroke work, considered negative. During the same time some pressures are acting on the other side of the piston, and for them also there will be a net work done equal to the corresponding algebraic sum. The work available for use during the complete two strokes, or one revolution, will be the sum of the net work done by the fluid on the two sides of the piston during that time, or the algebraic sum of two positive and two negative quantities of work. Methods of *analysis of the work* of compressed fluids in cylinders are consequently based on the action in *one end of a cylinder*, treated as if the other end did not exist.

Just how the high-pressure fluid from a source of supply, such as a boiler or an air compressor, is introduced into one end of a cylinder, how it is treated after it gets there, and how expelled, will determine the nature of the variation in pressure which acts on that side of the piston: these are subjects to be studied. To determine the work done in the cylinder end by the fluid, it is necessary to determine laws of pressure change with stroke, and these are fixed, first, by valve action controlling the distribution of the fluid with respect to the piston; and second, by the physical properties of the fluid in question.

It is necessary that the cylinder be fitted with a valve for getting fluid into a cylinder, isolating the charge from the source of supply and getting it out again; it may be that one valve will do, or that two or even more are desirable but this is a structural matter, knowledge of which is assumed here, and not concerned with the effects under investigation. The first step in the process is the admission of fluid from the source of supply to the cylinder at one end, which may continue for the whole, or be limited to a part

of the stroke. When admission ceases or supply is cut off before the end of the stroke there will be in the cylinder an isolated mass of fluid which will expand as the piston proceeds to the end. Thus the forward stroke, considering one side of the piston only, always consists of full pressure admission followed by expansion which may vary from zero to a very large amount; in fact the final volume of the fluid after expansion may be hundreds of times as great as at its beginning, when supply was cut off.

At the end of this forward stroke an exhaust valve is opened, which permits communication of the cylinder with the atmosphere in non-condensing steam and most compressed-air engines, or with another cylinder, or with a storage chamber, or with a condenser in the case of a steam engine in which the pressure approximates a perfect vacuum. If at the moment of exhaust opening the cylinder pressure is greater or less than the back pressure, there will be a more or less quick equalization either up or down before the piston begins to return, after which the return or exhaust stroke will proceed with some back-pressure resistance acting on the piston, which is generally though not always constant. This may last for the whole back stroke or for only a part, as determined by the closure of the exhaust valve. When the exhaust valve closes before the end of the return stroke the unexpelled steam will be trapped and compressed to a pressure depending partly on the point of the stroke when closure begins and the pressure at the time; and partly on the clearance volume of the cylinder into which the trapped steam is compressed. At any time near the end of the stroke the admission valve may be opened again, and this may occur, 1st, before compression has reached initial pressure, which will result in a sudden pressure rise in the cylinder before the end of the stroke equalizing it with the source of supply; 2d, just at or before the end of the stroke, which may result in a rise or a fall to equalize or no change at all, depending on whether compression has raised the cylinder pressure not quite equal to supply pressure, or to something greater, or to a value just equal to it; 3d, after the end of the stroke, which will result in a re-expansion of the steam previously under compression followed by a rise in pressure which may or may not be sudden.

It may be said that in general the following processes are carried out in cylinders of piston engines with more or less variation:

Forward stroke, constant-pressure admission followed by expansion.

Back stroke, constant-pressure exhaust followed by compression, while at both ends of the stroke there may or may not be a vertical line on a pressure-volume diagram representing a constant-volume change of pressure.

These processes will result in a cycle of pressure-volume changes which will be a closed curve made of more or less accurately defined phases, and the work of the cycle will be the area enclosed by the cyclic curve. There are causes of disturbance which make the phases take on peculiar characteristics. For example, the valve openings and closures may not take place as desired or as presupposed with respect to piston positions; leakage may occur, steam may condense during the operations in the cylinder, and water of condensation

may evaporate; the resistance through valves will always make the cylinder pressure during admission less than that in the supply chamber, and greater during exhaust than the atmosphere or than the pressure in exhaust receiver or condenser; and may, through the valve movements, make what might have been a constant-pressure straight line become a curve. By reason of these influences, encountered in real engines, there will be an almost infinite variety of indicator cards or pressure-volume cycles for such engines.

The various points of the stroke at which important events occur, important in their pressure-volume significance, have names, as do also the

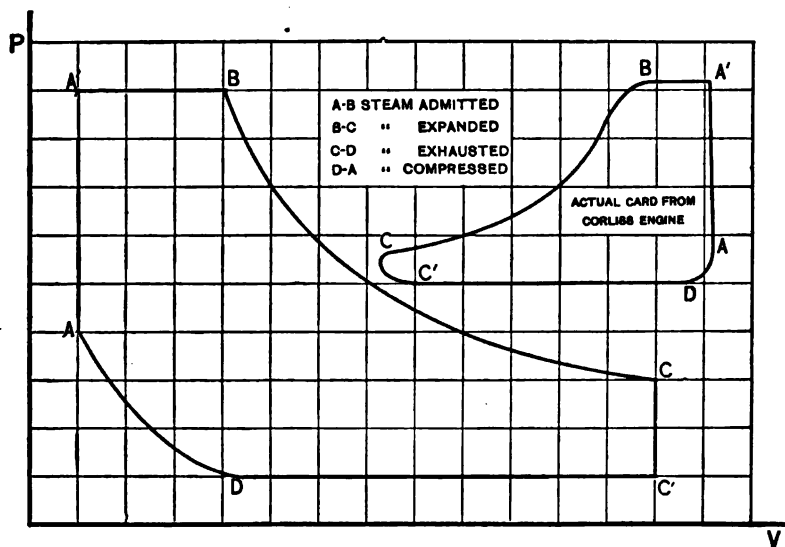


FIG. 35.—Diagram to Indicate Position of Admission, Cut-off, Release, Compression on Engine Indicator Card.

lines between the points, and these names are more or less commonly accepted and generally understood as follows: Letters refer to the diagram Fig. 35.

Point Names: Events of Cycle.

- A. *Admission* is that point of the stroke where the supply valve is opened.
- B. *Cut-off* is that point of the stroke where the supply valve is closed.
- C. *Release* is that point of the stroke where the exhaust valve is opened.
- D. *Compression* is that point of the stroke where the exhaust valve is closed.

Names of Lines, or Periods:

- A-B. *Admission* or *steam line* joins the points of admission and cut-off.
- B-C. *Expansion line* joins the points of cut-off and release.
- C-D. *Exhaust line* joins the points of release and compression if there is any, or admission if there is not.
- D-A. *Compression line* joins the points of compression and admission.

By reason of the interferences discussed, these points on actual indicator cards may be difficult to locate, one line merging into the next in so gradual a manner as to make it impossible to tell where the characteristic point lies, as will be apparent from Fig. 36, in which is reproduced a number of actual indicator cards. In such cases equivalent points must be located for study.

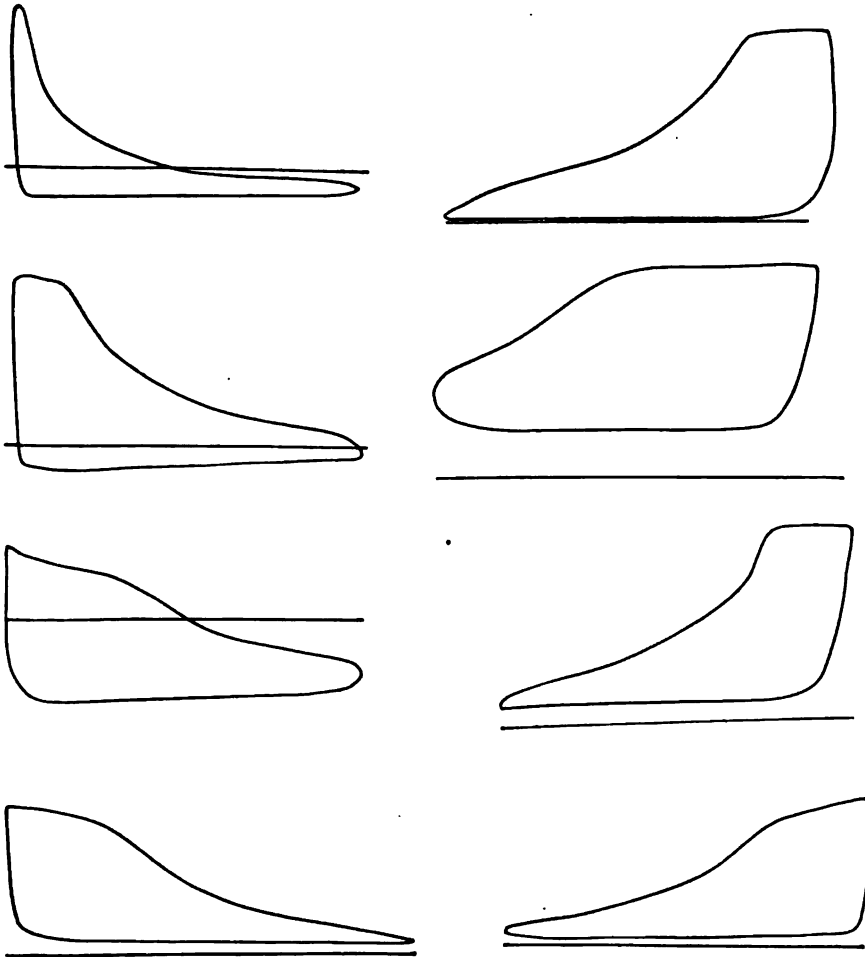


FIG. 36.—Actual Steam Engine Indicator Cards Showing Distortion of Lines and Uncertain Location of Characteristic Points.

These same terms, which it appears sometimes refer to points and sometimes to lines, are also used in other senses, for example, *cut-off* is commonly used to mean the fraction of stroke completed up to the point of cut-off, and *compression* that fraction of stroke remaining incomplete at the point of compression, while compression is also sometimes used to express the pressure attained at the end of the compression line. In general, there

is nothing in the use of these words to indicate just which of the various meanings is intended except the text, and experience will soon eliminate most of the possible chances of confusion.

Prob. 1. Draw a card in which admission and exhaust are late. Draw a card in which there is no compression; in which compression is very early; so that compression pressure is equal to admission pressure. Draw a card with 0 per cent cut-off; and cut-off = 100 per cent. Draw cards with same cut-off but with varying initial pressures.

Prob. 2. The following diagrams, Fig. 37, are reproductions of indicator cards actually taken from engines. Explain what features are peculiar to each and give the probable cause.

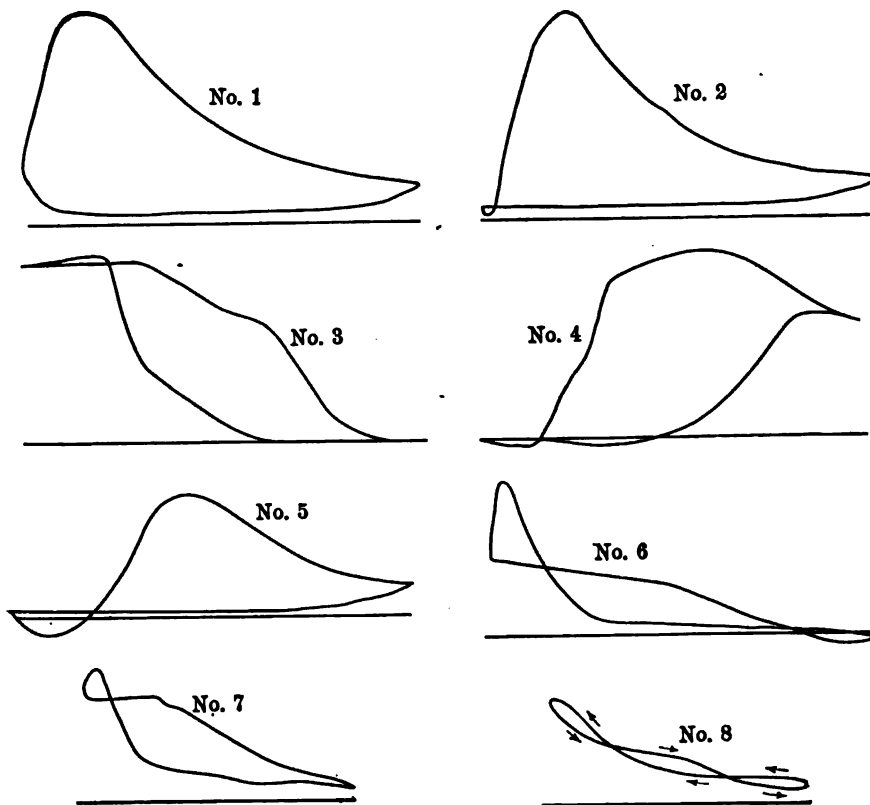


FIG. 37.—Indicator Diagrams from Steam Engines with Improperly Set Valve Gear.

For example, in No. 1 a line of pressure equalization between the end of the compression line and the beginning of the admission line inclines to the right instead of being perpendicular, as in a perfect diagram. This is due to the fact that admission does not occur until after the piston has begun to move outward, so that pressure rise does not occur at constant volume, but during a period of increasing volume.

38. Standard Reference Cycles or PV Diagrams for the Work of Expansive Fluids in a Single Cylinder; Simple Engines. To permit of the derivation of a formula for the work of steam, compressed air, or any other fluid in a

cylinder, the various pressure-volume changes must be defined algebraically. The first step is, therefore, the determination of the cycle or pressure-volume diagram representative of the whole series of processes and consisting of a number of well-known phases or single processes. These phases, ignoring all interferences due to leakage or improper valve action, will consist of constant-pressure and constant-volume lines representing fluid movement into or from the cylinder, combined with expansion and compression lines representing changes of condition of the fluid isolated in the cylinder. These *expansion and compression lines* represent strictly thermal phases, laws for which will be assumed here, but will be derived later in the part treating of the thermal analysis; however, all cases can be represented by the *general expression*

$$PV^s = C,$$

in which the character of the case is fixed by fixing the value of s . For all gases, and for vapors that do not contain liquid or do not form or evaporate any during expansion or compression, i.e., are continually superheated, the exponent s may have one of two *characteristic values*. The first is *isothermal* expansion and compression, and for this process s is the same for all substances and is equal to unity. The second is for *exponential* expansion or compression and for this process s will have values peculiar to the gas or superheated vapor under discussion, but it is possible that more than one substance may have the same value, as may be noted by reference to Table X in the Handbook of Tables, from which the values $s=1.406$ for air; and $s=1.3$ for superheated steam, and 1.3 also for ammonia expanding adiabatically, are selected for illustration.

When steam, or any other vapor not so highly superheated as to remain free from moisture during treatment, is expanded or compressed in cylinders, different values of s must be used to truly represent the process and, of course, there can be no isothermal value, since *there can be no change of pressure of wet vapors without a change of temperature*. For steam expanding adiabatically the value of s is not a constant, as will be proved later by thermal analysis, so that the exact solution of problems of adiabatic expansion of steam under ordinary conditions becomes impossible by pressure-volume analysis and can be solved only by thermal analysis. However, it is sometimes convenient or desirable to find a solution that is approximately correct, and for this an assumed average value for s may be taken. Rankine's average value is $s=1.111=\frac{1}{9}$ for adiabatic expansion of ordinarily wet steam, and while other values have been suggested from time to time this is as close as any and more handy than most. The value $s=1.035+.14\times(\text{the original dryness fraction})$, is given by Perry to take account of the variation in original moisture.

Steam during expansion adiabatically, tends to make itself wet, the condensation being due to the lesser heat content by reason of the work done; but if during expansion heat be added to steam originally just dry, to keep it so continuously as the expansion proceeds, it may be said to follow the *saturation law* of steam, for which $s=1.0646$. This is a strictly experimental value found

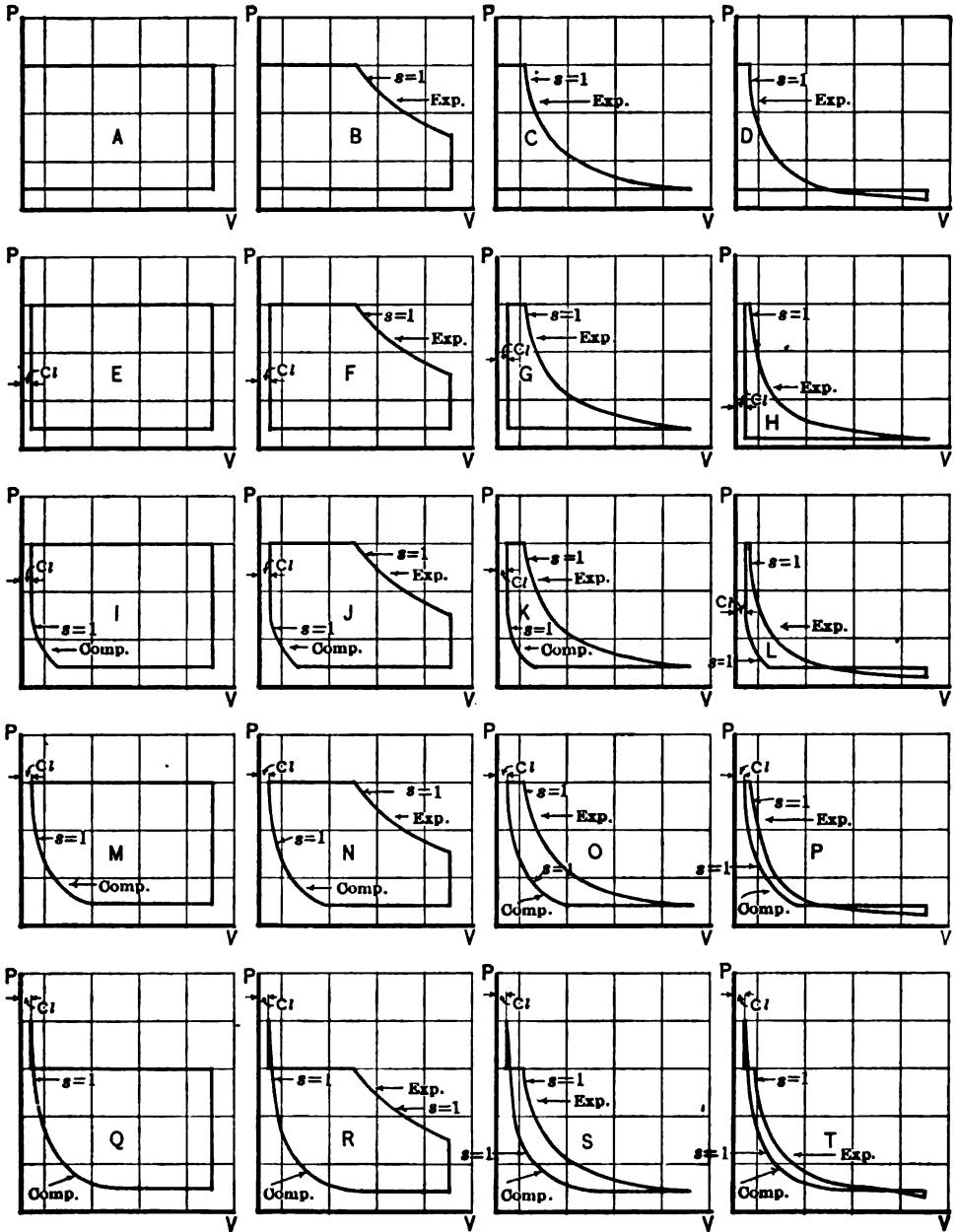


FIG. 38.—Standard Reference Cycles or Pressure-volume Diagrams for Expansive Fluid in Simple Engines.

by studying the volume occupied by a pound of just-dry steam at various pressures, quite independent of engines.

Direct observation of steam engine indicator cards has revealed the fact that while, in general, the pressure falls faster at the beginning of expansion and slower at the end than would be the case if $s=1$, yet the total work is about the same as if s had this value all along the curve. This law of expansion and compression, which may be conveniently designated as the *logarithmic law*, is almost universally accepted as representing about what happens in actual steam engine cylinders. Later, the thermal analysis will show a variation of wetness corresponding to $s=1$, which is based on no thermal hypothesis whatever, but is the result of years of experience with exact cards. Curiously

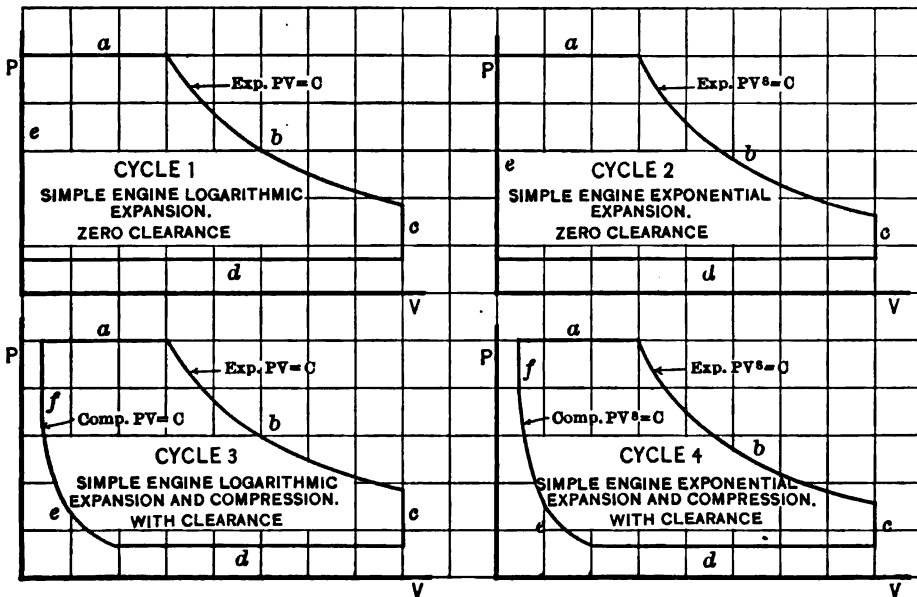


FIG. 39.—Simple Engine Reference, Cycles or PV Diagrams.

enough, this value of s is the same as that obtained from the thermal analysis of constant temperature or isothermal expansion for gases, but it fails entirely to represent the case of isothermal expansion for steam. That $s=1$ for isothermal gas expansion and actual steam cylinder expansion is a mere coincidence, a fact not understood by many authors of books on the subject in which it is spoken of as the isothermal curve for steam, and that it most positively is not. This discussion of the expansion or compression laws indicates that analysis falls into two classes; first, that for which $s=1$, which yields a logarithmic expression for work, and second, that for which s is greater or less than one, which yields an exponential expression for work; the former will be designated as the *logarithmic* and the latter as the *exponential* laws, for convenience.

The phases to be considered then, may be summed up as far as this analysis is concerned as:

1. Admission or exhaust, pressure constant, $P = \text{const.}$
2. Admission or exhaust, volume constant, $V = \text{const.}$
3. Expansion, $PV = \text{const.}$, when $s = 1$.
4. Expansion, $PV^s = \text{const.}$, when s is greater or less than 1.
5. Compression, $PV = \text{const.}$, when $s = 1$.
6. Compression $PV^s = \text{const.}$, when s is greater or less than 1.

Considering all the possible variations of phases, there may result any of the cycles represented by Fig. 38. These cycles have the characteristics indicated by the following table, in which any possible variation in the law of the expansion or compression is noted.

Cycle.	Clearance.	Expansion.	Compression.
<i>A</i>	Zero	Zero	Zero
<i>B</i>	Zero	Little	Zero
<i>C</i>	Zero	Complete	Zero
<i>D</i>	Zero	Over-expansion	Zero
<i>E</i>	Little	Zero	Zero
<i>F</i>	Little	Little	Zero
<i>G</i>	Little	Complete	Zero
<i>H</i>	Little	Over-expansion	Zero
<i>I</i>	Little	Zero	Little
<i>J</i>	Little	Little	Little
<i>K</i>	Little	Complete	Little
<i>L</i>	Little	Over-expansion	Little
<i>M</i>	Little	Zero	Complete
<i>N</i>	Little	Little	Complete
<i>O</i>	Little	Complete	Complete
<i>P</i>	Little	Over-expansion	Complete
<i>Q</i>	Little	Zero	Too much
<i>R</i>	Little	Little	Too much
<i>S</i>	Little	Complete	Too much
<i>T</i>	Little	Over-expansion	Too much

It is not necessary, however, to derive algebraic expressions for all these cases, since a few general expressions may be found involving all the variables in which some of them may be given a zero value and the resulting expression will apply to those cycles in which that variable does not appear. The resulting cycles, Fig. 39, that it is convenient to treat, are as follows:

SIMPLE ENGINE REFERENCE CYCLE OR PV DIAGRAMS

CYCLE 1. Simple Engine, Logarithmic Expansion without Clearance.

Phase (a) Constant-pressure admission.

“ (b) Expansion $PV = \text{const.}$ (may be absent).

“ (c) Constant-volume equalization of pressure with exhaust (may be absent).

“ (d) Constant-pressure exhaust.

“ (e) Constant = (zero) volume admission equalization of pressure with supply.

CYCLE II. Simple Engine, Exponential Expansion without Clearance.

Phase (a) Constant-pressure admission.

" (b) Expansion $PV^n = \text{const.}$ (may be absent).

" (c) Constant-volume equalization of pressure with exhaust (may be absent).

" (d) Constant-pressure exhaust.

" (e) Constant-(zero) volume admission equalization of pressure with supply.

CYCLE III. Simple Engine, Logarithmic Expansion and Compression with Clearance.

Phase (a) Constant-pressure admission.

" (b) Expansion $PV = \text{const.}$ (may be absent).

" (c) Constant-volume equalization of pressure with exhaust (may be absent).

" (d) Constant-pressure exhaust.

" (e) Compression $PV = \text{const.}$ (may be absent).

" (f) Constant-volume admission, equalization of pressure with supply (may be absent).

CYCLE IV. Simple Engine, Exponential Expansion and Compression with Clearance.

Phase (a) Constant-pressure admission.

" (b) Expansion $PV^n = \text{const.}$ (may be absent).

" (c) Constant-volume equalization of pressure with exhaust (may be absent).

" (d) Constant-pressure exhaust.

" (e) Compression $PV^n = \text{const.}$ (may be absent).

" (f) Constant-admission, equalization of pressure with supply (may be absent).

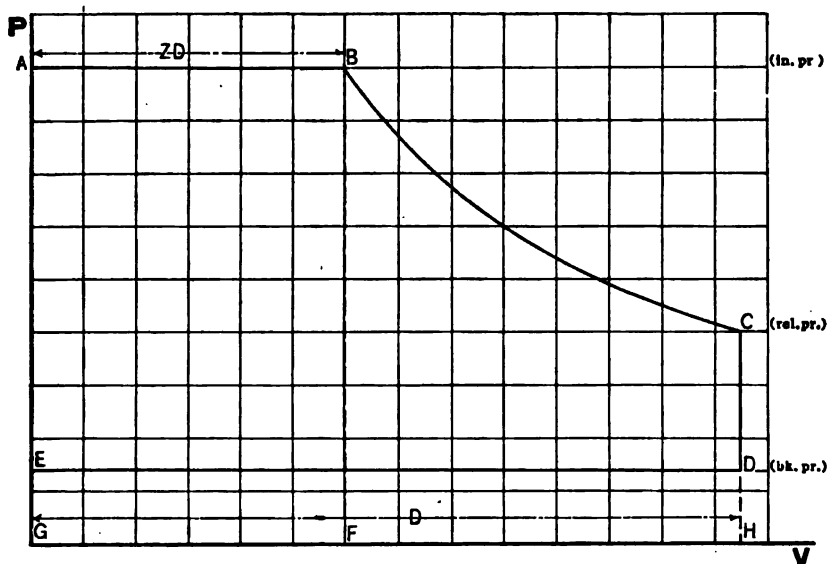


FIG. 40.—Work of Expansive Fluid in Single Cylinder with No Clearance. Logarithmic Expansion for Cycle I. Exponential for Cycle II.

39. Work of Expansive Fluid in Single Cylinder without Clearance. Logarithmic Expansion, Cycle I. Mean Effective Pressure, Horse-power and Consumption of Simple Engine. Referring to the diagram, Fig. 40, the *net work*, whether expansion be incomplete, perfect, or excessive, is the sum of

All these expressions, Eqs. (154) to (159), for the work of the cycle; the mean effective pressure; work per cubic foot of fluid supplied; cubic feet and pounds of fluid supplied per hour per I.H.P., are in terms of diagram point conditions and must be transformed so as to be read in terms of more generally defined quantities for convenience in solving problems. The first step is to introduce quantities representing supply and back pressures and the amount of expansion, accordingly:

Let (in.pr.) represent the initial or supply pressure p_b expressed in pounds per square inch;

(rel.pr.) represent the release pressure p_c , in pounds per square inch;

(bk.pr.) represent the back pressure p_d , in pounds per square inch;

R_v represent the ratio of expansion defined as the ratio of largest to smallest volume on the expansion line $\left(\frac{V_c}{V_b}\right)$, or $\left(\frac{V_d}{V_b}\right)$ which is, of

course, equal to the ratio of supply to release pressure $\left(\frac{p_b}{p_c}\right)$, when the logarithmic law is assumed;

D represent the displacement in cubic feet which is V_d or V_c when no clearance is assumed;

Let Z represent the fraction of stroke or displacement completed up to cut-off so that ZD represents the volume V_b admitted to the cylinder.

In this case when clearance is zero, $Z = \frac{1}{R_v}$.

Work of the cycle in foot-pounds

$$W = 144 \left[(\text{in.pr.}) \frac{1 + \log_e R_v}{R_v} - (\text{bk.pr.}) \right] D \quad (a) \quad \left. \vphantom{W = 144} \right\} \dots (160)$$

$$= 144 [(\text{rel.pr.})(1 + \log_e R_v) - (\text{bk.pr.})] D \quad (b)$$

$$\text{m.e.p.} = (\text{rel.pr.})(1 + \log_e R_v) - (\text{bk.pr.}) \quad (a) \quad \left. \vphantom{\text{m.e.p.}} \right\} \dots (161)$$

$$= (\text{in.pr.}) \left(\frac{1 + \log_e R_v}{R_v} \right) - (\text{bk.pr.}) \quad (b)$$

$$= (\text{in.pr.}) Z \left(1 + \log_e \frac{1}{Z} \right) - (\text{bk.pr.}) \quad (c)$$

$$\text{Work per cu.ft. supplied} = 144 [(\text{in.pr.})(1 + \log_e R_v) - (\text{bk.pr.}) R_v] \quad (a) \quad \left. \vphantom{\text{Work per cu.ft. supplied}} \right\} \dots (162)$$

$$= 144 \left[(\text{in.pr.}) \left(1 + \log_e \frac{1}{Z} \right) - \frac{(\text{bk.pr.})}{Z} \right] \quad (b)$$

$$\text{Cu.ft. supplied per hr. per I.H.P.} = \frac{13,750}{\text{m.e.p.}} \frac{1}{R_v} \quad (a) = \frac{13,750}{(\text{m.e.p.})} Z \quad (b) \quad (163)$$

$$\text{Lbs. supplied per hr. per I.H.P.} = \frac{13,750}{(\text{m.e.p.})} \frac{\delta_1}{R_v} \quad (a) = \frac{13,750}{(\text{m.e.p.})} Z \delta_1 \quad (b) \quad (164)$$

The indicated horse-power may be found by multiplying the work of the

cycle, Eq. (160), by the number of cycles performed per minute n and dividing the product by 33,000.

$$\text{I.H.P.} = \frac{Dn}{229.2} \left[(\text{in.pr.}) \left(\frac{1 + \log_e R_v}{R_v} \right) - (\text{bk.pr.}) \right] \quad \text{or} \quad \text{I.H.P.} = \frac{Dn(\text{m.e.p.})}{229.2} \quad (165)$$

In any of these expressions where R_v is the ratio of greatest to smallest volume during expansion, either R_p , ratio of greater to smaller pressures, or $\frac{1}{Z}$, the reciprocal of the cut-off, may be substituted, since the expressions apply only to the logarithmic law, and clearance is assumed equal to zero. When clearance is not zero; it is shown later that the cut-off as a fraction of stroke is not the reciprocal of R_p or R_v .

These expressions are perfectly general, but convenience in calculation will be served by deriving expressions for certain special cases. The first of these is the case of no expansion at all, the second that of complete expansion without over-expansion. This latter gives the most economical operation from the hypothetical standpoint, because no work of expansion has been left unaccomplished and at the same time no negative work has been introduced by over-expansion.

First Special Case. If there is no expansion, and no clearance, the diagram takes the form of a rectangle, and

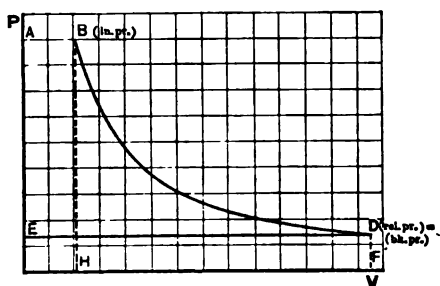


FIG. 41.—Second Special Case of Cycles I and II. Complete Expansion Without Over-expansion—Case of Best Cut-off.

$$W = 144D[(\text{in.pr.}) - (\text{bk.pr.})] \quad (166)$$

$$\text{m.e.p.} = (\text{in.pr.}) - (\text{bk.pr.}) \quad (167)$$

$$\begin{aligned} \text{Work per cu.ft. supplied} \\ = 144[(\text{in.pr.}) - (\text{bk.pr.})] \quad (168) \end{aligned}$$

$$\begin{aligned} \text{Cu.ft. supplied per hr. per I.H.P.} \\ = \frac{13,750}{(\text{in.pr.}) - (\text{bk.pr.})} \quad (169) \end{aligned}$$

$$\begin{aligned} \text{Lbs. supplied per hr. per I.H.P.} \\ = \frac{13,750 \delta_1}{(\text{in.pr.}) - (\text{bk.pr.})} \quad (170) \end{aligned}$$

Second Special Case. When the expansion is complete without over-expansion, no clearance, the points C and D, Fig. 41, coincide, and $(\text{rel.pr.}) = (\text{bk.pr.})$, hence $R_v = R_p = \frac{(\text{in.pr.})}{(\text{bk.pr.})} = \frac{1}{Z}$. This value of cut-off, Z , is known as best cut-off, as it is that which uses all the available energy of the fluid by expansion.

$$W = 144D(\text{in.pr.}) \frac{\log_e R_p}{R_p} = 144D(\text{bk.pr.}) \log_e R_p \quad (171)$$

$$(\text{m.e.p.}) = (\text{in.pr.}) \frac{\log_e R_p}{R_p} \quad (172)$$

$$\text{Work per cu.ft. supplied} = 144(\text{in.pr.}) \log_e R_p \quad (173)$$

$$\text{Cu.ft. supplied per hr. per I.H.P.} = \frac{13,750}{(\text{in.pr.}) \log_e R_P} \quad \dots \quad (174)$$

$$\text{Lbs. supplied per hr. per I.H.P.} = \frac{13,750 \delta_1}{(\text{in.pr.}) \log_e R_P} \quad \dots \quad (175)$$

Example 1. Method of calculating diagrams. Fig. 40 and Fig. 41.

Assumed data for Fig. 40. $P_a = P_b = 90$ lbs. per sq.in. abs. $V_a = V_e = 0$ cu.ft.

$$P_d = P_e = 14 \text{ lbs. per sq.in. abs.} \quad V_e = V_d = 13.5 \text{ cu.ft.} \quad V_b = 6 \text{ cu.ft.}$$

$$\text{To obtain point } C: \quad P_c = P_b \times \frac{V_b}{V_c} = 90 \times \frac{6}{13.5} = 40 \text{ lbs. per sq.in. abs.}$$

Assumed data for Fig. 41. $P_a = P_b = 90$ lbs. per sq.in. $V_a = V_e = 0$ cu.ft.

$$P_d = P_e = 14 \text{ lbs. per sq.in.} \quad V_d = 13.5 \text{ cu.ft.}$$

$$\text{To obtain point } B: \quad V_b = V_d \times \frac{P_d}{P_b} = 13.5 \times \frac{14}{90} = 2.1 \text{ cu.ft.}$$

Example 2. A simple double-acting engine admits steam at 100 lbs. per square inch absolute for $\frac{1}{2}$ stroke, allows it to expand to the end of the stroke and then exhausts it against a back pressure of 5 lbs. per square inch absolute. If the engine has no clearance, a 7×9 -in. cylinder and runs at 300 R.P.M., what is the horse-power and steam consumption when steam is expanding according to the logarithmic law? Note: 1 cu.ft. steam at 100 lbs. per square inch absolute weighs .2258 lb.

$$\text{From Eq. (161b), m.e.p.} = (\text{in.pr.}) \left(\frac{1 + \log_e R_V}{R_V} \right) - (\text{bk.pr.})$$

$$= 100 \frac{(1 + \log_e 4)}{4} - 5 = 100 \frac{(1 + 1.386)}{4} - 5 = 54.7 \text{ lbs.} \quad \therefore$$

$$\text{I.H.P.} = \frac{(\text{m.e.p.}) L a n}{33,000} = \frac{54.7 \times .75 \times 38.5 \times 600}{33,000} = 28;$$

$$\text{or directly from Eq. (165), I.H.P.} = \frac{D n (\text{m.e.p.})}{229.2}, \quad = \frac{.2 \times 600 \times 54.7}{229.2} = 28;$$

$$\text{Lbs. steam per I.H.P.} = \frac{13,750}{\text{m.e.p.}} \times \frac{\delta_1}{R_V}, \quad = \frac{13,750}{54.7} \times \frac{.2258}{4} = 14.13.$$

Therefore, steam per hour used by engine $= 14.13 \times 28 = 396$ lbs.

Prob. 1. A steam engine has a cylinder 12×18 ins. with no clearance. It runs at 200 R.P.M. and is double-acting. If the steam pressure be fixed at 100 lbs. per square inch absolute, and the back pressure at 10 lbs. per square inch abs., show how the horse-power and steam consumption will vary as cut-off increases. Take cut-off from $\frac{1}{4}$ to $\frac{3}{4}$ by eighths. Plot.

Prob. 2. Two engines of the same size and design as above are running on a steam pressure of 100 lbs. per square inch absolute, but one exhausts through a long pipe to the atmosphere, the total back pressure being 20 lbs. per square inch absolute, while the other exhausts into a condenser in which the pressure is but 3 lbs. per square inch absolute. If the cut-off is in each case $\frac{3}{4}$, how will the I.H.P. and steam used in the two cases vary?

Prob. 3. By finding the water rate and the horse-power in the two following cases, show the saving in steam and loss in power due to using steam expansively. A pump having a cylinder 9×12 ins. admits steam full stroke, while an engine of same size admits it but $\frac{1}{4}$ of the stroke; both run at the same speed and have the same back pressure.

Prob. 4. Steam from a 12×24 -in. cylinder is exhausted at atmospheric pressure (15 lbs. per square inch absolute) into a tank, from which a second engine takes steam. Neither engine has clearance. The first engine receives steam at 100 lbs. per square inch absolute and the cut-off is such as to give complete expansion. The second engine exhausts into a 24-in. vacuum and its cut-off is such that complete expansion occurs in its cylinder. Also the cylinder volume up to cut-off equals that of the first cylinder at exhaust. If the stroke is the same in both engines and the speed of each is 200 R.P.M., what is the diameter of the larger cylinder, the total horse-power developed, the total steam used, and the work per cubic foot of steam admitted to the first cylinder, the water rate of each engine and the total horse-power derived from each pound of steam?

Prob. 5. The steam pressure for a given engine is changed from 80 lbs. per square inch gage to 120 lbs. per square inch gage. If the engine is 12×16 ins., running 250 R.P.M. with a fixed cut-off of 25 per cent and no clearance, the back pressure being 15 lbs. per square inch absolute, what will be the horse-power and the water rate in each case?

NOTE: 1 cu.ft. of steam at 80 and 120 lbs. weighs .215 and .3 lb. respectively.

Prob. 6. By trial, find how much the cut-off should have been shortened to keep the H.P. constant when the pressure was increased and what effect this would have on the water rate.

Prob. 7. A certain type of automobile engine uses steam at 600 lbs. per square inch absolute pressure. The exhaust is to atmosphere. For a cut-off of $\frac{1}{4}$ and no clearance, what would be the water rate?

NOTE: for 600 lbs. $\delta_1 = 1.32$.

Prob. 8. Engines are governed by throttling the initial pressure or shortening the cut-off. The following cases show the effect of light load on economy. Both engines, 12×18 ins., running at 200 R.P.M., with 125 lbs. per square inch absolute initial pressure, and back pressure of 10 lbs. per square inch absolute. The load is sufficient to require full steam pressure at $\frac{1}{4}$ cut-off for each engine. Load drops to a point where the throttle engine requires but 50 lbs. per square inch absolute initial pressure with the cut-off still fixed at $\frac{1}{4}$. What is the original load and water rate, and new load and water-rate for each engine?

NOTE: δ for 125 lbs. absolute = .279 and for 50 lbs. = .117 lb.

Prob. 9. The guarantee for a simple engine 18×24 ins., running at 200 R.P.M., states that the I.W.R. when cut-off is $\frac{1}{4}$ will not exceed 15 lbs. if the initial pressure be 100 lbs. per square inch gage, and back pressure 5 lbs. per square inch absolute. If engine has no clearance, see if this would be possible.

40. Work of Expansive Fluid in Single Cylinder without Clearance. Exponential Expansion Cycle II. Mean Effective Pressure, Horse-power and Consumption of Simple Engines. Referring to the diagram, Fig. 40, the work is given by the same areas as for Cycle I, but its algebraic expression is different because s is greater than 1 and an exponential expansion results on integration instead of a logarithmic one.

In general, from Eq. (13a), Section 7, Chapter I,

$$W = P_b V_b + \frac{P_c V_c}{s-1} \left[\left(\frac{V_c}{V_b} \right)^{s-1} - 1 \right] - P_d V_d. \quad (176)$$

Putting this in terms of initial conditions by the relations

$$\left(\frac{P_b}{P_c} \right) = \left(\frac{V_c}{V_b} \right)^s = R_v^s \text{ and } V_c = V_d = V_b R_v,$$

also

$$P_c V_c = P_b V_b \left(\frac{V_b}{V_c} \right)^{s-1} = \frac{P_b V_b}{R_v^{s-1}},$$

there results $W = P_b V_b + \frac{P_b V_b}{(s-1)R_v^{s-1}} (R_v^{s-1} - 1) - P_d V_b R_v$

$$\begin{aligned} &= P_b V_b \left[1 + \frac{1}{(s-1)R_v^{s-1}} (R_v^{s-1} - 1) \right] - P_d V_b R_v. \\ W &= P_b V_b \left(\frac{s}{s-1} - \frac{1}{(s-1)R_v^{s-1}} \right) - P_d V_b R_v \quad (a) \\ &= 144D \left[\frac{(\text{in.pr.})}{R_v} \left(\frac{s}{s-1} - \frac{1}{(s-1)R_v^{s-1}} \right) - (\text{bk.pr.}) \right] \quad (b) \\ &= 144D \left[Z (\text{in.pr.}) \frac{s-Z^{s-1}}{s-1} - (\text{bk.pr.}) \right] \quad (c) \end{aligned} \quad (177)$$

which is the general equation for work of this cycle.

Dividing by V_b , the volume of fluid supplied,

$$\begin{aligned} \text{Work per cu.ft. supplied} &= P_b \left(\frac{s}{s-1} - \frac{1}{(s-1)R_v^{s-1}} \right) - P_d R_v \quad (a) \\ &= 144(\text{in.pr.}) \left(\frac{s}{s-1} - \frac{1}{(s-1)R_v^{s-1}} \right) - (\text{bk.pr.}) R_v \quad (b) \end{aligned} \quad (178)$$

Similarly, the *mean effective pressure* results from dividing the work by the displacement, $V_d = V_b R_v$.

$$\begin{aligned} \text{M.E.P.} &= \frac{P_b}{R_v} \left(\frac{s}{s-1} - \frac{1}{(s-1)R_v^{s-1}} \right) - P_d \quad (a) \\ \text{or } (\text{m.e.p.}) &= \frac{(\text{in.pr.})}{R_v} \left(\frac{s}{s-1} - \frac{1}{(s-1)R_v^{s-1}} \right) - (\text{bk.pr.}) \quad (b) \\ &= Z(\text{in.pr.}) \left(\frac{s-Z^{s-1}}{s-1} \right) - (\text{bk.pr.}) \quad (c) \end{aligned} \quad (179)$$

First special case of no expansion, when $R_v = 1$, gives the same diagram as in the previous section, and exactly the same set of formulas.

Second special case when the expansion is complete without over-expansion, is again represented by Fig. 41 and for it

$$P_d = P_b \left(\frac{V_b}{V_d} \right)^s = \frac{P_b}{R_v^s}.$$

Whence

$$W = P_b V_b \left(\frac{s}{s-1} - \frac{1}{(s-1)R_v^{s-1}} - \frac{1}{R_v^s} \right), \quad \text{or}$$

$$\left. \begin{aligned} \text{Work for complete expansion is } W &= P_b V_b \frac{s}{s-1} \left(1 - \frac{1}{R_v^{s-1}} \right) & (a) \\ &= 144 \frac{(\text{in.pr.}) D}{R_v} \frac{s}{s-1} \left(1 - \frac{1}{R_v^{s-1}} \right) & (b) \end{aligned} \right\}, \quad (180)$$

which is the general equation for the work of V_b or $\left(\frac{D}{R_v}\right)$ cubic feet of fluid when the economy is best or for best cut-off.

The work per cubic foot of fluid supplied for this case of *complete expansion* gives the maximum value for Eq. (178) and is obtained by dividing Eq. (180) by V_b or $\left(\frac{D}{R_v}\right)$.

$$\left. \begin{aligned} \text{Max. work per cu.ft. supplied} &= P_b \frac{s}{s-1} \left(1 - \frac{1}{R_v^{s-1}} \right) & (a) \\ &= 144 (\text{in.pr.}) \frac{s}{s-1} \left(1 - \frac{1}{R_v^{s-1}} \right) & (b) \end{aligned} \right\}, \quad (181)$$

which is the general equation for maximum work per cubic foot of fluid supplied.

The expression for *mean effective pressure becomes for this case of best cut-off*,

$$\left. \begin{aligned} \text{M.E.P.} &= \frac{P_b}{R_v} \frac{s}{s-1} \left(1 - \frac{1}{R_v^{s-1}} \right) & (a) \\ \text{or (m.e.p.)} &= \frac{(\text{in.pr.})}{R_v} \frac{s}{s-1} \left(1 - \frac{1}{R_v^{s-1}} \right) & (b) \end{aligned} \right\} \quad (182)$$

It is convenient to note that in using Eqs. (180), (181) and (182) it may be desirable to evaluate them without first finding R_v . Since

$$R_v = \frac{V_c}{V_b} = \left(\frac{P_b}{P_c} \right)^{\frac{1}{s}} = R_p^{\frac{1}{s}},$$

this substitution may be made, and $R_v^{s-1} = R_p^{\frac{s-1}{s}}$.

Example. Compare the horse-power and the steam consumption of a 9×12-in. simple double-acting engine with no clearance and running at 250 R.P.M. when initial pressure is 100 lbs. per square inch absolute and cut-off is $\frac{1}{4}$, if

- (a) steam remains dry and saturated throughout expansion,
- (b) remains superheated throughout expansion, and
- (c) if originally dry and suffers adiabatic expansion.

Back pressure is 10 lbs. per square inch absolute.

$$(\text{m.e.p.}) = \frac{(\text{in.pr.})}{R_v} \left(\frac{s}{s-1} - \frac{1}{(s-1)R_v^{s-1}} \right) - (\text{bk.pr.}).$$

$$\text{For case (a) } s=1.0646 \text{ and } (\text{m.e.p.}) = \frac{100}{4} \left(\frac{1.0646}{.0646} - \frac{1}{.0646 \times 4^{.0646}} \right) - 10 = 48.6;$$

$$(b) \ s=1.3 \text{ and } (\text{m.e.p.}) = \frac{100}{4} \left(\frac{1.3}{.3} - \frac{1}{.3 \times 4^{.3}} \right) - 10 = 43.5;$$

$$(c) \ s=1.111 \text{ and } (\text{m.e.p.}) = \frac{100}{4} \left(\frac{1.111}{.111} - \frac{1}{.111 \times 4^{.111}} \right) - 10 = 47.5.$$

$$\text{I.H.P.} = (\text{m.e.p.}) L a n = \frac{(\text{m.e.p.}) 1.0 \times 63.6 \times 500}{33,000} = .965 \text{ m.e.p.}$$

∴ I.H.P. for case (a) = 46.9, for case (b) = 42.0, for case (c) = 45.8.

$$\text{From Eq. (164), lbs. steam per hour per I.H.P.} = \frac{13,750}{\text{m.e.p.}} \times \frac{\delta_1}{R_v}$$

$$\therefore \text{For case (a) steam per hr.} = 46.9 \times \frac{13,750}{48.6} \times \frac{\delta_1}{4};$$

$$(b) \text{ steam per hr.} = 42 \times \frac{13,750}{43.5} \times \frac{\delta_1}{4};$$

$$(c) \text{ steam per hr.} = 45.8 \times \frac{13,750}{47.5} \times \frac{\delta_1}{4}.$$

Prob. 1. On starting a locomotive steam is admitted full stroke, while in running the valve gear is arranged for $\frac{1}{2}$ cut-off. If the engine were 18×30 ins., initial pressure 150 lbs. per square inch absolute, back pressure 15 lbs. per square inch absolute, what would be the difference in horse-power with the gear in normal running position and in the starting position for a speed of 20 miles per hour with 6-ft. driving wheels? Consider the steam to be originally dry and expanding adiabatically. What would be the difference in steam per horse-power hour for the two cases and the difference in total steam? Clearance neglected.

Prob. 2. Consider a boiler horse-power to be 30 lbs. of steam per hour; what must be the horse-power of a boiler to supply the following engine? Steam is supplied in a superheated state and remains so throughout expansion. Initial density of steam = .21 lbs. per cubic foot. Engine is 12×20 ins., double-acting, 200 R.P.M., no clearance, initial pressure 125 lbs. per square inch absolute, back pressure a vacuum of 26 ins. of mercury. Cut-off at maximum load $\frac{1}{2}$, no load, $\frac{1}{4}$. What per cent of rating of boiler will be required by the engine at no load?

Prob. 3. While an engine driving a generator is running, a short circuit occurs putting full load on engine, requiring a $\frac{1}{2}$ cut-off. A moment later the circuit-breaker opens and only the friction load remains, requiring a cut-off of but $\frac{1}{4}$. The engine being two-cylinder, double-acting, simple, 12×18 ins., running at 300 R.P.M., and having no clearance, what will be the rate at which it uses steam just before and just after circuit-breaker opens if the steam supplied is at 125 lbs. per square inch absolute and is just dry, becoming wet on expanding? Back pressure is 3 lbs. per square inch absolute.

Prob. 4. A pumping engine has two double-acting steam cylinders each 9×12 ins. and a fixed cut-off of $\frac{1}{2}$. It runs at 60 R.P.M. on 80 lbs. per square inch absolute steam pressure and atmospheric exhaust. Cylinder is jacketed so that steam stays dry throughout its expansion. How much steam will it use per hour? Neglect clearance.

Prob. 5. If an engine 10×14 ins. running at 250 R.P.M. has such a cut-off that complete expansion occurs for 90 lbs. per square inch absolute initial pressure, and at atmospheric (15 lbs. absolute) exhaust, what will be the horse-power and steam used per hour, steam being superheated at all times, and what would be the value for the horse-power and steam used if full stroke admission occurred?

Prob. 6. The steam consumption of an engine working under constant load is better than that of a similar one working under variable load. For a 16×24 ins. engine running at 250 R.P.M. on wet steam of 125 lbs. per square inch absolute and atmos-

pheric exhaust, find the horse-power and steam used per horse-power per hour for best condition. Take two lighter and three heavier loads and show by a curve how steam used per horse-power per hour will vary.

Prob. 7. For driving a shop a two-cylinder single-acting engine, 6×6 ins., running at 430 R.P.M., is used. The cut-off is fixed at $\frac{1}{2}$ and initial pressure varied to control speed. Plot a curve between horse-power and weight of steam per hour per horse-power for 20, 40, 60, 80, 100, 120 lbs. per square inch absolute initial pressure and atmospheric exhaust. Steam constantly dry. Clearance zero.

NOTE: δ_1 for above pressure equals .05, .095, .139, .183, .226, and .268 lbs. per cubic foot, respectively.

Prob. 8. Taking the loads found in Prob. 7, find what cut-off would be required to cause the engine to run at rated speed for each load if the initial steam pressure were 100 lbs. per square inch absolute, and the back pressure atmospheric. Plot a curve between horse-power and steam used per horse-power hour for this case.

Prob. 9. For working a mine-hoist a two-cylinder, double-acting engine is used in which compressed air is admitted $\frac{1}{2}$ stroke at 125 lbs. per square inch absolute and then

allowed to expand adiabatically and exhaust to atmosphere. If the cylinders are 18×24 ins. and speed is 150 R.P.M., find the horse-power and cubic feet of high pressure air needed per minute.

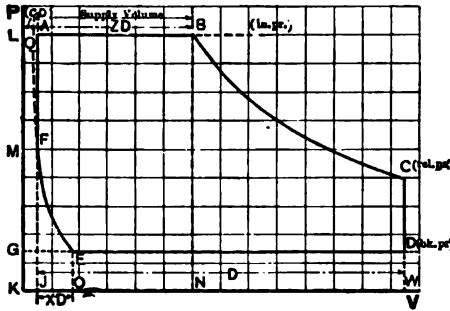


FIG. 42.—Work of Expansive Fluid in Single Cylinder with Clearance. Logarithmic Expansion for Cycle III. Exponential for Cycle IV.

41. Work of Expansive Fluid in Single Cylinder with Clearance. Logarithmic Expansion and Compression; Cycle III. Mean Effective Pressure, Horse-power, and Consumption of Simple Engines. As in previous cycles, the net work of the cycle is equal to the algebraic sum of the positive work done on the forward stroke and the

negative work on the return stroke. By areas, Fig. 42, this is,

$$\text{Work area} = JABN + NBCW - WDEO - OEFJ.$$

Expressed in terms of diagram points this becomes

$$W = P_b(V_b - V_a) + P_b V_b \log_e \frac{V_c}{V_b} - P_d(V_d - V_c) - P_d V_d \log_e \frac{V_e}{V_f}.$$

Expressing this in terms of displacement, in cubic feet D ; clearance as a fraction of displacement, c ; cut-off as a fraction of displacement, Z ; compression as a fraction of displacement, X ; initial pressure, in pounds per square inch (in.pr.), and exhaust or back pressure, in pounds per square inch (bk.pr.):

$$P_b = 144(\text{in.pr.}); \quad P_d = 144(\text{bk.pr.}); \quad (V_b - V_a) = ZD;$$

$$(V_d - V_c) = (1 - X)D; \quad V_b = D(Z + c); \quad V_e = D(X + c).$$

$$\frac{V_c}{V_b} = \frac{D(1+c)}{D(Z+c)} = \frac{1+c}{Z+c}, \quad \frac{V_e}{V_f} = \frac{D(X+c)}{Dc} = \frac{X+c}{c}.$$

Whence Work in ft.-lbs. per cycle is

$$W = \left\{ \begin{array}{l} 144D \left\{ (\text{in.pr.}) \left[Z + (Z+c) \log_e \frac{1+c}{Z+c} \right] \right. \\ \left. - (\text{bk.pr.}) \left[(1-X) + (X+c) \log_e \frac{X+c}{c} \right] \right\} \end{array} \right\} \dots \dots (183)$$

From Eq. (183), the *mean effective pressure*, pounds per square inch, follows by dividing by $144D$:

$$(\text{m.e.p.}) = \left\{ \begin{array}{l} (\text{in.pr.}) \left[Z + (Z+c) \log_e \frac{1+c}{Z+c} \right] \dots (\text{mean forward press.}) \\ - (\text{bk.pr.}) \left[(1-X) + (X+c) \log_e \frac{X+c}{c} \right] \dots (\text{mean bk.pr.}) \end{array} \right\} (184)$$

This is a general expression of very broad use in computing probable mean effective pressure for the steam engine with clearance and compression, or for other cases where it is practicable to assume the logarithmic law to hold. Chart 9, in the Handbook of Tables, will be found of assistance in evaluating this expression.

Indicated horse-power, according to expressions already given, may be found by either of the following equations:

$$\text{I.H.P.} = \frac{(\text{m.e.p.})La\pi}{33,000} = \frac{144(\text{m.e.p.})Dn}{33,000} = \frac{(\text{m.e.p.})Dn}{229.2},$$

where L is stroke in feet, a is effective area of piston, square inches, n is the number of cycles performed per minute and D the displacement, cubic feet.

It might seem that the work per cubic foot of fluid supplied could be found by dividing Eq. (183) by the admission volume, $(V_b - V_a)$, but this would be true only when no steam is needed to build up the pressure from F to A . This is the case only when the clearance is zero or when compression begins soon enough to carry the point F up to point A , i.e., when by compression the pressure of the clearance fluid is raised to the initial pressure.

It is evident that the fluid supplied may perform the two duties: first, building up the clearance pressure at constant or nearly constant volume, and second, filling the cylinder up to cut-off at constant pressure. To measure the steam supplied in terms of diagram quantities requires the fixing of the volume of live steam necessary to build up the pressure from F to A and adding it to the apparent admission volume $(V_b - V_a)$. This can be done by producing the compression line EF to the initial pressure Q , then LQ is the volume that the clearance steam would have at the initial pressure and QA the volume of live steam necessary to build up the pressure. The whole volume of steam admitted then is represented by \overline{QB} instead of \overline{AB} or by $(V_b - V_e)$ instead of by $(V_b - V_a)$; that is the supply volume $= (V_b - V_e)$.

$$\text{But} \quad V_e = V_c \frac{P_c}{P_e} = V_c \frac{(\text{bk.pr.})}{(\text{in.pr.})} = (X+c)D \frac{(\text{bk.pr.})}{(\text{in.pr.})}.$$

$$\text{Hence} \quad (\text{Sup. Vol.}) = D \left[(Z+c) - (X+c) \frac{(\text{bk.pr.})}{(\text{in.pr.})} \right], \dots \dots (185)$$

which is the cubic feet of fluid admitted at the initial pressure for the displacement of D cubic feet by the piston. Dividing by D there results

$$\frac{(\text{Sup.Vol.})}{D} = (Z+c) - (X+c) \frac{(\text{bk.pr.})}{(\text{in.pr.})}, \quad \dots \quad (186)$$

which is the ratio of admission volume to displacement or cubic feet of live steam admitted per cubic foot of displacement.

Dividing the work done by the cubic feet of steam supplied gives the economy of the simple engine in terms of volumes, or

$$\begin{aligned} \text{Work per cu.ft. of fluid supplied} &= \frac{W}{(\text{Sup.Vol.})} \\ &= 144 \frac{(\text{in.pr.}) \left[Z + (Z+c) \log_e \frac{1+c}{Z+c} \right] - (\text{bk.pr.}) \left[(1-X) + (X+c) \log_e \frac{X+c}{c} \right]}{(Z+c) - (X+c) \frac{(\text{bk.pr.})}{(\text{in.pr.})}}. \quad (187) \end{aligned}$$

It is more common to express economy of the engine in terms of the weight of steam used per hour per horse-power or the "water rate," which in more general terms may be called the consumption per hour per I.H.P.

Let δ_1 be the density or weight per cubic foot of fluid supplied, then the weight per cycle is (Sup. Vol.) δ_1 , and this weight is capable of performing W foot-pounds of work, or (Sup.Vol.) δ_1 lbs. per minute will permit of $\frac{W}{33,000}$ horse-power. But (Sup.Vol.) δ_1 lbs. per minute corresponds to 60 (Sup.Vol.) δ_1 lbs. per hour, whence the number of pounds per hour per horse-power is

$$\frac{60(\text{Sup.Vol.})\delta_1}{W/33,000},$$

which is the pounds consumption per hour per I.H.P., whence

$$\text{Consumption in lbs. per hr. per I.H.P.} = \frac{60 \times 33,000 (\text{Sup.Vol.}) \delta_1}{W}, \quad \dots \quad (188)$$

which is the general expression for consumption in terms of the cubic feet of fluid admitted per cycle, δ_1 initial density, and the work per cycle.

As work is the product of mean effective pressure in pounds per square foot (M.E.P.), and the displacement in cu.ft. or $W = (\text{M.E.P.})D$, or in terms of mean effective pressure, pounds per square inch, $W = 144 (\text{m.e.p.})D$, the consumption may also be written

$$\begin{aligned} \text{Consumption in lbs., per hr. per I.H.P.} &= \frac{60 \times 33,000 (\text{Sup.Vol.}) \delta_1}{144 (\text{m.e.p.}) D} \\ &= \frac{13,750}{(\text{m.e.p.})} \frac{(\text{Sup.Vol.}) \delta_1}{D} = \frac{13,750}{(\text{m.e.p.})} \left[(Z+c) - (X+c) \frac{(\text{bk.pr.})}{(\text{in.pr.})} \right] \delta_1, \quad \dots \quad (189) \end{aligned}$$

which gives the water rate in terms of the mean effective pressure, cut-off, clearance, compression, initial and back pressures and initial steam density.

It is sometimes more convenient to introduce the density of fluid at the back pressure δ_2 , which can be done by the relation (referring to the diagram),

$$P_e V_e = P_e V_e \text{ or } \frac{(\text{in.pr.})}{(\text{bk.pr.})} = \frac{V_e}{V_e} \frac{\delta_1}{\delta_2}, \text{ whence } \delta_2 = \delta_1 \frac{(\text{bk.pr.})}{(\text{in.pr.})}.$$

This on substitution gives: Consumption in lbs. per hr. per I.H.P.

$$= \frac{13,750}{(\text{m.e.p.})} [(Z+c)\delta_1 - (X+c)\delta_2]. \quad (190)$$

Since the step taken above of introducing δ_2 has removed all pressure or volume ratios from the expression, Eq. (190) is general, and not dependent upon the logarithmic law. It gives the consumption in terms of mean effective pressure, cut-off, clearance, compression and the density of steam at initial and back pressure, which is of very common use.

It cannot be too strongly kept in mind that all the preceding is true only when no steam forms from moisture water during expansion or compression or no steam condenses, which assumption is known to be untrue. These formulæ are, therefore, to be considered as merely convenient approximations, although they are almost universally used in daily practice.

Special Cases. First, no expansion and no compression but finite clearance would result in Fig. 43. For this case

$$W = 144D[(\text{in.pr.}) - (\text{bk.pr.})]. \quad (191)$$

$$(\text{m.e.p.}) = (\text{in.pr.}) - (\text{bk.pr.}) \quad (192)$$

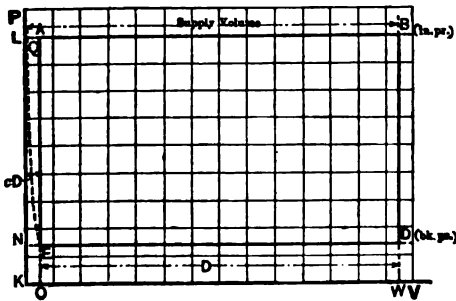


FIG. 43.—First Special Case of Cycles III and IV. Expansion and Compression both Zero but Clearance Finite.

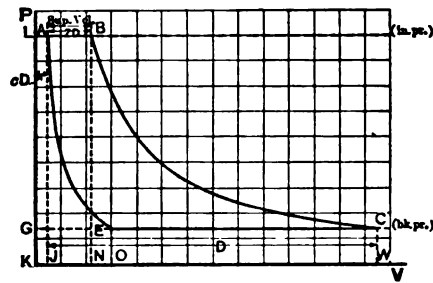


FIG. 44.—Second Special Case of Cycles III and IV. Perfect Expansion and Perfect Compression with Clearance.

The volume of fluid supplied per cycle is QB , or from Eq. (185) it is

$$(\text{Sup. Vol.}) = D \left[1 + c - c \frac{(\text{bk.pr.})}{(\text{in.pr.})} \right]. \quad (193)$$

$$\text{Consumption in lbs. per hr. per I.H.P.} = \frac{13,750}{(\text{in.pr.}) - (\text{bk.pr.})} \left[1 + c - c \frac{(\text{bk.pr.})}{(\text{in.pr.})} \right] \delta_1, \quad (194)$$

or in terms of initial and final densities,

$$\text{Consumption in lbs. per hr. per I.H.P.} = \frac{13,750}{(\text{in.pr.}) - (\text{bk.pr.})} [(1+c)\delta_1 - c\delta_2] \quad (195)$$

The second special case is that of complete expansion and compression, as indicated in Fig. 44. Complete expansion provides that the pressure at the end of expansion be equal to the back pressure, and complete compression that the final compression pressure be equal to the initial pressure.

Here $\frac{V_c}{V_b} = \frac{V_c}{V_a} = \frac{1+c}{Z+c} = \frac{X+c}{c} = \frac{(\text{in.pr.})}{(\text{bk.pr.})}$, and hence

$$\frac{V_c - V}{V_b - V_a} = \frac{\text{in.pr.}}{\text{bk.pr.}}, \text{ but } (V_c - V_a) = D \left[1 + c - c \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right) \right] \text{ and } (V_b - V_a) = ZD$$

$$\text{Hence by substitution } \frac{D \left[1 + c - c \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right) \right]}{ZD} = \frac{(\text{in.pr.})}{(\text{bk.pr.})},$$

$$\text{from which } Z = (1+c) \frac{(\text{bk.pr.})}{(\text{in.pr.})} - c. \quad (196)$$

$$\text{Again, } X = \frac{V_c - V_a}{V_c - V_b} = \frac{cD \left[\left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right) - 1 \right]}{D} = c \left[\left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right) - 1 \right]. \quad (197)$$

Eq. (196) gives the cut-off as a fraction of the displacement necessary to give complete expansion, while Eq. (197) gives the compression as a fraction of displacement to give complete compression, both in terms of clearance, initial pressure and back pressure, provided the logarithmic law applies to expansion and compression.

Substitution of the values given above in Eq. (183) gives, after simplification,

$$W = 144D[(1+c)(\text{bk.pr.}) - c(\text{in.pr.})] \log_e \frac{(\text{in.pr.})}{(\text{bk.pr.})}. \quad (198)$$

$$(\text{m.e.p.}) = [(1+c)(\text{bk.pr.}) - c(\text{in.pr.})] \log_e \frac{(\text{in.pr.})}{(\text{bk.pr.})}. \quad (199)$$

In this case the volume supplied is exactly equal to that represented by the admission line AB , and is equal to $(\text{Sup.Vol.}) = ZD. \quad (200)$

Hence, the consumption, in pounds fluid per hour per I.H.P. in terms of initial density, is

$$\text{Consumption in lbs. per hr. per I.H.P.} = \frac{13,750}{(\text{m.e.p.})} Z \delta_1, \text{ but}$$

$$\frac{Z}{\text{m.e.p.}} = \frac{(1+c) \frac{(\text{bk.pr.})}{(\text{in.pr.})} - c}{(\text{in.pr.}) \left[(1+c) \frac{(\text{bk.pr.})}{(\text{in.pr.})} - c \right] \log_e \frac{(\text{in.pr.})}{(\text{bk.pr.})}} = \frac{1}{(\text{in.pr.}) \log_e \frac{(\text{in.pr.})}{(\text{bk.pr.})}}, \text{ hence}$$

$$\text{Consumption in lbs. fluid per hr. per I.H.P.} = \frac{13,750 \delta_1}{(\text{in.pr.}) \log_e \frac{(\text{in.pr.})}{(\text{bk.pr.})}}. \quad (201)$$

This last equation is interesting in that it shows the consumption (or water rate, if it is a steam engine) is independent of clearance, and dependent only upon initial density, and on the initial and final pressures.

An expression may also be easily derived for the consumption in terms of initial and final density, but owing to its limited use, will not be introduced here.

Example 1. Method of calculating Diagrams Fig. 42 and Fig. 44.

Assumed data for Fig. 42:

$$\begin{aligned} P_e = P_a = P_b &= 90 \text{ lbs. per square inch abs.} & V_e = V_f &= .5 \text{ cu.ft.} \\ P_g = P_c = P_d &= 14 \text{ lbs. per square inch abs.} & V_d = V_c &= 13.5 \text{ cu.ft.} \\ P_f &= 50 \text{ lbs. per square inch abs.} & V_b &= 6 \text{ cu.ft.} \end{aligned}$$

To obtain point C: $P_c = P_b \times \frac{V_b}{V_c} = 90 \times \frac{6}{13.5} = 40 \text{ lbs. per sq.in. abs.}$

To obtain point E: $V_e = V_f \times \frac{P_f}{P_e} = .5 \times \frac{50}{14} = 1.78 \text{ cu.ft.}$

To obtain point Q: $V_q = V_f \times \frac{P_f}{P_e} = .5 \times \frac{50}{90} = .278 \text{ cu.ft.}$

Intermediate points from B to C and E to Q are found by assuming volumes and computing the corresponding pressures by relation $P_x V_x = P_b V_b$ or $P_x V_x = P_e V_e$.

Clearance is $\frac{V_a}{V_d - V_a} = \frac{.5}{13} = 3.8 \text{ per cent.}$ Cut-off is $\frac{V_b - V_a}{V_d - V_a} = \frac{5.5}{13} = 42.3 \text{ per cent.}$

Compression is $\frac{V_e - V_a}{V_d - V_a} = \frac{1.28}{13} = 9.9 \text{ per cent.}$

Assumed data for Fig. 44: $P_a = P_b = 90 \text{ lbs. per sq.in. absolute.}$ $V_a = .5 \text{ cu.ft.}$

$P_e = P_c = 14 \text{ lbs. per sq.in. absolute.}$ $V_d = 13.5 \text{ cu.ft.}$

To obtain point B: $V_b = V_c \frac{P_c}{P_b} = 13.5 \times \frac{14}{90} = 2.11 \text{ cu.ft.}$

To obtain point E: $V_e = V_a \times \frac{P_a}{P_e} = .5 \times \frac{90}{14} = 3.2 \text{ cu.ft.}$

Intermediate points from B to C and from A to E are to be found by assuming various volumes and finding the corresponding pressures from relation $P_x V_x = P_a V_a$ or $P_x V_x = P_b V_b$.

Example 2. What will be the horse-power of, and steam used per hour by the following engine: Cylinder, 12×18-in. double-acting, 200 R.P.M., 7 per cent clearance, initial pressure 85 lbs. per square inch gage, back pressure 15 lbs. per square inch absolute, and logarithmic expansion and compression.

(a) cut-off 50 per cent, compression 30 per cent, (b) complete expansion and compression, (c) no expansion or compression.

NOTE: δ for 85 lbs. gage = .23, δ_1 for 15 lbs. absolute = .038 lbs. per cu.ft.

(a) From Eq. (184)

$$\begin{aligned}
 (\text{m.e.p.}) &= (\text{in.pr.}) \left[Z + (Z+c) \log_e \frac{1+c}{Z+c} \right] - (\text{bk.pr.}) \left[(1-X) + (X+c) \log_e \frac{X+c}{c} \right], \\
 &= 100 \left[.5 + (.5+.07) \log_e \frac{1+.07}{.5+.07} \right] - 15 \left[(1-.3) + (.3+.07) \log_e \frac{.3+.07}{.07} \right] \\
 &= 86 - 20 = 66 \text{ lbs. sq.in.} \\
 \text{I.H.P.} &= \frac{(\text{m.e.p.}) \text{ Lan}}{33,000} = \frac{66 \times 1.5 \times 113.1 \times 400}{33,000} = 135.
 \end{aligned}$$

From Eq. (189) steam per hour per I.H.P. in pounds is

$$\begin{aligned}
 &\frac{13,750}{(\text{m.e.p.})} \left[(Z+c) - (X+c) \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right) \right] \delta_1, \\
 &= \frac{13,750}{66} \left[(.5+.07) - (.3+.07) \times \frac{15}{100} \right] \times .23 = 25 \text{ lbs.}
 \end{aligned}$$

Hence total steam per hour = $25 \times 135 = 3380$ lbs.

$$\begin{aligned}
 (b) \text{ From Eq. (199), } (\text{m.e.p.}) &= [(1+c)(\text{bk.pr.}) - c(\text{in.pr.})] \log_e \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right), \\
 &= [(1+.07) \times 15 - .07 \times 100] \log_e 6.67 = 17.2 \text{ lbs. sq.in.}
 \end{aligned}$$

$$\text{I.H.P.} = \frac{17.2 \times 1.5 \times 113.1 \times 400}{33,000} = 35.4.$$

$$\text{From Eq. (201), Steam per I.H.P. per hr.} = \frac{13,750 \delta_1}{(\text{in.pr.}) \log_e \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)} = \frac{13,750 \times .23}{100 \times 1.9} = 16.6,$$

Total steam per hour = $16.6 \times 35.4 = 588$ lbs.

$$(c) \text{ From Eq. (192) } (\text{m.e.p.}) = (\text{in.pr.}) - (\text{bk.pr.}) = 100 - 15 = 85 \text{ lbs. sq.in.}$$

$$\therefore \text{I.H.P.} = \frac{85 \times 1.5 \times 113.1 \times 400}{33,000} = 174.5.$$

From Eq. (195)

$$\text{Steam per I.H.P. per hour} = \frac{13,750}{(\text{m.e.p.})} [(1+c) \delta_1 - c \delta_2] = \frac{13,750}{85} [1.07 \times .23 - .07 \times .038] = 35.4.$$

Total steam per hour = $174.5 \times 35.4 = 6100$ lbs.

Prob. 1. What will be the horse-power and water rate of a 9×12 -in. simple engine having 5 per cent clearance, running at 250 R.P.M. on 100 lbs. per square inch absolute initial pressure and 5 lbs. per square inch absolute back pressure when the cut-off is $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$? Expansion follows the logarithmic law, and there is no compression.

NOTE: δ for 100 lbs. absolute = .23, δ for 5 lbs. absolute = .014.

Prob. 2. Will a pump with a cylinder 10×15 ins. and 10 per cent clearance give the same horse-power and have the same water rate as a pump with cylinders of the same size but with 20 per cent clearance, both taking steam full stroke? Solve for a case of 125 lbs. per square inch absolute initial pressure, atmospheric exhaust and a speed of 50 double strokes. No compression.

NOTE: δ for 125 lbs. absolute = .283, δ for 15 lbs. absolute = .038.

Prob. 3. Solve the above problem for an engine of the same size, using steam expansively when the cut-off is $\frac{1}{2}$ and R.P.M. 200, steam and exhaust pressure as in Prob. 2 and compression zero.

Prob. 4. Two engines, each 9×15 ins., are running on an initial pressure of 90 lbs. per square inch absolute, and a back pressure of one atmosphere. One has no clearance, the other 8 per cent. Each is cutting off so that complete expansion occurs. The speed of each is 200 and neither has any compression. What will be the horse-power and water rate?

NOTE: δ for 90 lbs. = .20, δ for 15 lbs. = .038.

Prob. 5. Find the horse-power and water rate of a 12×18 -in. double-acting engine with 8 per cent clearance, running at 150 R.P.M. on an initial steam pressure of 90 lbs. per square inch absolute and atmosphere exhaust for a fixed cut-off of $\frac{1}{2}$ and variable compression from 0 to the point where the pressure at the end of compression is equal to 125 per cent of the initial pressure. Plot the curves between compression and horse-power, and compression and water rate to show the effect of compression on the other two.

Prob. 6. A steam engine is running at such a load that the cut-off has to be $\frac{3}{4}$ at a speed of 150 R.P.M. The engine is 14×20 ins. and has no clearance. Initial pressure 100 lbs. per square inch absolute and back pressure 5 lbs. per square inch absolute. What would be the cut-off of an engine of the same dimensions but with 10 per cent clearance under similar conditions?

Prob. 7. The steam pressure is 100 lbs. per square inch gage and the back pressure is 26 ins. of mercury vacuum when the barometer is 30 ins. For a 16×22 -in. engine with 6 per cent clearance running at 125 R.P.M., cut-off at $\frac{1}{2}$ and 30 per cent compression, what will be the horse-power and the water rate? Should the steam pressure be doubled what would be the horse-power and the water rate? If it should be halved?

NOTE: δ for 100 lbs. gage = .257, δ for 26 ins. Hg. = .0057.

Prob. 8. While an 18×24 -in. simple engine with 4 per cent clearance at speed of 150 R.P.M. is running with a $\frac{1}{2}$ cut-off and a compression of $\frac{1}{2}$ on a steam pressure of 125 lbs. per square inch gage, and a vacuum of 28 ins. of mercury, the condenser fails and the back pressure rises to 17 lbs. per square inch absolute. What will be the change in the horse-power and water rate if all other factors stay constant? What would the new cut-off have to be to keep the engine running at the same horse-power and what would be the water rate with this cut-off?

NOTE: δ for 125 lbs. gage = .31, δ for 28 in. Hg. = .0029, δ for 17 lbs. absolute = .043.

Prob. 9. Under normal load an engine has a cut-off of $\frac{3}{4}$, while under light load the cut-off is but $\frac{1}{8}$. What per cent of the steam used at normal load will be used at light load for the following data? Cylinder 10×18 ins.; 7 per cent clearance; 200 R.P.M.; initial pressure 120 lbs. per square inch gage; back pressure 2 lbs. per square inch absolute; compression at normal load 5 per cent; at light load 25 per cent.

NOTE: δ for 120 lbs. gage = .30, δ for 2 lbs. absolute = .0058.

42. Work of Expansive Fluid in Single Cylinder with Clearance; Exponential Expansion and Compression, Cycle, IV. Mean Effective Pressure, Horse-power and Consumption of Simple Engines. As pointed out in several places, the logarithmic expansion of steam only approximates the truth in real engines and is the result of no particular logical or physically definable hypothesis as to the condition of the fluid, moreover its equations are of little or no value

for compressed air or other gases used in engine cylinders. All expansions that can be defined by conditions of physical state or condition of heat, including the adiabatic, are expressible approximately or exactly by a definite value of s , not unity, in the expression $PV^s = \text{const.}$ All these cases can then be treated in a group and expressions for work and mean effective pressure found for a general value of s , for which particular values belonging to, or following from any physical hypothesis can be substituted. The area under such expansion curves is given by Eq. (13) Chapter I, which applied to the work diagram, Fig. 42, in the same manner as for logarithmic expansion, gives the *net work*:

$$\left. \begin{aligned} W &= P_b(V_b - V_a) + \frac{P_b V_b}{s-1} \left[1 - \left(\frac{V_b}{V_c} \right)^{s-1} \right] && (\text{area } JABCWJ) \\ &- P_a(V_a - V_e) - \frac{P_a V_a}{s-1} \left[\left(\frac{V_a}{V_f} \right)^{s-1} - 1 \right] && (\text{area } WDEFJW) \end{aligned} \right\} \quad (202)$$

Introducing the symbols,

$$\begin{aligned} P_b &= 144(\text{in.pr.}), & V_b &= D(Z+c), & (V_b - V_a) &= ZD, \\ P_a &= 144(\text{bk.pr.}), & V_a &= D(X+c), & (V_a - V_e) &= D(1-X), \\ \left(\frac{V_b}{V_c} \right) &= \frac{Z+c}{1+c}, & \left(\frac{V_a}{V_f} \right) &= \frac{X+c}{c}. \end{aligned}$$

$$\begin{aligned} W &= 144D \left\{ (\text{in.pr.}) \left[Z + \frac{Z+c}{s-1} \left[1 - \left(\frac{Z+c}{1+c} \right)^{s-1} \right] \right] \right. \\ &\quad \left. - (\text{bk.pr.}) \left[(1-X) + \frac{X+c}{s-1} \left[\left(\frac{X+c}{c} \right)^{s-1} - 1 \right] \right] \right\} \quad (203) \end{aligned}$$

Eq. (203) gives the work in foot-pounds for D cubic feet of displacement in a cylinder having any clearance c , cut-off Z , and compression X , between two pressures, when the law of expansion is $PV^s = \text{const.}$ and s anything except unity, but constant.

The mean effective pressure, pounds per square inch, is obtained by dividing the expression for work by $144D$, giving

$$\begin{aligned} (\text{m.e.p.}) &= (\text{in.pr.}) \left\{ Z + \frac{Z+c}{s-1} \left[1 - \left(\frac{Z+c}{1+c} \right)^{s-1} \right] \right\} && (\text{mean for'd pr.}) \\ &- (\text{bk.pr.}) \left\{ (1-X) + \frac{X+c}{s-1} \left[\left(\frac{X+c}{c} \right)^{s-1} - 1 \right] \right\} && (\text{mean bk.pr.}) \end{aligned} \quad (204)$$

which is the general expression for mean effective pressure for this cycle.

It was pointed out in Section (41) that the cubic feet of fluid admitted at the initial pressure was not represented by \overline{AB} , Fig. 42, but by \overline{QB} , and the same is true for this case, so that the (Sup.Vol.) $= V_b - V_a$. But when the expansion and compression laws have the form $PV^s = c$

$$V_a = V_b \left(\frac{P_b}{P_a} \right)^{\frac{1}{s}} = D(X+c) \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}}.$$

$$\text{Whence} \quad (\text{Sup.Vol.}) = D \left[(Z+c) - (X+c) \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}} \right] \dots \dots \dots (205)$$

Eq. (205) gives for these cases what was given by Eq. (185) for the logarithmic law, the cubic feet of fluid supplied at the initial pressure for the displacement of D cubic feet in terms of cut-off, clearance, compression and the pressures. From this, by division there is found the volume of high pressure fluid per cu.ft. of displacement

$$\frac{(\text{Sup.Vol.})}{D} = (Z+c) - (X+c) \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}} \dots \dots \dots (206)$$

The consumption is given by the general expression already derived, Eq. (158), from which is obtained: Consumption lbs. per hr. per I.H.P.

$$= \frac{13,750}{(\text{m.e.p.})} \left[(Z+c) - (X+c) \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}} \right] \delta_1 \dots \dots \dots (207)$$

Eq. (207) gives the water rate or gas consumption in terms of mean effective pressure, initial and back pressure, cut-off, clearance, compression and initial fluid density. Introducing the density at the back pressure by the relation,

$$\delta_1 = \delta_2 \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)^{\frac{1}{s}}; \text{ an equation is obtained, which is identical with Eq. (190)}$$

and is, as previously observed, a general expression, no matter what the laws of expansion and compression, in terms of mean effective pressure, cut-off, clearance, compression and the initial and final steam density.

The *first special case of full admission, no compression*, might at first thought appear to be the same as in the preceding section, where the logarithmic law was assumed to hold, and so it is as regards work and mean effective pressure, Eq. (191) and (192), but referring to Fig. 43 it will be seen that since the exponential law is now assumed instead of the logarithmic, the point Q will be differently located (nearer to A than it was previously if s is greater than 1), and hence the supply volume \overline{QB} is changed, and its new value is

$$(\text{Sup.Vol.}) = D \left[1+c - c \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}} \right] \dots \dots \dots (208)$$

Hence, consumption stated in terms of initial density of the fluid δ_1 , is Consumption lbs. per hr. per I.H.P.

$$= \frac{13,750}{(\text{in.pr.}) - (\text{bk.pr.})} \left[1+c - c \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}} \right] \delta_1 \dots \dots \dots (209)$$

Stated in terms of initial and final densities, the expression is the same as given in Eq. (195).

The *second special case, complete expansion and compression* is again represented by Fig. 44. From the law of expansion it is evident that

$$V_b = V_c \left(\frac{P_c}{P_b} \right)^{\frac{1}{s}}, \text{ or stated in symbolic form, } D(Z+c) = D(1+c) \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}},$$

$$\text{whence} \quad Z = (1+c) \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}} - c. \quad \dots \quad (210)$$

$$\text{Again referring to Fig. 44, } X = \frac{V_c - V_a}{V_c - V_a} = \frac{cD \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)^{\frac{1}{s}} - cD}{D},$$

$$\text{from which} \quad X = c \left[\left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)^{\frac{1}{s}} - 1 \right]. \quad \dots \quad (211)$$

Eq. (210) gives the cut-off as a fraction of displacement necessary to give complete expansion, and (211), the compression fraction to give complete compression, both in terms of clearance, initial and back pressures, and the exponent s , in the equation of the expansion or compression line, $PV^s = \text{const.}$

The work of the cycle becomes for this special case, by substitution in Eq. (203),

$$W = 144D(\text{in.pr.})^{\frac{s}{s-1}} \left[(1+c) \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}} - c \right] \left[1 - \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{s-1}{s}} \right], \quad \dots \quad (212)$$

and the mean effective pressure, lbs. per sq.in., is

$$(\text{m.e.p.}) = (\text{in.pr.})^{\frac{s}{s-1}} \left[(1+c) \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}} - c \right] \left[1 - \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{s-1}{s}} \right]. \quad \dots \quad (213)$$

$$\text{The volume of fluid supplied is, (Sup.Vol.)} = ZD, \quad \dots \quad (214)$$

$$\text{hence, Consumption, lbs. per hr. per I.H.P.} = \frac{13,750}{\text{m.e.p.}} Z \delta_1, \text{ but}$$

$$\begin{aligned} \frac{Z}{(\text{m.e.p.})} &= \frac{\left[(1+c) \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}} - c \right]}{(\text{in.pr.})^{\frac{s}{s-1}} \left[(1+c) \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}} - c \right] \left[1 - \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{s-1}{s}} \right]} \\ &= \frac{1}{(\text{in.pr.})^{\frac{s}{s-1}} \left[1 - \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{s-1}{s}} \right]}, \end{aligned}$$

whence consumption lbs. fluid per hr. per I.H.P. is,

$$\frac{13,750 \times \delta_1}{(\text{in.pr.})^{\frac{s}{s-1}} \left[1 - \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{s-1}{s}} \right]}, \quad \dots \quad (215)$$

the expression for smallest consumption (or water rate if steam) of fluid for the most economical hypothetical cycle, which it may be noticed, is again independent of clearance.

The expressions for work and mean effective pressure are not, however, independent of clearance, and hence, according to the hypothetical cycles here considered, it is proved that large clearance decreases the work capacity of a cylinder of given size, but does not affect the economy, provided complete expansion and compression are attained, a conclusion similar to that in regard to clearance effect on compressor capacity and economy. Whether the actual performance of gas or steam engines agrees with this conclusion based only on hypothetical reasoning, will be discussed later.

Example 1. What will be the horse-power of, and steam used per hour by, the following engine: 12×18-in. double-acting, 200 R.P.M., 7 per cent clearance, initial pressure 85 lbs. per square inch gage, 160° superheat, back pressure 15 lbs. per square inch absolute, and expansion such that $s=1.3$.

(a) cut-off = 50 per cent; compression = 30 per cent; (b) complete expansion and compression; (c) no expansion or compression;

NOTE: δ for 85 lbs. gage = 0.18; for 15 lbs. absolute. = .038.

$$\begin{aligned} (a) \text{ From Eq. (204) } (\text{m.e.p.}) &= (\text{in.pr.}) \left\{ Z + \frac{Z+c}{s-1} \left[1 - \left(\frac{Z+c}{1+c} \right)^{s-1} \right] \right\} \\ &\quad - (\text{bk.pr.}) \left\{ (1-X) + \frac{X+c}{s-1} \left[\left(\frac{X+c}{c} \right)^{s-1} - 1 \right] \right\} \\ &= 100 \left\{ .5 + \frac{.57}{.3} \left[1 - \left(\frac{.57}{1.07} \right)^3 \right] \right\} - 15 \left\{ .7 + \frac{.37}{.3} \left[\left(\frac{.37}{.07} \right)^3 - 1 \right] \right\} = 59.8 \text{ lbs.sq.in.} \\ \text{I.H.P.} &= \frac{59.8 \times 1.5 \times 113.1 \times 400}{33,000} = 122. \end{aligned}$$

From Eq. (190) steam per hour per I.H.P.

$$= \frac{13,750}{(\text{m.e.p.})} \left[(Z+c)\delta_1 - (X+c)\delta_2 \right] = \frac{13,750}{59.8} [(.57) \times .18 - (.37) \times .038] = 20.9 \text{ lbs.}$$

∴ Steam per hour = 122 × 20.9 = 2560 lbs.

$$\begin{aligned} (b) \text{ From Eq. (213) } (\text{m.e.p.}) &= (\text{in.pr.}) \frac{s}{s-1} \left[(1+c) \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}} - c \right] \left[1 - \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{s-1}{s}} \right], \\ &= \frac{1.3}{.3} \times 100 \left[1.07 \times \left(\frac{15}{100} \right)^{\frac{1}{1.3}} - .07 \right] \left[1 - \left(\frac{15}{100} \right)^{\frac{.3}{1.3}} \right] = 26.2 \text{ lbs. sq.in.}, \\ \text{I.H.P.} &= \frac{26.2 \times 1.5 \times 113.1 \times 400}{33,000} = 54. \end{aligned}$$

From Eq. (215) steam used I.H.P. per hour is,

$$\frac{13,750\delta_1}{(\text{in.pr.}) \frac{s}{s-1} \left[1 - \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{s-1}{s}} \right]} = \frac{13,750 \times .18}{\frac{1.3}{.3} \times 100 \left[1 - \left(\frac{15}{100} \right)^{\frac{.3}{1.3}} \right]} = 16.5 \text{ lbs.,}$$

hence total steam per hour = 16.5 × 54 = 890 lbs.

(c) From Eq. (192) which holds for any value of s , m.e.p. = $100 - 15 = 85$ lbs. sq.in.

and
$$\text{I.H.P.} = \frac{85 \times 1.5 \times 113.1 \times 400}{33,000} = 174.5,$$

From Eq. (209) steam per I.H.P. hour

$$= \frac{13,750 \delta_1}{(\text{m.e.p.})} \left[1 + c - c \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}} \right] = \frac{13,750 \times .18}{85} \left[1 + .07 - .07 \times \left(\frac{15}{100} \right)^{\frac{1}{1.3}} \right] = 24.5 \text{ lbs.},$$

and total steam per hour = $174.5 \times 24.5 = 2475$ lbs.

Prob. 1. Two simple engines, each 12×18 ins., with 6 per cent clearance are operated at $\frac{1}{2}$ cut-off and with 20 per cent compression. One is supplied with air at 80 lbs. per square inch gage, and exhausts it to atmosphere; the other with initially dry steam at same pressure, which becomes wet on expansion, is also exhausted to atmosphere. For a speed of 200 R.P.M. what is the horse-power of each engine and the cubic feet of air and of steam supplied per horse-power hour?

Prob. 2. A crank-and-flywheel two-cylinder, double-acting, pumping engine is supplied with dry steam and the expansion is such that it remains dry until exhaust. The cylinder size is 24×36 ins., cut-off to give perfect expansion, clearance 5 per cent, compression to give perfect compression, initial pressure 50 lbs. per square inch absolute, back pressure 5 lbs. per square inch absolute. What is the horse-power and water rate? What would be the horse-power and water rate of a full-stroke pump of the same size and clearance but having no compression, running on the same pressure range and quality of steam.

NOTE: δ for 50 lbs. absolute = .12, δ for 15 lbs. absolute = .038.

Prob. 3. Should the cylinder of the following engine be so provided that the steam was always kept dry, would there be any change in the horse-power developed as compared with steam expanded adiabatically? How much? Cylinder 20×24 ins., initial pressure 125 lbs. per sq. in. gage, back pressure 26 ins. vacuum, standard barometer, clearance 6 per cent, cut-off $\frac{2}{3}$, compression 10 per cent, and speed 125 R.P.M.

Prob. 4. What will be the total steam used per hour by a 20×28 -in. double-acting engine running at 150 R.P.M. if the initial pressure be 125 lbs. per square inch absolute, back pressure one atmosphere, clearance 8 per cent, compression zero, for cut-off $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{1}{4}$, if steam expands adiabatically and is originally dry and saturated?

NOTE: δ for 125 lbs. absolute = .28, δ for 15 lbs. absolute = .038.

Prob. 5. An engine which is supplied with saturated steam is said to have an indicated water rate of 15 lbs. at $\frac{1}{2}$ cut-off and one of 25 lbs. at $\frac{1}{4}$ cut-off. See if this is reasonable for the following conditions: engine is 15×22 ins., 7 per cent clearance, no compression, initial pressure 100 lbs. per square inch gage, back pressure 28-in. vacuum, barometer 30 ins. and speed 180 R.P.M.

NOTE: δ for 100 lbs. gage = .257, δ for 28 in. Hg = .0029.

Prob. 6. The tank capacity of a locomotive is 4500 gals. The cylinders are 18×36 ins., initial pressure 200 lbs. per square inch gage, exhaust atmospheric, cut-off $\frac{1}{2}$, clearance 7 per cent, speed 200 R.P.M., no compression. The steam is dry at start and expansion adiabatic, how long will the water last if 40% condenses during admission?

NOTE: δ for 200 lbs. gage = .468, δ for 15 lbs. absolute = .038.

Prob. 7. To drive a hoist, an air engine is used, the air being supplied for $\frac{1}{2}$ stroke at 80 lbs. per square inch gage, expanded adiabatically and exhausted to atmos-

phere. If the clearance is 8 per cent and there is no compression how many cubic feet of air per hour per horse-power will be needed? What, with complete compression?

Prob. 8. A manufacturer rates his 44×42 -in. double-acting engine with a speed of 100 R.P.M. at 1000 H.P. when running non-condensing, initial pressure 70 lbs. per square inch gage and cut-off $\frac{1}{2}$. No clearance is mentioned and nothing said about manner of expansion. Assuming $s = 1.0646$, find on what clearance basis this rating is made.

Prob. 9. The water supply of a town is furnished by a direct-acting non-condensing pump with two cylinders, each 24×42 ins., with 10 per cent clearance, and no compression, initial pressure being 100 lbs. per square inch gage. What must be the size of the steam cylinder of a crank-and-flywheel pump with 6 per cent clearance to give the same horse-power on the same steam and exhaust pressures with a cut-off of $\frac{1}{2}$? Speed in each case to be 50 R.P.M.

CHAPTER V

WORK DISTRIBUTION IN MULTI-EXPANSION PISTON ENGINES

43. Action of Fluid in Multiple-expansion Cylinders. General Description of Structure and Processes. When steam, compressed air, or any other high pressure working fluid is caused to pass through more than one cylinder in series, so that the exhaust from the one is the supply for the next, the engine is, in general, a multiple-expansion engine, or more specifically, a compound when the operations are in two expansion stages, triple for three, and quadruple for four stages. It must be understood that while a compound engine is one in which the whole pressure-volume change from initial to back pressure takes place in two stages, it does not necessarily follow that the machine is a two-cylinder one, for the second stage of expansion may take place in two cylinders, in each of which, half of the steam is put through identical operations; this would make a three-cylinder compound. Similarly, triple-expansion engines, while they cannot have less than three may have four or five or six cylinders. Multiple expansion engines, most of which are compound, are of two classes with respect to the treatment and pressure-volume changes of the steam, first *without receiver*, and second, *with receiver*. A receiver is primarily a chamber large in proportion to cylinder volumes, placed between the high- and low-pressure cylinders of compounds or between any pair of cylinders in triple or quadruple engines; its purpose is to provide a reservoir of fluid so that the exhaust from the higher into it, or the admission to the lower from it, will be effected without a material change of pressure, and this will be accomplished as its volume is large in proportion to the charge of steam received by it or delivered from it. With a receiver of infinite size the exhaust line of a high-pressure cylinder discharging into it will be a constant-pressure line, as will also the admission line of the low-pressure cylinder. When, however, the receiver is of finite size high-pressure exhaust is equivalent to increasing the quantity of fluid in the receiver of fixed volume and must be accompanied by a rise of pressure, except when a low-pressure cylinder may happen to be taking out fluid at the same rate and at the same time. As the receiver becomes smaller in proportion to the cylinders, the pressure in it will rise more for each high-pressure exhaust and fall more for each low-pressure admission, but there will be a constant average value. The greatest possible change of pressure during high-pressure exhaust and low-pressure admission would occur when the receiver is of zero size, that is when there is none at all, in which case, the high- and low-pressure pistons must have synchronous movement, both starting and stopping at the same time, but moving either in

the same or opposite directions. When the pistons of the no-receiver compound engines move in the same direction at the same time, one end of the high-pressure cylinder must exhaust into the opposite end of the low; but with oppositely moving pistons, the exhaust from high will enter the similar end of the low. It is plain that a real receiver of zero volume is impossible, as the connecting ports must have some volume; and likewise that an infinite receiver is equally impracticable, so that any multiple-expansion real engine will have receivers of finite volume with corresponding pressure changes during the period when a receiver is in communication with a cylinder. The amount of these pressure changes will depend partly on the size of the receiver with respect to the cylinders, but also as well, on the relation between the periods of flow into receiver, by high-pressure exhaust, and out of it, by low-pressure admission, which latter factor will be fixed largely by crank angles, and partly by the settings of the two valves, relations which are often extremely complicated.

For the purpose of analysis it is desirable to treat the two limiting cases of no receiver and infinite receiver, because they yield formulas simple enough to be useful, while an exact simple solution of the general case is impossible. These simple expressions for hypothetical cases which are very valuable for estimates and approximations are generally close to truth for an actual engine especially if intelligently selected and used.

Receivers of steam engines may be simple tanks or temporary storage chambers or they may be fitted with coils or tubes to which live or high-pressure steam is supplied to heat the low-pressure, partly-expanded steam passing from cylinder to cylinder through the receiver. Such receivers are *reheating receivers*, and as noted, may heat the engine steam or may evaporate any moisture it might contain. As a matter of fact there can be no heating of the steam before all moisture is first evaporated, from which it appears that the action of such reheating receivers may be, and is quite complicated thermally, therefore a study of these conditions must be postponed till a thermal method of analysis is established. This will introduce no serious difficulty, as such reheating receivers assist the thermal economy of the whole system but little and have little effect on engine power, moreover they are now little used. *Reheating of* air or other gases, as well as *preheating them before admission* to the high-pressure cylinder is a necessary practice, when the supply pressure is high, to prevent freezing of moisture by the gases, which get very cold in expansion if it be carried far. This is likewise, however, a thermal problem, not to be taken up till later.

Multiple-expansion engines are built for greater economy than is possible in simple engines and the reasons are divisible into two classes, first mechanical, and second thermal. It has already been shown that by expansion, work is obtained in larger amounts as the expansion is greater, provided, expansion below the back pressure is avoided; since high initial and low back pressures necessarily permit the greater expansion, engines must be built capable of utilizing all that the steam or compressed gas may yield. If steam followed the logarithmic law of expansion, pressure falling inversely with volume

increase, then steam of 150 lbs. per square inch absolute expanding to 1 lb. per square inch absolute would require enough ultimate cylinder space to allow whatever volume of steam was admitted up to cut-off to increase 150 times. This would involve a *valve gear* and cylinder structure capable of admitting $\frac{1}{150} = .0067$ of the cylinder volume. It is practically impossible to construct a valve that will accurately open and close in this necessarily short equivalent portion of the stroke. This, however, is not the worst handicap even mechanically, because actual cylinders cannot be made without some *clearance*, usually more than 2 per cent of the displacement, and in order that any steam might be admitted at all, the clearance in the example would have to be less than 0.67 per cent of the total volume. These two mechanical or structural limitations, that of *admission valve gear* and that of *clearance limits*, supply the first argument for multiple-expansion engines, the structure of which is capable of utilizing any amount of expansion that high boiler pressure and good condenser vacuum make available. For, if neglecting clearance, the low-pressure cylinder had ten times the volume of the high, then the full stroke admission of steam to the high followed by expansion in the low would give ten expansions, while admission to the high for $\frac{1}{10}$ of its stroke would give 15 expansions in it, after which this final volume would increase in the low ten times, that is, to 150 times the original volume, and cylinder admission of $\frac{1}{15}$ or 6.7 per cent of the cylinder volume is quite possible with ordinary valve gears, even with clearance, which in reasonably large engines may be not over 2 per cent of the whole cylinder volume.

It is evident that the higher the initial and the lower the back pressures the greater the expansion ratio will be for complete expansion, and since in steam practice boiler pressures of 225 lbs. per square inch gage, or approximately 240 lbs. per square inch absolute, and vacuum back pressures as low as one or even half a pound per square inch are in use, it should be possible whether desirable or not, to expand to a final volume from 250 to 500 times the initial in round numbers. This is quite impossible in simple engine cylinders, and as it is easy with multiple expansion there is supplied another mechanical argument for *staging*. Sufficient expansion for practical purposes in locomotives, and land engines under the usually variable load of industrial service, is available for even these high pressures by compounding, but when the loads are about constant, as in waterworks pumping engines, and marine engines for ship propulsion, triple-expansion is used for pressures in excess of about 180 lbs. gage.

Use of very high initial and very low back pressures in simple engines, will result in a possibility of great unbalanced forces on a piston, its rods, pins and crank, when acting on opposite sides, and a considerable fluctuation in tangential turning force at the crank pin. Compounding will always reduce the unbalanced force on a piston, and when carried out in cylinders each of which has a separate crank, effects a very considerable improvement in turning effort. So that, not only does multiple-expansion make it possible to utilize to the fullest extent the whole range of high initial and low back pressures, but it may result in a better force distribution in the engine, avoiding shocks,

making unnecessary excessively strong pistons and rods, and equalizing turning effort so that the maximum and minimum tangential forces do not depart too much from the mean.

The second, or thermal reason for bothering with multiple-expansion complications in the interest of steam economy, is concerned with the prevention of steam loss by *condensation and leakage*. It does not need any elaborate analysis to show that low-pressure steam will be cooler than high-pressure steam and that expanding steam in a cylinder has a tendency to cool the cylinder and piston walls, certainly the inner skin at least, so that after expansion and exhaust they will be cooler than after admission; but as admission follows exhaust, hot live steam will come into contact with cool walls and some will necessarily condense, the amount being smaller the less the original expansion; hence in any one cylinder of a multiple-expansion engine the condensation may be less than in a simple engine with the same range of steam pressures and temperatures. Whether all the steam condensation during admission added together will equal that of the simple engine or not is another question. There is no doubt, however, that as the multiple expansion engine can expand usefully to greater degree than a simple engine, and so cause a lower temperature by expansion, it thus has a greater chance to *re-evaporate some of the water of initial condensation* and so get some work out of the extra steam so evaporated, which in the simple engine might have remained as water, incapable of working until exhaust opened and lowered the pressure, when, of course, it could do no good. It is also clear that steam or compressed-air leakage in a simple engine is a direct loss, whereas in a compound, high-pressure cylinder leakage has at least a chance to do some work in the low-pressure cylinder. The exact analysis of the thermal reasons for greater economy is complicated and is largely concerned with a study of steam condensation and re-evaporation, but the fact remains that multiple-expansion engines are capable of greater economy than simple. The thermal analysis must also consider the influence of the reheating receiver, the steam-jacketed working cylinder, and the use of superheated steam, their effects on the possible work per pound of steam and the corresponding quantity of heat expended to secure it, and for air and compressed gas the parallel treatment of preheating and reheating.

To illustrate the action of steam in multiple-expansion engines some indicator cards are given for a few typical cases in Figs. 45 to 48, together with the combined diagrams of pressure-volume changes of the fluid in all cylinders to the same scale of pressures and volumes, which makes the diagram appear quite different from the indicator cards. Fig. 45 shows four sets of cards taken from an engine of the compound no-receiver type, namely, a Vauclain compound locomotive. In this machine there are two cylinders, one high pressure and one low, on each side, the steam from the high pressure exhausting directly into the low-pressure cylinder so that the only receiver space is made up of the clearance and connecting passages between the cylinders. Starting with set A, the cards show a high-pressure cut-off of 76 per

cent in the case of set A decreasing to 54 per cent in the case of set D. The letters A, B, C and D refer in each case to admission, cut-off, release and compression, the use of primed letters denoting the low-pressure cylinder.

In set A the high-pressure admission line AFB may be considered as made up of two parts, the part AF representing pressure rise at constant volume, which is the admission of steam to the clearance space at dead center to raise the pressure from that at the end of compression to that of boiler pressure. From F to B admission occurred at constant pressure, steam filling the cylinder volume as the piston moved outward. At B cut-off or closure of the steam valve occurred and the steam in the cylinder expanded. At C , release or opening of the exhaust valve of the high-pressure cylinder occurred and the admission valve of the low-pressure cylinder opened, the steam dropping in pressure until the pressure in both high- and low-pressure clearance became equal, and then expanding in both cylinders, as the exhaust from the high and admission to

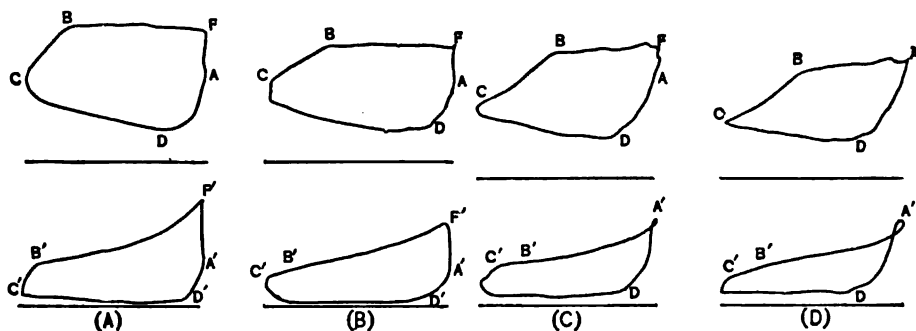


FIG. 45.—Set of Indicator Cards from Vaucain Locomotive Illustrating the No-receiver Compound Steam Engine.

the low occurred; the exhaust line CD of the high pressure and the admission line $F'B'$ of the low pressure are identical except for the slight pressure drop in the passages between the high- and the low-pressure cylinders. At D the high-pressure exhaust valve closed and compression of the steam, trapped in the high-pressure cylinder, occurred to point A , thus closing the cycle. From point B' in the low-pressure cylinder, which corresponds to D in the high pressure, no more steam was admitted to the low-pressure cylinder. What steam there was in the low expanded to the point C' when the exhaust valve opened and the pressure dropped to the back pressure; the steam was exhausted at nearly constant back pressure to D' , when the exhaust valve closed and the steam trapped in the cylinder was compressed to A' , at which point steam was again admitted and the cycle repeated.

In set B the cycle of operation is exactly the same as in set A. In set C the cycle is the same as in A, but there are one or two points to be especially noted, as they are not present in set A. The admission line of the high-pressure cylinder is not a constant-pressure, but rather a falling-pressure line, owing to throttling of the steam, or "*wire drawing*," as it is called, through the

throttle valve or steam ports, due to the higher speed at which this card was taken. It will also be noticed that the compression pressure is higher in this case, due to earlier closing of the exhaust valve, which becomes necessary with the type of valve gear used, as the cut-off is made earlier. In the low-pressure card it will be seen that the compression pressure is greater than the admission pressure and hence there is a pressure drop instead of rise on admission. In set D the peculiarities of C are still more apparent, the compression in high-pressure cylinder being equal to admission pressure, and above it in the low pressure cylinder. The wire drawing is also more marked, as the speed was still higher when this set of cards was taken.

In Fig. 46, one set of the cards of Fig. 45 is redrawn on cross-section paper and then combined. Cards taken from the different cylinders of a multiple-expansion engine will in nearly all cases have the same length but they will have two different pressure scales, in as much as that indicator spring

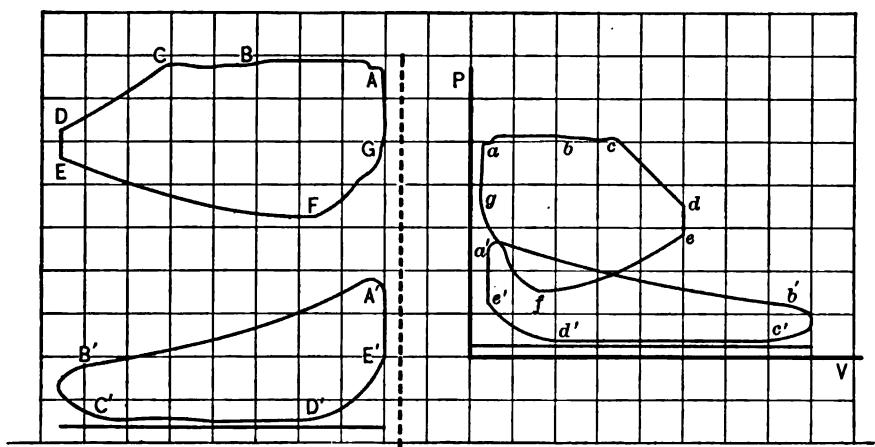


FIG. 46.—Diagram to Show Method of Combining the High- and Low-pressure Cylinder Indicator Cards of the No-receiver Compound Engine.

will be chosen for each cylinder which will give the greatest height of card consistent with safety to the instrument. To properly compare the cards they must be reduced to the same pressure scale, and also to the same volume scale. As the lengths represent volumes, the ratio of the two volume scales will be as that of the cylinder volumes, or diameters squared. Hence, the length of the high-pressure card must be decreased in this ratio or the low increased. As a rule it is found more convenient to employ the former method. When the cards have been reduced to a proper scale of pressures and volumes the clearance must be added to each in order that the true volume of the fluid may be shown. The cards may be placed with the atmospheric lines and zero volume lines coinciding and will then appear in their true relation. In this case the cylinder ratio was 1.65, the indicator springs 100 lbs. and 70 lbs. respectively and clearance 5 per cent in each cylinder.

The steps in combining the cards were as follows: The zero volume lines were first drawn perpendicular to the atmospheric line and at a distance from the end of the card equal to the length of the card times the clearance. PV axes were laid off and a line drawn parallel to the zero-pressure line at a distance above it equal to 14.7 lbs. to scale of combined diagram. This scale was taken to be that of the high-pressure diagram. A number of points $A'B'C'$, etc., were then chosen on the low-pressure card, and the corresponding points $a'b'c'$, etc., plotted by making the distances of a', b' , etc., from the zero-volume line equal to those of A', B' , etc., and the distances of the new points above the atmosphere .7 the distances of the original. By joining the points as plotted, the new diagram for the low-pressure card was formed. The high-pressure card was then redrawn by taking a number of points A, B, C , etc., and plotting new points a, b, c ,

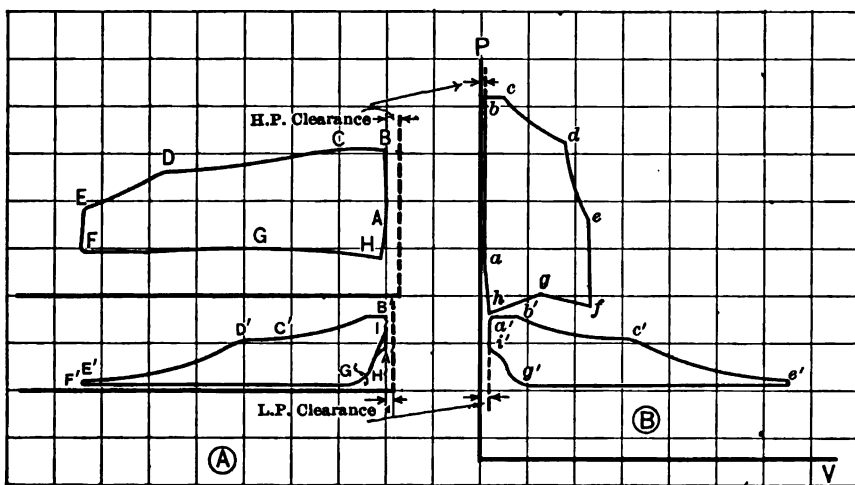


FIG. 47.—Indicator Cards from a Compound Engine with a Receiver, as Taken and as Combined.

etc., so that the distances of a, b, c , etc., from the zero-volume line were $\frac{1}{1.65}$ the distances of A, B, C , etc., while the distances of new points above the atmospheric line were the same as for the original points.

In Fig. 47 are shown two cards from a compound steam engine with receiver. Diagram A shows the cards as taken, but transferred to cross-section paper for ease in combining, and with the zero-volume axis added. On the high-pressure card admission occurred practically at constant volume, piston being at rest at dead center, at A, bringing the pressures in the cylinder up to the initial pressure at B. Admission continued from B to C at nearly constant pressure, the piston moving slowly with correspondingly small demand for steam and consequently little wire drawing. From C to D the piston is moving more rapidly and there is in consequence more wire drawing, admission being no longer at constant pressure. At D the steam valve closes and expansion occurs, to E, where release occurs, the pressure falling to that in the receiver. From

F to *G* exhaust occurs with increase of pressure due to the steam being forced into the receiver (receiver+decreasing H.P. cyl.vol.), while from *G* to *H* the pressure falls, owing to the low-pressure cylinder taking steam from the receiver; consequently volume of receiver (receiver+increasing L.P. cyl.vol.+decreasing H.P. cyl.vol.), increases. At *H* exhaust closes, a very slight compression occurring from *H* to *A*.

On the low-pressure card, admission occurred at *A'* and continued to *B'* at constant volume, the piston being on dead center as from *A* to *B* in high-pressure cylinder. From *B'* to *C'* admission occurred with falling pressure due to increase in receiver volume (receiver+increasing L.P. cyl.vol.), and from *C'* to *D'* admission still took place, but with less rapidly falling pressure, as high-pressure cylinder is now exhausting and receiver volume (receiver+increasing L.P. cyl.vol.+decreasing H.P. cyl.vol.), was receiving, as well as delivering some steam. At *D'* admission ceased and expansion took place to *E'* where release occurred, the pressure falling to the back pressure and continuing from *F'* to *G'*, where the exhaust valve closed and compression took place to *A'*, thus completing the cycle. At *H'* leakage past the exhaust valve was so great as to cause the curve to fall off considerably from *H'* to *A'*, instead of continuing to be a true compression curve, ending at *I*, as it should have done. The combined diagrams are shown in B.

In Fig. 48 are shown a set of three cards from a triple-expansion pumping engine with large receivers and cranks at 120°. In diagram A the cards are shown with the same length and with different pressure scales as taken, but with the zero volume line added and transferred to cross-section paper. On the high-pressure card admission occurred at *A*, causing a constant-volume pressure rise to *B*, the piston being at rest with the crank at dead center. From *B* to *C* admission occurred at nearly constant pressure to *C*, where steam was cut off and expansion took place to *D*. At this point release occurred, the pressure dropping at constant volume to *E* with the piston at rest. From *E* exhaust took place with slightly increasing pressure, since the intermediate cylinder was taking no steam, the intermediate piston being beyond the point of cut-off. The pressure rise is slight, however, due to the size of the receiver, which is large compared to the cylinder. At two-thirds of the exhaust stroke, point *F*, the back pressure became constant, for at this point the speed of the intermediate piston increased and the receiver pressure equalized. At *G* exhaust closed and a slight pressure rise occurred to *A*, due to the restricted passage of the closing exhaust valve. On the intermediate card admission occurred at *A'*, the pressure rising to *B'*. From *B'* the admission was at nearly constant pressure to *X* while the piston speed was low and then at a falling pressure to *C'*. Pressure was falling, since the steam was supplied from a finite receiver into which no steam was flowing during intermediate admission. At *C'* cut-off occurred and steam expanded to *D'*, where release took place, and the steam was exhausted. As in the case of the high-pressure cylinder the back pressure was rising for two-thirds of the stroke, since the steam was being compressed into the receiver or rather into a volume made

up of receiver and intermediate cylinder volume, which is, of course, a decreasing one, since the cylinder volume is decreasing. At two-thirds of the stroke the low-pressure cylinder begins to take steam and the receiver volume is now increased, inasmuch as it was made up of the receiver portion of the intermediate cylinder and a portion of the low-pressure cylinder, and the low-pressure cylinder volume increased faster than intermediate decreased for the same amount of piston travel. At G' exhaust closed and a slight compression occurred to A' , thus completing the cycle.

On the low-pressure card admission occurred at A'' and the pressure rose at constant volume to B'' , and then admission continued first at constant pressure and then falling, as in the intermediate cylinder, to the point of cut-off

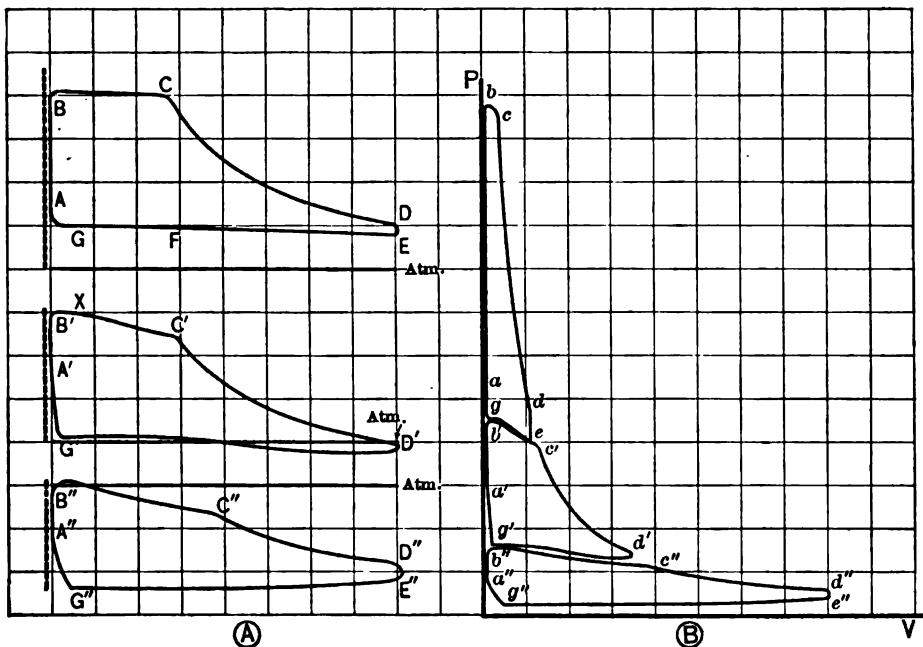


FIG. 48.—Indicator Cards from a Triple-expansion Engine with Receiver.

at C'' . From here expansion took place to D'' . At this point the exhaust valve opened, the pressure fell nearly to back pressure at E'' , and the steam was exhausted at practically constant back pressure to G'' , where the exhaust valve closed and there was compression to A'' , thus completing the cycle. The combined diagram is shown in B.

Prob. 1. In Fig. 49 are shown four sets of indicator cards from compound engines. The cylinder sizes and clearances are given below. Explain the cylinder events and the shape of lines for each card and form a combined diagram for each set.

No. 1. From a four-valve Corliss engine, 26×48 ins., with 3 per cent clearance in each cylinder.

No. 2. From a single-valve engine, $12 \times 20 \times 12$ ins., with 3.3 per cent clearance in high-pressure cylinder and 9 per cent in low.

No. 3. From a four-valve Corliss engine $22 \times 44 \times 60$ ins., with 2 per cent clearance in the high-pressure cylinder and 6 per cent in low.

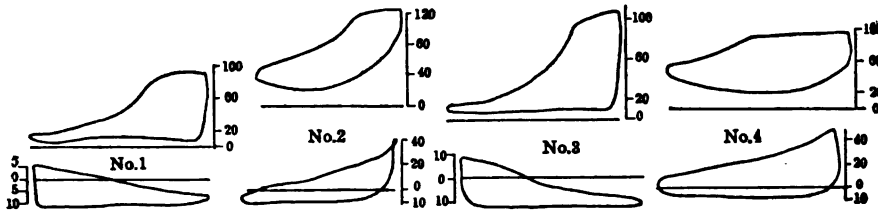
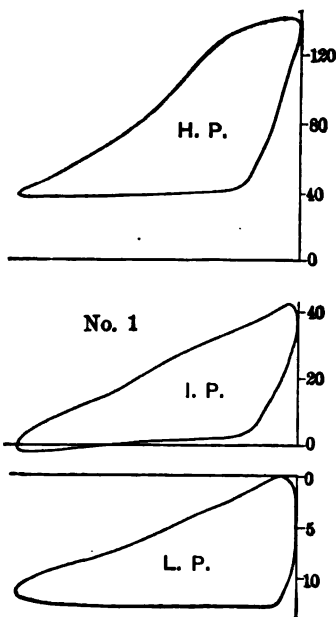


FIG. 49.—Four Sets of Compound-engine Indicator Cards.

No. 4. From a single-valve engine, $18 \times 30 \times 16$ ins., with 3.0 per cent clearance in the high-pressure cylinder and 8 per cent in the low.

Prob. 2. In Fig. 50 are shown two sets of indicator cards from triple-expansion marine engines. The cylinder sizes and clearances are given below. Explain the cylinder events and the shape of the lines of each card and form a combined diagram of each set.



No. 1. From the engine of a steam-ship, cylinders $21.9 \times 34 \times 57$ ins. $\times 39$ ins. with 6 per cent clearance in each and fitted with simple slide valves.

No. 2. From the engine of the steamer "Aberdeen," with cylinders $30 \times 45 \times 70 \times 54$ ins., and with 4 per cent clearance in the high, 7 per cent in the intermediate, and 8 per cent in the low.

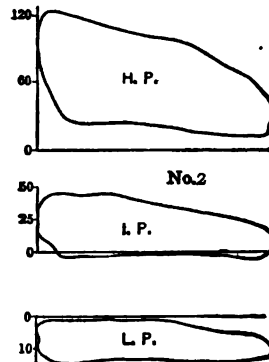


FIG. 50.—Two Sets of Triple-expansion Engine Indicator Cards.

44. Standard Reference Cycles or PV Diagrams for the Work of Expansive Fluids in Two-cylinder Compound Engines. The possible combinations of admission with all degrees of expansion for forward strokes and of exhaust with all degrees of compression for back strokes, with and without clearance, in each of the two cylinders of the compound engine, that may have any volume relation one to the other and any size of receiver between, and finally, any sort

of periodicity of receiver receipt and discharge of fluid, all make possible a very large number of cycles. In order that analysis of these conditions of working may be kept within reasonable space, it is necessary to proceed as indicated for compressors and simple engines, concentrating attention on such type forms as yield readily to analytical treatment for which the formulas are simple even if only approximate with respect to actual engines, but keeping in mind the possible value of the formulas, since those must be rejected as useless that teach no principles or fail to assist in solving problems.

The work that fluids under pressure can do by losing that pressure is no different in compound than in simple engines, *if the fluid has a chance to realize its possibilities*. If the structure is such as will not interfere with the completeness of the expansion, and no fluid is wasted in filling dead spaces without

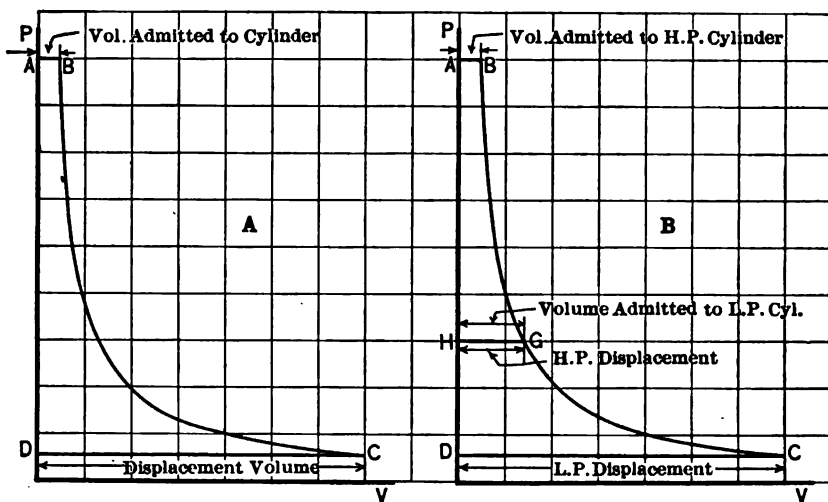


FIG. 51.—Diagram to Show Equality of Work for Expansion in One-cylinder Simple and in Two-cylinder Compound Engines for the Same Rate of Expansion.

working, then the work per cubic foot or per pound of fluid is the same for simple, compound and triple engines. Furthermore, there is a horse-power equivalence between the simple and compound, if, in the latter case the steam admitted up to cut-off may be assumed to be acting only in the low-pressure cylinder, that is, ignoring the high-pressure cylinder except as it serves as a cut-off means or meter. This should be clear from a comparison of Figs. 51, A and B. In Fig. 51A, representing the case of the simple engine without clearance and with complete expansion, the volume admitted, \overline{AB} , expands to the back pressure on reaching the full cylinder volume \overline{DC} , and exhausts at constant back pressure, the work being represented by the area \overline{ABCD} . It should be clear that no difference will result in the work done if a line be drawn across the work area as in Fig. 51B, all work done above the line \overline{HG} to be developed in the high-pressure cylinder and that below, in the low. This is merely equivalent to saying that a volume of steam \overline{AB} is admitted to the high-pressure

cylinder, expands completely to the pressure at G on reaching the full high-pressure cylinder volume, after which it exhausts at constant pressure (into a receiver of infinite capacity), this same amount being subsequently admitted without change of pressure to the low-pressure cylinder, when it again expands completely. Thus, it appears that the working of steam or compressed air in two successive cylinders instead of one will in no way change the maximum amount of work a cubic foot supplied can do, the compounding merely makes it easier to obtain this maximum. In simple engine cases, Fig. 51A, the cut-off in per cent of stroke is $100 \times \frac{AB}{DC}$, which is a very small value, leaving but little time to open and close the admission valve, whereas in the compound case the per cent cut-off in the high-pressure cylinder is $100 \times \frac{AB}{HG}$, and in the low-pressure cylinder, $100 \times \frac{HG}{DC}$, which are quite large enough ratios to be easily managed with ordinary valve gears.

Compounding does, however, introduce possibilities of loss not present in the single-stage expansion, if the dimensions or adjustments are not right, which may be classed somewhat improperly as *receiver losses*, and these are of two kinds, one due to incomplete expansion in the high and the other to over-expansion there. Thus, in Fig. 52, if $ABCEFGDA$ represent a combined compound diagram for the case of complete expansion in the high-pressure cylinder continued in the low without interruption but incomplete there, DC represents the volume in the low-pressure cylinder at cut-off, and at the same time the total high-pressure cylinder volume.

If now, the low-pressure cut-off be made to occur later, Fig. 53, then the volume that the steam would occupy when expansion began in the low-pressure cylinder is represented by $\overline{D'C'}$. This adjustment could not, of course, change the high-pressure total volume \overline{DC} , so that at release in the high-pressure cylinder the pressure would drop abruptly to such a value in the receiver as corresponds to filling the low pressure up to its cut-off, and work be lost equal to area $\overline{CC'C''}$.

A shortening of the low-pressure cut-off will have an equally bad effect by introducing negative work as indicated in Fig. 54, in which the L.P. cut-off volume is reduced from \overline{DC} to $\overline{D'C'}$. Expansion in the high pressure proceeds as before till the end of the stroke, at which time it has a volume \overline{DC}

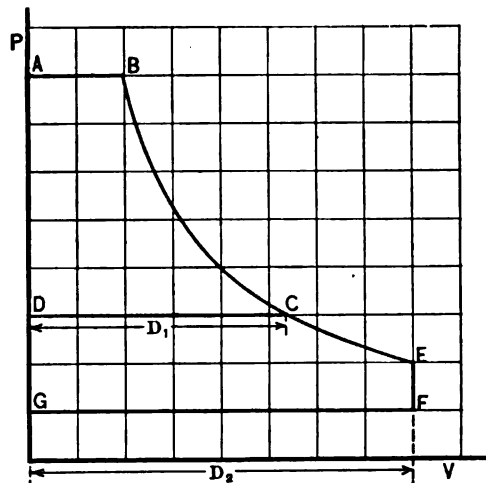


FIG. 52.—Diagram to Show Correct Low-pressure Cut-off for No-receiver Loss.

greater than the low pressure can receive, $\overline{D'C'}$, hence the receiver pressure must rise to such a value as will reduce the volume the required amount, introducing the negative work $\overline{CC'C''}$. Changes of low-pressure cut-off may, therefore, introduce negative work, change the receiver pressure and, of course, modify the distribution of work between high and low, but these same effects

might also have resulted from changes of high-pressure cut-off or of cylinder ratio.

For such conditions as have been assumed it seems that compounding does not increase the work capacity of fluids, but may make it easier to realize this capacity, *introducing at the same time certain rather rigid relations between cut-offs and cylinder volumes as necessary conditions to its attainment*. It can also be shown that the same proposition is true when there is clearance and compression, that is, in real cylinders, and when the receiver is real or not infinite in size, or when the exhaust of high and admission of low, are not constant-pressure lines. The former needs no direct proof, as inspection of previous diagrams makes it clear, but the latter requires some discussion.

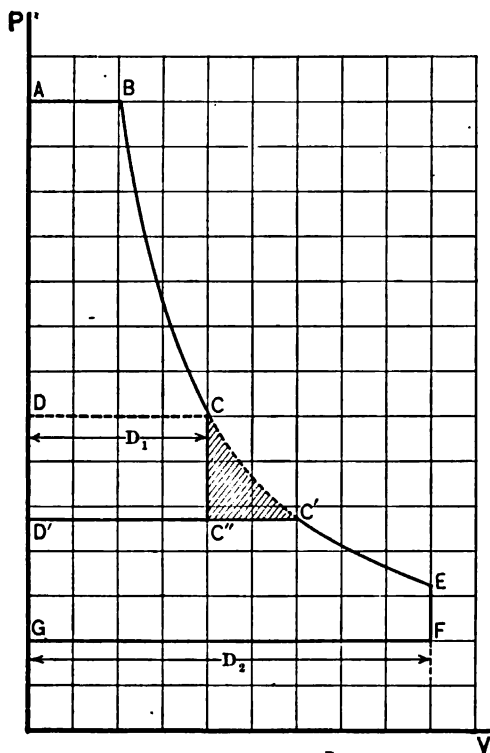


FIG. 53.—Diagram to Show Effect of Lengthening L.P. Cut-off Introducing a Receiver Loss Due to Incomplete High-pressure Cylinder Expansion.

its exhaust, and at other times with the low-pressure cylinder during admission, and these two events may take place at entirely independent times, be simultaneous as to time, or overlap in various ways. Suppose the *periods* to be *independent* and there be *no cylinder clearance*, then at the beginning of high-pressure exhaust two separate volumes of fluid come together, the contents of both the high-pressure cylinder and the receiver, and this double volume is compressed by the H.P. piston into the receiver, in which case the high-pressure exhaust would take place with rising pressure. Following this will come low-pressure admission, during which the volume of fluid in the receiver expands into the low-pressure cylinder up to its cut-off, and if the same volume is thus taken out of the receiver as entered it previously, low-pressure admission will take place with falling pressure, the line representing it exactly coinciding with that for the high-pressure exhaust. Independence of H.P.

A real receiver of finite size is at times in communication with the high-pressure cylinder during

cylinder exhaust and L.P. cylinder admission, as to time, may result in a cycle such as represented in Fig. 55 for the case of no cylinder clearance. On this diagram the receiver line is DC , an expansion or compression line referred to a second axis of volumes KJ , placed away from the axis of purely cylinder volumes by the distance \overline{LD} , equal to the receiver volume to scale. All diagram points are referred to the axis AI except those on the line DC .

This same case of time independence of H.P. exhaust and L.P. admission yields quite a different diagram when the cylinder clearance is considered. Such a case is represented by the diagram, Fig. 56, which also serves to illustrate the effect of incomplete expansion and compression as to equalization of receiver with cylinder pressures. At high-pressure release the volume of fluid in the H.P. cylinder is \overline{ML} and its pressure is \overline{LR} . This is about to come into communication with the receiver volume \overline{ON} from which the low-pressure cylinder finished taking fluid and which is, therefore, at the same pressure as the L.P. cylinder cut-off \overline{KS} . The question, therefore, is—what will be the pressure at P in both H.P. cylinder and receiver when \overline{LM} cu.ft. of fluid at \overline{LR} pressure combines with \overline{ON} cu.ft. at pressure \overline{KS} , and together occupy a volume $\overline{ON} + \overline{LM}$. By hypothesis the pressure after mixture is

$$\frac{(\text{first volume} \times \text{its pressure}) + (\text{second volume} \times \text{its pressure})}{\text{sum of volumes}}$$

From this or the graphic construction following, the point P is located. If the high-pressure expansion had continued to bring LQ to the receiver pressure \overline{KS} , it would reach it at X . At this hypothetical point there would be a volume \overline{NX} in the H.P. cylinder to add to the volume \overline{ON} in the receiver at the same pressure, resulting in \overline{OX} cu.ft. This fluid would have a higher pressure at the lesser volume of receiver and H.P. cylinder, and the value is found by a compression line through X , $XPAT$ referred to the receiver axis. This same line is also the exhaust of the H.P. cylinder from P to A . A similar situation exists at admis-

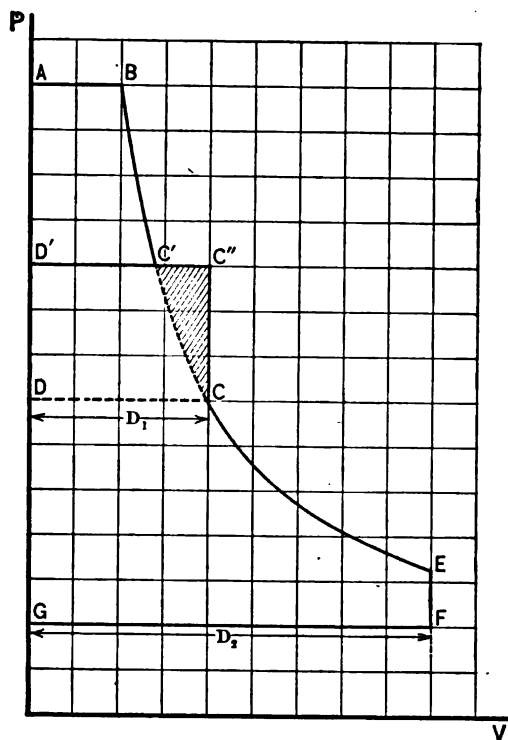


FIG. 54.—Diagram to Show Effect of Shortening L.P. Cut-off, Introducing a Receiver Loss Due to Over-expansion in the High-pressure Cylinder.

sion to the L.P. cylinder as to pressure equalization and location of admission line. At the end of the L.P. compression there is in the L.P. cylinder \overline{FE} cu.ft. at pressure \overline{EH} , to come into communication with the receiver volume \overline{CB} cu.ft. at pressure \overline{BG} , that at which H.P. exhaust ended. Producing the L.P. compression line to I , the volume \overline{BI} is found, which, added to receiver, results in no pressure change. An expansion line, referred to the receiver axis through I , fixes the equalized pressure J and locates the L.P. admission line JK , which, it must be observed, does not coincide with the H.P. exhaust.

So far only *complete independence* of the time of H.P. exhaust and L.P. admission have been considered, and it is now desirable to examine the diagrammatic representation of the results of *complete coincidence*. Such cases occur in practice with the ordinary tandem compound stationary steam engine, and twin-cylinder single-crosshead Vauclain compound steam locomotive. In the

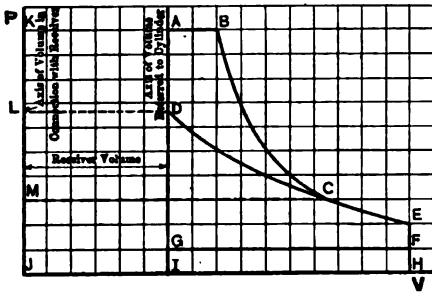


FIG. 55.—Diagram to Illustrate Variable Receiver Pressure for the Case of Independent High-pressure Exhaust and Low-pressure Admission with Zero Clearance.

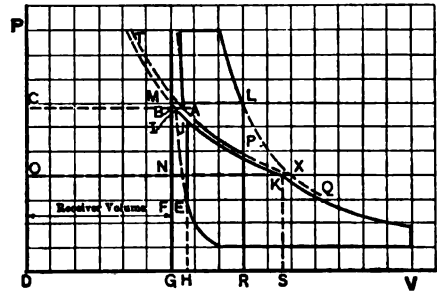


FIG. 56.—Diagram to Illustrate Variable Receiver Pressure for the Case of Independent High-pressure Exhaust and Low-pressure Admission with Clearance.

latter structure both pistons move together, a single valve controlling both cylinders, exhaust from high taking place directly into low, and for exactly equal coincident time periods. The diameter of the low-pressure cylinder being greater than the high, the steam at the moment of release suffers a drop in pressure in filling the low-pressure clearance, unless, as rarely happens, the pressure in the low is raised by compression to be just equal to that at H.P. release. After pressure equalization takes place, high-pressure exhaust and low-pressure admission events are really together a continuation of expansion, the volume occupied by the steam at any time being equal to the difference between the two-cylinder displacements and clearances up to that point of the stroke. Before this period of communication, that is, between high-pressure cut-off and release, the volume of the expanding fluid is that of the high-pressure displacement up to that point of the stroke, together with the high-pressure clearance. After the period of communication the volume of the expanding fluid is that of the low-pressure cylinder up to that point of the stroke, together with the low-pressure clearance, plus the high-pressure displacement not yet swept out, and the high-pressure clearance.

These fluid processes cannot be clearly indicated by a single diagram,

because a diagram drawn to indicate volumes of fluid will not show the volumes in the cylinders without distortion. If there be *no clearance*, Fig. 57 will assist in showing the way in which two forms of diagram for this purpose are derived. Referring to Fig. 57A, the volume \overline{AB} admitted to the high-

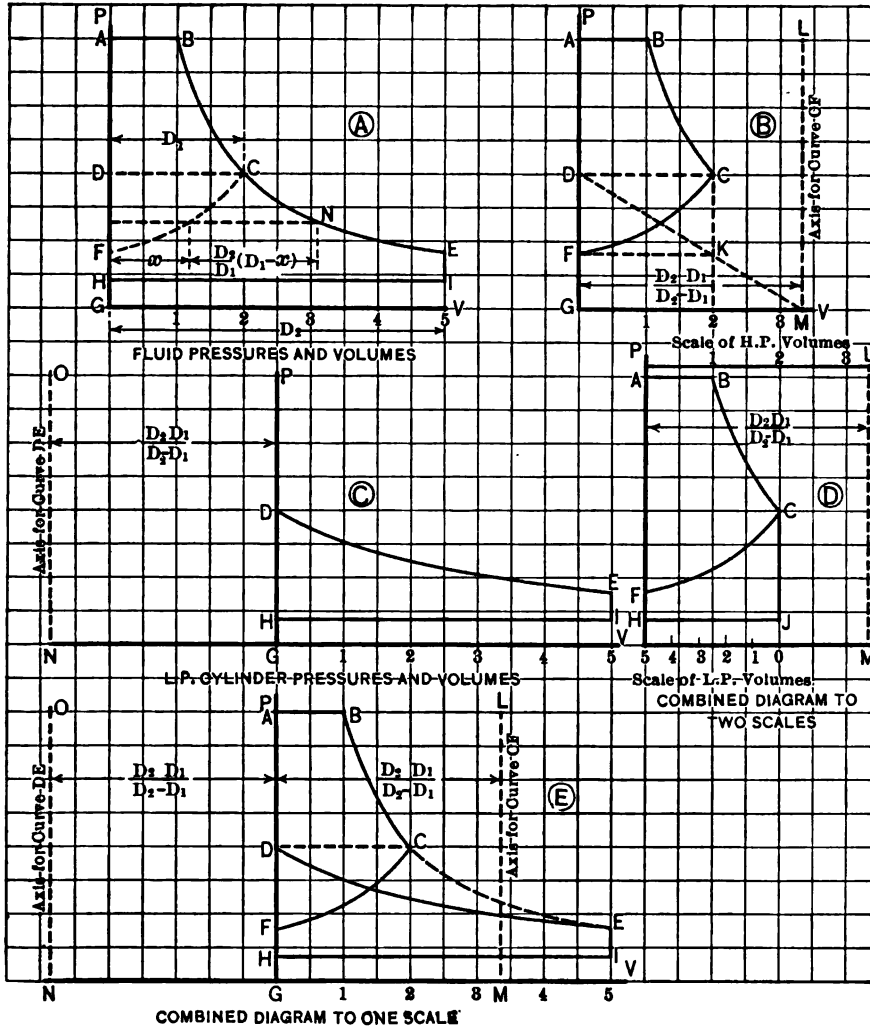


FIG. 57.—Diagram to Illustrate the Compound Engine Cycle with No-receiver, and Exact Coincidence of H.P. Exhaust and L.P. Admission Periods. No Clearance.

pressure cylinder expands until it occupies the whole H.P. cylinder volume \overline{DC} . At this point expansion proceeds in low and high together, with decreasing volumes in high and increasing in low until the low-pressure cylinder volume is attained at E. The line BCE then indicates the pressures and volumes of the fluid expanding, but *does not clearly show the volume in either cylinder*

between C and E , with the corresponding pressure. It is certain, however, that when the volume in the H.P. cylinder becomes zero the pressure must have fallen to a value the same as that in the low when the fluid completely fills it, or $P_f = P_s$.

As the high-pressure piston returns, on the exhaust stroke, the low-pressure piston advances an equal distance, on its admission stroke, sweeping through a greater volume than the high pressure, in the ratio of low-pressure to high-pressure displacements. If at any point of the stroke the volume remaining in the high-pressure cylinder be x , and the high- and low-pressure displacements be respectively D_1 , and D_2 , then $(D_1 - x)$ is the volume swept out by high-pressure piston, x the volume remaining in it, and $\frac{D_2}{D_1}(D_1 - x)$ the volume swept in by the low-pressure piston. Then the total volume still in the two cylinders is, for a point between C and E ,

$$V = x + \frac{D_2}{D_1}(D_1 - x) = D_2 - x\left(\frac{D_2}{D_1} - 1\right).$$

Since the equation of the curve CE is, $PV = P_c V_c = P_c D_1$, the value of V may be substituted, giving $P\left[D_2 - x\left(\frac{D_2}{D_1} - 1\right)\right] = P_c D_1 = \text{constant}$. Dividing by $\left(\frac{D_2}{D_1} - 1\right)$ this becomes $P\left[\frac{D_2}{\left(\frac{D_2}{D_1} - 1\right)} - x\right] = P\left[\frac{D_2 D_1}{D_2 - D_1} - x\right] = \text{a new constant}$, so

that if a new axis LM be laid off on B , $\overline{GV} = \frac{D_2 D_1}{D_2 - D_1}$ from the axis GP , any point on FC will be distant from the new axis LM an amount $\frac{D_2 D_1}{D_2 - D_1} - x$; as the product of this distance and the pressure P , is constant, the curve FC is an equilateral hyperbola referred to the axis LM . Therefore Fig. 57B is the pressure-volume representation of the entire cycle of the high-pressure cylinder.

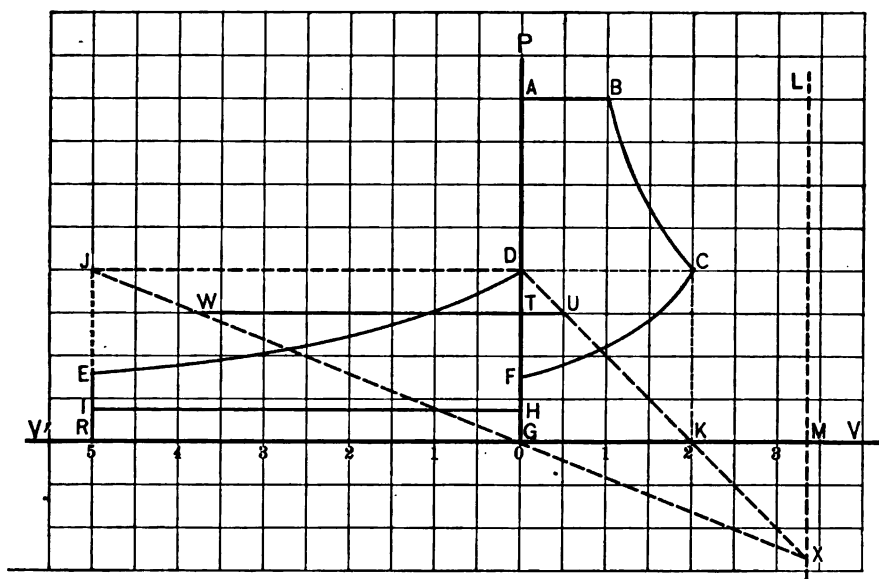
In Fig. 57C is shown the corresponding pressure-volume diagram for the low-pressure cylinder, for which it may similarly be shown that the curve DE may be plotted to an axis NO at a distance to the left of the axis GP equal to the same quantity,

$$GN = \frac{D_2 D_1}{D_2 - D_1} \dots \dots \dots (216)$$

These diagrams, 57A, B and C may be superposed, as in Fig. 57E, giving one form of combined diagram used for this purpose, and the one most nearly comparable with those already discussed. In this diagram, the area $ABCF A$ represents the work of the high-pressure cylinder, and \overline{DEIHD} , the work of the low-pressure cylinder. Together, they equal the work of the enclosing figure $\overline{ABEIH A}$, and hence the work of the low-pressure cylinder must also be represented by the area \overline{FCEIHF} .

It is not difficult to show that if a vertical, CJ , be dropped from the point C to the exhaust line HJ , the figure \overline{CFHJ} , in Fig. 57D is similar to \overline{DEIH} ,

Since it has been shown that the curves CF and DE are of the same mathematical form (hyperbolic) as the expansion line CE , it should be noted that they may be plotted in the same way after having found the axis. The location of this axis may be computed as given above, or may be found graphically by the method given in connection with the subject of clearance, Chapter I, and shown in Fig. 55*B*. Knowing two points that lie on the curve,



C and F , the rectangle $CDFK$ is completed. Its diagonal, DK , extended, cuts the horizontal axis GV in the point M , which is the base of the desired axis ML .

If now the axis *NO* and the figure *DEIH*, part of which is referred to this axis, be reversed about the axis *GP*, Fig. 57*C*, *NO* will coincide with *ML*, Fig. 57*D*, and Fig. 58 results. Note that the axis here may be found graphically, in a very simple way. Draw the vertical *CK* from *C* to the axis *GV* and the horizontal *DJ* to the vertical *IE* extended. \overline{DC} is then the high-pressure displacement and \overline{DJ} the low-pressure displacement. Draw the two diagonals *DK* and *JG*, extending them to their intersection *X*. By similar triangles it may be shown that a horizontal line, *UW*, will have an intercept between these two lines, *JG* and *DK*, equal to the volume of fluid present in the two cylinders

combined. The intersection X is the point at which this volume would become zero if the mechanical process could be continued unchanged to that point, and is, therefore, on the desired axis ML extended. T being the intersection of WU with the axis GP , when the volume UT is present in the high-pressure cylinder, TW gives the volume in the low-pressure cylinder.

When *clearance* and *compression* are considered, this diagram is changed in many respects, and is shown in Fig. 59. The axes OP , OV and OV' are laid out, with OZ equal to the clearance and ZK , the displacement, of the high-pressure cylinder, and OQ and QY , clearance and displacement of the low-pressure cylinder.

It is necessary to know high-pressure cut-off, $\frac{AB}{ZK}$; high-pressure compression,

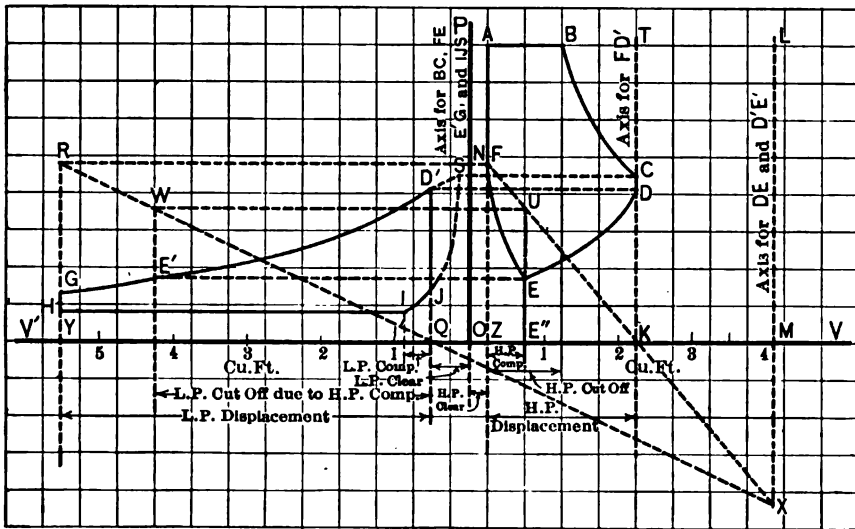


FIG. 59.—Diagram to Show Volume of Steam in the Cylinders of the No-receiver Compound Engine at any Point of the Stroke for the Case of Clearance and Coincidence of H.P. Exhaust with L.P. Admission.

$\frac{ZE''}{ZK}$; and low-pressure compression, $\frac{I'Q}{YQ}$, in addition to the initial and back pressures, in order to lay out the diagram. The drop in pressure CD at release is due to the coming together of (volume V_c at pressure P_c), with (volume V_j at pressure P_j). If the volume V_j (measured from axis OP) were decreased sufficiently to raise the pressure in the low-pressure clearance to the pressure at C , the volume would become V_s , as indicated at the point S , and the volumes now combined in the hypothetical condition would occupy the volume SC .

Increasing this volume to $D'D$, that actually occupied after communication, the pressure would fall along the curve SD' , which is constructed on KV' and KT' as axes. The pressure of D is then laid out equal to the pressure at D' . To find the axis, ML , for the curves DE and $D'E'$, from any convenient point N on ZA , draw the line NK extended to X . Extend HG to R , at a height equal

to that of N , and draw RQX ; through the intersection draw the desired axis XML . The fraction of stroke completed at E' in the low pressure at cut-off must be equal to that completed at E in the high pressure at compression, and may be laid out graphically by projecting up from E to the point U on the line NK and horizontally from U to W on the line RQ . Projecting down from W to the curve at E' locates the point of effective cut-off in the low-pressure cylinder. After the supply from the high-pressure cylinder has been cut off at E' , the expansion is that of the volume in the low-pressure cylinder and its clearance, and hence the curve $E'G$ is constructed on OP as an axis.

While in this last case a zero receiver volume has been assumed, there is nothing to prevent a receiver volume being interposed between H.P. and L.P. so that common expansion takes place with a volume greater than assumed by so much as this volume, the effect of which is to decrease the slope of DE and $D'E'$. Such receivers usually consist of the spaces included in a valve body and connecting passages and may be treated generally as increased L.P. clearances.

The most common of all relations between H.P. exhaust and L.P. admission is, of course, that of *partial coincidence of periods*, as it is thus with all cross-compound and triple-expansion engines having separate cranks for the individual cylinders. For these there is no simple fixed relation between the periods, for, while crank angles are generally fixed in some comparatively simple relation, such as 90° , 180° and 270° for compounds and 120° for triple-expansion engines, they are sometimes set at other angles for better balance or for better turning effort. Even if the angles were known the receiver line would have to be calculated point by point. When the H.P. cylinder begins to discharge into a receiver for, say, a cross compound with cranks at 90° , steam is compressed into the receiver, and so far the action is the same as already considered for independence of periods, but at near mid-stroke the low-pressure admission opens while high-pressure exhaust continues. This will cause the receiver pressure to stop rising and probably to fall until the low pressure cuts off, which may occur before the H.P. exhaust into the receiver ceases. If it does, the receiver pressure will again rise. Exact determination of such complex receiver lines is not often wanted, and when needed is best obtained graphically, point by point. Its value lies principally in fixing exactly the *work distribution between cylinders*, which is not of great importance except for engines that are to work at constant load nearly all the time, such as is the case with city water works pumping, and marine engines. While equations could be derived for these cases, they are not worth the trouble of derivation, because they are too cumbersome; therefore graphic methods are to be substituted or approximations made.

Four kinds of approximation are available, as follows, all of which ignore partial coincidence of periods:

1. Receiver pressure constant at some mean value and clearance ignored.
2. Receiver pressure constant at some constant value and clearance considered with compression zero or complete.

3. Receiver pressure fluctuates between fixed limits as determined by an assumed size, clearance ignored.

4. Receiver pressure fluctuates between fixed limits as determined by an assumed size, clearance considered, with compression zero or complete.

These are not all of equal difficulty in solution, and the one to be used is that nearest the truth as to representation of conditions, which is usually the most difficult, provided time permits or the information is worth the trouble. Quickest work is accomplished with assumption (1) and as this is most often used in practical work it indicates that its results are near enough for most purposes.

This discussion leads, therefore, to the analytical study of the following cycles:

INFINITE RECEIVER, ZERO CYLINDER CLEARANCE.

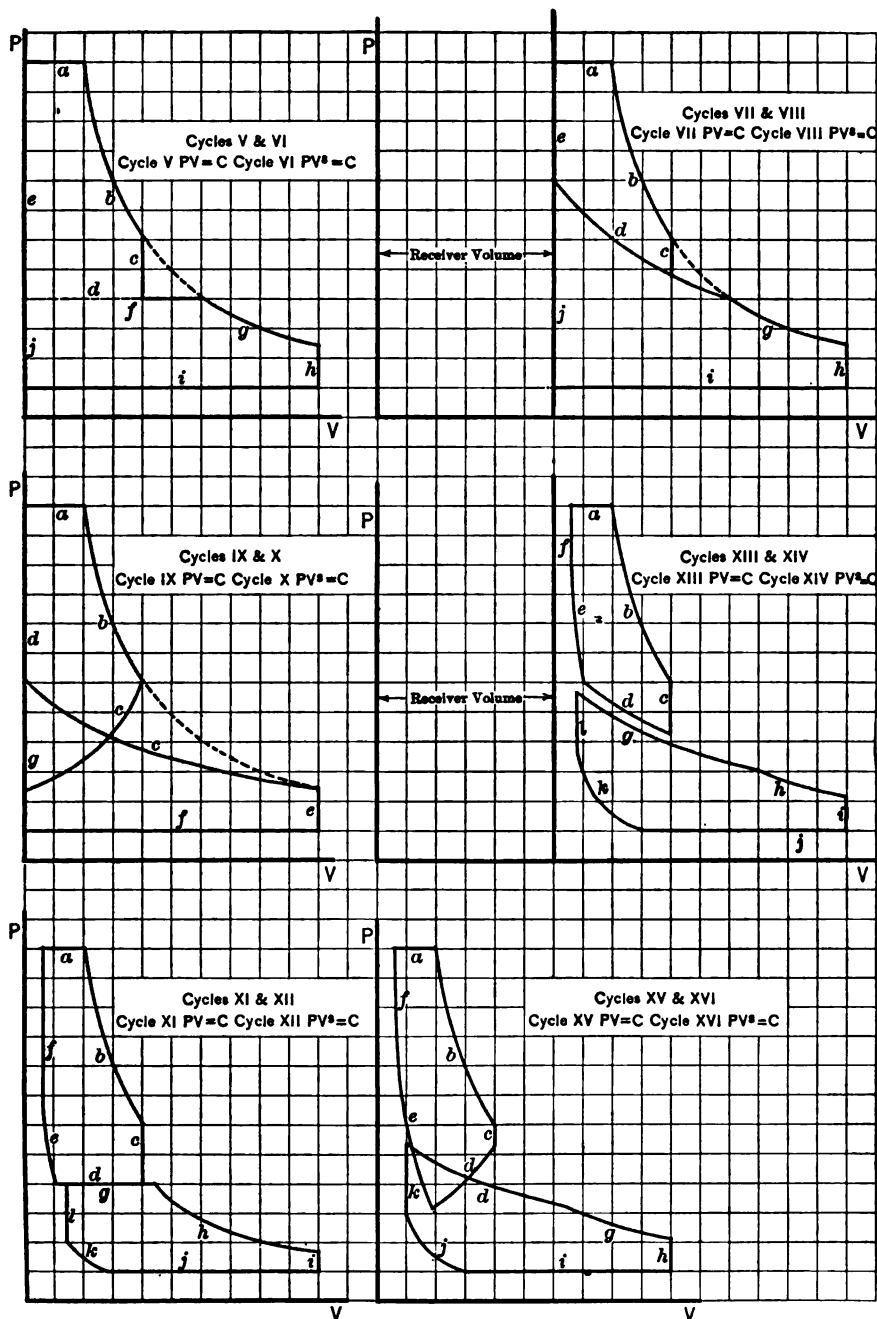
CYCLES V AND VI (Fig. 60).

H.P. CYLINDER EVENTS	(a)	Admission at constant supply pressure to H.P. cylinder.
	(b)	Expansion in H.P. cyl. (may be zero) by law $PV=c$ for (V); $PV^{\gamma}=c$ for (VI).
	(c)	Equalization of H.P. cyl. pressure with receiver pressure at constant volume (may be zero).
	(d)	Exhaust into infinite receiver at constant pressure from H.P. cylinder.
	(e)	Equalization of H.P. cylinder pressure with supply pressure at constant zero volume.
L.P. CYLINDER EVENTS	(f)	Admission from receiver at constant receiver pressure to L.P. cylinder.
	(g)	Expansion in L.P. cylinder (may be zero) by law $PV=c$ for (V); $PV^{\gamma}=c$ for (VI).
	(h)	Equalization of L.P. cylinder pressure with back pressure at constant volume (may be zero).
	(i)	Exhaust at constant back pressure for L.P. cylinder.
RELATIONS BETWEEN H.P. AND L.P. CYLINDER EVENTS	(j)	Equalization of L.P. cylinder pressure with receiver pressure at constant zero volume.
	(1)	H.P. exhaust and L.P. admission independent as to time, coincident as to representation (except as to length).
	(2)	H.P. expansion line produced coincides as to representation with L.P. expansion line.
	(3)	The length of the constant pressure receiver line up to the H.P. expansion line produced is equal to the length of the L.P. admission line.

FINITE RECEIVER, ZERO CYLINDER CLEARANCE.

CYCLES VII AND VIII, (Fig. 60).

H.P. CYLINDER EVENTS	(a)	Admission at constant supply pressure to H.P. cylinder.
	(b)	Expansion in H.P. cylinder (may be zero) by law $PV=c$ for (VII); $PV^{\gamma}=c$ for (VIII).

FIG. 60.—Compound Engine Standard Reference Cycles or PV Diagrams.

H.P. CYLINDER EVENTS	(c)	Equalization of H.P. cylinder pressure with receiver pressure at constant volume (may be zero) with a change of receiver pressure toward that at H.P. cylinder release (may be zero).
	(d)	Exhaust into finite receiver from H.P. cylinder at rising pressure equivalent to compression of fluid in H.P. cylinder and receiver into receiver by law $PV=c$ for (VII) and $PV^{\gamma}=c$ for (VIII).
	(e)	Equalization of H.P. cylinder pressure with supply pressure at constant zero volume.
L.P. CYLINDER EVENTS	(f)	Admission from receiver to L.P. cylinder at falling pressure equivalent to expansion of fluid in receiver into receiver and L.P. cylinder together by law $PV=c$ for (VII), $PV^{\gamma}=c$ for (VIII).
	(g)	Expansion in L.P. cylinder (may be zero) by law $PV=c$ for (VII); $PV^{\gamma}=c$ for (VIII).
	(h)	Equalization of L.P. cylinder pressure with back pressure at constant volume (may be zero).
	(i)	Exhaust at constant back pressure for L.P. cylinder.
	(j)	Equalization of L.P. cylinder pressure with receiver pressure at constant zero volume to value resulting from H.P. exhaust.
RELATION BETWEEN H.P. AND L.P. CYLINDER EVENTS	(1)	H.P. exhaust and L.P. admission independent as to time, coincident as to representation, except as to length.
	(2)	H.P. expansion line produced coincides as to representation with L.P. expansion line.
	(3)	The length of the receiver pressure line up to the H.P. expansion line produced is equal to the length of the L.P. admission line.

NO RECEIVER, ZERO CYLINDER CLEARANCE.

CYCLES IX AND X, (Fig. 60).

H.P. CYLINDER EVENTS	(a)	Admission at constant-supply pressure to H.P. Cylinder.
	(b)	Expansion in H.P. cylinder (may be zero) by law $PV=c$ for (IX); $PV^{\gamma}=c$ for (X).
BOTH H.P. AND L.P. SIMULTANEOUSLY	(c)	Transference of fluid from H.P. to L.P. cylinder with simultaneous continuation of expansion until all fluid is in L.P. cylinder and expanded to its full volume by law $PV=c$ for (IX); $PV^{\gamma}=c$ for (X).
H.P. CYLINDER EVENTS	(d)	Equalization of H.P. cylinder pressure to the pressure of supply.
L.P. CYLINDER EVENTS	(e)	Equalization of L.P. cylinder pressure with back pressure at constant volume (may be zero).
	(f)	Exhaust at constant back pressure for L.P. cylinder.
	(g)	Equalization of L.P. cylinder pressure to the pressure in H.P. cylinder at the end of its expansion.

INFINITE RECEIVER, WITH CYLINDER CLEARANCE.

CYCLES XI AND XII, (Fig. 60).

H.P. CYLINDER EVENTS	<ul style="list-style-type: none"> (a) Admission at constant-supply pressure to H.P. cylinder. (b) Expansion in H.P. cylinder (may be zero) by law $PV=c$ for (XI); $PV^{\gamma}=c$ for (XII). (c) Equalization of H.P. cylinder pressure with receiver pressure at constant volume (may be zero) pressure. (d) Exhaust into infinite receiver at constant pressure from H.P. cylinder. (e) Compression in H.P. cylinder to clearance volume (may be zero) by law $PV=c$ for (XI); $PV^{\gamma}=c$ for (XII). (f) Equalization of H.P. cylinder pressure with supply pressure at constant-clearance volume, may be zero.
L.P. CYLINDER EVENTS	<ul style="list-style-type: none"> (g) Admission from receiver at constant-receiver pressure to L.P. cylinder. (h) Expansion in L.P. cylinder (may be zero) by law $PV=c$ for (XI); $PV^{\gamma}=c$ for (XII). (i) Equalization of L.P. cylinder pressure with back pressure at constant volume (may be zero). (j) Exhaust at constant back pressure for L.P. cylinder. (k) Compression in L.P. cylinder to clearance volume by law $PV=c$ for (XI); $PV^{\gamma}=c$ for (XII) (may be zero). (l) Equalization of L.P. cylinder pressure with receiver pressure at constant-clearance volume without change of receiver pressure (may be zero).
RELATIONS BETWEEN H.P. AND L.P. CYLINDER EVENTS	<ul style="list-style-type: none"> (1) H.P. exhaust and L.P. admission independent as to time, coincident as to representation except as to length. (2) L.P. expansion line does not coincide as to representation with H.P. expansion line produced by reason of clearance influence except in one special and unusual case. (3) The length of the constant-receiver pressure line intercepted between H.P. compression line and H.P. expansion line produced is equal to the same intercept between L.P. expansion line and L.P. compression line produced. This is equivalent to the condition that the volume taken in by low is equal to expelled by the high reduced to the same pressure.

FINITE RECEIVER, WITH CYLINDER CLEARANCE.

CYCLES XIII AND XIV, (Fig. 60).

H.P. CYLINDER EVENTS	<ul style="list-style-type: none"> (a) Admission at constant supply pressure to H.P. cylinder. (b) Expansion in H.P. cylinder (may be zero) by law $PV=c$ for (XIII); $PV^{\gamma}=c$ for (XIV). (c) Equalization of H.P. cylinder pressure with receiver pressure at constant volume (may be zero) toward that at H.P. cylinder release (may be zero).
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H.P. CYLINDER EVENTS	<ul style="list-style-type: none"> (d) Exhaust into finite receiver from H.P. cylinder at rising pressure equivalent to compression of fluid in H.P. cylinder and receiver into receiver by law $PV=c$ for (XIII); $PV^s=c$ for (XIV). (e) Compression in H.P. cylinder to clearance volume (may be zero) by law $PV=c$ for (XIII); $PV^s=c$ for (XIV). (f) Equalization of H.P. cylinder pressure with supply pressure at constant-clearance volume.
L.P. CYLINDER EVENTS	<ul style="list-style-type: none"> (g) Admission from receiver to L.P. cylinder at falling pressure equivalent to expansion of fluid in receiver into receiver and L.P. cylinder together by $PV=c$ for (XIII); $PV^s=c$ for (XIV). (h) Expansion in L.P. cylinder (may be zero) by law $PV=c$ for (V); $PV^s=c$ for (VI). (i) Equalization of L.P. cylinder pressure with back pressure at constant volume (may be zero). (j) Exhaust at constant back pressure for L.P. cylinder. (k) Compression in L.P. cylinder to clearance volume by law $PV=c$ for (XI); $PV^s=c$ for (XII) (may be zero). (l) Equalization of L.P. cylinder pressure with receiver pressure at constant-clearance volume with change of receiver pressure in direction of L.P. compression pressure (may be zero).
RELATIONS BETWEEN H.P. AND L.P. CYLINDER EVENTS	<ul style="list-style-type: none"> (1) H.P. exhaust and L.P. admission independent as to time, representation and length. (2) L.P. expansion line does not coincide as to representation with H.P. expansion line produced by reason of clearance influence except in one special and unusual case. (3) The high-pressure exhaust and low-pressure admission lines do not coincide as to representation by reason of clearance influences. (4) There is a relation between the lengths of the L.P. admission and H.P. exhaust lines, but not a simple one.

NO RECEIVER, WITH CYLINDER CLEARANCE.

CYCLES XV AND XVI (Fig. 60).

H.P. CYLINDER EVENTS	ALSO L.P. EVENT	<ul style="list-style-type: none"> (a) Admission at constant-supply pressure to H.P. cylinder. (b) Expansion in H.P. cylinder (may be zero) according to law $PV=c$ for (XV); $PV^s=c$ for (XVI). (c) Equalization of pressures in H.P. cylinder after expansion with that in L.P. after compression at constant volume (may be zero). (d) Transference of fluid from H.P. to L.P. cylinder until all fluid is in L.P. cylinder and expanded to its full volume by same law as (b). (e) Compression in H.P. cylinder to clearance volume (may be zero) by law $PV=c$ for (XV); $PV^s=c$ for (XVI). (f) Equalization of pressure in H.P. cylinder with supply at constant-clearance volume (may be zero).
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L.P.
CYLINDER
EVENTS

- (g) Expansion in L.P. cylinder may be zero by law $PV=c$ for (XV); $PV^s=c$ for (XVI).
- (h) Equalization of pressure in L.P. cylinder with back pressure, at constant volume (may be zero).
- (i) Exhaust at constant pressure for L.P. cylinder.
- (j) Compression in L.P. cylinder to clearance, may be zero by law $PV=c$ for (XV); $PV^s=c$ for (XVI).
- (k) Equalization of L.P. cylinder pressure with H.P. cylinder pressure.

Cycle XVII, Fig. 61, for the triple expansion is defined in the same way as the corresponding case of compounds Cycle V, with appropriate alterations in wording to account for a third or intermediate cylinder between high- and low-pressure cylinders and an additional receiver. Thus, high-pressure cylinder exhausts into first, and intermediate cylinder into second receiver: intermediate cylinder receives its supply from first, and low-pressure cylinder from second receiver. This being so, it is unnecessary to write out the cylinder events, as these may be noted from their relation to the corresponding compound case.

45. Compound Engine with Infinite Receiver, Logarithmic Law. No Clearance, Cycle V. General Relations between Pressures, Dimensions, and Work. It must be understood that the diagrams representing this cycle, Fig. 62, indicating (A) *incomplete expansion* and (B) *over-expansion* in both cylinders, may just as well stand for over, complete, or incomplete expansion in all possible combinations in the two cylinders. Applying the principles already derived for calculating the work areas,

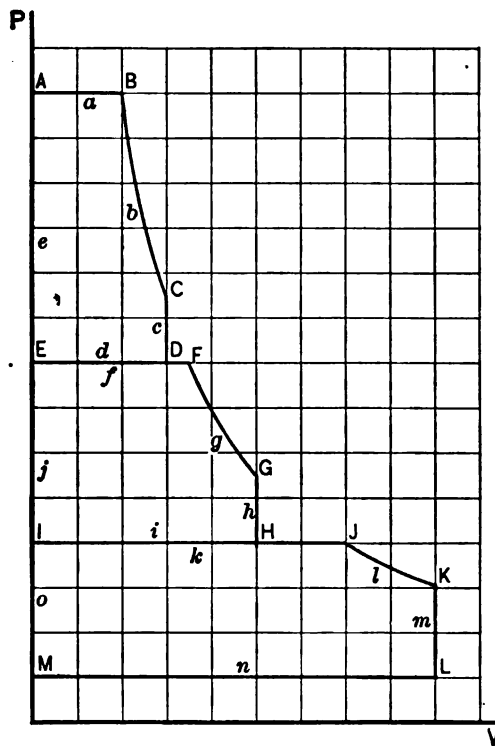


FIG. 61.—Triple-expansion Engine Standard Reference Diagram or PV Cycle for Infinite Receiver.

$$\text{High-pressure cylinder work, } W_H = P_b V_b \left(1 + \log_e \frac{V_c}{V_b} \right) - P_d V_d, \quad \dots \quad (217)$$

$$\text{Low-pressure cylinder work, } W_L = P_e V_e \left(1 + \log_e \frac{V_f}{V_e} \right) - P_g V_g, \quad \dots \quad (218)$$

$$\text{Total work, } W = P_b V_b \left(1 + \log_e \frac{V_c}{V_b} \right) + P_c V_c \left(1 + \log_e \frac{V_f}{V_c} \right) - P_d V_d - P_e V_e, \quad (219)$$

pressure being in pounds per square foot, and volumes in cubic feet.

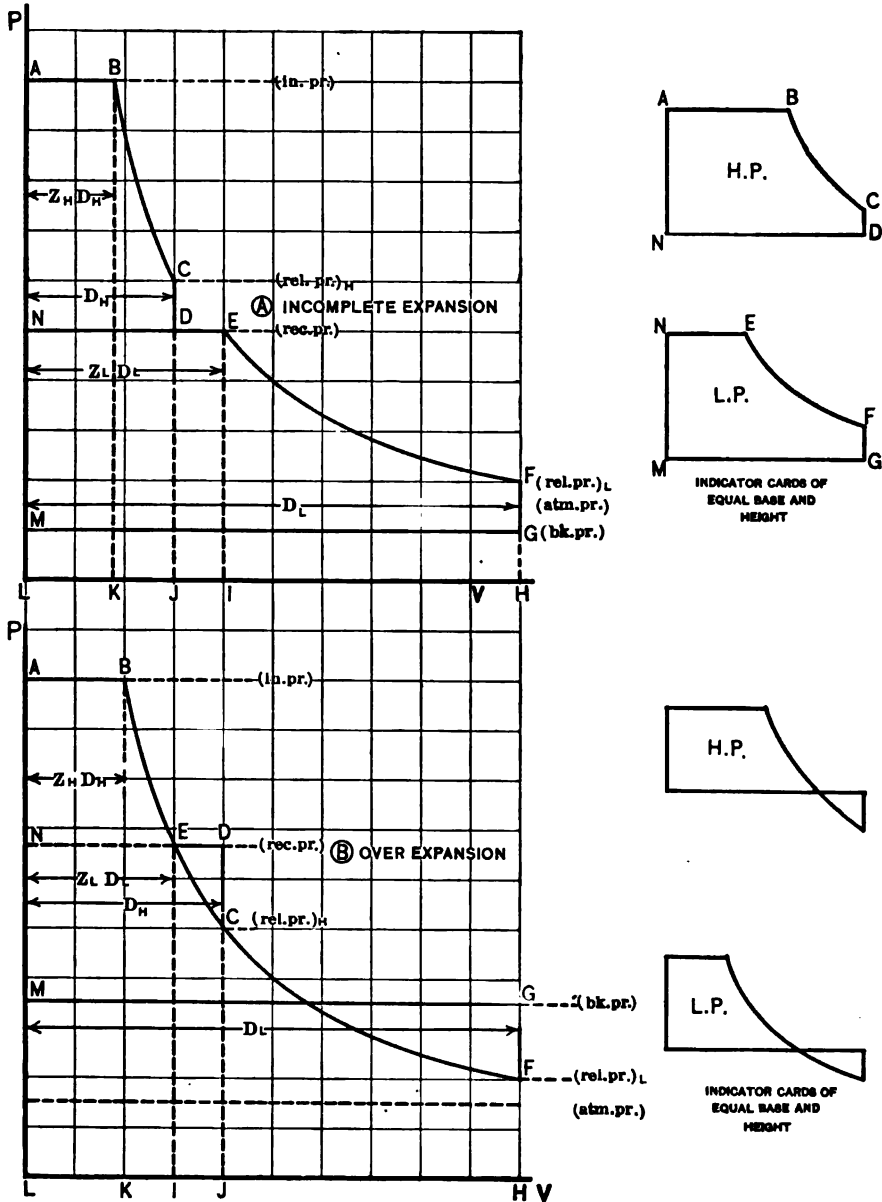


FIG. 62.—Work of Expansive Fluid in Compound Engine with Infinite Receiver, Zero Clearance. Cycle V, Logarithmic Expansion and Cycle VI, Exponential Expansion.

In these expressions the receiver pressure $P_e = P_d$ is unknown, but determinate, as it is a function of initial pressure and certain volumes; giving it the value

$$P_e = P_d = P_b \frac{V_b}{V_e},$$

is merely satisfying the condition that the point E at which expansion begins in the low-pressure cylinder must lie in the expansion line of the high. Substituting this value there results

$$\begin{aligned} W &= P_b V_b \left(1 + \log_e \frac{V_e}{V_d}\right) + P_b V_b \left(1 + \log_e \frac{V_f}{V_e}\right) - P_b V_b \frac{V_d}{V_e} - P_e V_e \\ &= P_b V_b \left[2 + \log_e \frac{V_e}{V_b} + \log_e \frac{V_f}{V_e} - \frac{V_d}{V_e}\right] - P_e V_e. \quad (220) \end{aligned}$$

This is a perfectly general expression for the *work* of the fluid expanding to any degree in two cylinders in succession when the clearance is zero and receiver volume infinite, in terms of initial and back pressures, pounds per square foot, the volumes occupied by the fluid in both cylinders at cut-off, and at full stroke in cubic feet. Dividing this by the volume of the low-pressure cylinder V_b gives the *mean effective pressure* referred to the low-pressure cylinder, from which the horse-power may be determined without considering the high-pressure cylinder at all. Hence, in the same units as are used for P_b and P_e ,

$$(\text{M.E.P. referred to L.P.}) = P_b \frac{V_b}{V_e} \left[2 + \log_e \frac{V_e}{V_b} + \log_e \frac{V_f}{V_e} - \frac{V_d}{V_e}\right] - P_e. \quad (221)$$

Proceeding as was done for simple engines, the *work per cubic foot of fluid supplied* is found by dividing Eq. (220) by the volume admitted to the high-pressure cylinder V_b , whence,

$$\text{Work per cu.ft. supplied} = P_b \left[2 + \log_e \frac{V_e}{V_b} + \log_e \frac{V_f}{V_e} - \frac{V_d}{V_e}\right] - P_e \frac{V_d}{V_b}. \quad (222)$$

Also applying the *consumption* law with respect to horse-power,

$$\text{Cu. ft. supplied per hour per I.H.P.} = \frac{13,750}{(\text{m.e.p. ref. to low})} \frac{V_b}{V_e}. \quad (223)$$

$$\text{Lbs. supplied per hr. per I.H.P.} = \frac{13,750}{(\text{m.e.p. ref. to low})} \frac{V_b}{V_e} \delta_1. \quad (224)$$

These last five equations, while characteristic, are not convenient for general use in their present form, but are rendered so by substituting general symbols for initial and back pressures, displacement, cut-off, and amount of expansion for each cylinder.

Let (in.pr.) = initial or supply pressure, pounds per square inch = $\frac{P_b}{144}$;

(rel.pr.)_H = release pressure, in H.P. cylinder pounds per square inch = $\frac{P_e}{144}$;

(rel.pr.)_L = release pressure in L.P. cylinder, pounds per square inch = $\frac{P_f}{144}$;

Let (rec.pr.) = receiver pressure, pounds per square inch = $\frac{P_a}{144} = \frac{P_e}{144}$;

(bk.pr.) = back pressure, pounds per square inch = $\frac{P_e}{144}$;

R_H = ratio of expansion in high-pressure cylinder = $\frac{V_c}{V_b}$;

R_L = ratio of expansion in low-pressure cylinder = $\frac{V_f}{V_e}$;

R_V = ratio of expansion for whole expansion = $\frac{V_f}{V_b}$;

D_H = displacement of high-pressure cylinder = $V_d = V_c$;

D_L = displacement of low-pressure cylinder = $V_f = V_e$;

R_C = cylinder ratio = $\frac{D_L}{D_H} = \frac{V_e}{V_d}$;

Z_H = fraction of displacement completed up to cut-off in high-pressure cylinder, so that $Z_H D_H = V_b = \frac{D_H}{R_H}$;

Z_L = fraction of displacement completed up to cut-off in low-pressure cylinder, so that $Z_L D_L = V_e = \frac{D_L}{R_L}$.

Substitution of these general symbols in Eqs. (220) to (224) gives another set of five equations in useful form for direct substitution of ordinary data as follows:

Work of cycle

$$\begin{aligned} &= 144 D_H (\text{in.pr.}) \left[\frac{1}{R_H} \left(2 + \log_e R_H + \log_e R_L \right) - \frac{R_L}{R_C R_H} \right] - 144 (\text{bk.pr.}) D_L \quad (a) \\ &= 144 D_L \left\{ (\text{in.pr.}) \frac{Z_H}{R_C} \left(2 + \log_e \frac{1}{Z_H} + \log_e \frac{1}{Z_L} - \frac{1}{R_C Z_L} \right) - (\text{bk.pr.}) \right\} \quad (b) \\ &= 144 D_L \left\{ (\text{in.pr.}) \frac{1}{R_H R_C} \left(2 + \log_e R_H + \log_e R_L - \frac{R_L}{R_C} \right) - (\text{bk.pr.}) \right\} \quad (c) \end{aligned} \quad (225)$$

(m.e.p.) lbs. per sq.in. referred to L.P. cyl.

$$\begin{aligned} &= (\text{in.pr.}) \frac{1}{R_H R_C} \left(2 + \log_e R_H + \log_e R_L - \frac{R_L}{R_C} \right) - (\text{bk.pr.}) \quad (a) \\ &= (\text{in.pr.}) \frac{Z_H}{R_C} \left(2 + \log_e \frac{1}{Z_H} + \log_e \frac{1}{Z_L} - \frac{1}{Z_L R_C} \right) - (\text{bk.pr.}) \quad (b) \end{aligned} \quad (226)$$

Work per cu.ft. supplied

$$\begin{aligned} &= 144 \left[(\text{in.pr.}) \left(2 + \log_e R_H + \log_e R_L - \frac{R_L}{R_C} \right) - (\text{bk.pr.}) R_C R_H \right] \quad (a) \\ &= 144 \left[(\text{in.pr.}) \left(2 + \log_e \frac{1}{Z_H} + \log_e \frac{1}{Z_L} - \frac{1}{Z_L R_C} \right) - (\text{bk.pr.}) \frac{R_C}{Z_H} \right] \quad (b) \end{aligned} \quad (227)$$

Cu.ft. supplied per hr. per I.H.P.

$$= \frac{13,750}{(\text{m.e.p. ref. to L.P.}) R_H R_C} \quad (a); \quad = \frac{13,750}{(\text{m.e.p. ref. to L.P.}) R_C} \frac{Z_H}{R_C} \quad (b). \quad (228)$$

From this the weight in pounds supplied per I.H.P. results directly from multiplication by the density of the fluid.

To these characteristic equations for evaluating work, mean pressure, economy and consumption in terms of the initial and final pressures and cylinder dimensions, there may be added a series defining certain *other general relations* of value in fixing the cycle for given dimensions and initial and final pressures, and in predicting dimensions for specified total work to be done and its division between high- and low-pressure cylinders.

Returning to the use of diagram points and translating into the general symbols as each expression is derived, there results,

$$\text{Receiver pressure} = P_d = P_e = P_b \frac{V_b}{V_e}, \quad \therefore$$

$$(\text{rec.pr.}) = (\text{in.pr.}) \frac{Z_H D_H}{Z_L D_L} = (\text{in.pr.}) \frac{Z_H}{R_C Z_L} \quad (a); \quad = (\text{in.pr.}) \frac{R_L}{R_C R_H} \quad (b). \quad (229)$$

$$\text{High-pressure cylinder release pressure} = P_c = P_b \frac{V_b}{V_c}, \quad \therefore$$

$$(\text{rel.pr.})_H = (\text{in.pr.}) \frac{1}{R_H} \quad (a); \quad = (\text{in.pr.}) Z_H \quad (b). \quad (230)$$

$$\text{Low-pressure cylinder release pressure} = P_f = P_e \frac{V_e}{V_f}, \quad \therefore$$

$$(\text{rel.pr.})_L = (\text{in.pr.}) \frac{1}{R_C R_H} \quad (a); \quad = (\text{in.pr.}) \frac{Z_H}{R_C} \quad (b); \quad = \frac{(\text{in.pr.})}{R_V} \quad (c); \quad = \frac{(\text{rel.pr.})_H}{R_C} \quad (d). \quad (231)$$

Division of work between cylinders may be made anything desired for a given load by suitably proportioning cylinders, and equations giving the necessary relations to be fulfilled can be set down. It is quite common for designers to assume equal division of work for the most commonly recurring or average load, or that corresponding to some high pressure cut-off or low-pressure terminal pressure, generally the latter. Therefore, a general expression for dimensional relations to be fulfilled for equal division of work is useful. On the other hand, for an engine the dimensions of which are determined, it is often necessary to find the work division for the imposed conditions, so that the following equations are of value.

$$\text{From Eqs. (217) and (218) noting that } P_d = P_e = P_b \frac{V_b}{V_e},$$

$$\begin{aligned} \frac{\text{High-press. cylinder work}}{\text{Low-press. cylinder work}} &= \frac{P_b V_b \left[\left(1 + \log_e \frac{V_c}{V_b} \right) - \frac{V_d}{V_e} \right]}{P_b V_b \left(1 + \log_e \frac{V_f}{V_e} \right) - P_e V_e} = \frac{\left(1 + \log_e \frac{V_c}{V_b} \right) - \frac{V_d}{V_e}}{\left(1 + \log_e \frac{V_f}{V_e} \right) - \frac{P_e V_e}{P_b V_b}} \\ &= \frac{1 + \log_e R_H - \frac{R_L}{R_C}}{1 + \log_e R_L - \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right) R_C R_H} \quad (a); \quad = \frac{1 + \log_e \frac{1}{Z_H} - \frac{1}{R_C Z_L}}{1 + \log_e \frac{1}{Z_L} - \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right) \frac{R_C}{Z_H}} \quad (b). \quad (232) \end{aligned}$$

This is a general expression for work division between the cylinders in terms of (a) ratio of expansion in each cylinder, initial and back-pressure

ratio and cylinder ratio, or, in terms of (b) cut-off in each, associated with cylinder and pressure ratios.

This expression Eq. (232) is less frequently used in its general form, as above, than in special forms in which the work of the two cylinders is made equal, or the expression made equal to unity. The conditions thus found for *equal division of work* between cylinders may be expressed either (a) in terms of initial and back pressures, release pressure of low-pressure cylinder and ratio of L.P. admission volume to H.P. displacement, and cylinder ratio; or (b) cut-off in high- and low-pressure cylinders, initial and back pressures and cylinder ratio. Still another expression giving equality of work may be found (c) when the cylinder ratio is made such that equality of work is obtained at all loads by equalizing high and low cut-offs.

(a) To find the first set of conditions, equate Eqs. (217) and (218) from the first part of this section; by simplification there results,

$$\log_e \frac{V_c}{V_b} - \frac{V_c}{V_c} = \log_e \frac{V_f}{V_s} - \frac{P_g}{P_f}, \quad \text{or} \quad \log_e \frac{V_c}{V_b} \frac{V_s}{V_f} = \frac{V_c}{V_s} - \frac{P_g}{P_f}; \quad \therefore \frac{V_c}{V_b} \frac{V_s}{V_f} = e^{\left(\frac{V_c}{V_s} - \frac{P_g}{P_f}\right)}.$$

Introducing the usual symbols and noting that

$$\frac{\text{Low-pressure admission volume}}{\text{High-pressure displacement volume}} = \frac{V_s}{V_c} = x = Z_L R_C = \frac{(\text{rel.pr.})_H}{(\text{rec.pr.})};$$

$$\text{Therefore,} \quad R_C = \left[e^{\left[\frac{(\text{bk.pr.})}{(\text{rel.pr.})_L} - \frac{1}{x} \right] \frac{(\text{in.pr.})}{(\text{rel.pr.})_L} x} \right]^{\frac{1}{2}} \quad \dots \quad (233)$$

This is of value when a given release pressure is to be reached in the low-pressure cylinder and with a particular value of low-pressure cut-off volume as fixed by x in terms of high-pressure cylinder displacement.

(b) Again for equal division of work, make Eq. (232) equal to unity,

$$\text{whence,} \quad 1 + \log_e \frac{1}{Z_H} - \frac{1}{R_C Z_L} = 1 + \log_e \frac{1}{Z_L} - \frac{(\text{bk.pr.})}{(\text{in.pr.})} \frac{R_C}{Z_H},$$

which may be reduced to the following, solving for R_C ,

$$R_C = \frac{\left[\left(\log_e \frac{Z_L}{Z_H} \right)^2 + 4 \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right) \frac{1}{Z_H Z_L} \right]^{\frac{1}{2}} - \log_e \frac{Z_L}{Z_H}}{\frac{2}{Z_H} \frac{(\text{bk.pr.})}{(\text{in.pr.})}} \quad \dots \quad (234)$$

Equal division of work for given initial and back pressure is to be obtained by satisfying these complex relations Eq. (234) between the two cut-offs, or their equivalent ratios of expansion in connection with a given cylinder ratio, or the relation between pressures and volumes in Eq. (233), equally complex.

(c) An assumption of *equal cut-off* in both cylinders gives results which are of interest and practical value, although it is a special case. Eq. (234) then becomes, when $Z_H = Z_L$ or $R_H = R_L$, $R_C = \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)^{\frac{1}{2}} \quad \dots \quad (235)$

For the case of most economical operation, that of complete and perfect expansion in both cylinders, there may be set down the four characteristic Eqs. (225) to (228) with suitable modifications to meet the case. These become,

$$\text{Work of cycle} = 144(\text{in.pr.}) D_L \frac{\log_e \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)}{\left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)} = 144(\text{in.pr.}) D_L \frac{\log_e R_v}{R_v} \quad (240)$$

$$(\text{m.e.p.}) (\text{ref. to L.P.}) = \frac{W}{144 D_L} = (\text{in.pr.}) \frac{\log_e \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)}{\left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)} = (\text{in.pr.}) \frac{\log_e R_v}{R_v} \quad (241)$$

$$\text{Work per cu.ft. supplied} = 144(\text{in.pr.}) \log_e \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right) = 144(\text{in.pr.}) \log_e R_v \quad (242)$$

$$\text{Cu.ft. per hr. per I.H.P.} = \frac{13,750}{(\text{m.e.p. ref. to L.P.}) \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)} (a); = \frac{13,750}{(\text{m.e.p. ref. to L.P.})} \frac{1}{R_v} (b). \quad (243)$$

For equal division of work with complete expansion in both cylinders, the ratios of Eqs. (238) and (239) become

$$R_H = R_L = \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)^{\frac{1}{2}} (a); \quad Z_H = Z_L = \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{1}{2}} (b), \quad (244)$$

and this is evidently a case to which Eqs. (235) and (236) apply without change.

Example. 1. Method of calculating diagram, Fig. 62. Assumed data for Case A:

$$\begin{array}{ll} P_a = P_b = 100 \text{ lbs. per sq.in. abs.} & V_a = V_n = V_m = 0 \text{ cu.ft.} \\ P_n = P_d = P_e = 50 \text{ lbs. per sq.in. abs.} & V_c = V_s = .6 \text{ cu.ft.} \\ P_m = P_f = 10 \text{ lbs. per sq.in. abs.} & V_f = V_g = 2 \text{ cu.ft.} \\ P_c = 60 \text{ lbs. per sq.in. abs.} & V_e = .8 \text{ cu.ft.} \end{array}$$

$$\text{To obtain point } B: \quad V_b = V_c \times \frac{P_c}{P_b} = .6 \times \frac{60}{100} = .36 \text{ cu.ft.}$$

$$\text{To obtain point } F: \quad P_f = P_e \times \frac{V_e}{V_f} = 50 \times \frac{.8}{2} = 20 \text{ lbs. per sq.in.}$$

To construct the indicator cards:

Lay off ND of the PV diagrams to equal the length of the card, and NA perpendicular to it at N to equal the height of the card. Cut-off equals $AB + ND$. From A on card lay off this ratio times the length of the card. From D on the card lay off a perpendicular equal to CD of the PV diagram reduced by the same proportion as AN of the card is to AN of the diagram. Join the points B and C by a curve through points located from intermediate points on the PV diagram. The low-pressure card is constructed in same manner.

Example. 2. A 12- and 18×24-in. steam engine without clearance runs on 150 lbs. per square inch absolute initial pressure, 10 lbs. per square inch absolute back pressure, and has a speed of 125 R.P.M. What will be (a) the horse-power for $\frac{1}{2}$ cut-off in H.P.

cylinder, (b) pounds of steam per I.H.P. hour, (c) terminal pressures, (d) L.P. cut-off for continuous expansion, (e) work done in each cylinder.

NOTE: δ for 150 lbs. = .332.

(a) From Eq. (226) (m.e.p.) referred to L.P. cylinder is

$$(\text{in.pr.}) \frac{1}{R_H R_C} \left(2 + \log_e R_H + \log_e R_L - \frac{R_L}{R_C} \right) - (\text{bk.pr.}).$$

In this case $R_H = 2$, $R_C = \left(\frac{18}{12} \right)^2 = 2.25$, $R_L = 2.25$,

since vol. of L.P. cyl. at cut-off must be equal to the entire volume of the high for continuous expansion, hence

$$(\text{m.e.p.}) = 150 \times \frac{1}{2 \times 2.25} \times (2 + .69 + .81 - 1) - 10 = 73.3 \text{ lb. sq. inch,}$$

and $\text{I.H.P.} = \frac{(\text{m.e.p.}) \text{ Lan}}{33,000} = 282.$

(b) From Eq. (228) Cu.ft. per hour per I.H.P. = $\frac{13,750 Z_H}{(\text{m.e.p.}) R_C} = \frac{13,750}{73.3} \times \frac{.5}{2.25} = 41.7,$

(c) From Eq. (230) $(\text{rel.pr.})_H = (\text{in.pr.}) Z_H = 150 \times \frac{1}{2} = 75 \text{ lbs. sq.in.}$

and from Eq. (231), $(\text{rel.pr.})_L = \frac{(\text{rel.pr.})_H}{R_C} = \frac{75}{2.25} = 33.3 \text{ lbs. sq.in.}$ $Z_L = \frac{1}{2.25} = .444.$

(e) From Eq. (232)

$$\frac{\text{H.P. work}}{\text{L.P. work}} = \frac{1 + \log_e \frac{1}{Z_H} - \frac{1}{R_C Z_L}}{1 + \log_e \frac{1}{Z_L} - \frac{(\text{bk.pr.}) R_C}{(\text{in.pr.}) Z_H}} = \frac{1 + .69 - \frac{1}{2.25 \times .44}}{1 + .81 - \frac{10}{150} \times \frac{2.25}{.5}} = \frac{.69}{1.51} = .456,$$

or $\text{H.P. work} = .456 \times \text{L.P. work}$, also $\text{H.P. work} + \text{L.P. work} = 282 \text{ I.H.P.}$

Hence $\text{H.P. work} = 88 \text{ I.H.P.}$ and $\text{L.P. work} = 194 \text{ I.H.P.}$

Prob. 1. What must be the cylinder diameters of a cross-compound engine to run on 100 lbs. per square inch absolute steam pressure, 18 ins. of mercury vacuum and to develop 150 H.P. at a speed of 200 R.P.M. with $\frac{1}{2}$ cut-off in each cylinder, if cylinder ratio is 3 and stroke is 18 ins.? Engine is double-acting and assumed to have no clearance.

Prob. 2. What will be the release pressure in each cylinder and the receiver pressure of the engine of Prob. 1? If cut-off were reduced to $\frac{1}{3}$ in H.P. cylinder, how would these pressures be affected and to what extent? How would the horsepower change?

Prob. 3. A 15- and 22 \times 30-in. infinite receiver engine has no clearance, a speed of 150 R.P.M., initial pressure 125 lbs. per square inch gage. What will be the horsepower and steam consumption for a H.P. cut-off of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$? Also that value which will give complete expansion in high-pressure cylinder? Low-pressure cut-off to be fixed at $\frac{1}{2}$.

NOTE: δ for 150 lbs. gage = .363.

Prob. 4. What will be the release and receiver pressures, and the work done in each cylinder for Prob. 3?

Prob. 5. An 18 and 24×30-in. infinite receiver engine is to be operated so as to give complete expansion in both cylinders. What will be the cut-off to accomplish this and what horse-power will result if the initial pressure is 100 lbs. and back pressure 10 lbs. per square inch absolute? Speed equals 100 r.p.m.

Prob. 6. Draw the PV diagram for following cases. Cylinder ratio 1 to 2.5, (in.pr.), 100 lbs. per square inch absolute, (bk.pr.), 20 lbs. per square inch absolute, H.P. cut-off (a) = $\frac{1}{4}$, (b) = $\frac{1}{2}$, (c) = $\frac{3}{4}$. L.P. cut-off (a) = $\frac{1}{4}$, (b) = $\frac{1}{8}$, (c) = $\frac{1}{2}$.

Prob. 7. For the following conditions find the horse-power, steam used per hour, receiver pressure and release pressures. Engine, 10- and 15×24-in. 150 R.P.M., 125 lbs. per square inch gage initial pressure, 2 lbs. per square inch absolute, back pressure, $\frac{1}{2}$ cut-off in high-pressure cylinder, $\frac{1}{8}$ cut-off in low-pressure cylinder, with infinite receiver.

NOTE: δ for 125 lbs. = .311.

Prob. 8. An infinite receiver engine is to develop 150 H.P. at 200 R.P.M. when initial pressure is 150 lbs. per square inch absolute. Cylinder ratio is 1 to 3 and back pressure is one atmosphere. What must be its size if the stroke is equal to the low-pressure cylinder diameter for $\frac{1}{2}$ cut-off in the high-pressure cylinder, and $\frac{1}{2}$ cut-off in the low-pressure cylinder?

Prob. 9. Find by trial the cut-offs at which work division will be equal for an infinite receiver engine with a cylinder ratio of 2.5, an initial pressure of 100 lbs. per square inch absolute and a back pressure of 5 lbs. per square inch absolute?

46. Compound Engine with Infinite Receiver, Exponential Law. No Clearance, Cycle VI. General Relations between Pressures, Dimensions, and Work. Again referring to Fig. 62, which may be used to represent this cycle also, the work of each cylinder may be expressed as follows, by the assistance of Eq. (177) derived in Section 40.

$$W_H = 144 D_H \left[Z_H (\text{in.pr.}) \left(\frac{s - Z_H^{s-1}}{s-1} \right) - (\text{rec.pr.}) \right] \quad \dots (245)$$

$$W_L = 144 D_L \left[Z_L (\text{rec.pr.}) \left(\frac{s - Z_L^{s-1}}{s-1} \right) - (\text{bk.pr.}) \right], \quad \dots (246)$$

where Z_H , the cut-off in the high pressure = $\frac{V_b}{V_a}$ and Z_L , low-pressure cut-off = $\frac{V_c}{V_d}$.

In combining these into a single equation for the total work, the term for receiver pressure (rec.pr.) should be eliminated. Referring to Fig. 62,

$$(\text{rec.pr.}) = P_a = P_c = P_b \left(\frac{V_b}{V_c} \right)^s = (\text{in.pr.}) \left(\frac{Z_H}{R_C Z_L} \right)^s, \quad \dots (247)$$

$$\text{hence } W = 144 D_L \left\{ (\text{in.pr.}) \left[\left(\frac{Z_H}{R_C} \right) \left(\frac{s - Z_H^{s-1}}{s-1} \right) - \frac{1}{R_C} \left(\frac{Z_H}{R_C Z_L} \right)^s \right. \right. \\ \left. \left. + Z_L \left(\frac{Z_H}{R_C Z_L} \right)^s \left(\frac{s - Z_L^{s-1}}{s-1} \right) \right] - (\text{bk.pr.}) \right\}, \quad \dots (248)$$

a rather complex expression which permits of little simplification, but offers no particular difficulty in solution.

Mean effective pressure referred to the low-pressure cylinder may be obtained by dividing Eq. (248) by $144D_L$. See Eq. (241).

Work per cubic foot fluid supplied may be found by dividing Eq. (248) by the supply volume, which in terms of low-pressure displacement is

$$(\text{Sup. Vol.}) = D_L \frac{Z_H}{R_C} \quad (249)$$

The consumption of fluid, cubic feet per hour per indicated horse-power is obtained from the same expression as given for the logarithmic law in Eq. (223). Multiplying this by δ_1 , the initial density of the fluid, pounds per cubic foot, gives consumption, pounds fluid per hour per I.H.P.

The receiver pressure has already been determined in Eq. (247). Release pressure of the high-pressure cylinder is $(\text{rel.pr.})_H = (\text{in.pr.})Z_H^s$, . . . (250)

and for the low-pres. cylinder, $(\text{rel.pr.})_L = (\text{in.pr.}) \left(\frac{Z_H}{R_C} \right)^s (a); = \frac{(\text{in.pr.})}{R_V^s} (b)$. (251)

where R_V is the ratio of maximum volume in the low pressure, to volume at cut-off in the high, and equals $\frac{R_C}{Z_H} = \frac{(\text{in.pr.})}{(\text{bk.pr.})}$.

The distribution of work between the high- and low-pressure cylinders may be found as follows, by means of Eqs. (245) and (246), eliminating (rec.pr.) through Eq. (247)

$$\frac{W_H}{W_L} = \frac{Z_H \left(\frac{s - Z_H^{s-1}}{s-1} \right) - \left(\frac{Z_H}{R_C Z_L} \right)^s}{R_C \left[Z_L \left(\frac{Z_H}{R_C Z_L} \right)^s \left(\frac{s - Z_L^{s-1}}{s-1} \right) - \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right) \right]} \quad (252)$$

Equality of work in the two cylinders will be obtained if this expression is equal to unity, but this involves a very complex relation between high- and low-pressure cut-offs, cylinder ratio and ratio of initial and back pressures. It is found that the simple conditions for equality in the case of logarithmic law will not give equality of work for the exponential law. There is, however, a case under this law which yields itself to analysis, that of complete expansion in both cylinders, without over-expansion. The conditions for equality of work for this case will be treated after deriving work and mean effective pressure for it.

Complete expansion, without over-expansion, in both cylinders may be represented by Fig. 63, in which

$$Z_H = \frac{\overline{AB}}{\overline{NC}}, \quad Z_L = \frac{\overline{NC}}{\overline{ME}}, \quad \text{and since } \overline{NC} = D_H \text{ and } \overline{ME} = D_L, \quad R_C = \frac{D_L}{D_H} = \frac{\overline{ME}}{\overline{NC}} = \frac{1}{Z_L}.$$

The true ratio of expansion $= R_V = \frac{\overline{ME}}{\overline{AB}} = \frac{1}{Z_H Z_L} = \frac{R_C}{Z_H}$, but this is also equal to $\left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)^{\frac{1}{s}}$ due to the law of the curve, $P_b V_b^s = P_s V_s^s$.

By means of Eq. (180) in Section (40) the work of the two cylinders may be evaluated,

$$\left. \begin{aligned} W_H &= 144(\text{in.pr.}) D_H Z_H \frac{s}{s-1} (1 - Z_H^{s-1}) \quad (a) \\ &= 144(\text{in.pr.}) D_L \frac{Z_H}{R_C} \frac{s}{s-1} (1 - Z_H^{s-1}) \quad (b) \end{aligned} \right\} \dots \dots (253)$$

$$W_L = 144(\text{bk.pr.}) D_L \frac{s}{s-1} \left(\frac{1}{Z_L^{s-1}} - 1 \right), \text{ but since } Z_L = \frac{1}{R_C},$$

$$\left. \begin{aligned} W_L &= 144(\text{bk.pr.}) D_L \frac{s}{s-1} (R_C^{s-1} - 1) \quad (a) \\ &= 144(\text{in.pr.}) D_L \frac{s}{s-1} \left(\frac{Z_H}{R_C} \right)^s (R_C^{s-1} - 1) \quad (b) \end{aligned} \right\} \dots \dots (254)$$

The total work is evidently the same as that of a cylinder equal in size to the low-pressure cylinder with a cut-off equal to $\frac{Z_H}{R_C}$, working between the given (in.pr.) and (bk.pr.) and may be stated by reference to Eq. (180), Section 40, or by taking the sum of W_H and W_L given previously,

$$W = 144(\text{in.pr.}) D_L \frac{Z_H}{R_C} \frac{s}{s-1} \left\{ (1 - Z_H^{s-1}) + \left(\frac{Z_H}{R_C} \right)^{s-1} (R_C^{s-1} - 1) \right\},$$

$$\text{which reduces to } W = 144(\text{in.pr.}) D_L \frac{Z_H}{R_C} \frac{s}{s-1} \left\{ 1 - \left(\frac{Z_H}{R_C} \right)^{s-1} \right\} \dots \dots (255)$$

For this case of complete expansion in both cylinders, the ratio of high- to low-pressure work is given by division and cancellation,

$$\frac{W_H}{W_L} = \frac{1 - Z_H^{s-1}}{\left(\frac{Z_H}{R_C} \right)^{s-1} (R_C^{s-1} - 1)} = \frac{\left(\frac{R_C}{Z_H} \right)^{s-1} - R_C^{s-1}}{R_C^{s-1} - 1} \dots \dots (256)$$

Equality of work, obtained by placing this expression equal to unity, provides the condition that

$$2R_C^{s-1} - 1 = \left(\frac{R_C}{Z_H} \right)^{s-1} = \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)^{\frac{s-1}{s}}, \text{ or } R_C = \left\{ \frac{\left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)^{\frac{s-1}{s}} + 1}{2} \right\}^{\frac{1}{s-1}}, \quad (257)$$

for equal work and complete expansion, and

$$Z_H = \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right) R_C^{\frac{1}{s}}; \quad = \left\{ \frac{\left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{s-1}{s}} + 1}{2} \right\}^{\frac{1}{s-1}} \dots \dots (258)$$

Since $Z_L = \frac{1}{R_C}$ for complete expansion, and (in Fig. 63) $P_c V_c^s = P_r V_r^s$, the receiver pressure, P_r , is (rec.pr.) = (bk.pr.) $\left(\frac{V_c}{V_r} \right)^s = (\text{bk.pr.}) R_C^s$, . . . (259)

in which R_C will have the value given above if work is equally distributed.

Example 1. What will be (a) the horse-power, (b) consumption, (c) work ratio, (d) receiver and release pressures for the following conditions? Engine 12 and 18×24 ins., running at 125 R.P.M. on initial air pressure of 150 lbs. per square inch absolute, and back pressure of 10 lbs. per square inch absolute, with $\frac{1}{2}$ cut-off in high-pressure cylinder and continuous expansion in low-pressure cylinder. Exponent of expansion curve = 1.4 for compressed air, infinite receiver.

(a) By dividing Eq. (241) by $144D_L$ there is obtained (m.e.p.)

$$= (\text{in.pr.}) \left[\left(\frac{Z_H}{R_C} \right) \left(\frac{s - Z_H^{s-1}}{s-1} \right) - \frac{1}{R_C} \left(\frac{Z_H}{R_C Z_L} \right)^s + Z_L \left(\frac{Z_H}{R_C Z_L} \right)^s \left(\frac{s - Z_L^{s-1}}{s-1} \right) \right] - (\text{bk.pr.}),$$

which, on substituting values from above, gives for (m.e.p.) 63 lbs. per sq. inch. Hence, the indicated horse-power = 242.

(b) From Eq. (223) compressed air per hour per I.H.P. = $\frac{13,750 Z_H}{\text{m.e.p. } R_C}$ cu.ft.,

which, on substitution, gives $\frac{13,750}{63} \times \frac{.5}{2.25} = 48.3$ cu.ft.

(c) Substituting in Eq. (252),

$$\frac{W_H}{W_L} = \frac{.5 \left(\frac{1.4 - .5^4}{.4} \right) - \left(\frac{.5}{2.25 \times \frac{1}{2.25}} \right)^{1.4}}{2.25 \left[\frac{1}{2.25} \left(\frac{.5}{2.25 \times \frac{1}{2.25}} \right)^{1.4} \left(\frac{1.4 - \left(\frac{1}{2.25} \right)^4}{.4} \right) - \frac{10}{150} \right]} = .294.$$

and $W_H + W_L = 242$ I.H.P. Hence $W_H = 56$ I.H.P. and $W_L = 184$ I.H.P.

(d) From Eq. (247) (rec.pr.) = (in.pr.) $\left(\frac{Z_H}{R_C Z_L} \right)^s = 150 \left(\frac{.5}{2.25 \times \frac{1}{2.25}} \right)^{1.4} = 57$ lbs. per sq.in.

From Eq. (250) (rel.pr.)_H = (in.pr.) Z_H^s , = $150 \times (.5)^{1.4} = 57$ lbs. per sq.in.

From Eq. (251) (rel.pr.)_L = (in.pr.) + R_v^s , = $150 + 21.85 = 6.85$ lbs. per sq.in.

These values may be compared with those of Ex. 2, Section 45, which were for the same data with logarithmic expansion.

Prob. 1. What will be the horse-power and steam used per hour by the following engine under the conditions given? Cylinders 18 and 30×48 ins., speed 100 R.P.M., initial pressure 150 lbs. per square inch absolute, back-pressure 10 lbs. per square inch absolute, steam continually dry. Cut-off at first $\frac{1}{2}$ in high-pressure and $\frac{1}{2}$ in low, and then $\frac{1}{2}$ in each; infinite receiver.

Prob. 2. The very large receiver of a compound pumping engine is fitted with safety valve which is to be set to blow at 25 per cent above ordinary pressure. The cylinder ratio is 1 to 3.5, and cut-offs are $\frac{1}{2}$ in high and $\frac{1}{2}$ in low. If initial pressure is 125 lbs. per sq.in. gage, at what pressure must valve be set? What vacuum must be carried in the condenser to have complete expansion in low-pressure cylinder? Superheated steam.

Prob. 3. A compound engine is to be designed to work on superheated steam of 125 lbs. per square inch absolute initial pressure, and on an 18-inch vacuum. The

load which it is to carry is 150 horse-power and piston speed is to be 500 ft. per minute at 200 R.P.M. Load is to be equally divided between cylinders and there is to be complete expansion in both cylinders. What must be cylinder sizes, and what cut-offs will be used for an infinite receiver?

Prob. 4. How will the economy of the two following engines compare? Each is 14 and 20×24 ins., and runs at 200 R.P.M., on compressed air of 100 lbs. per square inch gage pressure, with 15 lbs. per square inch absolute exhaust pressure. Low-pressure cut-off of each is $\frac{1}{2}$; high pressure of one is $\frac{1}{2}$, the other, $\frac{3}{4}$. Infinite receivers.

Prob. 5. A compound engine 12 and 18×24 ins. is running at 200 R.P.M. on superheated steam of 100 lbs. per square inch absolute pressure and exhausting to a condenser in which pressure is 10 lbs. per square inch absolute. The cut-off is $\frac{1}{2}$ in high-pressure cylinder and $\frac{1}{2}$ in low-pressure cylinder. Compare the power and steam consumption under this condition with corresponding values for wet steam under same conditions of pressure and cut-off and infinite receiver.

Prob. 6. The initial pressure of an engine is 150 lbs. per square inch absolute, the back pressure one atmosphere, the cylinder ratio 3. As operated, both cut-offs are at $\frac{1}{2}$. What will be the receiver pressure, high-pressure release pressure, and low-pressure release pressure? What will be the new values of each if (a) high-pressure cut-off is made $\frac{1}{4}$, (b) $\frac{3}{4}$, without change of anything else, (c) if low pressure cut-off is made $\frac{1}{4}$, (d) $\frac{3}{4}$, without change of anything else? Infinite receiver, $s=1.3$.

Prob. 7. In the above problem for $\frac{1}{2}$ cut-off in each cylinder how will the release and receiver pressures change if (a) initial pressure be raised 25 per cent, (b) lowered 25 per cent, (c) back pressure raised 25 per cent, (d) lowered 25 per cent?

Prob. 8. How many pounds of initially-dry steam per hour will be required to supply an 18-in. and 24×30-in. engine running at $\frac{1}{2}$ cut-off in each cylinder if speed be 100 R.P.M., initial pressure 100 lbs. per square inch gage and back pressure 5 lbs. per square inch absolute? Expansion to be adiabatic and receiver infinite.

NOTE: δ for 100 lbs. = .26.

47. Compound Engine with Finite Receiver. Logarithmic Law. No Clearance, Cycle VII. General Relations between Dimensions and Work when H.P. Exhaust and L.P. Admission

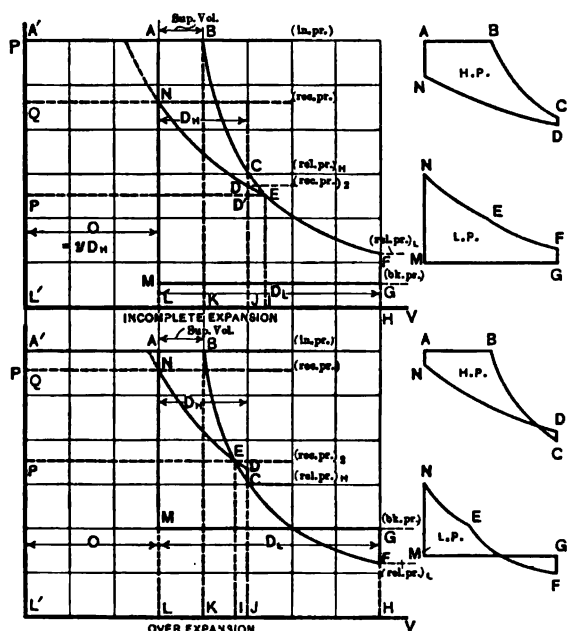


FIG. 64.—Work of Expansive Fluid in Compound Engines with Finite Receiver, Zero Clearance. Cycle VII, Logarithmic Expansion; Cycle VIII, Exponential Expansion.

are not Coincident. The diagrams, Fig. 64, while showing only two degrees of expansion, that of over and under in both cylinders, suffice for the

derivation of equations applicable to all degrees in either cylinder. Volumes measured from the axis AL are those occupied by the fluid in either cylinder alone, while fluid volumes entirely in the receiver, or partly in receiver, and in either cylinder at the same time are measured from the axis $A'L'$. No confusion will result if all volumes represented by points be designated by the (V) with a subscript, and to these a constant, O , representing the receiver volume, be added when part of the fluid is in the receiver. Then, high-pressure cylinder work is

$$W_H = P_b V_b \left(1 + \log_e \frac{V_c}{V_b} \right) - P_n O \log_e \left(\frac{V_c + O}{O} \right). \quad (260)$$

Low-pressure cylinder work is

$$W_L = P_n O \log_e \left(\frac{V_c + O}{O} \right) + P_s V_s \log_e \frac{V_f}{V_s} - P_s V_s. \quad (261)$$

$$\begin{aligned} \text{Total work} \quad W = P_b V_b \left(1 + \log_e \frac{V_c}{V_b} \right) + P_n O \log_e \left(\frac{V_c + O}{O} \right) \\ + P_s V_s \log_e \frac{V_f}{V_s} - P_n O \log_e \left(\frac{V_c + O}{O} \right) - P_s V_s. \end{aligned} \quad (262)$$

These expressions include some terms not known as initial data, but they may be reduced by the following relations:

$$P_b V_b = P_s V_s = P_s V_f, \text{ and } P_n O = P_s (V_s + O) = P_b V_b \left(\frac{V_s + O}{V_s} \right). \text{ Hence}$$

$$\begin{aligned} W = P_b V_b \left[1 + \log_e \frac{V_c}{V_b} + \left(\frac{V_s + O}{V_s} \right) \log_e \left(\frac{V_c + O}{O} \right) \right. \\ \left. + \log_e \frac{V_f}{V_s} - \left(\frac{V_s + O}{V_s} \right) \log_e \left(\frac{V_c + O}{O} \right) \right] - P_s V_s. \end{aligned} \quad (263)$$

Dividing this expression by the low-pressure cylinder displacement, V_s , the result will be the mean effective pressure referred to the low-pressure cylinder.

A similar division but with the volume supplied, V_b as the divisor, gives the work per cu.ft. supplied. Also as in previous cases,

$$\text{Cu.ft. supplied per hr. per I.H.P.} = \frac{13,750}{(\text{m.e.p. ref. to L.P.})} \times \frac{V_b}{V_s}. \quad (264)$$

The weight per hour per I.H.P. follows from Eq. (264) by introducing the density as a multiplier.

While these equations may be used for the solution of problems, it is much better to transform them by introducing dimensional relations as in the previous cases developed.

Let $(\text{rec.pr.})_1$ = maximum receiver pressure P_n , which is also the initial admission pressure for the low-pressure cylinder;

Let (rec.pr.)₂ = minimum receiver pressure P_2 , which is the terminal admission pressure for the low-pressure cylinder and that at which expansion begins there;

$$y = \frac{\text{receiver volume}}{\text{high-pressure cyl. displ.}} = \frac{O}{V_c} = \frac{O}{D_H}, \text{ and, } \frac{O}{D_L} = \frac{D_H y}{D_L} = \frac{y}{R_C}.$$

Other symbols necessary are unchanged from the meaning imposed in Section (45).

Substitution in Eq. (263) gives the following expression in a form for direct substitution of ordinary data:

$$\begin{aligned} \text{Work of cycle} &= 144(\text{in.pr.})Z_H D_H \left\{ 1 + \log_e \frac{1}{Z_H} + \log_e \frac{1}{Z_L} \right. \\ &+ \left(1 + \frac{y}{Z_L R_C} \right) \left[\log_e \left(1 + \frac{Z_L R_C}{y} \right) - \log_e \left(1 + \frac{1}{y} \right) \right] \left. \right\} - 144(\text{bk.pr.})D_L \quad (a) \\ &= 144(\text{in.pr.})\frac{D_H}{R_H} \left\{ 1 + \log_e R_H + \log_e R_L + \left(1 + \frac{R_L y}{R_C} \right) \left[\log_e \left(1 + \frac{R_C}{R_L y} \right) \right. \right. \\ &\quad \left. \left. - \log_e \left(1 + \frac{1}{y} \right) \right] \right\} - 144(\text{bk.pr.})D_L \quad (b) \end{aligned} \quad (265)$$

Dividing this expression by 144 and noting that the low-pressure cylinder displacement $D_L = D_H R_C$, an expression for the mean-effective pressure is obtained which reduces to

$$\begin{aligned} (\text{m.e.p. ref. to L.P.}) &= (\text{in.pr.})\frac{Z_H}{R_C} \left\{ 1 + \log_e \frac{1}{Z_H} + \log_e \frac{1}{Z_L} \right. \\ &\quad \left(1 + \frac{y}{Z_L R_C} \right) \left[\log_e \left(1 + \frac{Z_L R_C}{y} \right) - \log_e \left(1 + \frac{1}{y} \right) \right] \left. \right\} - (\text{bk.pr.}) \quad (a) \\ &= (\text{in.pr.})\frac{1}{R_H R_C} \left\{ 1 + \log_e R_H + \log_e R_L + \left(1 + \frac{R_L y}{R_C} \right) \left[\log_e \left(1 + \frac{R_C}{R_L y} \right) \right. \right. \\ &\quad \left. \left. - \log_e \left(1 + \frac{1}{y} \right) \right] \right\} - (\text{bk.pr.}) \quad (b) \end{aligned} \quad (266)$$

The work per cubic foot supplied may be obtained in the same way by dividing Eq. 265 by the value for (sup.vol.) given in Eq. 249, noting as before from Eq. 239 that $D_H R_C = D_L$. Hence

$$\begin{aligned} \text{Work per cu.ft. supplied} &= 144 (\text{in.pr.}) \left\{ 1 + \log_e \frac{1}{Z_H} + \log_e \frac{1}{Z_L} \right. \\ &+ \left(1 + \frac{y}{Z_L R_C} \right) \left[\log_e \left(1 + \frac{Z_L R_C}{y} \right) - \log_e \left(1 + \frac{1}{y} \right) \right] \left. \right\} - 144(\text{bk.pr.})\frac{R_C}{Z_H} \quad (a) \\ &= 144(\text{in.pr.}) \left\{ 1 + \log_e R_H + \log_e R_L + \left(1 + \frac{R_L y}{R_C} \right) \left[\log_e \left(1 + \frac{R_C}{R_L y} \right) \right. \right. \\ &\quad \left. \left. - \log_e \left(1 + \frac{1}{y} \right) \right] \right\} - 144(\text{bk.pr.})R_C R_H \quad (b) \end{aligned} \quad (267)$$

Cu.ft. supplied per hr. per I.H.P. may be obtained from Eq. (264),

$$= \frac{13,750}{(\text{m.e.p. ref. to L. P.})} \times \frac{Z_H}{R_C} \quad (a); \quad = \frac{13,750}{(\text{m.e.p. ref. to L. P.})} \times \frac{1}{R_H R_C} \quad (b). \quad (268)$$

It is desirable at this point to introduce a series of expressions fixing the relations between the dimensions, the cycle that may follow, and the fluctuations in the receiver pressure, and for the selection of cylinder and receiver dimensions for a required output of work and its division between cylinders.

In doing this it will be convenient to start with diagram points and finally substitute general symbols in each case. There will first be established the *maximum and minimum receiver pressures and the fluctuations*.

Maximum receiver pressure

$$P_* = P_e \frac{(V_e + O)}{O} = \left(\frac{P_b V_b}{V_e} \right) \left(\frac{V_e + O}{O} \right) = P_b \left(\frac{V_b}{O} + \frac{V_b}{V_e} \right).$$

$$\therefore (\text{rec.pr.})_1 = (\text{in.pr.}) \left(\frac{Z_H D_H}{O} + \frac{Z_H D_H}{Z_L D_L} \right) = (\text{in.pr.}) \left(\frac{Z_H}{y} + \frac{Z_H}{Z_L R_C} \right) \quad (a)$$

$$= (\text{in.pr.}) \left(\frac{1}{R_H y} + \frac{R_L}{R_H R_C} \right) \quad (b) \quad (269)$$

Minimum receiver pressure $P_* = P_b \frac{V_b}{V_e}$; therefore

$$(\text{rec.pr.})_2 = (\text{in.pr.}) \frac{Z_H D_H}{Z_L D_L} = (\text{in.pr.}) \frac{Z_H}{Z_L R_C} \quad (a); \quad = (\text{in.pr.}) \frac{R_L}{R_H R_C} \quad (b). \quad (270)$$

Fluctuation in receiver pressure $= (P_* - P_e) = P_b \frac{V_b}{O} \therefore$

$$(\text{rec.pr.})_1 - (\text{rec.pr.})_2 = (\text{in.pr.}) \frac{Z_H D_H}{O} = (\text{in.pr.}) \frac{Z_H}{y} \quad (a); \quad = (\text{in.pr.}) \frac{1}{R_H y} \quad (b). \quad (271)$$

It is interesting to note that the *minimum receiver pressure* is exactly the same as the value of the constant-receiver pressure for *infinite receiver*, so that limiting the size of receiver does not affect the point *E*, but only raises point *N* higher, tending to throw more work on the L.P. cylinder for the same valve setting.

The *two release pressures* P_e and P_f can be evaluated as in the case of the infinite receiver, as both these points lie on the common expansion line, which is not at all affected by the receiver-pressure changes, and the values are the same as for the infinite receiver given in Eqs. (230) and (231).

Division of work between the cylinders cannot, as pointed out, be the same as for the infinite receiver, the tendency being to throw more work on the low as the receiver becomes smaller, assuming the cut-off to remain the same. As, therefore, equal division was obtainable in the case of infinite receiver with equal cut-offs when the cylinder ratio was equal to the square root of initial over back pressure, it is evident that a finite receiver will require unequal cut-offs with the same cylinder ratio. As increase of low-pressure admission period, or cut-off fraction, lowers the receiver pressure, it follows that with the finite receiver the low-pressure cut-off must be greater than the high for equal work division, and it is interesting to examine by analysis the ratio between them to determine if it should be constant or variable,

For equal work division Eqs. (260) and (261) should be equal, hence by diagram points

$$\begin{aligned}
 P_1 V_1 \left(1 + \log_e \frac{V_c}{V_b}\right) - P_2 O \log_e \left(\frac{V_c + O}{O}\right) &= P_2 O \log_e \left(\frac{V_c + O}{O}\right) + P_1 V_1 \log_e \frac{V_f}{V_e} - P_1 V_1 \\
 P_1 V_1 \left(1 + \log_e \frac{V_c}{V_b}\right) - P_1 V_1 \left(\frac{V_c + O}{V_e}\right) \log_e \left(\frac{V_c + O}{O}\right) & \\
 &= P_1 V_1 \left(\frac{V_c + O}{V_e}\right) \log_e \left(\frac{V_c + O}{O}\right) + P_1 V_1 \log_e \frac{V_f}{V_e} - P_1 V_1 \\
 1 + \log_e R_H - \left(1 + \frac{R_L y}{R_c}\right) \log_e \left(1 + \frac{1}{y}\right) &= \left(1 + \frac{R_L y}{R_c}\right) \log_e \left(1 + \frac{R_c}{R_L y}\right) + \log_e R_L \\
 &\quad - \left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right) R_H R_c,
 \end{aligned}$$

hence for equal division of work, the following relations must be satisfied:

$$(\log_e R_H - \log_e R_L) = \log_e \frac{R_H}{R_L} = \left(1 + \frac{R_L y}{R_c}\right) \log_e \left[\left(1 + \frac{R_c}{R_L y}\right) \left(1 + \frac{1}{y}\right)\right] - \left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right) R_H R_c - 1. \quad (272)$$

It will be shown later that when expansion is complete in both cylinders and work equal, the high-pressure cylinder cut-off, or the equivalent ratio of expansion, bears a constant relation to that of the low, according to

$$\frac{R_H}{R_L} = a^2, \quad \dots \quad (273)$$

in which a is a constant depending only on the size of the receiver. It will also be shown that the cylinder ratio is a constant function of the initial and back pressures and the receiver volume for equal division of work according to

$$R_c = \frac{1}{a} \left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)^{\frac{1}{4}}, \quad \dots \quad (274)$$

in which (a) is the same constant as in Eq. (273). It is important to know whether these same values will also give equal division for this general case. Substituting them in Eq. (272)

$$2 \log_e a = \left(1 + \frac{R_L y a}{\sqrt{\frac{\text{in.pr.}}{\text{bk.pr.}}}}\right) \log_e \left[\left(1 + \frac{\sqrt{\frac{\text{in.pr.}}{\text{bk.pr.}}}}{R_L y a}\right) \left(1 + \frac{1}{y}\right)\right] - \frac{a R_L}{\sqrt{\frac{\text{in.pr.}}{\text{bk.pr.}}}} - 1.$$

Here there is only one variable, R_L , the evaluation of which can be made by inspection, for if

$$R_L = \frac{1}{a} \sqrt{\frac{\text{in.pr.}}{\text{bk.pr.}}}, \quad \text{the equation will become}$$

$$2 \log_e a = (1 + y) \log_e \left(1 + \frac{1}{y}\right)^2 - 2 = 2(1 + y) \log_e \left(1 + \frac{1}{y}\right) - 2,$$

$$\text{or} \quad a = e^{[(1+y) \log_e (1 + \frac{1}{y}) - 1]}, \quad \text{which is a constant.} \quad (275)$$

As only one constant value of low-pressure ratio of expansion or cut-off satisfies the equation for equal division of work when there is a fixed ratio between the values for high and low—that necessary for equal division with complete expansion in both—it is evident that equal division of work between the two cylinders cannot be maintained at all values of cut-off by fixing the ratio between them. As the relation between these cut-offs is a matter of some interest and as it cannot be derived by a solution of the general equation it is given by the curve, Fig. 65, to scale, the points of which were calculated.

A special case of this cycle of sufficient importance to warrant derivation of equations because of the simplicity of their form and consequent value in estimating when exact solutions of a particular problem are impossible, is the case of complete and perfect expansion in both cylinders. For it the following equations of condition hold, referring to Fig. 64,

$$P_c = P_d = P_e; V_d = V_e = V_f; P_f = P_g;$$

which when fulfilled yield the diagram, Fig. 66. These equations of condition are equivalent to fixing the cut-off in both high- and low-pressure cylinders, and the volume of the high- with respect to the low-pressure volume. Accordingly, the volume relations are as given in Eqs. (238) and (239) in which it will be noted that for the low-pressure cylinder the cut-off volume must equal the whole high-pressure volume, or $D_H = Z_L D_L$.

Substituting these equations of condition in the characteristic set, Eqs. (265) to (268), the following expression for most economical operation is obtained:

$$\left. \begin{aligned} W &= 144(\text{bk.pr.}) D_L \log_e \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right) & (a) \\ &= 144(\text{in.pr.}) D_L \frac{\log_e \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)}{\left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)} = 144(\text{in.pr.}) D_L \frac{\log_e R_V}{R_V} & (b) \end{aligned} \right\} \quad (276)$$

$$(\text{m.e.p. ref. to L.P.}) = \frac{W}{144 D_L} = (\text{in.pr.}) \frac{\log_e R_V}{R_V} \quad (277)$$

$$\text{Work per cu.ft. supplied} = \frac{W}{Z_H D_H} = 144(\text{in.pr.}) \log_e R_V \quad (278)$$

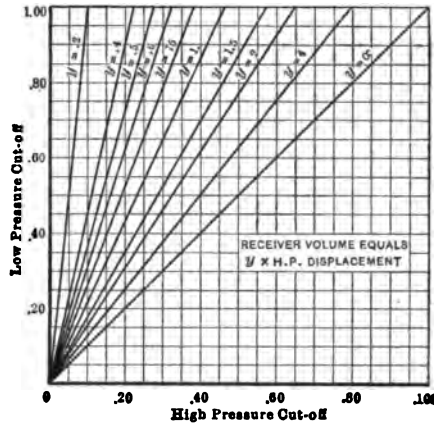


FIG. 65.—Diagram to Show Relation of High- and Low-Pressure Cut-offs for Equal Work in the Two Cylinders of a Finite-receiver Compound Engine with Zero Clearance and Logarithmic Law.

From Eq. (283) the cylinder ratio can be found in terms of a , and the ratio of expansion. Referring to Fig. 66,

$$\begin{aligned} \frac{R_H}{R_L} &= \frac{\frac{V_s}{V_b}}{\frac{V_f}{V_s}} = \frac{V_s^2}{V_b V_f} = a^2, \quad \text{therefore, } V_s = a\sqrt{V_b V_f}, \\ R_C &= \frac{V_f}{V_s} = \frac{V_f}{a\sqrt{V_b V_f}} = \frac{1}{a}\sqrt{\frac{V_f}{V_b}} = \frac{1}{a}\sqrt{\frac{\text{in.pr.}}{\text{bk.pr.}}} = \frac{1}{a}\sqrt{R_v}, \quad \dots (284) \end{aligned}$$

whence the cylinder ratio is equal to a constant depending on the receiver size, multiplied by the value for the infinite receiver, i.e., the square root of the initial divided by back pressure.

The high-pressure cylinder ratio of expansion is

$$R_H = \frac{V_s}{V_b} = a\sqrt{\frac{V_b V_f}{V_b}} = a\sqrt{\frac{V_f}{V_b}} = a\sqrt{\frac{\text{in.pr.}}{\text{bk.pr.}}} = a\sqrt{R_v}, \quad \dots (285)$$

and the corresponding value for the low-pressure cylinder is

$$R_L = \frac{V_f}{V_s} = \frac{V_f}{a\sqrt{V_b V_f}} = \frac{1}{a}\sqrt{\frac{V_f}{V_b}} = \frac{1}{a}\sqrt{\frac{\text{in.pr.}}{\text{bk.pr.}}} = \frac{1}{a}\sqrt{R_v}. \quad \dots (286)$$

For convenience in calculation, Table I of values of a and a^2 is added for various sizes of receivers.

TABLE I.

Receiver Vol. H.P. Cyl. Disp. $-\gamma$	$a = e^{[(1+\gamma)\log_e(1+\frac{1}{\gamma})-1]}$	a^2
.5	1.915	2.64
.75	1.624	3.67
1.0	1.474	2.17
1.5	1.322	1.75
2.0	1.243	1.55
2.5	1.198	1.437
3.0	1.164	1.359
4.0	1.1223	1.262
5.0	1.0973	1.204
7.0	1.0690	1.143
10.0	1.0478	1.098
14.0	1.0366	1.068
20.0	1.0228	1.046
Infinite	1.0	1.0

In the handbook of tables, there is presented a diagram, Chart No. 10, which gives the relation between cylinder and receiver volumes, cylinder ratio, and high-and low-pressure cut-offs graphically.

The corresponding values of maximum and minimum receiver pressure for equal division of work for this case of best economy are

$$(\text{rec.pr.})_1 = (\text{bk.pr.}) \frac{1}{a} \sqrt{\frac{\text{in.pr.}}{\text{bk.pr.}}} \left(\frac{1}{\gamma} + 1 \right) = \frac{\sqrt{(\text{in.pr.})(\text{bk.pr.})}}{a} \left(\frac{1}{\gamma} + 1 \right). \quad (287)$$

$$(\text{rec.pr.})_2 = (\text{bk.pr.}) \frac{1}{a} \sqrt{\frac{\text{in.pr.}}{\text{bk.pr.}}} = \frac{\sqrt{(\text{in.pr.})(\text{bk.pr.})}}{a} \quad (288)$$

$$(\text{rec.pr.})_1 - (\text{rec.pr.})_2 = \frac{\sqrt{(\text{in.pr.})(\text{bk.pr.})}}{ay} \quad (289)$$

Example 1. Method of calculating Diagram, Fig. 64.

Assumed data for case A:

$$P_a = P_b = 120 \text{ lbs. per sq.in. abs.} \quad P_m = P_f = 10 \text{ lbs. per sq.in. abs.}$$

$$V_a = V_n = V_m = 0 \text{ cu.ft.}; \quad V_b = .4 \text{ cu.ft.}; \quad V_c = 1 \text{ cu.ft.}; \quad O = 1.2 \text{ cu.ft.} \quad V_e = .8 \text{ cu.ft.}$$

To find point C: $P_c = P_b \frac{V_b}{V_c} = \frac{120 \times .4}{.8} = 60 \text{ lbs. per sq.in.}$

To obtain point E: $P_e = P_b \frac{V_b}{V_e} = \frac{120 \times .4}{1} = 48 \text{ lbs. per sq.in.}$

To obtain point D: $P_e(V_e + O) = P_d(V_c + O) \quad \text{or} \quad P_d = \frac{48 \times 2.2}{2} = 53 \text{ lbs. per sq.in.}$

To obtain point N: $P_n(O + V_n) = P_e(V_e + O) \quad \text{or} \quad P_n = \frac{48 \times 2.2}{1.2} = 88 \text{ lbs. per sq.in.}$

To obtain point F: $P_f = \frac{P_e V_e}{V_f} = \frac{48 \times 1}{2} = 24 \text{ lbs. per sq.in.}$

Example 2. Find (a) the horse-power, (b) steam used per hour, (c) the release and receiver pressures for a 12- and 18×24-in. engine with receiver twice as large as the low-pressure cylinder when the initial pressure is 150 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute, speed 125 R.P.M., and cut-offs $\frac{1}{2}$ in high-pressure and such a value in the low pressure as to give complete expansion, and clearances zero.

(a) From Eq. (266) which on substituting the above values gives

$$(\text{m.e.p.}) = \frac{150}{2 \times 2.25} \left\{ 1 + .69 + .81 + \frac{(1 + 2.25 \times 4.5)}{2.25} (.8 - .8) \right\} - 10 = 73.3 \text{ lbs.,}$$

hence I.H.P. = 282.

(b) From Eq. (268) we have

$$\text{Cu.ft. steam per hour per horse-power} = \frac{13,750}{(\text{m.e.p.})} \times \frac{1}{R_H R_C} = \frac{13,750}{73.3} \times \frac{1}{2 \times 2.25} = 41.7.$$

(c) From Eqs. (269) and (270) for maximum and minimum receiver pressures respectively:

$$(\text{in.pr.}) \left(\frac{1}{R_H} + \frac{R_L}{R_H R_C} \right) \quad \text{and} \quad (\text{in.pr.}) \frac{R_L}{R_H R_C};$$

$$\text{maximum receiver pressure} = 150 \left(\frac{1}{2 \times 4.5} + \frac{2.25}{2 \times 2.25} \right) = 91.5 \text{ lbs. per sq. inch;}$$

$$\text{minimum receiver pressure} = 150 \times \frac{2.25}{2 \times 2.25} = 75 \text{ lbs. per sq. inch;}$$

From Eqs. (230) and (231) for release pressures

high-pressure cylinder release pressure = $150 \times .5 = 75$ lbs. per sq. inch;

low-pressure cylinder release pressure = $\frac{150}{.444} = 33.9$ lbs. per sq. inch.

These results may be compared with those of Example 2 of Section 45 and Example 1 of 46 which are derived for same engine, with data to fit the special cycle described in the particular section.

NOTE: In all the following problems clearance is to be neglected.

Prob. 1. A 12- and 18×24-in. engine has a receiver equal to 5 times the volume of the high-pressure cylinder. It is running on an initial pressure of 150 lbs. per square inch gage and exhausts to the atmosphere. It has a speed of 150 R.P.M. and the cut-offs are $\frac{1}{10}$ and $\frac{1}{2}$ in high- and low-pressure cylinders respectively. What is the horsepower and the steam used in cubic feet per hour?

Prob. 2. What will be the release pressures, and variation of receiver pressure for an engine in which the cylinder ratio is 3, cut-offs $\frac{2}{3}$ and $\frac{1}{2}$, in high and low; initial pressure is 100 lbs. per square inch absolute, and receiver 2 times low-pressure cylinder volume?

Prob. 3. Show whether or not the following engine will develop equal cylinder work for the conditions given. Cylinder diameters, 15 and 22 in., initial pressure 135 lbs. per square inch gage, back pressure 10 lbs. per square inch absolute, cut-offs $\frac{1}{2}$ and $\frac{2}{3}$, receiver volume 4 times high-pressure cylinder, strokes equal.

Prob. 4. For the same conditions as above, what low-pressure cut-off would give equal work?

Prob. 5. What will be the most economical load for a 16- and 24×30-in. engine running at 125 R.P.M. on 150 lbs. per square inch absolute initial pressure and atmospheric back pressure? What will be the economy at this load?

Prob. 6. What will be the release and receiver pressures for the above engine if the receiver has a volume of 15 cu.ft.?

Prob. 7. Find the cut-offs and cylinder ratio for equal work division and complete expansion when initial pressure is 150 lbs. per square inch absolute and back pressure is 10 lbs. per square inch absolute, receiver four H.P. volumes.

Prob. 8. A 14- and 20×20-in. engine, has a receiver volume equal to 5 times the H.P. cylinder and runs at $\frac{2}{3}$ cut-off on the high-pressure cylinder and $\frac{1}{2}$ cut-off on the low, with steam pressure of 100 lbs. per square inch gage and back pressure of 5 lbs. per square inch absolute. Will this engine have complete expansion and equal work distribution? If not, what changes must be made in the cut-off or initial pressure?

Prob. 9. What must be the size of an engine to give 200 I.H.P. at 150 R.P.M. on an initial steam pressure of 150 lbs. per square inch absolute, and 10 lbs. per square inch absolute back pressure, if the piston speed is limited to 450 ft. per minute and complete expansion and equal work distribution is required? Receiver is to be 6 times the volume of high-pressure cylinder and H.P. stroke equal to diameter.

48. Compound Engine with Finite Receiver. Exponential Law, No Clearance. Cycle VIII. General Relations between Pressures, Dimensions, and Work, when High-pressure Exhaust and Low-pressure Admission are Independent. The diagram Fig. 64 may be used to represent this cycle, as well as cycle VII, by conceiving a slight change in the slope of the expansion

and receiver lines. Using the same symbols as those of the preceding section, and the expression for work as found in Section 7, Chapter I,

$$W_H = P_b V_b + \frac{P_b V_b}{s-1} \left[1 - \left(\frac{V_b}{V_c} \right)^{s-1} \right] - \frac{P_n O}{s-1} \left[1 - \left(\frac{O}{V_c + O} \right)^{s-1} \right]$$

$$= 144 D_H \left\{ (\text{in.pr.}) Z_H + \frac{(\text{in.pr.}) Z_H}{s-1} (1 - Z_H^{s-1}) - \frac{(\text{rec.pr.})_1 y}{s-1} \left[1 - \left(\frac{y}{y+1} \right)^{s-1} \right] \right\}$$

but $(\text{rec.pr.})_1 = P_n = P_b \left(\frac{V_b}{V_c} \right)^s \left(\frac{V_c + O}{O} \right)^s,$

or $(\text{rec.pr.})_1 = (\text{in.pr.}) \left(\frac{Z_H}{R_c Z_L} \right)^s \left(\frac{y + Z_L R_c}{y} \right)^s = (\text{in.pr.}) \left(\frac{Z_H}{y} \right)^s \left(\frac{y}{R_c Z_L} + 1 \right)^s,$

and the last term in the equation for W_H within the bracket may therefore

be written $\frac{(\text{in.pr.}) Z_H}{s-1} \left(\frac{y}{Z_H} \right) \left(\frac{Z_H}{y} \right)^s \left(\frac{y}{R_c Z_L} + 1 \right)^s \left[1 - \left(\frac{y}{y+1} \right)^{s-1} \right],$

or $\frac{(\text{in.pr.}) Z_H}{s-1} \left(\frac{Z_H}{y} \right)^{s-1} \left(\frac{y}{R_c Z_L} + 1 \right)^s \left[1 - \left(\frac{y}{y+1} \right)^{s-1} \right],$

hence by simplifying the first two terms also,

$$W_H = 144 (\text{in.pr.}) \frac{Z_H D_H}{s-1} \left\{ s - Z_H^{s-1} - \left(\frac{Z_H}{y} \right)^{s-1} \left(\frac{y}{R_c Z_L} + 1 \right)^s \left[1 - \left(\frac{y}{y+1} \right)^{s-1} \right] \right\}. \quad (290)$$

Work of the low-pressure cylinder may be expressed in terms of pressure and volumes at N , E , and G , but it is convenient to use instead of the pressure at N or at E , its equivalent in terms of the point B . The pressure at N is that of the $(\text{rec.pr.})_1$ and when multiplied by the receiver volume $y D_H$, it becomes $(\text{rec.pr.})_1 y D_H$

$$= (\text{in.pr.}) D_H Z_H \left(\frac{y}{Z_H} \right) \left(\frac{Z_H}{y} \right)^s \left(\frac{y}{R_c Z_L} + 1 \right)^s = (\text{in.pr.}) D_H Z_H \left(\frac{Z_H}{y} \right)^{s-1} \left(\frac{y}{R_c Z_L} + 1 \right)^s.$$

At E the product of pressure and volume is

$$(\text{rec.pr.})_2 \times Z_L D_L = (\text{in.pr.}) Z_H D_H \left(\frac{Z_H}{R_c Z_L} \right)^{s-1}.$$

Using these quantities, the following equation gives the work of the low-pressure cylinder:

$$W_L = 144 (\text{in.pr.}) \frac{Z_H D_H}{s-1} \left\{ \left(\frac{Z_H}{y} \right)^{s-1} \left(\frac{y}{R_c Z_L} + 1 \right)^s \left[1 - \left(\frac{y}{y + R_c Z_L} \right)^{s-1} \right] \right.$$

$$\left. + \left(\frac{Z_H}{Z_L R_c} \right)^{s-1} [1 - Z_L^{s-1}] \right\} - 144 (\text{bk.pr.}) D_L, \quad (291)$$

and the total work by adding (W_H) and (W_L) is,

$$W = 144 (\text{in.pr.}) \frac{Z_H D_H}{s-1} \left\{ s - Z_H^{s-1} + \left(\frac{Z_H}{y} \right)^{s-1} \left(\frac{y}{R_c Z_L} + 1 \right)^s \left[\left(\frac{y}{y+1} \right)^{s-1} \right. \right.$$

$$\left. \left. - \left(\frac{y}{y + R_c Z_L} \right)^{s-1} \right] + \left(\frac{Z_H}{Z_L R_c} \right)^{s-1} (1 - Z_L^{s-1}) \right\} - 144 (\text{bk.pr.}) D_L, \quad (292)$$

This Eq. (292) is the general expression for work of the zero clearance compound engine with exponential expansion, no clearance, and finite receiver. From this the following expressions are derived:

$$(\text{m.e.p. ref. to L.P.}) = \frac{(\text{in.pr.})}{s-1} \frac{Z_H}{R_C} \left\{ s - Z_H^{s-1} + \left(\frac{Z_H}{y} \right)^{s-1} \left(\frac{y}{R_C Z_L} + 1 \right)^s \left[\left(\frac{y}{y+1} \right)^{s-1} - \left(\frac{y}{y+R_C Z_L} \right)^{s-1} \right] + \left(\frac{Z_H}{Z_L R_C} \right)^{s-1} (1 - Z_L^{s-1}) \right\} - (\text{bk.pr.}) \quad (293)$$

Work per cu.ft. supplied

$$= 144 \frac{(\text{in.pr.})}{s-1} \left\{ s - Z_H^{s-1} + \left(\frac{Z_H}{y} \right)^{s-1} \left(\frac{y}{R_C Z_L} + 1 \right)^s \left[\left(\frac{y}{y+1} \right)^{s-1} - \left(\frac{y}{y+R_C Z_L} \right)^{s-1} \right] + \left(\frac{Z_H}{Z_L R_C} \right)^{s-1} (1 - Z_L^{s-1}) \right\} - 144 (\text{bk.pr.}) \frac{R_C}{Z_H} \quad (294)$$

Cu.ft. supplied per hr. per I.H.P. may be obtained from Eq. (268).

$$(\text{rec.pr.})_2 = (\text{in.pr.}) \left(\frac{Z_H}{Z_L R_C} \right)^s; \quad (295)$$

$$(\text{rec.pr.})_1 = (\text{in.pr.}) \left(\frac{Z_H}{Z_L R_C} \right)^s \left(1 + \frac{R_C Z_L}{y} \right)^s; \quad (296)$$

$$(\text{rel.pr.})_H = (\text{in.pr.}) Z_H^s; \quad (297)$$

$$(\text{rel.pr.})_L = (\text{rec.pr.})_2 Z_L^s = (\text{in.pr.}) \left(\frac{Z_H}{R_C} \right)^s \quad (298)$$

If work is equally divided between the cylinders, W_H , Eq. (290) will equal W_L , Eq. (291), from which a rather complex equation will result, but this does not yield directly to solution and will not be given here.

When *expansion is complete in both cylinders*,

$$Z_L = \frac{1}{R_C} \quad \text{and} \quad \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right) = \left(\frac{Z_H}{R_C} \right)^s.$$

Introducing these values in the general expression Eq. (292) for work of this cycle, it may be reduced to the following:

$$W = 144 (\text{in.pr.}) Z_H D_H \frac{s}{s-1} \left[1 - \left(\frac{Z_H}{R_C} \right)^{s-1} \right] \quad (299)$$

$$\text{Hence,} \quad (\text{m.e.p. ref. to L.P.}) = (\text{in.pr.}) \frac{Z_H}{R_C} \frac{s}{s-1} \left[1 - \left(\frac{Z_H}{R_C} \right)^{s-1} \right] \quad (300)$$

$$\text{Work per cu.ft. supplied} = 144 (\text{in.pr.}) \frac{s}{s-1} \left[1 - \left(\frac{Z_H}{R_C} \right)^{s-1} \right] \quad (301)$$

$$\text{Cu.ft. supplied per hr. per I.H.P.} = \frac{13,750}{(\text{m.e.p. ref. to L.P.})} \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}} \quad (302)$$

If the work is equally divided and complete expansion is maintained in both cylinders an expression may be obtained which, when simplified, reduces

$$\text{to the form,} \quad 2 Z_H^{s-1} \left\{ 1 + \frac{(y+1)^s}{y^{s-1}} \left[1 - \left(\frac{y}{y+1} \right)^{s-1} \right] \right\} = \frac{s}{R_C v^s} - s, \quad (303)$$

where R_v is the ratio of maximum low-pressure volume, to the high-pressure volume at cut-off, $R_v = \frac{R_c}{Z_H}$, and the value of R_v may be found from original data,

$$R_v = \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)^{\frac{1}{\gamma}} \quad (304)$$

Eq. (303) may easily be solved for Z_H , from which the required cylinder ratio may be found from $R_c = Z_H R_v$ (305)

This is the cylinder ratio which gives equal work in the two cylinders and complete expansion in both, when used with the value found for the high-pressure cut-off Z_H , the assumed initial and back pressures, and the assumed ratio, γ , of receiver volume of high-pressure displacement.

Example. Find (a) the horse-power, (b) steam used per hour, (c) the release and receiver pressures of a 12- and 18×24-in. engine, with a receiver twice as large as the low-pressure cylinder when the initial pressure is 150 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute, speed 125 R.P.M., and cut-offs $\frac{1}{2}$ in the high and such a value in the low as to give complete expansion. For comparison with other problems in which air is used assume exponent for expansion curve = 1.4.

(a) Substituting the above values in Eq. (293), gives (m.e.p. ref. to L.P.)

$$\frac{150}{.4} \times \frac{.5}{2.25} \left\{ 1.4 - .5^4 + \left(\frac{.5}{4.5} \right)^4 \left(\frac{4.5}{2.25 \times \frac{1}{2.25}} + 1 \right)^{1.4} \left[\left(\frac{4.5}{5.5} \right)^4 - \left(\frac{4.5}{4.5 + 2.25 \times \frac{1}{2.25}} \right)^4 \right] + \left(\frac{.5}{\frac{1}{2.25} \times 2.25} \right)^4 \left[1 - \left(\frac{1}{2.25} \right)^4 \right] \right\} - 10,$$

or (m.e.p.) = 57.5 lbs. per sq.in., hence I.H.P. = 221.

$$\begin{aligned} \text{(b) From Eq. (268) cubic feet of steam per hour per horse-power} &= \frac{13,750 Z_H}{\text{m.e.p. } R_c'} \\ &= \frac{13,750}{57.5} \times \frac{.5}{2.25} = 53.2 \text{ cu.ft.,} \end{aligned}$$

hence total pounds per hour will be $53.2 \times 221 \times .332 = 3910$.

Eqs. (295) to (298) on substitution of the proper numerical values, become:

$$\begin{aligned} (\text{rec.pr.})_2 &= 150 \times (.5)^{1.4} &&= 57 \text{ lbs., per sq. inch,} \\ (\text{rec.pr.})_1 &= 150 \times \left(\frac{.5}{1} \right)^{1.4} \left(1 + \frac{1}{4.5} \right)^{1.4} &&= 75 \text{ lbs.} \quad " \\ (\text{rel.pr.})_H &= 150 \times (.5)^{1.4} &&= 57 \text{ lbs.} \quad " \\ (\text{rel.pr.})_L &= 57 \times \left(\frac{1}{2.25} \right)^{1.4} &&= 32.1 \text{ lbs.} \quad " \end{aligned}$$

NOTE: In all the following problems clearance is assumed to be zero.

Prob. 1. A 12×18×24-in. engine is running on superheated steam of 150 lbs. per square inch absolute pressure, and exhausts to the atmosphere. If the speed is 100 R.P.M., high-pressure cut-off $\frac{1}{2}$, low-pressure cut-off $\frac{1}{2}$, and receiver volume 10 cu.ft., what horse-power will be developed and what steam used per hour?

Prob. 2. What would be the effect on the power and the economy of (a) changing to wet steam in the above? (b) to compressed air?

Prob. 3. What would be the receiver and the release pressures for each case?

Prob. 4. Will there be equal work distribution between the two cylinders?

Prob. 5. It is desired to obtain complete expansion in a $14 \times 22 \times 36$ -in. engine running on fluid which gives a value for s of 1.2. Initial pressure is 100 lbs. per square inch gage, and back pressure 5 lbs. per square inch absolute. What must be the cut-offs and what power will be developed at 500 ft. piston speed? Receiver = $3 \times$ H.P. volume.

Prob. 6. How large must the receiver be for the above engine in order that the pressure in it shall not fluctuate more than 5 lbs. per sq. inch?

Prob. 7. An engine is to run on steam which will give a value of $s = 1.1$, and develop 500 horse-power at 100 R.P.M. Piston speed is not to exceed 500 ft. per minute. Steam pressure 150 lbs. per square inch absolute, back pressure 5 lbs. per square inch absolute. Complete expansion and equal work distribution for this load are to be accomplished. What will be the cylinder sizes and the high-pressure cut-off if the receiver is to be 3 times the high-pressure cylinder volume?

Prob. 8. What will be the steam used per hour by the engine of Prob. 7, and what will be the variation in the receiver pressure?

Prob. 9. If the high-pressure cut-off were halved, how would the power and economy be affected?

49. Compound Engine without Receiver, Logarithmic Law. No Clearance, Cycle IX. General Relations between Dimensions and Work when High-pressure Exhaust and Low-pressure Admission are Coincident. Such a peculiar case as this admits of but little modification of the cycle compared with the receiver cases, because the low-pressure expansion is necessarily a direct continuation of the high pressure without any possible break. There can be no over-expansion in the high nor can expansion there be incomplete, as there is, properly speaking, no back pressure with which to compare the high-pressure cylinder terminal pressure. There may, however, be over- and incomplete expansion in the low-pressure cylinder. It might appear that the high-pressure cylinder negative work was equal to the low-pressure admission work, as each is represented by the area below DC , Fig. 67A, but this is not the case, since the diagram is drawn to two different scales of volumes, showing the pressure-stroke relation between high and low. This is apparent from the diagram, Fig. 67C, showing fluid volumes in each cylinder to a single scale on which \overline{ABCD} is the work done in the high-pressure cylinder, $\overline{ABD'EF}$ the whole work, whence $\overline{DCD'EF}$ is the part done in the low-pressure cylinder. There is no low-pressure cut-off or even admission as ordinarily considered. *The cycle, so far as the work to be done is concerned, is the same as for a simple engine, and the only reason for introducing formulas for overall work, work per cubic foot supplied (m.e.p. referred to low), and fluid consumption, is to put them into form for immediate substitution of dimensional relations.* Because of the absence of cut-off in the low, the distribution of work between high and low will depend solely on the cylinder ratio and high-pressure cut-off, for, the earlier the high-pressure cut-off, and the larger the high-pressure cylinder, the greater

the fraction of the total work that will be done in it since there is only a fixed amount available, and the less there will be left to be done in the low.

The diagrams of the two cylinders are plotted to combined axes in Fig. 67D. The points *Q* and *R* at equal heights. *KN* is the L.P. displacement, and *KG* that of the H.P. It has been shown in Section 44, that the expan-

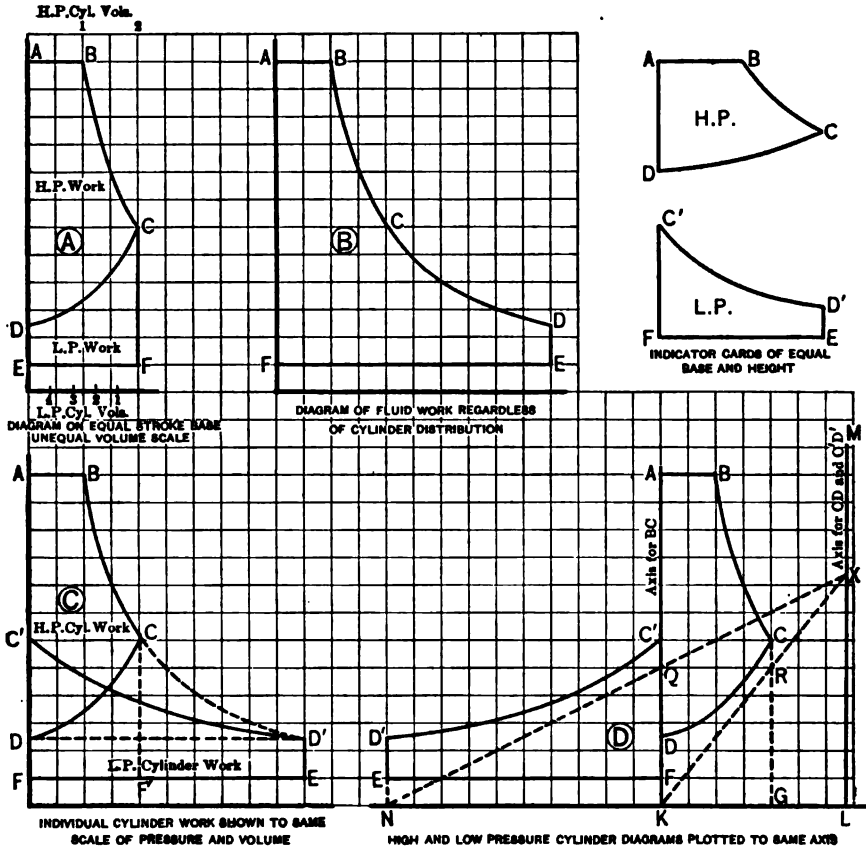


FIG. 67.—Work of Expansion in the No-receiver Compound Engine, Zero Clearance; Cycle IX, Logarithmic Expansion; Cycle X, Exponential Expansion, Coincident Piston Movement.

sion lines *CD* and *C'D'* may be plotted to the axes *LN* and *LXM*, the point *X* being the intersection of *NQ* and *KR* extended, and that the distance

$$\overline{KL} = \frac{D_L D_H}{D_L - D_H} = D_H \frac{R_C}{R_C - 1}, \quad \dots \quad (306)$$

and
$$\overline{GL} = \frac{D_L D_H}{D_L - D_H} - D_H = \frac{D_H^2}{D_L - D_H} = D_H \frac{1}{R_C - 1}; \quad \dots \quad (307)$$

also
$$\overline{NL} = \frac{D_L D_H}{D_L - D_H} + D_L = \frac{D_L^2}{D_L - D_H}. \quad \dots \quad (308)$$

Hence the work area under CD is

$$W_{cd} = \overline{GL} \times P_c \log_e \frac{\overline{KL}}{\overline{GL}} = 144(\text{rel.pr.})_H \frac{D_H^2}{D_L - D_H} \log_e \frac{D_L}{D_H},$$

but $(\text{rel.pr.})_H = (\text{in.pr.})_{Z_H}$,

$$\text{hence } W_H = 144(\text{in.pr.})_{Z_H} D_H \left\{ 1 + \log_e \frac{1}{Z_H} - \frac{1}{R_C - 1} \log_e R_C \right\}. \quad (309)$$

Again the work area under $C'D'$ is

$$W_{c'd'} = \overline{KL} \times P_c \log_e \frac{\overline{NL}}{\overline{KL}} = 144(\text{rel.pr.})_H \frac{D_L D_H}{D_L - D_H} \log_e \frac{D_L}{D_H},$$

$$\text{hence } W_L = 144(\text{in.pr.})_{Z_H} D_H \left(\frac{R_C}{R_C - 1} \right) \log_e R_C - 144(\text{bk.pr.})_{D_L}, \quad (310)$$

Adding Eqs. (309) and (310), and noting that

$$\left(\frac{R_C}{R_C - 1} - \frac{1}{R_C - 1} \right) = 1, \text{ and } \log_e \frac{1}{Z_H} + \log_e R_C = \log_e \frac{R_C}{Z_H} = \log_e R_v,$$

$$\text{the total work, } W = 144(\text{in.pr.})_{Z_H} D_H \left(1 + \log_e \frac{R_C}{Z_H} \right) - 144(\text{bk.pr.})_{D_L}, \quad (311)$$

which shows by its similarity to the work of the simple engine that, as before stated, the total work is the same for this cycle as it would be if the entire expansion were made to take place in a single cylinder.

Dividing the total work by the low-pressure cylinder volume and the high-pressure admission volume in turn,

$$\begin{aligned} (\text{m.e.p. ref. to L.P.}) &= (\text{in.pr.})_{Z_H} \left(1 + \log_e \frac{R_C}{Z_H} \right) - (\text{bk.pr.}) \quad (a) \\ &= (\text{in.pr.})_{Z_H} \frac{1}{R_C} [1 + \log_e (R_H R_C)] - (\text{bk.pr.}) \quad (b) \end{aligned} \quad (312)$$

$$\begin{aligned} \text{Work per cu.ft. supplied} &= 144(\text{in.pr.})_{Z_H} \left(1 + \log_e \frac{R_C}{Z_H} \right) - (\text{bk.pr.})_{R_C} \quad (a) \\ &= 144(\text{in.pr.})_{Z_H} \frac{1}{R_H} [1 + \log_e (R_H R_C)] - (\text{bk.pr.})_{R_C} \quad (b) \end{aligned} \quad (313)$$

The cu.ft. supplied per hr. per I.H.P. may be obtained as before from Eq. (268).

For equal division of work there can obviously be only one setting of the high-pressure cut-off for a given cylinder ratio, and any change of load to be met by a change of initial pressure or of high-pressure cut-off will necessarily unbalance the work. Equating the high-pressure and low-pressure work expressions, Eqs. (309) and (310), and noting the relation which exists between Z_H and R_C , as shown in Eq. (305),

$$1 + \log_e \frac{R_v}{R_C} - \frac{R_C + 1}{R_C - 1} \log_e R_C + \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right) R_v = 0,$$

but $\log_e \frac{R_v}{R_c} - \frac{R_c+1}{R_c-1} \log_e R_c = \log_e \left[\left(\frac{R_v}{R_c} \right) \left(\frac{1}{R_c^{\frac{R_c+1}{R_c-1}}} \right) \right] = \log_e \frac{R_v}{R_c^{\frac{2R_c}{R_c-1}}}$

hence $\log_e \frac{R_c^{\frac{2R_c}{R_c-1}}}{R_v} = 1 + \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right) R_v. \quad (314)$

With this formula it is possible to find the necessary ratio of cylinder displacements for given initial and back pressures and for given ratio of expansion R_v .

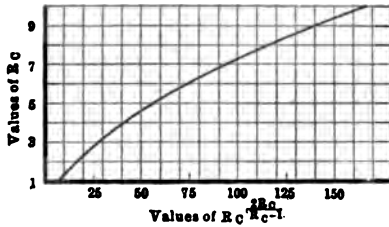


FIG. 68.—Curve to Show Relation between Values of R_c and $R_c^{\frac{2R_c}{R_c-1}}$ for Use in Solving Eq. (314), Giving-Cylinder Ratio in Terms of Ratio of Expansion for the No-receiver Compound Engine without Clearance.

For convenience in solving this, a curve is given in Fig. 68 to find value of R_c when $R_c^{\frac{2R_c}{R_c-1}}$ has been found.

The case of complete expansion with this cycle results from the condition

$$P_d = P_e \quad \text{or} \quad (\text{rel.pr.})_L = (\text{bk.pr.})$$

$$\text{or} \quad R_v = \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right),$$

which when applied to Fig. 67, transforms the diagrams to the form Fig. 69.

$$\text{It also follows that } (\text{bk.pr.}) D_L = (\text{in.pr.}) Z_H D_H \quad \text{and} \quad \frac{R_c}{Z_H} = \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right).$$

These conditions will, of course, reduce the total work Eq. (311) to the common value for all cycles with logarithmic expansion and likewise those for mean effective pressure, work per cubic foot supplied, and consumption. For the equal division of work under this condition, Eq. (314), becomes

$$R_c^{\frac{2R_c}{R_c-1}} = 7.39R, \quad (315)$$

since $\left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right) R_v = 1$ and R may represent ratio of expansion or ratio of initial to back pressures, these being equal. Fig. 70 gives a curve showing the

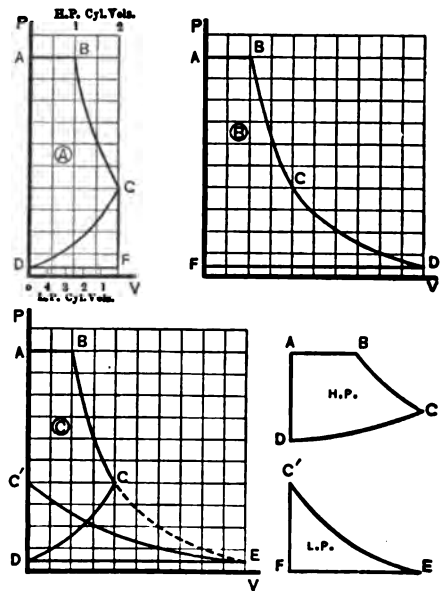


FIG. 69.—Special Case of Cycles IX and X. Complete Expansion in both Cylinders of the No-Receiver Compound Engine, Zero Clearance.

relation between cylinder ratio and ratio of expansion established by the above condition.

Example 1. Method of calculating Diagram, Fig. 67.

A. As described in the text this diagram is drawn to two-volume scales, so that there may be two volumes for one point.

Assumed data:

$$P_a = P_b = 120 \text{ lbs. per sq.in. abs.} \quad V_a = V_c = V_d = V_e = V_f = 0 \text{ cu.ft.} \quad V_b = 1 \text{ cu.ft.}$$

$$P_e = P_f = 10 \text{ lbs. per sq.in. abs.} \quad V_e = 2 \text{ cu.ft.} \quad V_d = V_g = 5 \text{ cu.ft.}$$

$$\text{To locate point C:} \quad P_c = \frac{P_b V_b}{V_c} = \frac{120 \times 1}{2} = 60 \text{ lbs. per sq.in.}$$

$$\text{To locate point D:} \quad P_d = \frac{P_e V_e}{V_d} = \frac{60 \times 2}{5} = 24 \text{ lbs. per sq.in.}$$

To locate intermediate points from C to D. The volume at any intermediate point is (the volume of low-pressure cylinder up to that point)+(volume of high-

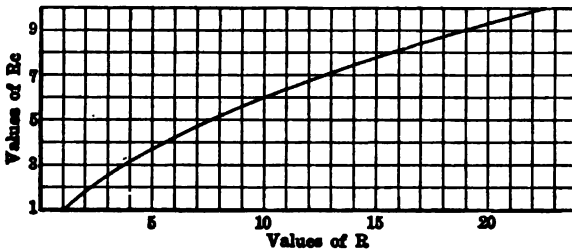


FIG. 70.—Curve to Show Relation between Values of R_c , the Cylinder Ratio and R the Ratio of Initial to Back Pressure for Complete Expansion in the No-receiver Compound Engine without Clearance—Eq. (315).

pressure cylinder from that point to end of stroke), e.g., at $\frac{1}{4}$ stroke the volume in low, is $.75 \times 5$, and the volume in the high is $.25 \times 2$, or total 4.25, and the pressure at that point is found by the PV relation as above.

B. Assumed data: $P_a = P_b = 120$ lbs. per sq.in. abs. $V_a = V_f = 0$ cu.ft.

$$P_e = P_f = 10 \text{ lbs. per sq.in. abs.} \quad V_d = V_e = 5 \text{ cu.ft.} \quad V_b = 1 \text{ cu.ft.}$$

$$\text{To locate point D:} \quad P_d = \frac{P_b V_b}{V_d} = \frac{120 \times 1}{5} = 24 \text{ lbs. per sq.in.}$$

Intermediate points from B to D found by assuming volumes and computing pressures from the PV relation as above.

C. Figure $ABCD$ constructed as in A. Figure $C'D'EF$ is figure $CDEF$ of A to the same pressure scale but to a volume scale 2.5 times as large.

D. Figures constructed as in C.

To draw indicator cards. The volume and pressure scales are chosen and from diagram A, a distance \overline{AB} is laid off to the volume scale, \overline{AD} is then laid off equal to \overline{AD} of diagram A to the pressure scale. Point C is located to these scales and joined to B and D by drawing curves through the intermediate points plotted from the PV diagram to the scales of the card. For the low-pressure card EF is laid off to the volume scale, and FC' and ED' to pressure scale. C' and D' are then joined in same manner as C and D for high-pressure card.

Example 2. Find (a) the horse-power, and (b) steam used per hour for a $12 \times 18 \times 24$ -in. engine with no clearance when initial pressure is 150 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute, speed 125 R.P.M., cut-off in the high-pressure cylinder is $\frac{1}{4}$, there being no receiver.

(a) From Eq. (312) we have by substitution,

$$(\text{m.e.p.}) = 150 \times \frac{1}{2 \times 2.25} \times (1 + .8) - 10 = 50 \text{ lbs. sq.in.}, \text{ hence I.H.P.} = 192.$$

(b) From Eq. (268) we have

$$\text{Cubic feet of steam per I.H.P. per hour} = \frac{13,750}{(\text{m.e.p.})} \times \frac{1}{R_H R_C} = \frac{13,750}{50} \times \frac{1}{2 \times 2.25} = 61.2,$$

hence the weight of steam used per hour will be $61.2 \times .332 \times 192 = 3890$ pounds.

Example 3. What will be the cylinder ratio and the high-pressure cut-off to give equal work distribution for a ratio of expansion of 6, an initial pressure of 150 lbs. per square inch absolute and back pressure of 10 lbs. per square inch absolute?

Ratio of back to initial pressures is .067 and $R_V = 6$, hence from Eq. (314)

$$\log_e \frac{R_C^{2R_C} - 1}{6} = 1.40, \text{ or } R_C^{2R_C - 1} = 24.36, \text{ and from Fig. 68, } R_C = 2.8.$$

From the relation $Z_H = \frac{R_C}{R_V}$ we have high-pressure cut-off $= \frac{2.8}{6} = .446$.

Prob. 1. A compound locomotive has no receiver and runs on an initial pressure of 175 lbs. per square inch gage and atmospheric exhaust. The cylinders are 18 and 30×42 in. The steam pressure may be varied, as may also the cut-off to a limited degree. For a speed of 200 R.P.M. and a constant cut-off of $\frac{1}{4}$, find how the power will vary with initial pressure, and for constant initial pressure equal to boiler pressure show how the power at the same speed will vary from $\frac{1}{4}$ cut-off to full stroke.

Prob. 2. Show how the steam used per h.p. hour will vary in above problem.

NOTE: δ for 175 lbs. = .416.

Prob. 3. With the cut-off at $\frac{1}{4}$, what should the initial pressure be to give equal work distribution?

Prob. 4. With full boiler pressure and $\frac{1}{4}$ cut-off what would be terminal pressure in the low-pressure cylinder?

Prob. 5. What must be size of cylinders for a tandem compound engine, with negligible receiver volume, to run at 125 R.P.M. with complete expansion and equal work distribution on an initial pressure of 125 lbs. per square inch gage and a back pressure of 5 lbs. per square inch absolute, when carrying a load of 500 horse-power, the piston speed to be less than 500 ft. per minute?

Prob. 6. What will be the steam used by the above engine in pounds per hour?

NOTE: δ for 125 lbs. = .31.

Prob. 7. A builder gives following data for a tandem compound steam engine. Check the horse-power and see if the work is equally divided at the rated load. Cylinders 10 ins. and $17\frac{1}{2} \times 15$ ins., initial pressure 125 lbs., speed 250 R.P.M., horse-power 155. Neglect the receiver volume.

Prob. 8. Another manufacturer gives for his engine the following, check this: Cylinders 20 and 32×18 ins., initial pressure 100 lbs., atmospheric exhaust, speed 200 R.P.M., horse-power 400. Neglect the receiver volume.

50. Compound Engine without Receiver, Exponential Law, Cycle X. General Relations between Dimensions and Work when High-pressure Exhaust and Low-pressure Admission are Coincident. Referring to Fig. 67D it is desirable first to evaluate the work areas $CDKG$ and $C'D'NK$.

Combining previous expression for work done, in high-pressure cylinder, exponential law, Eq. (290), with value for GL and KL as given in Eqs. (306) and (307), and noting that $(\text{rel.pr.})_H = (\text{in.pr.})Z_H^s$, there is obtained,

$$W_H = 144(\text{in.pr.}) \frac{D_H Z_H}{s-1} \left[s - Z_H^{s-1} - Z_H^{s-1} \left(\frac{1 - \left(\frac{1}{R_C} \right)^{s-1}}{R_C - 1} \right) \right] \quad (316)$$

and
$$W_L = 144(\text{in.pr.}) \frac{D_H Z_H^s}{s-1} R_C \left[\frac{1 - \left(\frac{1}{R_C} \right)^{s-1}}{R_C - 1} \right] - 144(\text{bk.pr.}) D_L \quad (317)$$

It is to be expected that the sum of high- and low-pressure work will be of a form similar to that which would be obtained if all work were performed in a single cylinder of a displacement equal to that of the low-pressure, hence adding, W_H and W_L and reducing,

$$W = 144(\text{in.pr.}) \frac{D_H Z_H}{s-1} \left\{ s - Z_H^{s-1} + Z_H^{s-1} \left[1 - \left(\frac{1}{R_C} \right)^{s-1} \right] \right\} - 144(\text{bk.pr.}) D_L$$

Substituting $\frac{Z_H}{R_C} = \frac{1}{R_V}$, $W = 144(\text{in.pr.}) \frac{D_H Z_H}{s-1} \left[s - \left(\frac{1}{R_V} \right)^{s-1} \right] - 144(\text{bk.pr.}) D_L$; (318)

(m.e.p. ref. to L.P.) $= \frac{W}{144 D_L} = \frac{(\text{in.pr.})}{s-1} \frac{Z_H}{R_C} \left[s - \left(\frac{1}{R_V} \right)^{s-1} \right] - (\text{bk.pr.})$. . . (319)

Work per cu.ft. supplied $= 144 \frac{(\text{in.pr.})}{s-1} \left[s - \left(\frac{1}{R_V} \right)^{s-1} \right] - 144(\text{bk.pr.}) \frac{R_C}{Z_H}$. . . (320)

Cu.ft. supplied per hr. per I.H.P. $= \frac{13,750}{(\text{m.e.p. ref. to L.P.}) \frac{1}{R_V}}$ (321)

Conditions for *equal division of work* between high- and low-pressure cylinders may be obtained by equating Eqs. (316) and (317).

Combining and substituting as before,

$$\frac{R_C + 1}{R_C - 1} [R_C^{s-1} - 1] + R_C^{s-1} = R_V^{s-1} \left[s + \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right) R_V (s-1) \right], \quad (322)$$

which is not a simple relation, but can be solved by trial.

The assumption of *complete expansion* in the low-pressure cylinder (it is always complete in high, for this cycle), leads to the following relations:

$$\left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right) = R_V^s, \text{ hence } 144(\text{bk.pr.}) D_L = 144(\text{in.pr.}) \frac{R_C}{R_V^s} D_H.$$

From Eq. (316), $W_H = 144(\text{in.pr.}) \frac{D_H Z_H}{s-1} \left[s - \left(\frac{1}{R_V} \right)^{s-1} - \frac{R_C}{Z_H} \frac{s-1}{R_V^s} \right]$,

but $\frac{R_C}{Z_H} = R_V$, and $\frac{R_V s - 1}{Z_H R_C^s} = \frac{s-1}{R_V^{s-1}}$, and, $\frac{1}{R_V} = \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}}$,

therefore, $W = 144(\text{in.pr.}) D_H Z_H \frac{s}{s-1} \left[1 - \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{s-1}{s}} \right] \dots (323)$

(m.e.p. ref. to L.P.) = $(\text{in.pr.}) \frac{Z_H}{R_C} \frac{s}{s-1} \left[1 - \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{s-1}{s}} \right] \dots (324)$

The expression for *equality of work* Eq. (322) becomes, for this case of complete expansion, $\frac{R_C + 1}{R_C - 1} (R_C^{s-1} - 1) + R_C^{s-1} = s R_V^{s-1} + (s-1)$, $\dots (325)$

by which it is not difficult to find the ratio of expansion R_V , which gives equality of work for given values of s , and R_C , the cylinder ratio. Values for R_V for various values of R_C and s may be found by the curves of Fig. 71.

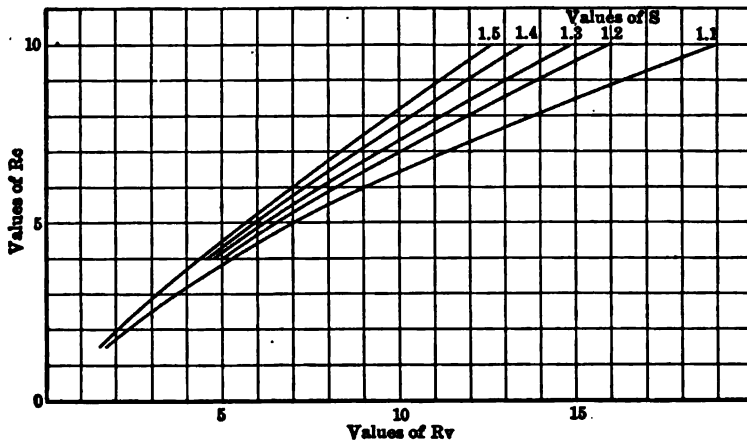


FIG. 71.—Curves to Show Relation between R_C the Cylinder Ratio, and R_V the Ratio of Expansion, for Various Values of (s) , Applied to the No-receiver Compound Engine without Clearance, when the Expansion is not Logarithmic.

Example 1. Find (a) the horse-power, and (b) the steam used per hour for a 12- and 18×24-in. engine with no receiver when the initial pressure is 150 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute, speed 125 R.P.M., cut-off in high-pressure cylinder is $\frac{1}{3}$, there being no receiver and steam having expansion, such that $s = 1.3$.

From Eq. (319) by substituting the above values,

$$(\text{m.e.p.}) = \frac{150}{.3} \times \frac{.5}{2.25} \left[1.3 - \left(\frac{.1}{4.5} \right) \right] - 10 = 63.3 \text{ lbs. per sq.in.}$$

hence the indicated horse-power = 243.

(b) From Eq. (321) the steam used per hour in cu.ft. per horse-power is

$$\frac{13,750}{\text{m.e.p.} \cdot R_C} \quad \text{hence} \quad \frac{13,750}{63.3} \times \frac{.5}{2.25} = 48.2 \text{ cu.ft.,}$$

or pounds per hour total, is, $48.2 \times 243 \times .332 = 3880$.

Example 2. What will be the high-pressure cut-off and cylinder ratio to give equal work distribution and complete expansion for an initial pressure of 150 lbs. per square inch absolute, and back pressure of 10 lbs. per square inch absolute?

From relation $R_V^s = \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)$, $R_V = 6.9$ and from this, by the curve of Fig. 71,

$R_C = 5.4$. For complete expansion $Z_H = \frac{R_C}{R_V} = \frac{5.4}{6.9} = .8$.

Prob. 1. A tandem compound engine without receiver has cylinders 18- and 30×42-ins. and runs at 200 R.P.M. What will be the horse-power developed at this speed if the initial pressure is 175 lbs. per square inch gage, back pressure atmosphere, high-pressure cut-off $\frac{1}{2}$, and s has a value of (a) 1.1, (b) 1.3? Compare the results with Prob. 1 of Sec. 49.

Prob. 2. What will be the weight of steam used per H.P. per hour for the two cases of the above problem? Compare these results with those of Prob. 2, Section 14.

NOTE: $\delta = .416$.

Prob. 3. What must be the cut-off in a 10- and 15×20-in. compressed-air engine running on 100 lbs. per square inch gage initial pressure and atmospheric back pressure, to give complete expansion, and what will be the horse-power per 100 ft. per minute piston speed, s being 1.4?

Prob. 4. It is desired to run the following engine at its most economical load. What will this load be and how much steam will be needed per hour?

Cylinders 20- and 32×18-ins., speed 150 R.P.M., steam pressure 100 lbs. per square inch gage, atmospheric exhaust, dry saturated steam.

Prob. 5. Should the load increase 50 per cent in Prob. 4, how would the cut-off change and what would be the effect on the amount of steam used?

Prob. 6. What would be the gain in power and what the economy of the engine of Prob. 4 if superheated steam were used, for which $s = .3$?

Prob. 7. In a 14- and 20×24-in. engine will the work be equally divided between the cylinders for the following conditions? If not, what per cent will be done in each? Steam pressure 100 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute, $s = 1.2$, cut-off = $\frac{1}{2}$.

Prob. 8. What would be the work and steam used by the above engine if there were complete expansion and equal distribution?

51. Compound Engine with Infinite Receiver. Logarithmic Law. With Clearance and Compression, Cycle XI. General Relations between Pressures, Dimensions and Work. In terms of point pressures and volumes, Fig. 72, the work of the two cylinders may be written down at once as if each were independent of the other, the connection between them being fixed first by making the back pressure of the high equal to the initial pressure of the low, or to the receiver pressure; and second by making the volume admitted to the low equal to that discharged from the high reduced to the same pressure. This last condition may be introduced in either of two ways:

$$(a) \overline{EM} = \overline{NH},$$

$$(b) [(PV \text{ on H.P. expansion line} - (PV \text{ on H.P. comp. line})] \\ = [(PV \text{ on L.P. expansion line} - (PV \text{ on L.P. comp. line})]$$

Without introducing the last relation

$$W_H = P_s V_s \left(1 + \log_e \frac{V_c}{V_s} \right) - P_f V_f \left(1 + \log_e \frac{V_c}{V_f} \right) - (P_s - P_f) V_s - P_s (V_s - V_c); \quad (326)$$

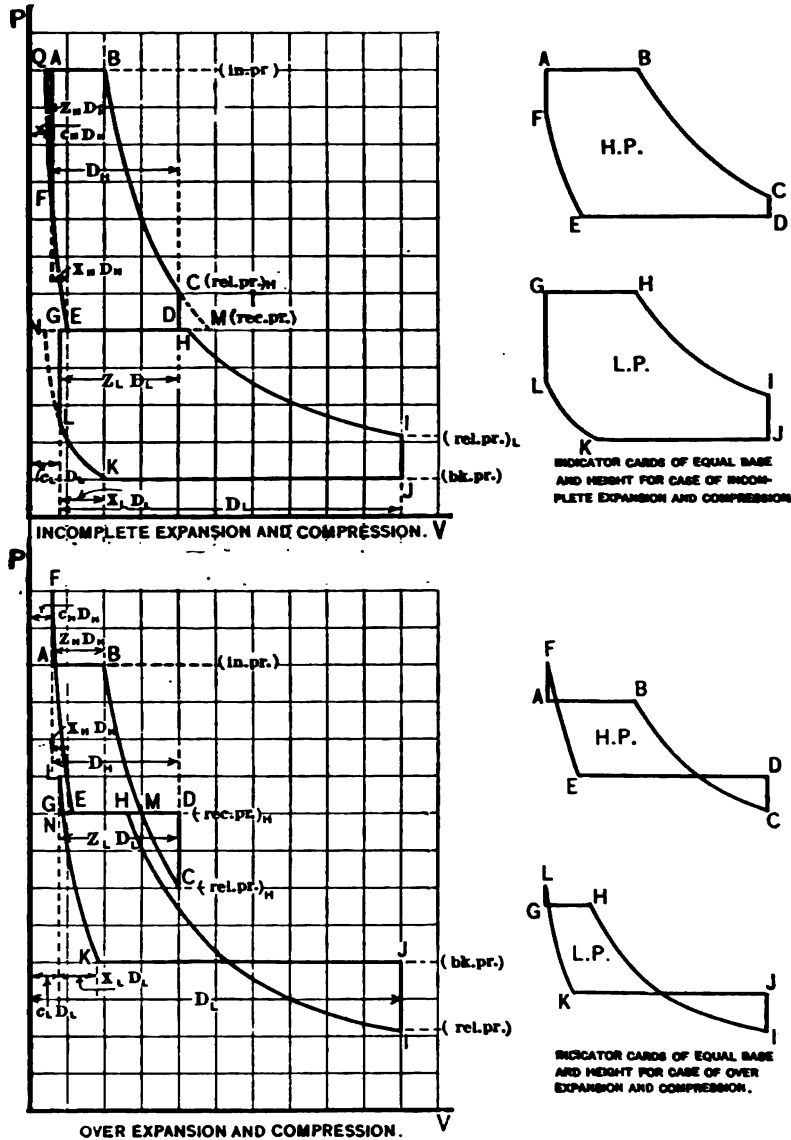


FIG. 72.—Work of Expansive Fluid in Compound Engines with Infinite Receiver, with Clearance, Cycle XI Logarithmic, and Cycle XII Exponential Expansion, and Compression.

$$W_L = P_s V_s \left(1 + \log_e \frac{V_c}{V_s} \right) - P_f V_f \left(1 + \log_e \frac{V_c}{V_f} \right) - (P_s - P_f) V_s - P_f (V_s - V_c). \quad (327)$$

$$W = W_H + W_L = P_b V_b \log_e \frac{V_c}{V_b} + P_h V_h \log_e \frac{V_i}{V_h} - P_c V_c \log_e \frac{V_c}{V_f} - P_k V_k \log_e \frac{V_k}{V_i} \\ + P_b V_b - P_f V_f - P_a V_a + P_f V_f - P_d V_d + P_c V_c \\ + P_h V_h - P_i V_i - P_g V_g + P_i V_i - P_j V_j + P_k V_k.$$

The second condition is

$$P_b V_b - P_c V_c = P_h V_h - P_k V_k; \text{ or } P_b V_b + P_k V_k = P_c V_c + P_h V_h,$$

or
$$P_b V_b + P_k V_k + P_c V_c + P_h V_h = 2(P_b V_b + P_k V_k). \quad (328)$$

Substituting,
$$W = P_b V_b \log_e \frac{V_c}{V_b} + P_h V_h \log_e \frac{V_i}{V_h} - P_c V_c \log_e \frac{V_c}{V_f} - P_k V_k \log_e \frac{V_k}{V_i} \left. \vphantom{W = P_b V_b \log_e \frac{V_c}{V_b} + P_h V_h \log_e \frac{V_i}{V_h} - P_c V_c \log_e \frac{V_c}{V_f} - P_k V_k \log_e \frac{V_k}{V_i}} \right\} (329) \\ + 2(P_b V_b + P_k V_k) - P_a V_a - P_g V_g - P_d V_d - P_j V_j.$$

This expression, Eq. (329) contains, however, the receiver pressure which is related to the release pressure by

$$(\text{rec.pr.}) = P_g = P_c = P_d = P_h = P_i \frac{V_i}{V_h} = (\text{rel.pr.})_L \frac{\text{L.P. max.vol.}}{\text{L.P. cut-off vol.}}$$

Introducing this

$$W = P_b V_b \log_e \frac{V_c}{V_b} + P_i V_i \log_e \frac{V_i}{V_h} - P_i \frac{V_i}{V_h} V_c \log_e \frac{V_c}{V_f} - P_k V_k \log_e \frac{V_k}{V_i} \left. \vphantom{W = P_b V_b \log_e \frac{V_c}{V_b} + P_i V_i \log_e \frac{V_i}{V_h} - P_i \frac{V_i}{V_h} V_c \log_e \frac{V_c}{V_f} - P_k V_k \log_e \frac{V_k}{V_i}} \right\} (330) \\ + 2(P_b V_b + P_k V_k) - P_a V_a - P_i \frac{V_i}{V_h} V_g - P_i \frac{V_i}{V_h} V_d - P_j V_j$$

In addition to the usual symbols the following should be considered in connection with Eq. (329) and (330):

Z = cut-off as fraction of stroke, so that $Z_H D_H$ is the displacement volume up to cut-off;

c = clearance volume divided by displacement, so that $c_H D_H$ is the clearance volume and $(Z_H + c_H) D_H$ is the volume in the high-pressure cylinder at cut-off;

X = that fraction of the stroke during which compression is taking place so that $(X_H + c_H) D_H$ is the volume in the high-pressure cylinder when compression begins.

Applying the general symbols to Eq. (329),

$$W = 144 \left\{ \begin{aligned} & (\text{in.pr.})(Z_H + c_H) D_H \log_e \left(\frac{1 + c_H}{Z_H + c_H} \right) \\ & + (\text{rec.pr.})(Z_L + c_L) D_L \log_e \left(\frac{1 + c_L}{Z_L + c_L} \right) \\ & - (\text{rec.pr.})(X_H + c_H) D_H \log_e \left(\frac{X_H + c_H}{c_H} \right) \\ & - (\text{bk.pr.})(X_L + c_L) D_L \log_e \left(\frac{X_L + c_L}{c_L} \right) \\ & + 2(\text{in.pr.})(Z_H + c_H) D_H - (\text{in.pr.}) c_H D_H - (\text{rec.pr.})(1 + c_H) D_H \\ & + 2(\text{bk.pr.})(X_L + c_L) D_L - (\text{rec.pr.}) c_L D_L - (\text{bk.pr.})(1 + c_L) D_L. \end{aligned} \right. \quad (331)$$

This expression gives the work in terms of initial, receiver and back pressures, the valve periods, cut-off and compression, the clearances and cylinder displacements.

Substitution of the symbols in Eq. (330) will give another equivalent expression in terms of the same quantities except that low-pressure cylinder release pressure will take the place of receiver pressure.

It is sometimes more convenient to involve the cylinder ratio and low-pressure displacement and the ratios of expansion instead of cut-offs. This may be done by means of the following:

$$R_C = \frac{D_L}{D_H}, \quad \text{and} \quad R_H = \frac{V_c}{V_b} = \frac{1+c_H}{Z_H+c_H}; \quad R_L = \frac{V_t}{V_h} = \frac{1+c_L}{Z_L+c_L}. \quad (332)$$

It should be noted here that the *ratio of expansion* in each cylinder is *no longer the reciprocal of its cut-off*, as was the case when clearance was zero, nor is the whole ratio of expansion equal to the product of the two separate ones because the low-pressure cylinder expansion line is not a continuation of that in the high. Making these substitutions for cylinder and expansion ratios, and combining with Eq. (330) the work becomes:

$$W = \frac{1}{144D_L} \left\{ \begin{aligned} &(\text{in.pr.})(1+c_H) \frac{1}{R_H R_C} \log_e R_H + (\text{rel.pr.})_L (1+c_L) \log_e R_L \\ &- (\text{rel.pr.})_L (X_H+c_H) \frac{R_L}{R_C} \log_e \left(\frac{X_H+c_H}{c_H} \right) - (\text{bk.pr.})(X_L+c_L) \log_e \left(\frac{X_L+c_L}{c_L} \right) \\ &+ 2(\text{in.pr.})(1+c_H) \frac{1}{R_H R_C} - (\text{in.pr.}) \frac{c_H}{R_C} - (\text{rel.pr.})_L \frac{R_L}{R_C} (1+c_H) \\ &+ 2(\text{bk.pr.})(X_L+c_L) - (\text{rel.pr.})_L R_L c_L - (\text{bk.pr.})(1+c_L) \end{aligned} \right\}. \quad (333)$$

This reduces to Eq. (225) of Section 45 by making clearance and compression zero.

From any of the expressions for work, the usual expressions for (m.e.p.) referred to low-pressure cylinder, work per cubic foot supplied, and consumption per hour per I.H.P. can be found; as these are somewhat lengthy they are not derived but merely indicated as follows:

$$(\text{m.e.p. ref. to L.P.}) = \frac{W}{144D_L}. \quad (334)$$

$$\text{Work per cu.ft. supplied} = \frac{W}{D_H \left[(Z_H+c_H) - (X_H+c_H) \left(\frac{\text{rec.pr.}}{\text{in.pr.}} \right) \right]}. \quad (335)$$

Cu.ft. sup. per hr. I.H.P.

$$\begin{aligned} &= \frac{13,750}{(\text{m.e.p. ref. to L.P.})} \left[(Z_H+c_H) - (X_H+c_H) \left(\frac{\text{rec.pr.}}{\text{in.pr.}} \right) \right] \frac{1}{R_C} \\ &= \frac{13,750}{(\text{m.e.p. ref. to L.P.})} \left[(Z_L+c_L) - (X_L+c_L) \left(\frac{\text{bk.pr.}}{\text{rec.pr.}} \right) \right] \left(\frac{\text{rec.pr.}}{\text{in.pr.}} \right). \quad (336) \end{aligned}$$

As the receiver pressure is related to the initial and back pressures and to the relation between the amount taken out of the receiver to that put in, which is a function of the compression as well as the cut-off and cylinder ratio, it is expressed only by a complicated function which may be derived from the equivalence of volumes in the high and low, reduced to equal pressure.

$$P_b V_b - P_s V_s = P_h V_h - P_k V_k, \text{ or } P_h V_h + P_s V_s = P_b V_b + P_k V_k = P_h (V_h + V_s).$$

$$\text{Therefore, } P_h = P_b \frac{V_b}{V_h + V_s} + P_k \frac{V_k}{V_h + V_s}. \text{ Introducing symbols,}$$

$$\begin{aligned} (\text{rec.pr.}) &= (\text{in.pr.}) \frac{(Z_H + c_H) D_H}{(Z_L + c_L) D_L + (X_H + c_H) D_H} + (\text{bk.pr.}) \frac{(X_L + c_L) D_L}{(Z_L + c_L) D_L + (X_H + c_H) D_H}, \\ (\text{rec.pr.}) &= (\text{in.pr.}) \frac{(Z_H + c_H)}{(Z_L + c_L) R_C + (X_H + c_H)} + (\text{bk.pr.}) \frac{(X_L + c_L) R_C}{(Z_L + c_L) R_C + (X_H + c_H)}. \quad (337) \end{aligned}$$

This Eq. (337) gives the receiver pressure in terms of initial and back pressures, the two clearances and compressions, the cylinder ratio and the cut-off in each cylinder.

Proceeding in a similar way, the release pressures can be found in terms of initial data,

$$P_c = P_b \frac{V_c}{V_b}, \text{ or } (\text{rel.pr.})_H = (\text{in.pr.}) \left(\frac{Z_H + c_H}{1 + c_H} \right) (a) = (\text{in.pr.}) \frac{1}{R_H} (b) \quad (338)$$

$$\text{And } P_i = P_h \frac{V_h}{V_i} = P_b \left(\frac{\frac{V_b}{V_i}}{1 + \frac{V_c}{V_h}} \right) + P_k \left(\frac{\frac{V_k}{V_i}}{1 + \frac{V_c}{V_h}} \right), \text{ or}$$

$$\begin{aligned} (\text{rel.pr.})_L &= (\text{in.pr.}) \left\{ \frac{\left[\frac{(Z_H + c_H)}{(1 + c_L) R_C} \right]}{1 + \frac{(X_H + c_H)}{(Z_L + c_L) R_C}} + (\text{bk.pr.}) \frac{\left[\frac{(X_L + c_L)}{(1 + c_L)} \right]}{1 + \frac{(X_H + c_H)}{(Z_L + c_L) R_C}} \right\} (a) \\ &= (\text{in.pr.}) \left\{ \frac{\left[\frac{(1 + c_H)}{(1 + c_L)} \frac{1}{R_H R_C} \right]}{1 + \frac{(X_H + c_H) R_L}{(1 + c_L) R_C}} + (\text{bk.pr.}) \frac{\left[\frac{(X_L + c_L)}{(1 + c_L)} \right]}{1 + \frac{(X_H + c_H) R_L}{(1 + c_L) R_C}} \right\} (b) \quad (339) \end{aligned}$$

These three pressures all reduce to those of Eqs. (229), (230), (231), Section 45, when clearance and compression are zero.

Equal work in both cylinders is, of course, possible, but it may be secured by an almost infinite variety of combinations of clearance, compression and cut-off in the two cylinders for various ratios of expansion; it is, therefore, not worth while setting down the equation of condition to be satisfied, but reference may be had to Eqs. (326) and (327), which must be made equal to each other, and the result combined with the equation of cylinder relations.

There are certain special cases of this cycle for which equations expressing important relations are simpler, and they are for that reason worth investigating. Those that will be examined are

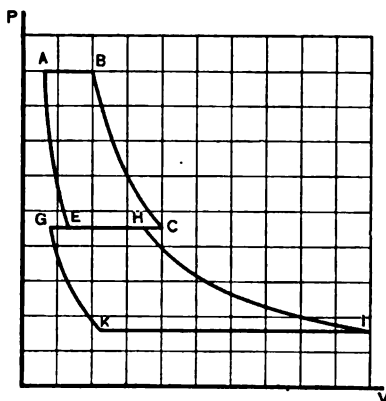


FIG. 73.—Special Case of Cycles XI and XII, Complete Expansion and Compression in both Cylinders, of Compound Engine with Clearance and Infinite Receiver.

(a) Complete expansion and compression in both cylinders, Fig. 73.

(b) Complete expansion in both cylinders with no compression, any clearance, Fig. 74.

(c) Any amount of expansion and compression but equal in both cylinders, equal clearance percentages and a cylinder ratio equal to the square root of the ratio of initial to back pressures, Fig. 75.

Case (a). When both expansion and compression are complete in both cylinders, Fig. 73,

$$W_H = 144(\text{in.pr.})Z_H D_H \log_e \left(\frac{(\text{in.pr.})}{(\text{rec.pr.})} \right), \quad (340)$$

$$W_L = 144(\text{rec.pr.})Z_L D_L \log_e \left(\frac{(\text{rec.pr.})}{(\text{bk.pr.})} \right), \quad (341)$$

but

$$Z_H D_H (\text{in.pr.}) = Z_L D_L (\text{rec.pr.})$$

$$\text{and } \log_e \left(\frac{(\text{in.pr.})}{(\text{rec.pr.})} \right) + \log_e \left(\frac{(\text{rec.pr.})}{(\text{bk.pr.})} \right) = \log_e \left(\frac{(\text{in.pr.})}{(\text{bk.pr.})} \right) = \log_e \left(\frac{(\text{in.pr.})}{(\text{bk.pr.})} \right),$$

$$\text{hence } W = 144(\text{in.pr.})Z_H D_H \log_e \left(\frac{(\text{in.pr.})}{(\text{bk.pr.})} \right). \quad (342)$$

$$(\text{m.e.p. ref. to L.P.}) = (\text{in.pr.}) \frac{Z_H}{R_c} \log_e \left(\frac{(\text{in.pr.})}{(\text{bk.pr.})} \right). \quad (343)$$

$$\text{Work per cu.ft. supplied} = 144(\text{in.pr.}) \log_e \left(\frac{(\text{in.pr.})}{(\text{bk.pr.})} \right). \quad (344)$$

$$\text{Consumption, cu.ft. per hr. per I.H.P.} = \frac{13,750}{(\text{m.e.p. ref. to L.P.}) R_c} \frac{Z_H}{R_c}. \quad (345)$$

Equality of work in high- and low-pressure cylinders is obtained by making $\left(\frac{(\text{in.pr.})}{(\text{rec.pr.})} \right) = \left(\frac{(\text{rec.pr.})}{(\text{bk.pr.})} \right) = \left(\frac{(\text{in.pr.})}{(\text{bk.pr.})} \right)^{1/2}$, or $(\text{rec.pr.}) = [(\text{in.pr.})(\text{bk.pr.})]^{1/2}$. (346)

It is desirable to know that clearances and displacements will permit equal work and complete expansion and compression.

$$(\text{rec.pr.}) = (\text{in.pr.}) \frac{V_b}{V_c} = (\text{in.pr.}) \left(\frac{Z_H + c_H}{1 + c_H} \right) = (\text{bk.pr.}) \frac{V_1}{V_2} = (\text{bk.pr.}) \left(\frac{1 + c_L}{Z_L + c_L} \right),$$

$$\text{hence } \left(\frac{(\text{in.pr.})}{(\text{rec.pr.})} \right) = \frac{1 + c_H}{Z_H + c_H} = \left(\frac{(\text{in.pr.})}{(\text{bk.pr.})} \right)^{1/2}, \quad \left(\frac{(\text{rec.pr.})}{(\text{bk.pr.})} \right) = \frac{1 + c_L}{Z_L + c_L} = \left(\frac{(\text{in.pr.})}{(\text{bk.pr.})} \right)^{1/2},$$

$$\text{or calling } \left(\frac{(\text{in.pr.})}{(\text{bk.pr.})} \right) = R_P, \quad Z_H = \frac{1+c_H-c_H R_P^{1/2}}{R_P^{1/2}}, \text{ and, } Z_L = \frac{1+c_L-c_L R_P^{1/2}}{R_P^{1/2}}. \quad (347)$$

Equating discharge of high and intake of low-pressure cylinders,

$$Z_H D_H R_P^{1/2} = Z_L D_L, \text{ or } \frac{D_L}{D_H} = R_C = \frac{Z_H}{Z_L} R_P^{1/2}.$$

Inserting in this the values just found for Z_H and Z_L ,

$$R_C = \frac{1+c_H-c_H R_P^{1/2}}{1+c_L-c_L R_P^{1/2}} R_P^{1/2}, \quad \dots \quad (348)$$

which is the required relation between cylinder sizes, clearances and ratio of pressures, which, together with cut-offs given in Eq. (347), will give equal work and complete expansion and compression. The compression in the high-pressure cylinder is such that $c_H + X_H = c_H R_P^{1/2}$.

$$X_H = c_H (R_P^{1/2} - 1), \text{ and for L.P. cylinder, } X_L = c_L (R_P^{1/2} - 1). \quad \dots \quad (349)$$

Case (b). With complete expansion and no compression, both cylinders, any clearance, Fig. 74,

$$W_H = 144 D_H \left[(\text{in.pr.}) (Z_H + c_H) \log_e \left(\frac{(\text{in.pr.})}{(\text{rec.pr.})} \right) - c_H [(\text{in.pr.}) - (\text{rec.pr.})] \right] \quad (350)$$

$$W_L = 144 D_L \left[(\text{rec.pr.}) (Z_L + c_L) \log_e \left(\frac{(\text{rec.pr.})}{(\text{bk.pr.})} \right) - c_L [(\text{rec.pr.}) - (\text{bk.pr.})] \right] \quad (351)$$

with the added requirement that the high-pressure discharge volume, \overline{EC} = low pressure admission volume \overline{FH} , or

$$D_H = D_L \left[Z_L + c_L - c_L \left(\frac{(\text{bk.pr.})}{(\text{rec.pr.})} \right) \right], \quad \dots \quad (352)$$

$$\text{and} \quad (\text{rec.pr.}) = (\text{bk.pr.}) \frac{V_t}{V_h} = (\text{bk.pr.}) \frac{1+c_L}{Z_L+c_L}, \quad \dots \quad (353)$$

$$\therefore \left(\frac{(\text{bk.pr.})}{(\text{rec.pr.})} \right) = \frac{Z_L+c_L}{1+c_L}, \text{ which substituted in Eq. (352) and rearranging gives}$$

$$R_C = \frac{1}{Z_L+c_L-c_L \left(\frac{Z_L+c_L}{1+c_L} \right)}, \quad \dots \quad (354)$$

Eq. (354) indicates that for this special case of complete expansion and no compression the cylinder ratio required to give this case is determined entirely by

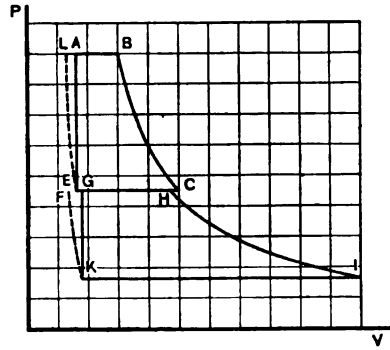


FIG. 74.—Special Case of Cycles XI and XII. Complete Expansion and Zero Compression in both Cylinders of Compound Engine with Clearance and Infinite Receiver.

the L.P. cut-off and clearance. If the cylinder ratio and clearance are fixed, the required cut-off in the L.P. cylinder can be found by solving Eq. (354) for Z_L ,

$$Z_L = \frac{1+c_L}{R_C} - c_L, \quad (355)$$

$$\text{from Eq. (353), (rec.pr.)} = (\text{bk.pr.}) \left[\frac{1+c_L}{\frac{1+c_L}{R_C} - c_L + c_L} \right] = (\text{bk.pr.}) R_C \quad . . . (356)$$

Cut-off in the high-pressure cylinder is determined by clearance, initial pressure and receiver pressure, which in turn depends on low-pressure cut-off and clearance Eq. (353), or may be reduced to cylinder ratio and low-pressure clearance by Eq. (355), as follows

$$\frac{V_c}{V_b} = \frac{1+c_H}{Z_H+c_H} = \left(\frac{(\text{in.pr.})}{(\text{rec.pr.})} \right) = \left(\frac{(\text{in.pr.})}{(\text{bk.pr.})} \right) \frac{Z_L+c_L}{1+c_L},$$

$$\text{hence} \quad Z_H = \frac{(1+c_H)(1+c_L)}{R_P(Z_L+c_L)} - c_H. \quad \text{Eliminate } Z_L \text{ by Eq. (355).}$$

$$\text{Therefore} \quad Z_H = \frac{(1+c_H)(1+c_L)}{R_P \left(\frac{1+c_L}{R_C} - c_L + c_L \right)} - c_H = (1+c_H) \frac{R_C}{R_P} - c_H. \quad . . . (357)$$

Since the high- and low-pressure cut-offs are functions of cylinder and clearance dimensions, and of R_P —the ratio of initial and back pressures—the work of high- and low-pressure cylinders may be expressed entirely in terms of these quantities.

$$W_H = 144 D_H (\text{in.pr.}) \left\{ (1+c_H) \frac{R_C}{R_P} \log_e \left(\frac{R_P}{R_C} \right) - c_H \left(1 - \frac{R_C}{R_P} \right) \right\}. \quad . . . (358)$$

$$W_L = 144 D_H R_C (\text{bk.pr.}) [(1+c_L) \log_e R_C - c_L (R_C - 1)] \quad . . . (359)$$

Hence, total work by addition is

$$W = 144 D_H (\text{in.pr.}) \frac{R_C}{R_P} \left\{ (1+c_H) \log_e \left(\frac{R_P}{R_C} \right) - c_H \left(\frac{R_P}{R_C} - 1 \right) + (1+c_L) \log_e R_C - c_L (R_C - 1) \right\} \quad . . (360)$$

Expressions might be easily written for mean effective pressure referred to the L.P. cylinder, work per cubic foot fluid supplied, and consumption, but will be omitted for brevity. It is important to note, however, the volume of fluid used per cycle is not \overline{AB} , but is \overline{LB} , Fig. 74, and is,

$$(\text{Sup.Vol.}) = D_H \left[(Z_H+c_H) - c_H \frac{(\text{rec.pr.})}{(\text{in.pr.})} \right] = D_H \left[(Z_H+c_H) - c_H \frac{R_C}{R_P} \right]. \quad . . (361)$$

Equality of work, secured by equating Eqs. (358) and (359) gives

$$(1+c_H) \log_e \left(\frac{R_P}{R_C} \right) - c_H \left(\frac{R_P}{R_C} - 1 \right) = (1+c_L) \log_e R_C - c_L (R_C - 1). \quad . . (362)$$

This equation may be satisfied in an infinite number of ways. One case worth noting is that of equal clearances, when it is evident that if $c_H = c_L$, and

$\frac{R_P}{R_C} = R_C, \therefore R_C = \sqrt{R_P}$. This last condition is the same as that which satisfied Case (a) with complete compression.

Case (c), Fig. 75, assumes that $c_H = c_L = c$, $Z_H = Z_L = Z$, $X_H = X_L = X$,

and
$$R_C = \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)^{\frac{1}{2}} = \sqrt{R_P}$$

and corresponds to the first special case considered in Section 45, which lead in the no-clearance case to equality of high- and low-pressure work.

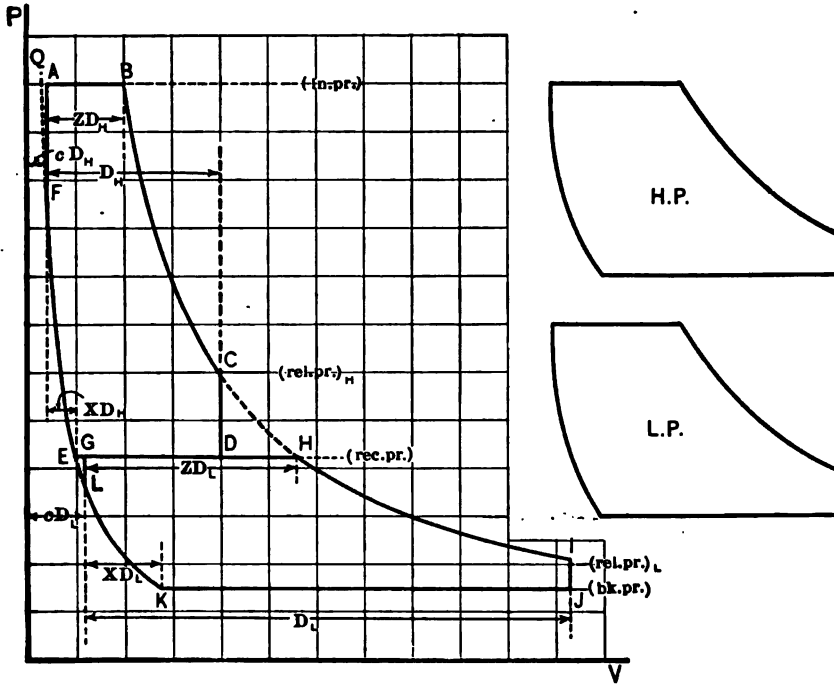


FIG. 75.—Special Case of Cycles XI and XII, Equal Per Cent Clearance in Each Cylinder of Compound Engine with Infinite Receiver and Cylinder Ratio Equal to the Square Root of Initial Divided by Back Pressure.

The assumptions already made are sufficient to determine the receiver pressure. By Eq. (337)

$$\begin{aligned} (\text{rec.pr.}) &= (\text{in.pr.}) \frac{Z+c}{(Z+c) \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)^{\frac{1}{2}} + X+c} + (\text{bk.pr.}) \frac{(X+c) \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)^{\frac{1}{2}}}{(Z+c) \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)^{\frac{1}{2}} + X+c} \\ &= \frac{(Z+c) \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)^{\frac{1}{2}} + X+c}{(Z+c) \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)^{\frac{1}{2}} + X+c} [(\text{in.pr.})(\text{bk.pr.})]^{\frac{1}{2}} = [(\text{in.pr.})(\text{bk.pr.})]^{\frac{1}{2}}. \quad (363) \end{aligned}$$

The work of the high-pressure cylinder may now be evaluated.

$$W_H = 144D_H(\text{in.pr.}) \left\{ Z + c \left[1 + \log_e \left(\frac{1+c}{Z+c} \right) \right] - c \right\} \\ - 144D_H[(\text{in.pr.})(\text{bk.pr.})]^{1/2} \left\{ (X+c) \log_e \left(\frac{X+c}{c} \right) + 1 - X \right\}. \quad (364)$$

The low-pressure cylinder work may be similarly stated,

$$W_L = 144D_L(\text{in.pr.})(\text{bk.pr.})^{1/2} \left\{ Z + c \left[1 + \log_e \left(\frac{1+c}{Z+c} \right) \right] - c \right\} \quad (365) \\ - 144D_L(\text{bk.pr.}) \left\{ (X+c) \log_e \left(\frac{X+c}{c} \right) + 1 - X \right\},$$

$$\text{but } D_L[(\text{in.pr.})(\text{bk.pr.})]^{1/2} = D_H R_c [(\text{in.pr.})(\text{bk.pr.})]^{1/2}$$

$$= D_H \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)^{1/2} [(\text{in.pr.})(\text{bk.pr.})]^{1/2} = D_H(\text{in.pr.}),$$

$$\text{and similarly, } D_L(\text{bk.pr.}) = D_H [(\text{in.pr.})(\text{bk.pr.})]^{1/2}.$$

With these substitutions the value of *low-pressure work*, W_L , Eq. (365), becomes *equal to high pressure work*, Eq. (364), hence the total work

$$W = 2W_H = 2W_L. \quad (366)$$

Example. 1. Method of calculating Diagrams, Fig. 72.

Assumed data:

$$P_c = P_a = P_b = 120 \text{ lbs. per square inch abs.} \quad V_a = V_f = .12 \text{ cu.ft.} \\ P_m = P_n = P_g = P_e = P_d = P_h = 50 \text{ lbs. per square inch abs.} \quad V_b = .4 \text{ cu.ft.} \\ P_k = P_j = 10 \text{ lbs. per square inch abs.} \quad V_c = V_d = .8 \text{ cu.ft.} \\ V_g = V_i = .16 \text{ cu.ft.} \quad V_t = V_j = 2 \text{ cu.ft.} \quad V_e = .2 \text{ cu.ft.} \quad V_k = .4 \text{ cu.ft.} \quad (V_h - V_n) = (V_m - V_e).$$

The above may be expressed in initial pressure, terms of cut-off, and other factors, but as the relation of the lettered points to these terms is shown on the diagram, they will not be given here, since they may readily be found from values of the lettered points.

$$\text{To locate point C: } P_c = P_b \frac{V_b}{V_c} = \frac{120 \times .4}{.8} = 60 \text{ lbs. per sq.in.}$$

$$\text{To locate point F: } P_f = \frac{P_e V_e}{V_f} = \frac{50 \times .2}{.12} = 83.3 \text{ lbs. per sq.in.}$$

$$\text{To locate point Q: } V_e = \frac{P_e V_e}{P_q} = \frac{10}{120} = .083 \text{ cu.ft.}$$

$$\text{To locate point L: } P_t = \frac{P_k V_k}{V_t} = \frac{10 \times .4}{.16} = 25 \text{ lbs. per sq.in.}$$

$$\text{To locate point N: } V_n = \frac{P_k V_k}{P_n} = \frac{4}{50} = .08 \text{ cu.ft.}$$

To locate point H:

$$(V_h - V_n) = (V_m - V_e), \text{ or } V_h = V_m + V_n - V_e = .96 + .08 - .2 = .84 \text{ cu.ft.,}$$

since $P_m V_m = P_b V_b$, $V_m = \frac{48}{50} = .96$ cu.ft.

To locate point I: $P_i = \frac{P_h V_h}{V_i} = \frac{50 \times .84}{2} = 21$ lbs. per sq.in.

Example 2. Find (a) the horse-power, (b) steam used per hour, and (c) receiver and release pressures for a 12- and 18×24-in. engine with infinite receiver, 6 per cent clearance in high-pressure cylinder, and 4 per cent clearance in low-pressure cylinder, when initial pressure is 150 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute, speed 125 R.P.M., cut-off in high-pressure cylinder is $\frac{1}{2}$, low-pressure cut-off is such as to give complete H.P. expansion, and compression is 15 per cent in high and complete in low.

(a) For complete high-pressure expansion the receiver pressure must be equal to the high-pressure release, and to maintain the receiver pressure constant the low-pressure cylinder must take as much steam per stroke as the high-pressure discharges. With initial pressure and cut-off as given, the release pressure for the high-pressure cylinder may be found from the relation (in.pr.) $(c_H + Z_H) = (\text{rel.pr.})_H (c_H + 1)$ or $150 \times (.56) = (\text{rel.pr.})_H (1.06)$, or $(\text{rel.pr.})_H = 79.3$ lbs. Since there is 15 per cent compression in high-pressure cylinder there is exhausted each stroke 85 per cent of its volume. Also since compression in low-pressure cylinder is complete, the low-pressure clearance is full of steam at the receiver pressure at the beginning of the stroke. Hence the low-pressure displacement up to cut-off must equal $.85 D_H$ or L.P. cut-off $= .85 D_H$, divided by cylinder ratio, or $.85 \div 2.25 = .378$. Since compression is complete, the per cent compression may be found from the relation $c_L \times (\text{rec.pr.}) = (c_L + X_L) (\text{bk.pr.})$, or $.04 \times 79.3 = (.04 + X_L) 10$ or $X_L = .28$.

From Eq. (334), (m.e.p.) referred to low-pressure cylinder is obtained by dividing by $144 D_L$, and on substituting the above values it becomes,

$$\begin{aligned} & 150(1+.06) \left(\frac{1}{2 \times 2.25} \right) \log_e 1.9 + 30(1+.04) \log_e 2.64 - 30(.15+.06) \left(\frac{2.64}{2.25} \right) \log_e \left(\frac{.15+.06}{.06} \right) \\ & - 10(.28+.04) \log_e \left(\frac{.28+.04}{.04} \right) + 2 \times 150(1+.06) \left(\frac{1}{1.9 \times 2.25} \right) - 150 \times \frac{.06}{2.25} \\ & - 30 \times \frac{2.64}{2.25} (1+.06) + 2 \times 10(.28+.04) - 30 \times 2.64 \times .04 - 10(1+.04) = 60.5 \text{ lbs. per sq.in.,} \\ & \text{hence} \quad \text{I.H.P.} = 235. \end{aligned}$$

(b) From Eq. (336) by substituting the above values

$$\text{Cu.ft. steam per hour per horse-power} = \frac{13,750}{60.5} \left[\left((.38+.04) - (.28+.04) \frac{10}{79.3} \right) \frac{79.3}{150} \right] = 45.5,$$

or pounds per hour will be 3550.

(c) Release pressure for high-pressure cylinder has been shown to be 79.3 lbs. and may be checked by Eq. (338), as follows:

$$(\text{rel.pr.})_H = 150 \left(\frac{.5+.06}{1.06} \right) = 79.3 \text{ lbs.}$$

Receiver pressure has already been shown to be equal to this quantity and may be checked by Eq. (337)

$$(\text{rec.pr.})_H = \frac{150 \times (.5+.06)}{(.378+.04)2.25 + (.28+.04)} + \frac{10 \times (.28+.04)2.25}{(.378+.04)2.25 + (.28+.04)} = 79.3 \text{ lbs.}$$

Low-pressure release pressure is found from Eq. (339) to be

$$(\text{rel.pr.})_L = 150 \left[\frac{1 + .06}{1 + .04} \times \frac{1}{2 \times 2.25} \right] + 10 \left[\frac{.28 + .04}{1 + .04} \right] = 30 \text{ lbs.}$$

$$1 + \frac{(.15 + .06)2.64}{(1 + .04)2.25}$$

Prob. 1. What will be the horse-power and steam used by the following engine for the data as given?

Engine 20 and 28×36 ins., running at 100 R.P.M., clearance 5 per cent in high pressure, 3 per cent in low. From cards H.P. cut-off = .3, L.P. = .4; H.P. compression, .1; L.P., .2. Gages show (in.pr.) to be 150 lbs., (rec.pr.) 60 lbs., (bk.pr.) 26 ins. Hg (barometer = 30 ins.).

Prob. 2. What must be the cut-offs and the cylinder ratio of an engine to give equal work and complete expansion and compression for 200 lbs. per square inch absolute initial pressure and atmospheric exhaust, if clearance is 5 per cent in the high- and 3 per cent in the low-pressure cylinder? What will be the horse-power for an engine with a low-pressure cylinder 24×36 ins., running at 100 R.P.M. for this case?

Prob. 3. Should there be no compression, how would the results of Prob. 2 be altered?

Prob. 4. What will be the total steam used by engines of Probs. 2 and 3?

Prob. 5. For an 11- and 20×24-in. engine with 5 per cent clearance in each cylinder, $\frac{1}{4}$ cut-off in each cylinder, and 20 per cent compression in each cylinder, what will be the horse-power and the steam consumption when the speed is 125 R.P.M., the initial pressure 150 lbs. per square inch gage, and back pressure at atmosphere?

52. Compound Engine with Infinite Receiver. Exponential Law, with Clearance and Compression, Cycle XII. General Relation between Pressures, Dimensions, and Work. Referring to Fig. 72, of the preceding section, which will represent this cycle by a slight change of slope of the expansion and compression lines, the high-pressure work may be expressed in terms of dimensions, ratios and pressures. Since this must contain receiver pressure as a factor, and since that is not an item of original data, it is convenient first to state receiver pressure in terms of fundamental data:

$$V_m - V_s = V_h - V_r. \text{ But } V_m = V_b \left(\frac{\text{in.pr.}}{\text{rec.pr.}} \right)^{\frac{1}{s}} \text{ and } V_r = V_k \left(\frac{\text{bk.pr.}}{\text{rec.pr.}} \right)^{\frac{1}{s}}$$

$$\text{Whence, } V_b \left(\frac{\text{in.pr.}}{\text{rec.pr.}} \right)^{\frac{1}{s}} + V_k \left(\frac{\text{bk.pr.}}{\text{rec.pr.}} \right)^{\frac{1}{s}} = V_h + V_s, \text{ or}$$

$$(\text{rec.pr.}) = (\text{in.pr.}) \left(\frac{V_b + V_k \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}}}{V_h + V_s} \right)^s, \text{ or in terms of dimensions and pressures,}$$

$$(\text{rec.pr.}) = (\text{in.pr.}) \left(\frac{Z_H + c_H + R_C(X_L + c_L) \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}}}{R_C(Z_L + c_L) + X_H + c_H} \right)^s \dots (367)$$

The high-pressure work may be stated as follows:

$$W_H = 144D_H \left\{ (\text{in.pr.}) \frac{(Z_H + c_H)}{s-1} \left[s - \left(\frac{Z_H + c_H}{1 + c_H} \right)^{s-1} \right] - (\text{rec.pr.})(1 - X_H) \right. \\ \left. - (\text{rec.pr.}) \frac{(X_H + c_H)}{s-1} \left[\left(\frac{X_H + c_H}{c_H} \right)^{s-1} - 1 \right] - (\text{in.pr.})c_H \right\} \quad (368)$$

By substituting values of (rec.pr., from Eq. (367) an equation will be obtained which gives the work, W_H , in terms of dimensions, initial and back pressures.

The work in the low-pressure cylinder will be:

$$W_L = 144D_L \left\{ (\text{rec.pr.}) \frac{Z_L + c_L}{s-1} \left[s - \left(\frac{Z_L + c_L}{1 + c_L} \right)^{s-1} \right] - (\text{bk.pr.})(1 - X_L) \right. \\ \left. - (\text{bk.pr.}) \frac{X_L + c_L}{s-1} \left[\left(\frac{X_L + c_L}{c_L} \right)^{s-1} - 1 \right] - (\text{rec.pr.})c_L \right\} \quad (369)$$

In the same way as stated for W_H an expression may be obtained for the work, W_L , in terms of dimensions and cylinder pressures.

The expression for *total work* need not be written here, as it is simply the sum of Eqs. (368) and (369) in terms of receiver pressure and the exponent of the expansion, s .

The volume of high-pressure fluid supplied per cycle is \overline{QB} , Fig. 72, which may be expressed either in terms of high-pressure or of low-pressure points, thus:

$$(\text{Sup.Vol.}) = D_H \left[Z_H + c_H - (X_H + c_H) \left(\frac{\text{rec.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}} \right] \quad (a) \\ = D_L \left[(Z_L + c_L) \left(\frac{\text{rec.pr.}}{\text{bk.pr.}} \right)^{\frac{1}{s}} - (X_L + c_L) \right] \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}} \quad (b) \quad (370)$$

The following quantities will be indicated, and may be evaluated by substitution from the preceding:

$$(\text{m.e.p. ref. to L.P.}) = \frac{W}{144D_L} = \frac{W}{144D_H R_C} \quad (371)$$

$$\text{Work per cu.ft. fluid supplied} = \frac{W}{(\text{Sup.Vol.})} \quad (372)$$

$$\text{Consumption cu.ft. per hr. per I.H.P.} = \frac{13,750}{(\text{m.e.p. ref. to L.P.})} \frac{(\text{Sup.Vol.})}{D_L} \quad (373)$$

Equal division of work between high- and low-pressure cylinders requires that Eqs. (368) and (369), be made equal. The latter will give an expression showing the required relation between dimensions and initial and final pressures, cut-off and compression in high- and low-pressure cylinders. *In this expression there are so many variables that an infinite number of combinations may be made to give equality of work.*

It is desirable to examine the results of assuming special conditions such as those of the previous section, the most important of which is that of *complete expansion and compression in both cylinders*, which is represented by Fig. 73.

$$W_H = 144(\text{in.pr.})Z_H D_H \frac{s}{s-1} \left[1 - \left(\frac{\text{rec.pr.}}{\text{in.pr.}} \right)^{\frac{s-1}{s}} \right] \quad (374)$$

$$W_L = 144(\text{rec.pr.})Z_L D_L \frac{s}{s-1} \left[1 - \left(\frac{\text{bk.pr.}}{\text{rec.pr.}} \right)^{\frac{s-1}{s}} \right] \quad (375)$$

but $Z_L D_L = Z_H D_H \left(\frac{\text{in.pr.}}{\text{rec.pr.}} \right)^{\frac{1}{s}}$, hence, by substitution and adding W_H and W_L ,

$$W = 144(\text{in.pr.})D_H Z_H \frac{s}{s-1} \left[1 - \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{s-1}{s}} \right] \quad (376)$$

The receiver pressure may be found as follows. In Fig. 73, $\overline{EC} = \overline{GH}$:

$$\overline{EC} = D_H \left[1 + c_H - c_H \left(\frac{\text{in.pr.}}{\text{rec.pr.}} \right)^{\frac{1}{s}} \right],$$

$$\overline{GH} = \overline{KI} \left(\frac{\text{bk.pr.}}{\text{rec.pr.}} \right)^{\frac{1}{s}} = D_L \left[1 + c_L - c_L \left(\frac{\text{rec.pr.}}{\text{bk.pr.}} \right)^{\frac{1}{s}} \right] \left(\frac{\text{bk.pr.}}{\text{rec.pr.}} \right)^{\frac{1}{s}}.$$

$$\text{Equating } 1 + c_H - c_H \left(\frac{\text{in.pr.}}{\text{rec.pr.}} \right)^{\frac{1}{s}} = R_c \left[1 + c_L - c_L \left(\frac{\text{rec.pr.}}{\text{bk.pr.}} \right)^{\frac{1}{s}} \right] \left(\frac{\text{bk.pr.}}{\text{rec.pr.}} \right)^{\frac{1}{s}} \quad (377)$$

When this is solved for receiver pressure it results in an equation of the second degree, which is somewhat cumbersome, and will not be stated here. Eq. (377) is, however, used later to find R_c .

If work is to be equally distributed between high- and low-pressure cylinders, Eq. (374) will equal Eq. (375), hence, for equal division of work,

$$(\text{rec.pr.}) = (\text{in.pr.}) \left[\frac{1 + \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{s-1}{s}}}{2} \right]^{\frac{s}{s-1}} \quad (378)$$

which, if satisfied, will give equality of work in the two cylinders, for this case of perfect compression and expansion.

In Fig. 76 is given a set of curves for use in determining the value of the ratio of (rec.pr.) to (in.pr.) as expressed by Eq. (378).

When (rec.pr.) has been found by Eq. (482) it is possible by means of (481) and the clearances to find R_c . The events of the stroke must have the following values to maintain complete and perfect compression and expansion.

$$Z_H = (1 + c_H) \left(\frac{\text{rec.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}} - c_H; \quad (379) \quad Z_L = (1 + c_L) \left(\frac{\text{bk.pr.}}{\text{rec.pr.}} \right)^{\frac{1}{s}} - c_L; \quad (380)$$

$$X_H = c_H \left[\left(\frac{\text{in.pr.}}{\text{rec.pr.}} \right)^{\frac{1}{s}} - 1 \right]; \quad (381) \quad X_L = c_L \left[\left(\frac{\text{rec.pr.}}{\text{bk.pr.}} \right)^{\frac{1}{s}} - 1 \right] \quad (382)$$

Example. Find (a) the horse-power, (b) compressed air used per hour, and (c) receiver and release pressures for a 12- and 18×24-in. engine with infinite receiver, 6 per cent clearance in the high-pressure cylinder, and 4 per cent in the low-pressure cylinder, when initial pressure is 150 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute, speed 125 R.P.M., cut-off in high-pressure cylinder $\frac{1}{2}$, low-pressure cut-off such as to give complete expansion in high-pressure cylinder, compression in high-pressure cylinder 15 per cent, and complete in low. Expansion such that $s = 1.4$.

(a) As in example of Sec. 51, receiver pressure equals high-pressure release pressure, and low-pressure volume at cut-off must equal volume of air exhausted from high pressure.

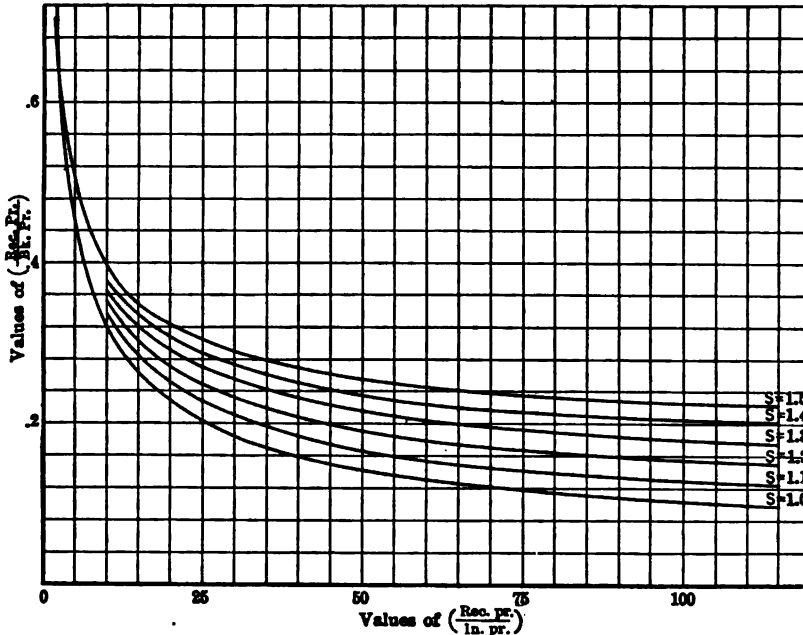


FIG. 76.—Curves to Show Receiver Pressure to Give Equal Work Distribution when Expansion and Compression are Complete in both Cylinders of the Compound Engine with Infinite Receiver, with Clearance when Expansion and Compression are not Logarithmic.

Release pressure may be found from relation $(in.pr.)(c_H + Z_H)^s = (rel.pr.)_H(c_H + D_H)^s$, or $150(.06 + .5)^{1.4} = (rel.pr.)_H(.06 + 1)^{1.4}$, or $(rel.pr.) = 60$ lbs. As in the previous example, the low-pressure cut-off is .38, and the low-pressure compression may be found from the relation $c_L^s(rec.pr.) = (c_L + X_L)^s(bk.pr.)$, or $(.04)^{1.4} \times 60 = (.04 + X_L)^{1.4}$ (10), or $X_L = .09$.

From equations (368) and (369) with the proper values substituted for $(rec.pr.)$, and dividing by $144D_H R_C$, the following expression for $(m.e.p.)$ results in the present case:

$$\frac{150}{2.25} \left\{ \frac{.5 + .06}{.4} \left[1.4 - \left(\frac{.5 + .06}{1 + .06} \right)^{1.4} \right] - .06 \right\} - \frac{150}{2.25} \left\{ \frac{.5 + .06 + 2.25(.15 + .06) \left(\frac{10}{150} \right)^{.71}}{2.25(.38 + .04) + .15 + .06} \right\}^{1.4}$$

$$\left\{ \frac{.15 + .06}{.4} \left[\left(\frac{.15 + .06}{.06} \right)^{1.4} - 1 \right] + 1 - .15 \right\} + 150 \left\{ \frac{.5 + .06 + 2.25(.15 + .04) \left(\frac{10}{150} \right)^{.71}}{2.25(.38 + .04) + .15 + .06} \right\}^{1.4}$$

$$\left\{ \frac{.38+.04}{.4} \left[1.4 - \left(\frac{.38+.04}{1+.04} \right)^{1.4} \right] - .04 \right\} - .10 \left\{ \frac{.09+.04}{.4} \left[\left(\frac{.09+.04}{.04} \right)^{1.4} - 1 \right] + 1 - .09 \right\} = 55 \text{ lbs. per sq.in.,}$$

hence the horse-power is 214.

(b) From Eq. (373) with proper values substituted,

$$\text{Cu.ft. per I.H.P. hr.} = \frac{13,750}{55} \times \left[(.38+.04) \left(\frac{60}{10} \right)^{.71} - (.09+.04) \left(\frac{10}{130} \right)^{.71} \right] = 50,$$

or total air per hour will be $50 \times 214 = 10,700$ cu.ft.

(c) Release for the high-pressure cylinder has already been given as 60 lbs. and the receiver pressure the same. The latter quantity may be checked by equation (367) and will be found to be the same. The low-pressure release pressure may be found from the relation $(\text{rec.pr.})(Z_L + c_L)^{1.4} = (\text{rel.pr.})_L (1 + c_L)^{1.4}$, which on proper substitution

$$\text{gives} \quad (\text{rel.pr.})_L = 60 \left(\frac{.38+.04}{1.04} \right)^{\frac{1}{1.4}} = 27 \text{ lbs. per sq. inch.}$$

Prob. 1. What will be the horse-power and steam used per hour by an 18- and 24×30-in. engine with 5 per cent clearance in each cylinder and with infinite receiver running on 100 lbs. per square inch gage initial pressure, and 5 lbs. per square inch absolute back pressure, when the speed is 100 R.P.M. and the cut-off in high-pressure cylinder is $\frac{1}{2}$ and in low $\frac{1}{10}$?

NOTE: $s=1.3$ and $\delta=.2$.

Prob. 2. What must be the receiver pressure for equal work distribution when the initial pressure has the following values for a fixed back pressure of 10 lbs. per square inch absolute? 200, 175, 150, 125, 100, and 75 lbs. per square inch gage?

Prob. 3. For the case of 150 lbs. per square inch gage initial pressure and 14 lbs. per square inch absolute back pressure, what will be the required high-pressure cylinder size for an air engine with a low-pressure cylinder 18×24 ins., to give equality of work, clearance in both cylinders being 5 per cent?

Prob. 4. What will be the horse-power and air consumption of the above engine when running at a speed of 150 R.P.M., under the conditions of perfect expansion and compression?

53. Compound Engine with Finite Receiver. Logarithmic Law, with Clearance and Compression, Cycle XIII. General Relations between Pressures, Dimensions, and Work when H.P. Exhaust and L.P. Admission are Independent. As this cycle, Fig. 77, is made up of expansion and compression lines referred to the different origins together with constant pressure, and constant volume lines, the work for high- and low-pressure cylinders and for the cycle can be set down at once. These should be combined, however, with the relation noted for the case of infinite clearance which might be termed the condition for a steady state:

$$[(PV) \text{ on H.P. expansion line}] - [(PV) \text{ on H.P. compression line}]$$

$$[(PV) \text{ on L.P. expansion line} - PV \text{ on L.P. compression line}],$$

or,

$$P_b V_b - P_c V_c = P_h V_h - P_k V_k \quad (383)$$

Besides this there is a relation between H.P. exhaust and L.P. admission pressures, corresponding to the equality that existed for the infinite receiver, that may be set down as follows:

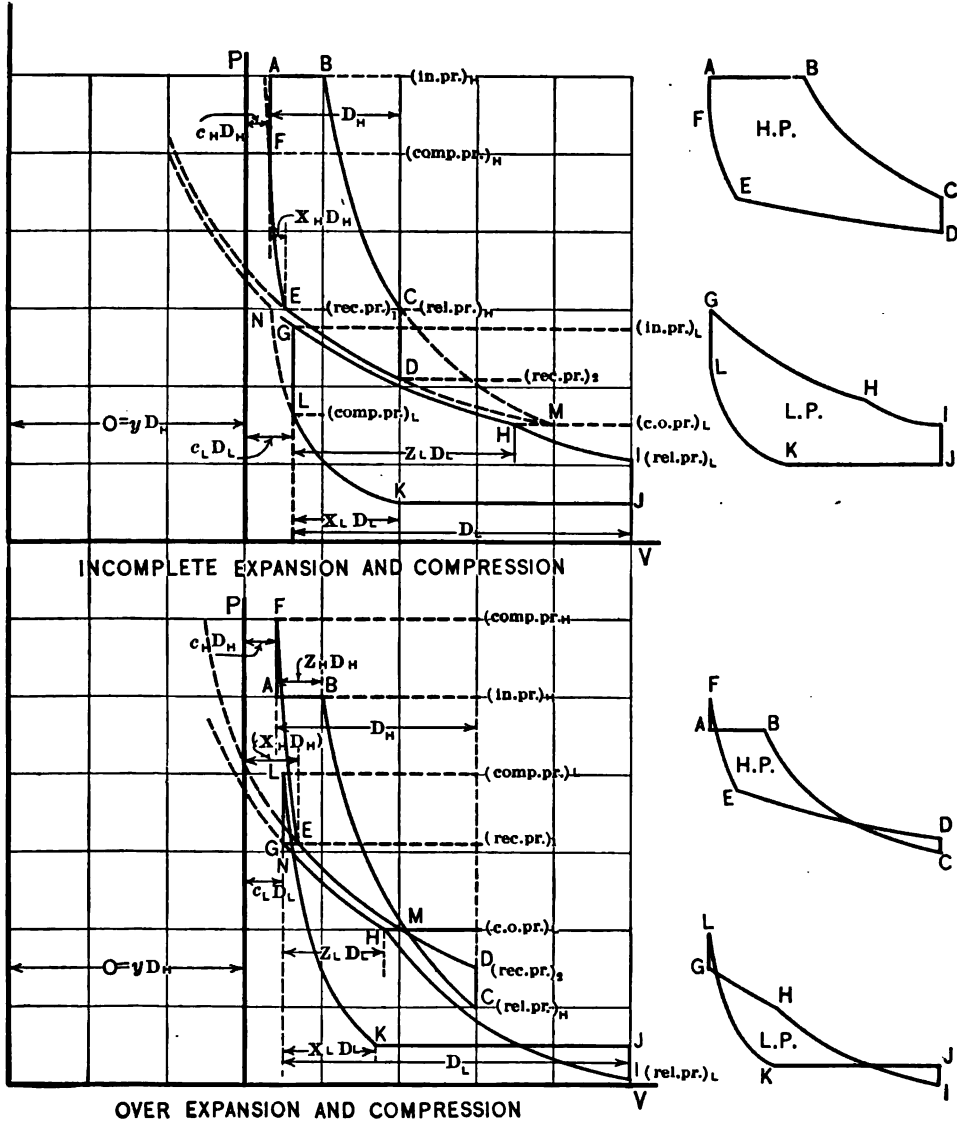


FIG. 77.—Work of Expansive Fluid in Compound Engine with Finite Receiver and with Clearance. Cycle XIII, Logarithmic Cycle XIV, Exponential Expansion, and Compression.

$$P_m V_m = P_b V_b, \text{ and } P_m (V_m + O) = P_d (V_d + O); \text{ therefore}$$

$$P_m V_m = P_b V_b = P_d (V_d + O) - P_m O, \text{ and } P_m = P_h; \therefore P_d = \frac{P_b V_b + P_h O}{V_d + O}. \quad (384)$$

Also $P_n V_n = P_k V_k$, and $P_n(V_n + O) = P_g(V_g + O)$; therefore

$$P_n V_n = P_k V_k = P_g(V_g + O) - P_n O, \text{ and } P_n = P_g, \therefore P_g = \frac{P_k V_k + P_n O}{V_g + O}. \quad (385)$$

These two expressions for the pressure at D and at G are not available in their present form, since they involve two unknown pressures—those at H and E , but two other equations of relation can be set down from which four equations, the four unknown pressures P_e , P_d , P_g and P_h , can be found. These other equations are

$$P_d(V_d + O) = P_e(V_e + O), \text{ or } P_d = P_e \left(\frac{V_e + O}{V_d + O} \right), \quad \dots \quad (386)$$

and $P_g(V_g + O) = P_h(V_h + O), \text{ or } P_g = P_h \left(\frac{V_h + O}{V_g + O} \right). \quad \dots \quad (387)$

Equating (384) to (386), there results $P_e = \frac{P_b V_b + P_h O}{V_e + O},$

Equating (385) to (387) $P_e = \frac{P_h(V_h + O) - P_k V_k}{O}.$

Hence $P_e = \frac{P_b V_b + P_h O}{V_e + O} = \frac{P_h(V_h + O) - P_k V_k}{O}.$

Therefore $P_h = \frac{P_b V_b O + P_k V_k (V_e + O)}{V_h(V_e + O) + V_e O}. \quad (a)$

Substitution will give

$$P_e = \frac{P_b V_b (V_h + O) + P_k V_k O}{(V_e + O)(V_h + O) - O^2} \quad (b) \quad \dots \quad (388)$$

$$P_g = \left[\frac{P_b V_b O + P_k V_k (V_e + O)}{V_h(V_e + O) + V_e O} \right] \left(\frac{V_h + O}{V_g + O} \right) \quad (c)$$

$$P_d = \left[\frac{P_b V_b (V_h + O) + P_k V_k O}{(V_e + O)(V_h + O) - O^2} \right] \left(\frac{V_e + O}{V_d + O} \right) \quad (d)$$

It will be found that the use of these pressures is equivalent to the application of the equation of condition given in Eq. (383), for substitution of them reduces to an identity, therefore the work of the two cylinders can be set down by inspection in terms of point pressures and volumes and the above pressures substituted. The result will be the work in terms of the pressures and cylinder dimensions. Thus

$$W_H = P_b V_b \left(1 + \log_e \frac{V_e}{V_b} \right) - P_f V_f \left(1 + \log_e \frac{V_e}{V_f} \right) - P_e V_e \log_e \left(\frac{V_d + O}{V_e + O} \right) - (P_a - P_f) V_a.$$

Therefore by substitution,

$$W_H = P_b V_b \left(1 + \log_e \frac{V_e}{V_b} \right) - P_a V_a - \left[\frac{P_b V_b (V_d + O) + P_k V_k O}{(V_e + O)(V_h + O) - O^2} \right] V_e \log_e \left(\frac{V_e}{V_a} \right) \left(\frac{V_d + O}{V_e + O} \right) \quad \dots \quad (389)$$

$$\begin{aligned}
W_L = & P_i V_i \left(1 + \log_e \frac{V_h + O}{V_i + O} \right) + P_h V_h \log_e \frac{V_i}{V_h} - P_i V_i \left(1 + \log_e \frac{V_k}{V_i} \right) - (P_i - P_i) V_i \\
= & \left[\left(\frac{P_h V_h O + P_k V_k (V_i + O)}{V_h (V_i + O) + V_i O} \right) \left(\frac{V_h + O}{V_i + O} \right) \right] V_i \log_e \left(\frac{V_h + O}{V_i + O} \right) \\
& + \left[\frac{P_h V_h O + P_k V_k (V_i + O)}{V_h (V_i + O) + V_i O} \right] V_h \log_e \frac{V_i}{V_h} \\
& - P_k V_k \log_e \frac{V_k}{V_i} - P_i (V_i - V_k) \quad \dots (390)
\end{aligned}$$

An expression for the total work, W_2 may be obtained by adding Eqs. (389) and (390).

While such an equation for the cyclic work will be in terms of initial data, it is not of very much value by reason of its complex form, and will therefore be omitted here. A still more involved equation for W will be obtained by substitution of the usual symbols, but *such equations as this are almost, if not quite, useless in the solution of problems requiring numerical answers in engine design, or in estimation of engine performance, and this fact justifies the conclusion that in cases of finite receivers graphic methods should be used rather than the analytic for design work. When estimates of power of a given engine are needed, this graphic work is itself seldom justifiable, as results of sufficient accuracy for all practical engine operation problems can be obtained by using the formulas derived for infinite receiver when reasonably large, and zero receivers when small and the pistons move together. It might also be possible to derive an expression for work with an equivalent constant-receiver pressure, that would give the same total work and approximately the same work division, but this case so seldom arises that it is also omitted here.*

Inspection of the work equations makes it clear that any attempt to find equations of condition for equal division of work for the general case must be hopeless. It is, however, worth while to do this for one special case, that of complete expansion and compression in both stages, yielding the diagram Fig. 78. This is of value in drawing general conclusions on the influence of receiver size by comparing with the similar case for the infinite receiver.

By referring to Fig. 78, it will be seen by inspection that cylinder sizes, clearances and events of the stroke must have particular relative values in order to give the condition assumed, i.e., complete expansion and compression. It is, therefore, desirable to state the expressions for work in terms which may be regarded as fundamental. For this purpose are chosen, initial pressure, (in.pr.), back pressure, (bk.pr.); high-pressure displacement, D_H ; cylinder ratio, R_C ; high-pressure clearance, c_H ; and ratio of receiver volume to high-pressure displacement, y . Call $\left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right) = R_P$.

It will be convenient first to find values of maximum receiver pressure, (rec.pr.)₁, and minimum, (rec.pr.)₂; high-pressure cut-off Z_H , and compression X_H ; low-pressure clearance c_L , cut-off Z_L , and compression, X_L , in terms of

these quantities. Nearly all of these are dependent upon the value of c_L and it will, therefore, be evaluated first.

$$\text{From the points } C \text{ and } J, \text{ Fig. 78, } (\text{rec.pr.})_2 = (\text{bk.pr.}) \frac{R_C(1+c_L)}{(1+c_H)} \dots (391)$$

$$\text{From } A \text{ and } E, \quad (\text{rec.pr.})_1 = (\text{in.pr.}) \frac{c_H}{R_C c_L}, \dots (392)$$

$$\text{and from } E \text{ and } C, \quad \frac{(\text{rec.pr.})_1}{(\text{rec.pr.})_2} = \frac{1+c_H+y}{R_C c_L + y} \dots (393)$$

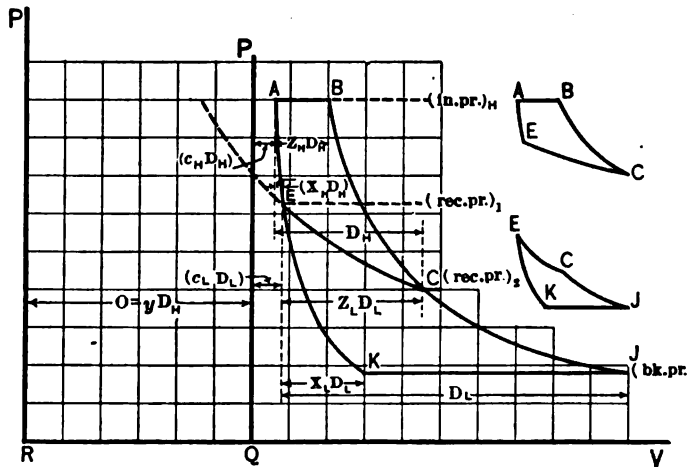


FIG. 78.—Special Case of Cycles XIII and XIV, Complete Expansion and Compression in both Cylinders of Compound Engine with Clearance and Finite Receiver.

Dividing Eq. (392) by Eq. (391) and equating to Eq. (393), multiplying out and arranging with respect to c_L , the relation to be fulfilled in order that complete expansion and compression may be possible is,

$$c_L^2 [R_C^2 (1+c_H+y)] + c_L [R_C^2 (1+c_H+y) - R_C R_P c_H (1+c_H)] - [y R_P c_H (1+c_H)] = 0 \dots (394)$$

$$\text{from which,} \quad c_L = \frac{(m^2 + 4ln)^{1/2} - m}{2l} \dots (395)$$

It is much simpler in numerical calculation to evaluate l , m , and n and insert their values in Eq. (395) than to make substitutions in Eq. (394), which would make a very cumbersome formula. $(\text{rec.pr.})_2$ and $(\text{rec.pr.})_1$ may now be evaluated from Eqs. (391) and (392) by use of the now known value of c_L .

High-pressure cut-off Z_H may be found from the relation of points B and J , Fig. 78,

$$R_P (Z_H + c_H) = R_C (1 + c_L), \quad \text{or} \quad Z_H = \frac{R_C}{R_P} (1 + c_L) - c_H \dots (396)$$

$$\text{Low-pressure cut-off, } Z_L, \text{ from } R_C (Z_L + c_L) = 1 + c_H, \quad \text{or} \quad Z_L = \frac{1 + c_H}{R_C} - c_L \dots (397)$$

High-pressure compression, $X_H = c_L R_C - c_H$ (398)

Low-pressure compression, X_L , by the use of points A and K ,

$$R_C(X_L + c_L) = R_P c_H, \text{ or } X_L = \frac{R_P}{R_C} c_H - c_L. \quad (399)$$

If c_L is regarded as part of the original data, though it is related to R_C , R_P , c_H and y as indicated in Eq. (384), the expressions for high- and low-pressure work may be stated as follows:

$$W_H = 144(\text{in.pr.}) D_H \left\{ \frac{R_C}{R_P} (1 + c_L) \left[1 + \log_e \frac{R_P}{R_C} \frac{(1 + c_H)}{(1 + c_L)} \right] - \frac{R_C}{R_P} \frac{(1 + c_L)}{(1 + c_H)} (1 + c_H + y) \log_e \left(\frac{1 + c_H + y}{R_C c_L + y} \right) - c_H \left[1 + \log_e \frac{c_L R_C}{c_H} \right] \right\}. \quad (400)$$

$$W_L = 144(\text{in.pr.}) D_H \left\{ \frac{R_C}{R_P} \frac{(1 + c_L)}{(1 + c_H)} \left[(1 + c_H + y) \log_e \left(\frac{1 + c_H + y}{R_C c_L + y} \right) + (1 + c_H) \log_e R_C \frac{1 + c_L}{1 + c_H} \right] - \left[\frac{R_C}{R_P} (1 + c_L) - c_H \right] - c_H \log_e \frac{R_P c_H}{R_C c_L} \right\}. \quad (401)$$

Adding these two equations gives the total work of the cycle which, however, may be greatly simplified by substituting the following values:

$$\log_e \frac{R_P (1 + c_H)}{R_C (1 + c_L)} + \log_e R_C \frac{1 + c_L}{1 + c_H} = \log_e R_P, \quad \text{and} \quad \log_e \frac{c_L R_C}{c_H} + \log_e \frac{R_P c_H}{R_C c_L} = \log_e R_P.$$

$$\begin{aligned} \text{Hence} \quad W &= 144(\text{in.pr.}) D_H \left[\frac{R_C}{R_P} (1 + c_L) - c_H \right] \log_e R_P. \\ &= 144(\text{in.pr.}) D_H Z_H \log_e R_P. \quad (402) \end{aligned}$$

From this may be obtained mean effective pressure referred to the low-pressure cylinder, work per cubic foot supplied, and consumption per hour per indicated horse-power, all leading to the same results as were found for the case of complete expansion and compression with infinite receiver (Section 51) and will not be repeated here.

To find the conditions of *equal division of work* between cylinders, equate Eqs. (400) and (401). This will result in an equation which may be simplified to the form,

$$1 - \frac{1 + c_H + y}{1 + c_H} \log_e \frac{1 + c_H + y}{R_C c_L + y} + \log_e \frac{R_P^{1/4} (1 + c_H)}{R_C (1 + c_L)} + \frac{R_P c_H}{R_C (1 + c_L)} \left[\log_e \frac{R_P^{1/4} c_H}{R_C c_L} - 1 \right] = 0. \quad (403)$$

This equation reduces to Eq. (283) of Section 47, when c_H and c_L are put equal to zero. In its present form, however, Eq. (403) is not capable of solution, and it again becomes apparent that for such cases the graphical solution of the problem is most satisfactory.

Example 1. Method of calculating Diagram, Fig. 77. Assumed data:

$$\begin{aligned} P_a = P_b &= 120 \text{ lbs. per square inch abs.} & V_j = V_i &= 2 \text{ cu.ft.} \\ P_m = P_h &= 30 \text{ lbs. per square inch abs.} & V_c = V_d = V_k &= .8 \text{ cu.ft.} \\ P_j &= 10 \text{ lbs. per square inch abs.} & V_g = V_l &= .24 \text{ cu.ft.} \\ V_e &= .2 \text{ cu.ft.} & V_a = V_f &= 12 \text{ cu.ft.} & V_o &= 1.2 \text{ cu.ft.} & V_b &= .4 \text{ cu.ft.} \end{aligned}$$

To locate point C: $P_c = \frac{P_b V_b}{V_c} = \frac{120 \times .4}{.8} = 60 \text{ lbs. per sq.in.}$

To locate point M: $V_m = \frac{P_b V_b}{P_m} = \frac{120 \times .4}{30} = 1.6 \text{ cu.ft.}$

To locate point D:

$$P_d(V_d + O) = P_m(V_m + O), \text{ or } P_d = 30 \frac{(1.6 + 1.2)}{.8 + 1.2} = 42 \text{ lbs. per sq.in.}$$

To locate point E:

$$P_e(V_e + O) = P_m(V_m + O), \text{ or } P_e = \frac{2.8}{.2 + 1.2} \times 30 = 60 \text{ lbs. per sq.in.}$$

To locate point F: $P_f = \frac{P_e V_e}{V_f} = \frac{60 \times .2}{.12} = 100 \text{ lbs. per sq.in.}$

To locate point L: $P_l = \frac{P_k V_k}{V_l} = \frac{10 \times .8}{.24} = 33.3 \text{ lbs. per sq.in.}$

To locate point N: $V_n = \frac{P_k V_k}{P_n} = \frac{10 \times .8}{60} = 13 \text{ cu.ft., since } P_n = P_e.$

To locate point G:

$$P_g(V_g + O) = P_n(V_n + O) \text{ or } P_g = 60 \frac{(.13 + 1.2)}{(.24 + 1.2)} = 55.5 \text{ lbs. per sq.in.}$$

To locate point H: $P_h(V_h + O) = P_g(V_g + O) V_h = \frac{P_g V_g + P_g O - P_h O}{P_h},$

or $V_h = \frac{.24 \times 55.5 + 55.5 \times 1.2 - 30 \times 1.2}{30} = 1.46 \text{ cu.ft.}$

To locate point I: $P_i = \frac{P_h V_h}{V_i} = \frac{1.46 \times 30}{2} = 21.9 \text{ lbs. per sq.in.}$

Prob. 1. Find the work done in the high-pressure cylinder and in the low-pressure cylinder of the following engine under the conditions given:

Engine 14 and 30×28 ins., 100 R.P.M., 5 per cent clearance in each cylinder, high-pressure cut-off $\frac{1}{16}$, low-pressure cut-off $\frac{1}{16}$, high-pressure compression $\frac{1}{16}$, low-pressure compression $\frac{1}{16}$, initial pressure 100 lbs. per square inch gage, back pressure 5 lbs. per square inch absolute, and receiver volume 3 times the high-pressure displacement. Logarithmic expansion.

Prob. 2. The following data are available: initial pressure 200 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute, engine 10×15×22 ins., with 5 per cent clearance in the high- and low-pressure cylinders, speed 100 R.P.M. What will be the cut-offs, and compression percentages to give complete expansion and compression? Logarithmic expansion.

Prob. 3. What will be the work done by the above engine working under these conditions?

Prob. 4. What must be the low-pressure clearance, cut-offs, and compression percentages, to give complete expansion and compression for a similar engine working under the same conditions as those of Prob. 2, but equipped with a receiver twice as large as the high-pressure cylinder?

54. Compound Engine with Finite Receiver, Exponential Law, with Clearance and Compression, Cycle XIV. General Relations between Pressures, Dimensions, and Work when H.P. Exhaust and L.P. Admission are Independent. It cannot be expected that the treatment of this cycle by formulas will give satisfactory results, since even with the logarithmic expansion law, Cycle XIII gave formulas of unmanageable form. For the computation of work done during the cycle, however, and for the purpose of checking pressures and work determined by graphical means, it is desirable to have set down the relations of dimensional proportions, initial and final pressures, and valve adjustments, to the receiver pressures, release pressures and work of the individual cylinders.

The conditions of a steady state, explained previously, require that

$$V_b - V_a = V_h \left(\frac{P_h}{P_b} \right)^{\frac{1}{s}} - V_k \left(\frac{P_k}{P_b} \right)^{\frac{1}{s}}, \quad (404)$$

(Fig. 77), that is, the quantity of fluid passing, per cycle, in the high-pressure cylinder must equal that passing in the low. Expressed in terms of dimensions,

$$\begin{aligned} D_H(c_H + Z_H) - D_H(c_H + X_H) \left[\frac{(\text{rec.pr.})_1}{(\text{in.pr.})} \right]^{\frac{1}{s}} \\ = D_H R_C(c_L + Z_L) \left[\frac{(\text{cut-off pr.})_L}{(\text{in.pr.})} \right]^{\frac{1}{s}} - D_H R_C(c_L + X_L) \left[\frac{(\text{bk.pr.})}{(\text{in.pr.})} \right]^{\frac{1}{s}} \end{aligned}$$

or, rearranging, and using $R_P = \frac{(\text{in.pr.})}{(\text{bk.pr.})}$,

$$\begin{aligned} c_H + Z_H + R_C(c_L + X_L) \left(\frac{1}{R_P} \right)^{\frac{1}{s}} = (c_H + X_H) \left[\frac{(\text{rec.pr.})_1}{(\text{in.pr.})} \right]^{\frac{1}{s}} \\ + R_C(c_L + Z_L) \left[\frac{(\text{cut-off pr.})_L}{(\text{in.pr.})} \right]^{\frac{1}{s}}, \quad . (405) \end{aligned}$$

an equation which contains two unknown pressures $(\text{rec.pr.})_1$ and $(\text{cut off pr.})_L$. To evaluate either, another equation must be found:

$$V_n = V_k \left(\frac{P_k}{P_n} \right)^{\frac{1}{s}}, \quad \text{where } P_n = P_s, \text{ so that } V_n = R_C D_H(c_L + X_L) \left[\frac{(\text{bk pr.})}{(\text{rec.pr.})_1} \right]^{\frac{1}{s}}. \quad (406)$$

Hence

$$P_h = P_n \left(\frac{V_n + O}{V_h + O} \right)^s, \quad \text{or}$$

$$\begin{aligned} (\text{cut-off pr.})_L = (\text{rec.pr.})_1 \left[\frac{y + R_C(c_L + X_L) \left[\frac{(\text{bk.pr.})}{(\text{rec.pr.})_1} \right]^{\frac{1}{s}}}{y + R_C(c_L + Z_L)} \right]^s \\ = \left[\frac{y(\text{rec.pr.})_1^{\frac{1}{s}} + R_C(c_L + X_L)(\text{bk.pr.})^{\frac{1}{s}}}{y + R_C(c_L + Z_L)} \right]^s. \quad (407) \end{aligned}$$

this constitutes a second equation between (cut-off pr.)_L and (rec.pr.)₁, which, used with Eq. (405) makes it possible to solve for the unknown. By substitution in Eq. (405) and rearranging,

$$(\text{rec.pr.})_1 = (\text{bk.pr.}) \left[\frac{R_P^{\frac{1}{s}} (c_H + Z_H) [y + R_C(c_L + Z_L)] + R_C y (c_L + X_L)}{(c_H + X_H) [y + R_C(c_L + Z_L)] + R_C y (c_L + Z_L)} \right]^s. \quad (408)$$

This expression is of great assistance even in the graphical construction of the diagram, as otherwise, with all events known, a long process of trial and error must be gone through with. It should also be noted that when $s=1$ this expression does not become indeterminate and can, therefore, be used to solve for maximum receiver pressure for Cycle XIII, as well as Cycle XIV.

Cut-off pressure of the low-pressure cylinder, which is same as the pressure at H or at M , Fig. 77, is now found most easily by inserting the value found by Eq. (408) for (rec.pr.)₁ in Eq. (407).

Enough information has been gathered now to set down the expressions for work.

$$W_H = 144 D_H \left\{ (\text{in.pr.}) \frac{c_H + Z_H}{s-1} \left[s - \left(\frac{c_H + Z_H}{1 + c_H} \right)^{s-1} \right] - c_H (\text{in.pr.}) \right. \\ \left. - \frac{(c_H + X_H)}{s-1} (\text{rec.pr.})_1 \left[\left(\frac{c_H + X_H}{c_H} \right)^{s-1} - 1 \right] \right. \\ \left. - \frac{(y + c_H + X_H)}{s-1} (\text{rec.pr.})_1 \left[1 - \left(\frac{y + c_H + X_H}{y + c_H + 1} \right)^{s-1} \right] \right\}. \quad (409)$$

$$W_L = 144 D_H \left\{ (\text{cut-off pr.})_L \frac{y + R_C(c_L + Z_L)}{s-1} \left[\left(\frac{y + R_C(c_L + Z_L)}{y + R_C c_L} \right)^{s-1} - 1 \right] \right. \\ \left. + (\text{cut-off pr.})_L \frac{y + R_C(c_L + Z_L)}{s-1} \left[1 - \left(\frac{y + R_C(c_L + Z_L)}{y + R_C(c_L + 1)} \right)^{s-1} \right] \right. \\ \left. - (\text{bk.pr.}) \frac{R_C(c_L + X_L)}{s-1} \left[\left(\frac{c_L + X_L}{c_L} \right)^{s-1} - 1 \right] - (\text{bk.pr.})(1 - X_L) R_C \right\}. \quad (410)$$

Addition of these two Eqs. (409) and (410) gives an expression for the total work W , and equating them gives conditions which must be fulfilled to give equality of work in the high- and low-pressure cylinders. Since these equations so obtained cannot be simplified or put into more useful form, there is no object in inserting them here, but if needed for any purpose they may be easily written. In finding the conditions of equal work, the volumes of (rec.pr.)₁ and (cut-off pr.)_L, must be inserted from Eqs. (407) and (408) in (409) and (410), in order to have the terms in the two equations consist of fundamental data. This, however, increases greatly the complication of the formula.

After finding the total work of the cycle, the mean effective pressure referred to the low pressure is obtained by dividing by $144 \times D_L$.

To assist in finding the work per cubic foot supplied and consumption, and the cubic feet or pounds per hour per I.H.P. it is important to know the volume of fluid supplied per cycle,

$$(\text{Sup. Vol.}) = \overline{QB} = D_H \left[(c_H + Z_H) - (c_H + X_H) \left(\frac{(\text{rec.pr.})_1}{(\text{in.pr.})} \right)^{\frac{1}{s}} \right]. \quad (411)$$

Example. Find the horse-power of and compressed air used by a 12- and 18×24-in. air engine running at 125 R.P.M., with a receiver volume twice as large as the low-pressure cylinder, 6 per cent clearance in the high-pressure cylinder, 4 per cent in the low, when the initial pressure is 150 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute, high-pressure cut-off $\frac{1}{2}$, low-pressure cut-off $\frac{1}{4}$, high-pressure compression 10 per cent, low-pressure compression 30 per cent, and expansion and compression follow the law $PV^{1.4}=c$.

From Eq. (408) (rec.pr.)₁ is found to be as follows when values for this problem are substituted:

$$(\text{rec.pr.})_1 = 10 \left[\frac{15^{.7}(.56)[4.5+2.25(.79)]+4.5 \times 2.25(.34)}{.16[4.5+2.25 \times .79]+4.5 \times 2.25 \times .79} \right]^s = 81.7 \text{ lbs. sq.in. absolute,}$$

and by using this in Eq. (407)

$$(\text{cut-off pr.})_L = \left(\frac{4.5 \times 81.7^{.71} + 2.25 \times .34 \times 10^{.71}}{4.5 + 2.25 \times .79} \right)^s = 53 \text{ lbs. sq.in. absolute.}$$

It is now possible by use of Eqs. (409) and (410) by addition and division by $144D_L$, to obtain (m.e.p.). Substituting the values found above and carrying out the process just mentioned,

$$\begin{aligned} (\text{m.e.p.}) = & \frac{1}{2.25} \left\{ 150 \times \frac{.56}{.4} \times \left[1.4 - \left(\frac{.56}{1.06} \right)^{.4} \right] - .06 \times 150 - \frac{.16}{.4} \times 82 \left[\left(\frac{.16}{.06} \right)^{.4} - 1 \right] \right. \\ & - \frac{4.5 + .16}{.4} \times 82 \left[1 - \left(\frac{4.5 + .16}{4.5 + 1.06} \right)^{.4} \right] + 53 \times \frac{4.5 + 2.25 \times .79}{.4} \left[\left(\frac{4.5 + 2.25 \times .79}{4.5 + 2.25 + .04} \right)^{.4} - 1 \right] \\ & + 53 \times \frac{4.5 + 2.25 \times .79}{.4} \left[1 - \left(\frac{4.5 + 2.25 \times .79}{4.5 + 2.25 \times 1.04} \right)^{.4} \right] - 10 \times \frac{2.25 \times .34}{.4} \left[\left(\frac{.34}{.04} \right)^{.4} - 1 \right] \\ & \left. - 10 \times 2.25 \times .7 \right\} = 51.5 \text{ lbs. sq.in.} \end{aligned}$$

hence the horse-power will be 200.

By means of Eq. (411) the supply volume may be found. This gives upon substitution of the proper values:

$$(\text{Sup. Vol.}) = D_H \left[.56 - 16 \times \left(\frac{82}{150} \right)^{.71} \right] = .46 D_H.$$

$$\text{Cubic feet per hour per I.H.P.} = \frac{13,750}{(\text{m.e.p.})} \times \frac{\text{Sup. Vol.}}{D_L} = \frac{13,750}{51.5} \times \frac{.46}{2.25} = 54.5,$$

hence the total volume of air per hour will be $54.5 \times 200 = 10,900$ cu.ft.

Prob. 1. What will be the receiver pressure and L.P. cut-off pressure for a cross-compound compressed air engine with 5 per cent clearance in each cylinder, running on 100 lbs. per square inch gage initial pressure and atmospheric exhaust, when the high-pressure cut-off is $\frac{1}{2}$, low-pressure $\frac{1}{4}$, high-pressure compression 15 per cent, low 25 per cent, and $s=1.4$? Receiver volume is twice the high-pressure cylinder volume.

Prob. 2. Find the superheated steam per hour necessary to supply a 14- and 21×28-in. engine with 5 per cent clearance in each cylinder and a receiver twice the size of the high-pressure cylinder when the initial pressure is 125 lbs. per square inch gage, back pressure 7 lbs. per square inch absolute, speed 100 R.P.M., high-pressure cut-off $\frac{1}{2}$, low-pressure $\frac{1}{4}$, high-pressure compression 15 per cent, low pressure 40 per cent and $s=1.3$. NOTE: $\delta=.3$.

Prob. 3. If the high-pressure cut-off is changed to $\frac{1}{2}$ without change of any other factor in the engine of Prob. 2, how will the horse-power, total steam per hour, and steam per horse-power per hour be affected? If it is changed to $\frac{2}{3}$?

Prob. 4. A boiler capable of supplying 5000 lbs. of steam per hour at rated load furnishes steam for a 12- and 18×24-in. engine running at 125 R.P.M. with 5 per cent clearance in each cylinder. The receiver is three times as large as the high-pressure cylinder, the initial pressure 150 lbs. per square inch gage, back pressure 5 lbs. per square inch absolute, the low-pressure cut-off fixed at $\frac{1}{2}$ and low-pressure compression fixed at 30 per cent. At what per cent of its capacity will boiler be working for these following cases, when $s=1.2$ for all and 20% of the steam condenses during admission?

- (a) high-pressure cut-off $\frac{1}{2}$, high-pressure compression 80 per cent,
- (b) high-pressure cut-off $\frac{1}{2}$, high-pressure compression 20 per cent,
- (c) high-pressure cut-off $\frac{2}{3}$, high-pressure compression 10 per cent.

NOTE: $\delta = .33$.

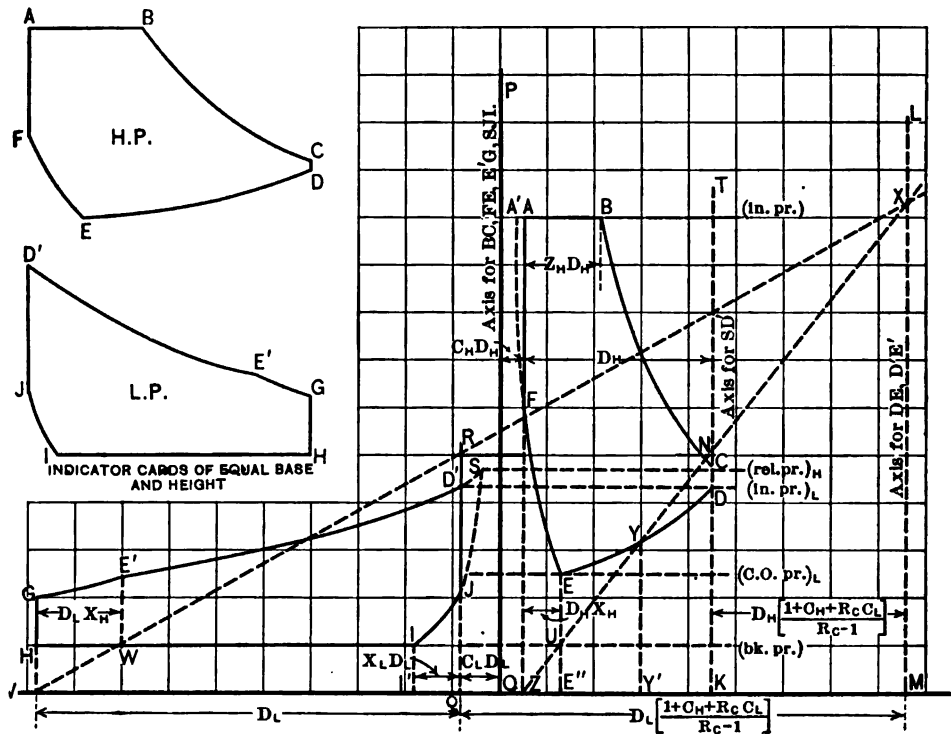


FIG. 79.—Work of Expansion in Compound Engine without Receiver and with Clearance. Cycle XV, Logarithmic Expansion; Cycle XVI, Exponential Expansion, High-pressure Exhaust and Low-pressure Admission Coincident.

55. Compound Engine without Receiver. Logarithmic Law, with Clearance and Compression, Cycle XV. General Relations between Pressures, Dimensions, and Work when H.P. Exhaust and L.P. Admission are Coincident. The graphical construction for this cycle has been described to some extent in connection with the first description of the cycle, given in Section 44 of this chapter, and is represented here by Fig. 79 in more detail.

To show that the expansion from D to E is the same as if volumes were measured from the axis ML , consider a point Y on DE . If the hypothesis is correct,

$$P_d \times \overline{KM} = P_v \times (\overline{KM} + \overline{Y'K}). \quad (412)$$

The true volume when the piston is at the end D of the stroke, is $D_H(1+c_H+R_{ccL})$, and at Y , the true volume is $D_H(1+c_H+R_{ccL})-D_H y+D_L y$, where y is the fraction of the return stroke that has been completed in both cylinders when the point Y has been reached. Then

$$P_d D_H(1+c_H+R_{ccL}) = P_v D_H(1+c_H+R_{ccL}) + P_v D_H(R_c+1)y.$$

Dividing through by (R_c-1) ,

$$P_d \left[\frac{D_H+c_H+R_{ccL}}{R_c-1} \right] = P_v \left[D_H \frac{1+c+R_{ccL}}{R_c-1} + D_H y \right]. \quad (413)$$

This equation may be observed to be similar in form to Eq. (412). Moreover, the last term within the bracket, $D_H y$, is equal to the corresponding term $Y'K$, in Eq. (412), hence,

$$\overline{KM} = D_H \left[\frac{1+c_H+R_{ccL}}{R_c-1} \right]. \quad (414)$$

Similarly, the distance \overline{QM} , or equivalent volume at D' is

$$\overline{QM} = D_L \left[\frac{1+c_H+R_{ccL}}{R_c-1} \right]. \quad (415)$$

Equations for the work done in each cylinder W_H and W_L may be obtained after evaluating the various quantities by methods previously given.

The total work found by adding W_H and W_L thus determined, leads to the following:

$$\begin{aligned} W = 144 D_H (\text{in.pr.}) & \left\{ (Z_H+c_H) \log_e \left(\frac{1+c_H}{Z_H+c_H} \right) \right. \\ & + Z_H + \left[\frac{Z_H+c_H+R_c(X_L+c_L)}{(\text{in.pr.})} \right] \log_e \left[\frac{1+c_H+R_{ccL}+(R_c-1)(1-X_H)}{1+c_H+R_{ccL}} \right] \\ & + \left[\frac{Z_H+c_H+R_c(X_L+c_L)}{(\text{in.pr.})} \right] \left[R_c(1+c_L-X_H) \times \right. \\ & \quad \left. \log_e \left(\frac{1+c_L}{1+c_L-X_H} \right) - (c_H+X_H) \log_e \left(\frac{c_H+X_H}{c_H} \right) \right] \left. \right\} \\ & - 144 D_L (\text{bk.pr.}) \left\{ 1-X_L+(X_L+c_L) \log_e \left(\frac{X_L+c_L}{c_L} \right) \right\}. \quad (416) \end{aligned}$$

This is the general expression for the work of the compound engine without receiver, with clearance and compression, when high-pressure exhaust and low-pressure admission are simultaneous and expansion and compression logarithmic, in terms of fundamental data regarding dimensions and valve periods.

From this the usual expressions for mean effective pressure, work per cubic

foot supplied, and consumption per hour per I.H.P., may be easily written provided the supply volume is known. This is given by

$$\begin{aligned}
 (\text{Sup. Vol.}) = \overline{A'B} &= D_H \left[(Z_H + c_H) - (c_H + X_H) \frac{(\text{cut-off pr.})_L}{(\text{in.pr.})} \right] \\
 &= D_H \left\{ Z_H + c_H - (c_H + X_H) \left[\frac{Z_H + c_H + R_C(X_L + c_L) \frac{(\text{bk.pr.})}{(\text{in.pr.})}}{1 + c_H + R_C c_L + (R_C - 1)(1 - X_H)} \right] \right\}. \quad (417)
 \end{aligned}$$

To find the conditions which must be fulfilled to give equal work in the two cylinders, equate the expressions for the work W_H and W_L , as heretofore shown.

While the expressions thus obtained will be perfectly general for this cycle, yet they are so complicated that their use is very limited, as in the case of some of the general expressions previously derived for other cycles, and they will therefore not be given here.

As in other cycles, it is desirable to investigate a special case, that of complete expansion and compression in both cylinders, Fig. 80. First it is nec-

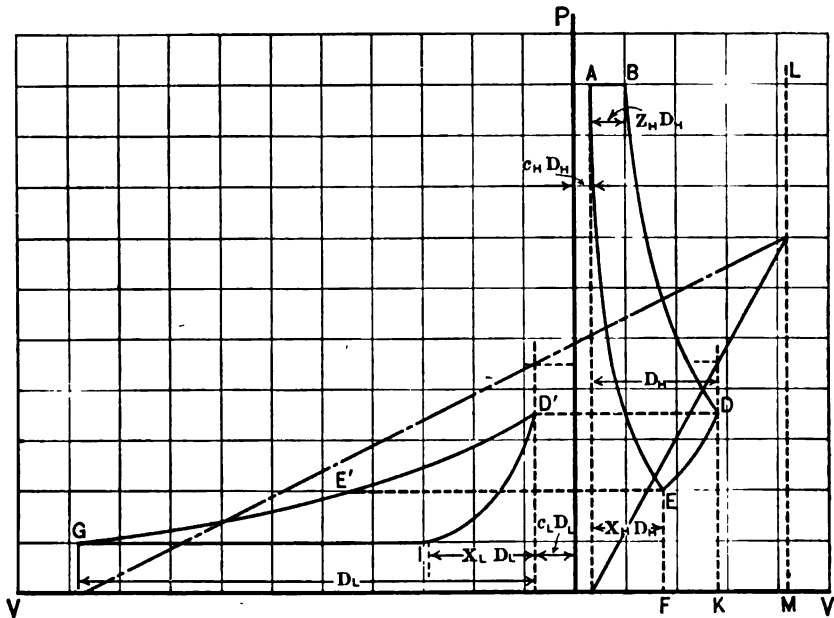


FIG. 80.—Special Case of Cycles XV and XVI. Complete Expansion and Compression in both Cylinders of Compound Engine without Receiver and with Clearance High-Pressure Exhaust and Low-Pressure Admission Coincident.

essary to determine what are regarded as fundamental data in this case, and then to evaluate secondary quantities in terms of these data. The following items are assumed to be known: (in.pr.), (bk.pr.), which is equal to (rel.pr.)_L, R_C , c_H , and c_L , and D_H , which are dimensions, and it is known that the pressure at the end of compression in L.P. is equal to (rel.pr.)_H.

Referring to diagram, the displacements, clearances, and the axis for the

common expansion, ML , can all be laid out, and the location of the points A and G determined.

The points E and E' are at the end of the common expansion within the two cylinders, and at beginning of high-pressure compression and of separate low-pressure expansion, hence $p_e = p_{e'}$.

$$\text{From the points } A \text{ and } E: p_e = (\text{in.pr.}) \frac{c_H}{X_H + c_H}.$$

$$\text{From the points } G \text{ and } E': p_{e'} = p_e = (\text{bk.pr.}) \frac{1 + c_L}{1 + c_L - X_H},$$

$$\text{equating,} \quad (\text{in.pr.}) \frac{c_H}{X_H + c_H} = (\text{bk.pr.}) \frac{1 + c_L}{1 + c_L - X_H},$$

$$\text{noting that} \quad R_P = \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right), \quad X_H = c_H \frac{(1 + c_L)(R_P - 1)}{1 + c_L + c_H R_P} \quad \dots \quad (418)$$

Substituting the value of X_H in either of the expressions for p_e , which is the low-pressure cut-off pressure,

$$(\text{cut-off pr.})_L = p_e = (\text{bk.pr.}) \left[\frac{1 + c_L + R_P c_H}{1 + c_L + c_H} \right] = p_{e'} \quad \dots \quad (419)$$

It may be noted here that the cylinder ratio does not enter into this, but only clearances and pressures. In the no-clearance case, it may be remembered that the point E or E' was not present, as it coincided with G .

Next, to find the high-pressure release pressure, p_d , by means of points E and D , and their relation to the axis ML , Fig. 80.

$$\begin{aligned} (\text{rel.pr.})_H = p_d = p_e \frac{FM}{KM} &= (\text{bk.pr.}) \left[\frac{1 + c_L + c_H R_C}{1 + c_L + c_H} \right] \left[\frac{1 - X_H + \frac{1 + c_H + R_C c_L}{R_C - 1}}{\frac{1 + c_H + R_C c_L}{R_C - 1}} \right] \\ &= (\text{bk.pr.}) \frac{R_C(1 + c_L) + R_P c_H}{1 + c_H + R_C c_L} \quad \dots \quad (420) \end{aligned}$$

Knowing the release pressure of the high-pressure cylinder, it is possible to find the high-pressure cut-off and compression necessary to give the required performance.

$$Z_H = (1 + c_H) \frac{(\text{rel.pr.})_H}{(\text{in.pr.})} - c_H = \frac{(1 + c_H)}{R_P} \left[\frac{R_C(1 + c_L) + R_P c_H}{1 + c_H + R_C c_L} \right] - c_H, \quad \dots \quad (421)$$

$$X_L = c_L \frac{(\text{rel.pr.})_H}{(\text{bk.pr.})} - c_L = c_L \left[\frac{R_C(1 + c_L) + R_P c_H}{1 + c_H + R_C c_L} - 1 \right] \quad \dots \quad (422)$$

The work of the two cylinders is as follows:

$$\begin{aligned} W_H &= 144 D_H (\text{in.pr.}) \left\{ \frac{(1 + c_H)}{R_P} \frac{R_C(1 + c_L) + R_P c_H}{1 + c_H + R_C c_L} \left[1 + \log_e \frac{R_P(1 + c_H + R_C c_L)}{R_C(1 + c_L) + R_P c_L} \right] - c_H \right. \\ &\quad - \left[\frac{R_C(1 + c_L) + R_P c_H}{R_P(R_C - 1)} \right] \log_e \left[\frac{R_C(1 + c_L) + R_P c_H}{1 + c_H + R_C c_L} \right] \left[\frac{1 + c_L + c_H}{1 + c_L + R_P c_H} \right] \\ &\quad \left. - c_H \log_e \left[\frac{R_P(1 + c_L + c_H)}{1 + c_L + R_P c_H} \right] \right\} \quad \dots \quad (423) \end{aligned}$$

$$\begin{aligned}
 W_L = 144 D_L (\text{bk.pr.}) \left\{ \left[\frac{R_C(1+c_L)+R_{PC_H}}{(R_C-1)} \right] \log_e \left[\frac{R_C(1+c_L)+R_{PC_H}}{1+c_H+R_{CC_L}} \right] \left[\frac{1+c_L+c_H}{1+c_L+R_{PC_H}} \right] \right. \\
 + (1+c_L) \log_e \left[\frac{1+c_L+R_{PC_H}}{1+c_L+c_H} \right] - c_L \left[\frac{R_C(1+c_L)+R_{PC_H}}{1+c_H+R_{CC_L}} \right] \log_e \left[\frac{R_C(1+c_L)+R_{PC_H}}{1+c_H+R_{CC_L}} \right] \\
 \left. - 1 + c_L \left[\frac{R_C(1+c_L)+R_{PC_H}}{1+c_H+R_{CC_L}} - 1 \right] \right\}. \quad \dots \dots \dots 424)
 \end{aligned}$$

These expressions, when added and simplified, give the following for total work per cycle,

$$W = 144 D_L (\text{bk.pr.}) \left\{ 1 - c_L \left[\frac{R_C(1+c_L)+R_{PC_H}}{1+c_H+R_{CC_L}} - 1 \right] \right\} \log_e R_P, \quad \dots \quad (425)$$

in which of course $D_H R_C$ may be used instead of D_L and $\frac{(\text{in.pr.})}{R_P}$ instead of (bk.pr.) and then $W = 144 D_H Z_H (\text{in.pr.}) \log_e R_P, \quad \dots \quad (426)$

Z_H having the value of Eq. (421).

Equality of work in the high- and low-pressure cylinders results, if W_H Eq. (423) equals W_L , Eq. (424), or if $2W_H = W$, or $2W_L = W$, all of which lead to equivalent expressions. Simplification of these expressions, however, does not lead to any direct solution, and hence the equations will not be given here.

Example. Find (a) the horse-power and (b) steam used per hour for a 12- and 18×24-in. tandem compound engine with no receiver, 6 per cent clearance in the high-pressure cylinder, and 4 per cent in the low, when the initial pressure is 150 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute, speed 125 R.P.M., high-pressure cut-off $\frac{1}{2}$, high-pressure compression 15 per cent and low-pressure compression is complete.

(a) Since the low-pressure compression is complete, the pressure at end of compression must be equal to the release pressure of the high. This latter quantity may be found from the relation $(\text{in.pr.})(Z_H+c_H) = (\text{rel.pr.})_H(1+c_H)$, or

$$(\text{rel.pr.})_H = 150 \frac{(.56)}{1.06} = 79.3 \text{ lbs. per sq.in. absolute.}$$

Low-pressure compression may be found from the relation $(\text{rel.pr.})_H(c_L) = (\text{bk.pr.})(c_L+X_L)$, or $X_L = .28$. (m.e.p.) may be found from Eq. (540) divided by $144 D_L$, which on substitution gives

$$\begin{aligned}
 \frac{150}{2.25} \left\{ (.5+.06) \log_e \frac{1.06}{.56} \right. \\
 + .5 + \left[.5+.06+2.25(.28+.04) \frac{10}{150} \right] \log_e \left[\frac{1+.06+2.25 \times .04+1.25 \times .85}{1+.06+2.25 \times .04} \right] \\
 + \left[\frac{.5+.06+2.25(.28+.04) \frac{10}{150}}{1+.06+2.25 \times .04+1.75+.85} \right] 2.25(1.04-.15) \log_e \frac{1+.04}{1+.04-.15} \\
 \left. - (.06+.15) \log_e \frac{.06+.15}{.06} \right\} - 10 \left\{ 1.28 + (.28+.04) \log_e \frac{.28+.04}{.04} \right\} = 69.7 \text{ lbs. sq. in.}
 \end{aligned}$$

and the horse-power will be 271.

(b) Since the consumption in cubic feet per hour per horse-power is equal to

$$\frac{13,750}{(\text{m.e.p.})} \times \frac{\text{Sup. Vol.}}{D_L},$$

and supply volume is given by Eq. (417), this becomes

$$\frac{13,750}{69.7} \times \frac{1}{2.25} \left\{ .56 - .21 \left(\frac{.56 + 2.25(.32) \frac{10}{150}}{1.06 + .09 + 1.25 \times .85} \right) \right\} = 44,$$

hence the consumption per hour will be $44 \times 271 \times .332 = 4000$ pounds.

Prob. 1. A Vaucain compound locomotive has cylinders 18 and 30×42 ins., with 5 per cent clearance in each and runs on a boiler pressure of 175 lbs. per square inch gage and atmospheric exhaust. The steam pressure may be varied as may also the cut-off to a limited degree. For a speed of 200 R.P.M., a cut-off $\frac{1}{2}$ and 10 per cent compression in each cylinder, find how the horse-power will vary with the initial pressures of 175, 150, 125, and 100 lbs. gage.

Prob. 2. When the cut-off is reduced to $\frac{1}{2}$ in the above engine, compression increases in the high-pressure cylinder to 20 per cent. For the case of 175 lbs. gage initial pressure find the change in horse-power.

Prob. 3. Find the steam used by the engine per hour for the first case of Prob. 1 and for Prob. 2.

Prob. 4. It is desired to run a 12- and 18×24 -in. no-receiver engine with 5 per cent clearance in each cylinder, under the best possible hypothetical economy conditions for an initial pressure of 200 lbs. per square inch absolute and atmospheric exhaust. To what cut-off and compression must the valves be set, and what horse-power will result for 100 R.P.M.?

56. Compound Engine without Receiver. Exponential Law, with Clearance and Compression, Cycle XVI. General Relations between Pressures, Dimensions, and Work, when H.P. Exhaust and L.P. Admission are Coincident. Again referring to Fig. 79, it may be observed that reasoning similar to that in Section 55 but using the exponential law, would show that the same formulas and graphical constructions will serve to locate the axes of the diagram, hence as before \overline{KM} may be obtained from Eq. (414) and \overline{QM} from Eq. (415). Release pressure in the high-pressure cylinder is

$$(\text{rel.pr.})_H = (\text{in.pr.}) \left(\frac{c_H + Z_H}{1 + c_H} \right)^s \quad (427)$$

Immediately after release the pressure is equalized in the high-pressure cylinder and the low-pressure clearance. The pressure after equalization termed $(\text{in.pr.})_L$, is found by the relation of the volume at S and that at D' . Fig. 79, measured from the axis KT .

$$(\text{in.pr.})_L = (\text{rel.pr.})_H \left[\frac{1 + c_H + R_C(c_L + X_L) \left(\frac{(\text{bk.pr.})}{(\text{rel.pr.})_H} \right)^{\frac{1}{s}}}{1 + c_H + R_C c_L} \right]^s,$$

which, by means of Eq. (427), becomes

$$(\text{in.pr.})_L = (\text{bk.pr.}) \left[\frac{R^{\frac{1}{s}} P(c_H + Z_H) + R_C(c_L + X_L)}{1 + c_H + R_C c_L} \right]^s \quad (428)$$

The expansion of the fluid goes on as it passes from the high-pressure cylinder to the greater volume in the low-pressure, as indicated by $D'E'$ and DE , and when the communicating valve closes, the pressure has become

$$(\text{cut-off pr.})_L = (\text{in.pr.})_L \left[\frac{\frac{1 + c_H + R_C c_L}{R_C - 1}}{\frac{1 + c_H + R_C c_L}{R_C - 1} + (1 - X_H)} \right]^s,$$

which, by means of Eq. (428) reduces to

$$(\text{cut-off pr.})_L = (\text{bk.pr.}) \left[\frac{R^{\frac{1}{s}} P(c_H + Z_H) + R_C(c_L + X_L)}{1 + c_H + R_C c_L + (1 - X_H)(R_C - 1)} \right]^s \quad (429)$$

After cut-off in the low pressure, expansion goes on in that cylinder alone to the end of the stroke, when release occurs at a pressure

$$(\text{rel.pr.})_L = (\text{cut-off pr.})_L \left(\frac{1 + c_L - X_H}{1 + c_L} \right)^s,$$

or by substitution from Eq. (429),

$$(\text{rel.pr.})_L = (\text{bk.pr.}) \left[\left(\frac{R^{\frac{1}{s}} P(c_H + Z_H) + R_C(c_L + X_L)}{1 + c_H + R_C c_L + (1 - X_H)(R_C - 1)} \right) \left(\frac{1 + c_L - X_H}{1 + c_L} \right) \right]^s \quad (430)$$

In terms of these quantities the work of the high- and low-pressure cylinders can be written out as follows:

$$\begin{aligned} W_H = 144 D_H \left\{ (\text{in.pr.}) \frac{c_H + Z_H}{s-1} \left[s - \left(\frac{c_H + Z_H}{1 + c_H} \right)^{s-1} \right] - c_H (\text{in.pr.}) \right. \\ \left. - \frac{(\text{bk.pr.})}{s-1} \left[\frac{R^{\frac{1}{s}} P(c_H + Z_H) + R_C(c_L + X_L)}{R_C - 1} \right]^s \left[1 - \left(\frac{1 + c_H + R_C c_L}{1 + c_H + R_C c_L + (1 - X_H)(R_C - 1)} \right)^{s-1} \right] \right. \\ \left. - \frac{(\text{bk.pr.})}{s-1} (c_H + X_H) \left[\frac{R^{\frac{1}{s}} P(c_H + Z_H) + R_C(c_L + X_L)}{1 + c_H + R_C c_L + (1 - X_H)(R_C - 1)} \right]^s \left[\left(\frac{c_H + X_H}{c_H} \right)^{s-1} - 1 \right] \right\}, \quad (431) \end{aligned}$$

$$\begin{aligned} \text{and } W_L = 144 D_L \left\{ \frac{(\text{bk.pr.})}{s-1} \left[\frac{R^{\frac{1}{s}} P(c_H + Z_H) + R_C(c_L + X_L)}{R_C - 1} \right]^s \right. \\ \times \left[1 - \left(\frac{1 + c_H + R_C c_L}{1 + c_H + R_C c_L + (1 - X_H)(R_C - 1)} \right)^{s-1} \right] \\ \left. + \frac{(\text{bk.pr.})}{s-1} (1 + c_L - X_H) \left[\frac{R^{\frac{1}{s}} P(c_H + Z_H) + R_C(c_L + X_L)}{1 + c_H + R_C c_L + (1 - X_H)(R_C - 1)} \right]^s \left[1 - \left(\frac{1 + c_L - X_H}{1 + c_L} \right)^{s-1} \right] \right. \\ \left. - \frac{(\text{bk.pr.})}{s-1} (c_L + X_L) \left[\left(\frac{c_L + X_L}{c_L} \right)^{s-1} - 1 \right] - (1 - X_L) (\text{bk.pr.}) \right\}. \quad (432) \end{aligned}$$

These are general expressions for work of high- and low-pressure cylinders for this cycle, and from them may be obtained the total work of the cycle, and mean effective pressure referred to the low-pressure cylinder; by equating them the relation may be obtained which must exist between dimensions, events, and pressures to give equal division of work. It would, however, be of no advantage to state these in full here, as they can be obtained from the above when needed.

The supply volume, cubic feet per cycle, is represented by $\overline{A'B}$, Fig. 79, and its value is found by referring to points B and E as follows:

$$\begin{aligned} (\text{Sup. Vol.}) &= D_H \left[(c_H + Z_H) - (c_H + X_H) \left(\frac{(\text{cut-off pr.})_L}{(\text{in. pr.})} \right)^{\frac{1}{s}} \right], \\ &= D_H \left[c_H + Z_H - \frac{(c_H + X_H)}{R^{\frac{1}{s_P}}} \left(\frac{R^{\frac{1}{s_P}}(c_H + Z_H) + R_C(c_L + X_L)}{1 + c_H + R_C c_L + (1 - X_H)(R_C - 1)} \right) \right]. \quad (433) \end{aligned}$$

Work per cubic foot supplied is found from Eqs. (431), (432), and (433).

$$\text{Work per cu.ft. supplied} = \frac{W_H + W_L}{(\text{Sup. Vol.})} \quad (434)$$

Consumption, cubic feet per hour per I.H.P., is found from mean effective pressure referred to L.P. cyl. and supply volume as follows:

$$\text{Consumption, cu.ft. per hr. per I.H.P.} = \frac{13,750}{(\text{m.e.p. ref. to L.P.})} \frac{(\text{Sup. Vol.})}{D_L} \quad (435)$$

This will give pounds consumption by introducing the factor of density.

Further than this, it will be found more practicable to use graphical methods instead of computations with this cycle.

Example. Find (a) the horse-power and (b) air consumption of a 12- and 18×24-in. no-receiver engine having 6 per cent clearance in the high pressure cylinder and 4 per cent in the low when the initial pressure is 150 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute, speed 125 R.P.M., high-pressure cut-off $\frac{1}{2}$, high-pressure compression 15 per cent, and low-pressure compression is complete.

(a) The per cent of low-pressure compression may be found as in the Example of Section 55, using the value of s in this case of 1.4. Then

$$(\text{in.pr.})(Z_H + c_H)^{1.4} = (\text{rel.pr.})_H(1 + c_H)^{1.4}, \quad \text{or} \quad (\text{rel.pr.})_H = 61.5 \text{ lbs. sq.inch absolute,}$$

and

$$(\text{rel.pr.})_H \times c_L^{1.4} = (\text{bk.pr.})(c_L + X_L)^{1.4}, \quad \text{or} \quad X_L = .11.$$

From the sum of Eqs. (431) and (432) divided by $144D_L$ and with proper values substituted, (m.e.p.) = 48.5 lbs.; hence the horse-power is 189.

(b) From Eqs. (433) the value for (Sup. Vol.) may be obtained, which when multiplied by $\frac{13,750}{\text{m.e.p.}}$, and divided by D_L , gives cubic feet air per hour per I.H.P.

$$\frac{13,750}{48.5} \times \frac{1}{2.25} \left[.06 + .5 - \frac{.21}{(15)^{.7}} \frac{15^{.7}(.56) + 2.25(.15)}{1 + .06 + 2.25 \times .04 + (1 - .15)(2.25 - 1)} \right] = 0.63 \text{ cu.ft. per hour per I.H.P.}$$

Prob. 1. If the locomotive of Prob. 1, Section 55, should be equipped with superheater so that the steam expanded in such a way that $s=1.3$, what would be the effect upon the horse-power for conditions of that problem and on the cylinder event pressures?

Prob. 2. A 30- and 42×54-in. no receiver steam pumping engine runs at 30 R.P.M. and has 3 per cent clearance in the high-pressure cylinder and 2 per cent in low. There is no compression in either cylinder. Initial pressure is 120 lbs. per square inch gage, and back pressure 28 ins. of mercury (barometer reading 30 ins.). The steam is such that the expansion exponent is 1.25. What will be the horse-power of, and the steam used by, the engine when the cut-off in the high is $\frac{1}{4}$?

Prob. 3. How much would the power change if the cut-off were shortened to $\frac{1}{8}$ and then to $\frac{1}{16}$, and what would be the effect of these changes in the economy?

57. Triple-Expansion Engine with Infinite Receiver. Logarithmic Law. No Clearance, Cycle XVII. General Relations between Pressures, Dimensions and Work. Fig. 81 represents the cycle of the triple-expansion engine

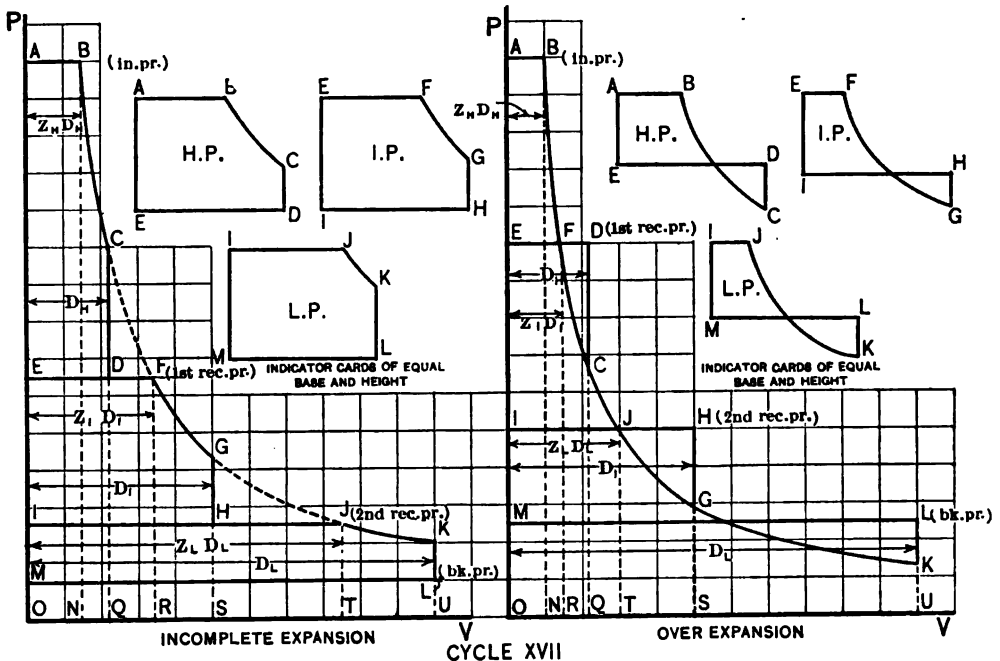


FIG. 81.—Work of Expansive Fluid in Triple-expansion Engine with Infinite Receiver and Zero Clearance. Cycle XVII, Logarithmic Expansion.

with infinite receiver, no clearance, showing one case of incomplete expansion in all cylinders, and another where overexpansion takes place in all cylinders. The reasoning which follows applies equally well to either case, and to any combination of under- or overexpansion in the respective cylinders.

It is desired to express the work of the respective cylinders and the total work in terms of dimensions, initial and back pressures, and the cut-offs of the respective cylinders. To do this, it is convenient first to express the

first receiver pressure (1st rec.pr.) and *second receiver pressure* (2d rec.pr.) in terms of these quantities. The subscript *i* refers to the intermediate cylinder.

$$P_f = P_b \frac{V_b}{V_f}, \quad \text{or} \quad (1\text{st rec.pr.}) = (\text{in.pr.}) \frac{Z_H D_H}{Z_I D_I}, \quad \dots \quad (436)$$

and
$$P_j = P_b \frac{V_b}{V_j}, \quad \text{or} \quad (2\text{d rec.pr.}) = (\text{in.pr.}) \frac{Z_H D_H}{Z_L D_L}. \quad \dots \quad (437)$$

Work of high-pressure cylinder is
$$W_H = P_b V_b \left(1 + \log_e \frac{V_c}{V_b} \right) - P_a V_a,$$
$$= 144(\text{in.pr.}) D_H \left\{ Z_H \left(1 + \log_e \frac{1}{Z_H} \right) - \frac{Z_H D_H}{Z_I D_I} \right\}. \quad \dots \quad (438)$$

Work of intermediate cylinder is
$$W_I = P_f V_f \left(1 + \log_e \frac{V_g}{V_f} \right) - P_b V_b,$$
$$= 144(\text{in.pr.}) D_H \left\{ Z_H \left(1 + \log_e \frac{1}{Z_I} \right) - \frac{D_I}{D_H} \frac{Z_H D_H}{Z_L D_L} \right\}$$
$$= 144(\text{in.pr.}) D_H \left\{ Z_H \left(1 + \log_e \frac{1}{Z_I} \right) - \frac{Z_H D_I}{Z_L D_L} \right\}. \quad \dots \quad (439)$$

Work of low-pressure cylinder is
$$W_L = P_j V_j \left(1 + \log_e \frac{V_k}{V_j} \right) - P_i V_i,$$
$$= 144(\text{in.pr.}) D_H Z_H \left(1 + \log_e \frac{1}{Z_L} \right) - 144(\text{bk.pr.}) D_L. \quad \dots \quad (440)$$

The total work by addition is
$$W = 144(\text{in.pr.}) D_H Z_H \left\{ \left(1 + \log_e \frac{1}{Z_H} \right) \right.$$
$$\left. + \left(1 + \log_e \frac{1}{Z_I} \right) + \left(1 + \log_e \frac{1}{Z_L} \right) - \frac{D_H}{Z_I D_I} - \frac{D_I}{Z_L D_L} \right\} - 144(\text{bk.pr.}) D_L$$
$$= 144(\text{in.pr.}) D_H Z_H \left\{ 3 + \log_e \frac{1}{Z_H Z_I Z_L} - \frac{D_H}{Z_I D_I} - \frac{D_I}{Z_L D_L} \right\} - 144(\text{bk.pr.}) D_L. \quad (441)$$

Mean effective pressure referred to the low-pressure cylinder is found by dividing W by $144 D_L$, and is therefore: (m.e.p. ref. to L.P.)

$$= (\text{in.pr.}) Z_H \frac{D_H}{D_L} \left\{ 3 + \log_e \frac{1}{Z_H Z_I Z_L} - \frac{D_H}{Z_I D_I} - \frac{D_I}{Z_L D_L} \right\} - (\text{bk.pr.}). \quad \dots \quad (442)$$

Work done per cubic foot supplied is equal to W divided by the supply volume \overline{AB} , or $Z_H D_H$: Work per cu.ft. supplied

$$= 144(\text{in.pr.}) \left\{ 3 + \log_e \frac{1}{Z_H Z_I Z_L} - \frac{D_H}{Z_I D_I} - \frac{D_I}{Z_L D_L} \right\} - 144(\text{bk.pr.}) \frac{D_L}{Z_H D_H}. \quad (443)$$

The volume of fluid supplied per hour per indicated horse-power is,

$$\text{Consumption, cu.ft. per hr. per I.H.P.} = \frac{13,750}{(\text{m.e.p. ref. to L.P.})} \frac{Z_H D_H}{D_L}. \quad \dots \quad (444)$$

The *weight* of fluid used per hour per indicated horse-power is found by multiplying this volume, Eq. (444), by the density of the fluid used.

The conditions which will provide for *equal division of work* between the three cylinders may be expressed in the following ways:

$$W_H = W_I = W_L. \quad \text{First let } W_H = W_I, \quad \therefore \log_e \frac{1}{Z_H} - \frac{1}{Z_I} \left(\frac{D_H}{D_I} \right) = \log_e \frac{1}{Z_I} - \frac{1}{Z_L} \left(\frac{D_I}{D_L} \right),$$

$$\text{hence} \quad \log_e \frac{Z_I}{Z_H} = \frac{1}{Z_I} \left(\frac{D_H}{D_I} \right) - \frac{1}{Z_L} \left(\frac{D_I}{D_L} \right). \quad \dots \dots \dots (445)$$

$$\text{Similarly from } W_H = W_L, \quad \log_e \frac{Z_L}{Z_H} = \frac{1}{Z_I} \left(\frac{D_H}{D_I} \right) - \frac{(\text{bk.pr.})}{(\text{in.pr.})} \left(\frac{D_L}{D_H} \right) \frac{1}{Z_L}. \quad \dots \dots (446)$$

These two equations, (445) and (446), show the necessary relations between

$$Z_H, \quad Z_I, \quad Z_L, \quad \left(\frac{D_I}{D_H} \right), \quad \left(\frac{D_L}{D_H} \right), \quad \text{and} \quad \left(\frac{(\text{in.pr.})}{(\text{bk.pr.})} \right),$$

in order that work shall be equally divided. Since there are six independent quantities entering (as above) and only two equations, there must be *four* of these quantities *fixed by conditions of the problem*, in order that the other two may be found. For instance, if the cylinder ratios, the pressure ratio, and one cut-off are known, the other two cut-offs may be found, though the solution is difficult.

Again, if *cut-offs are equal*, and the ratio of initial to back pressure is known, it is possible to find the cylinder ratios. This forms a special case which is of sufficient importance to require investigation.

$$\text{If } Z_H = Z_I = Z_L, \text{ Eq. (445) becomes, } \frac{D_I}{D_H} = \frac{D_L}{D_I}, \quad \dots \dots \dots (447)$$

$$\text{and Eq. (446) reduces to } \frac{D_I D_L}{D_H^2} = \frac{(\text{in.pr.})}{(\text{bk.pr.})}, \quad \dots \dots \dots (448)$$

$$\text{but from Eq. (447), } \frac{D_L}{D_I} \frac{D_I}{D_H} = \frac{D_L}{D_H} = \left(\frac{D_I}{D_H} \right)^2, \quad \text{and therefore } \frac{D_I D_L}{D_H^2} = \left(\frac{D_I}{D_H} \right)^3,$$

$$\text{and} \quad \frac{D_I}{D_H} = \frac{D_L}{D_I} = \left(\frac{(\text{in.pr.})}{(\text{bk.pr.})} \right)^{\frac{1}{3}}, \quad \dots \dots \dots (449)$$

$$\text{which, along with the condition assumed that } Z_H = Z_I = Z_L, \quad \dots \dots \dots (450)$$

constitute *one set of conditions that will make work equal* in the three cylinders. This is not an uncommon method of design, since by merely maintaining equal cut-offs, the work division may be kept equal.

The work done in any one cylinder under these conditions, Eqs. (449) and (450), is then

$$W_H = W_I = W_L = 144(\text{in.pr.})D_H \left\{ Z \left(1 + \log_e \frac{1}{Z} \right) - \frac{(\text{bk.pr.})^{\frac{1}{3}}}{(\text{in.pr.})^{\frac{1}{3}}} \right\}. \quad \dots \dots (451)$$

$$\text{and the total work, } W = 432(\text{in.pr.})D_H \left\{ Z \left(1 + \log_e \frac{1}{Z} \right) - \frac{(\text{bk.pr.})^{\frac{1}{3}}}{(\text{in.pr.})^{\frac{1}{3}}} \right\}. \quad \dots \dots (452)$$

in which Z represents the cut-off in each cylinder, all being equal.

A special case of the triple-expansion engine with infinite receiver and no clearance which demands attention is that of *complete expansion* in all cylinders, represented by Fig. 82. Here

$$Z_L = \frac{D_I}{D_L} = \frac{(\text{bk.pr.})}{(2\text{d rec.pr.})} \quad (453); \quad Z_I = \frac{D_H}{D_I} = \frac{(2\text{d rec.pr.})}{(1\text{st rec.pr.})} \quad (454). \quad (453) \quad (454)$$

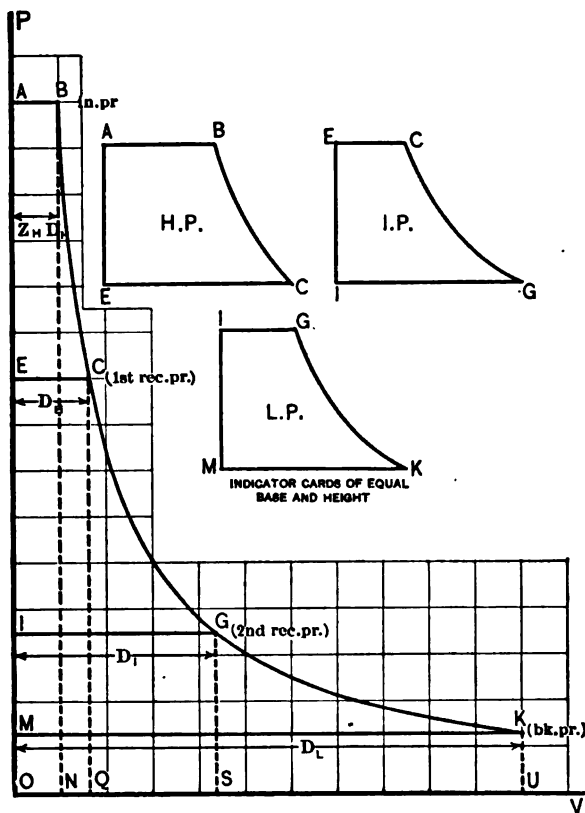


FIG. 82.—Special Case of Cycle XVIII Complete Expansion in Triple-expansion Engine with Infinite Receiver, Zero Clearance, Logarithmic Expansion.

and
$$Z_H = \frac{(1\text{st rec.pr.})}{(\text{in.pr.})} = \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right) \frac{D_L}{D_H}, \quad (455)$$

hence the receiver pressures are as follows: $(1\text{st rec.pr.}) = (\text{bk.pr.}) \frac{D_L}{D_H}, \quad (456)$

and
$$(2\text{d rec.pr.}) = (\text{bk.pr.}) \frac{D_L}{D_I}. \quad (457)$$

The work of the respective cylinders, expressed in terms of initial and back pressures and displacements, is then,

$$\begin{aligned} W_H &= 144(\text{in.pr.})D_H \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right) \frac{D_L}{D_H} \left[1 + \log_e \left(\frac{(\text{in.pr.})}{(\text{bk.pr.})} \frac{D_H}{D_L} \right) \right] - 144 \text{ bk.pr.} \left(\frac{D_L}{D_H} \right) D_H \\ &= 144(\text{bk.pr.})D_L \log_e \left(\frac{(\text{in.pr.})}{(\text{bk.pr.})} \frac{D_H}{D_L} \right). \quad (458) \end{aligned}$$

Similarly,
$$W_I = 144(\text{bk.pr.}) D_L \log_e \left(\frac{D_I}{D_H} \right), \quad \dots \quad (459)$$

and
$$W_L = 144(\text{bk.pr.}) D_L \log_e \left(\frac{D_L}{D_I} \right). \quad \dots \quad (460)$$

Total work, by addition, is

$$\begin{aligned} W &= 144(\text{bk.pr.}) D_L \left[\log_e \left(\frac{(\text{in.pr.}) D_H}{(\text{bk.pr.}) D_L} \right) + \log_e \frac{D_I}{D_H} + \log_e \frac{D_L}{D_I} \right] \\ &= 144(\text{bk.pr.}) D_L \log_e \frac{(\text{in.pr.})}{(\text{bk.pr.})} \quad \dots \quad (461) \end{aligned}$$

If for this special case of complete expansion *equality of work* is to be obtained, then from Eqs. (458), (459), and (460),

$$\frac{(\text{in.pr.}) D_H}{(\text{bk.pr.}) D_L} = \frac{D_I}{D_H} = \frac{D_L}{D_I}, \quad \dots \quad (462)$$

which is readily seen to be the same result as was obtained when all cut-offs were equalized, Eqs. (447) and (448). This case of complete expansion and equal work in all cylinders is a special case of that previously discussed where cut-offs are made equal. Hence for this case cut-offs are equal,

$$Z_H = Z_I = Z_L = \frac{D_H}{D_I} = \frac{D_I}{D_L} = \frac{D_L}{D_H} (\text{bk.pr.}). \quad \dots \quad (463)$$

Example. A triple-expansion engine 12- and 18- and 27×24-ins., with infinite receiver and no clearance, runs at 125 R.P.M. on an initial pressure of 150 lbs. per square inch absolute, and a back pressure 10 lbs. per square inch absolute. If the cut-offs in the different cylinders, beginning with the high, are $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$, what will be (a) the horse-power, (b) steam consumed per hour, (c) release and receiver pressures?

(a) From Eq. (442)

$$(\text{m.e.p.}) = 150 \times .5 \times \frac{1}{5.06} \left\{ 3 + \log_e 14.2 - \frac{8}{3 \times 2.25} - \frac{8}{3 \times 2.25} \right\} - 10 = 39 \text{ lbs. per sq.in.,}$$

hence
$$\text{I.H.P.} = \frac{39 \times 2 \times 573 \times 250}{33,000} = 338.$$

(b) From Eq. (444),

$$\text{Cubic feet per horse-power per hour} = \frac{13,750}{(\text{m.e.p.})} \frac{Z_H D_H}{D_L} = \frac{13,750}{39} \times .5 \times \frac{1}{5.06} = 34.9,$$

hence total pounds per hour will be, $34.9 \times .338 \times .332 = 3920$.

(c) From Eq. (436) 1st (rec.pr.) $= 150 \times \frac{.5}{.375 \times 2.25} = 89 \text{ lbs. per sq.in. absolute.}$

From Eq. (437) 2d (rec.pr.) $= 150 \times \frac{.5}{.375 \times 5.06} = 3.75 \text{ per sq.in. absolute.}$

High-pressure release pressure may be found from relation $(\text{in.pr.}) Z_H D_H = (\text{rel.pr.})_H D_H \therefore (\text{rel.pr.})_H = 75 \text{ lbs.}$ Similarly 1st (rec.pr.) $Z_I D_I = (\text{rel.pr.})_I D_I$, or $(\text{rel.pr.})_I = 33.4$. Similarly 2d (rec.pr.) $Z_L D_L = (\text{rel.pr.})_L D_L$, or $(\text{rel.pr.})_L = 14.8$.

Prob. 1. What would be the horse-power and steam used per hour by a 10- and 16- and 25×20-in. infinite receiver, no-clearance engine, running at 185 R.P.M. on an initial pressure of 180 lbs. per square inch gage and atmospheric exhaust? Cut-offs .4, .35, and .3.

Prob. 2. The following data are reported for a test of a triple engine:

Size 20×33×52×42 ins., speed 93 R.P.M., initial pressure 200 lbs. per square inch gage, back pressure one atmosphere, H.P. cut-off .5, horse-power 1600, steam per horse-power per hour 17 lbs. Check these results, using cut-offs in other cylinders to give approximately even work distribution.

Prob. 3. What change in cylinder sizes would have to be made in the above engine to have equal work with a cut-off of $\frac{1}{2}$ in each cylinder, keeping the high pressure the same size as before?

Prob. 4. What would be the horse-power of a triple-expansion engine whose low-pressure cylinder was 36×30 ins., when running on 150 lbs. per square inch absolute initial pressure and 10 lbs. per square inch absolute back pressure, with a cut-off in each cylinder of .4 and equal work distribution? Make necessary assumptions.

Prob. 5. A triple engine 18×24×36×30 ins., running at 100 R.P.M. on an initial pressure of 200 lbs. per square inch absolute and back pressure of 20 lbs. per square inch absolute, is to be run at such cut-offs as will give complete expansion in all cylinders. What will these be, what receiver pressures will result, what horse-power can be produced under these conditions, and how much steam will be needed per hour?

NOTE: δ for 200 lbs. = .437.

CHAPTER VI

PISTON ENGINES. WORK, POWER AND CONSUMPTION. GRAPHICAL METHODS.

58. Multiple-Expansion Engine. General Case. Any Relation between Cylinder and Receiver. Determination of Pressure-Volume Diagram and Work. It is possible to arrange multiple-expansion engines in an almost infinite variety of ways with respect to the pressure-volume changes of the fluid that take place in their cylinders and receivers. There may be two or three cylinder compounds of equal or unequal strokes, pistons moving together by connection to one piston rod, or separate piston rods with a common cross-head or even with completely independent main parts and cranks at 0° , and 180° , displaced with either one leading; or the pistons may not move together, being connected to separate cranks at any angle apart, and any order of lead. Moreover, there may be receivers of large or small size, and there may be as a consequence almost any relation between H.P. discharge to receiver and low-pressure receipt from it, any amount of fluid passing to correspond to engine load demands and consequently any relation of cut-offs, compressions, and receiver-pressure fluctuations. Triple and quadruple engines offer even greater varieties of combination of related factors, so that problems of practical value cannot be solved by analytical methods with anything like the same facility as by graphic means, and in some cases not at all.

These problems that demand solution are of two classes:

1. To find the work distribution and total work for cylinders of given dimensions, clearances, receiver volumes and mechanical connection or movement relation, with given initial and back pressures, and given valve gear at any setting of that valve gear or at a variety of settings.
2. To find the cylinder relations to give any proportion of the total work in any cylinder at any given valve setting, or any fraction of initial pressure, or any value of release pressure or total number of expansions.

The essential differences between these two classes of problems is that in the first the cylinder dimensions are given, while in the second they are to be found. In general, however, the same methods will do for both with merely a change in the order, and in what follows the dimensions of cylinders, valve periods, receiver volume, initial and back pressures will be assumed and the diagrams found. By working to scale these diagrams will give the work by evaluation of their area, by means either of cross-section paper directly, on which strips can be measured and added, or by the planimeter. Thus

will high- and low-pressure work be evaluated through the foot-pound equivalent per square inch of diagram, and the total work or the equivalent mean effective pressure found by the methods of mean ordinates referred to the pressure scale of ordinates.

In the finding of the pressure-volume diagram point by point there is but one common principle to be applied, and that is: For a given mass the product of pressure and volume is to be taken as constant (for nearly all steam problems, which is almost the sole application of this work) and *when two masses come together at originally different pressures and mix, the product of the resulting pressure and the new volume, is equal to the sum of the PV products of the two parts before mixture.* At the beginning of operations in the high-pressure cylinder, a known volume of steam is admitted at a given pressure and its pressure and volume are easily traced up to the time when it communicates with the receiver in which the pressure is unknown, and there difficulty is encountered, but this can be met by working from the other end of the series of processes. The low-pressure cylinder, having a known compression volume at the back pressure, there will be in it at the time of opening to receiver a known volume, its clearance, at a known compression pressure. The resulting receiver pressure will then be that for the mixture. These two receiver pressures are not equal ordinarily, but are related by various compressions and expansions, involving high- and low-pressure cylinder partial displacements, grouped with receiver volumes in various ways.

Take for an illustrative example the case of a two-cylinder, single-acting, cross-compound engine with slide valves, cylinder diameters $12\frac{1}{2}$ and 20 ins. with 24-in. stroke for both. High-pressure clearance is 10 per cent, low-pressure clearance 8 per cent. Receiver volume 4000 cu.ins. High-pressure crank following by 90° . Find the mean effective pressure for the high- and low-pressure cylinders, for a cut-off of 50 per cent in the high, and 60 per cent in the low, a compression of 10 per cent in the high and 20 per cent in the low, initial pressure 105 lbs. per square inch gage, back pressure 5 lbs. per square inch absolute, expansion according to logarithmic law.

On a horizontal line SZ , Fig. 83, lay off the distances

TU = low-pressure cylinder displacement volume in cubic inches to scale.

UV = low-pressure cylinder clearance volume in cubic inches to scale.

VW = receiver volume in cubic inches to scale.

WX = high-pressure cylinder clearance volume in cubic inches to scale.

XY = high-pressure cylinder displacement volume in cubic inches to scale.

Through these points draw verticals produced above and below, $T'T''$, $U'U''$, $V'V''$, $W'W''$, $X'X''$, $Y'Y''$. Then will $W'W$ and WZ be PV coordinates for the high pressure diagram in the quadrant $W'WZ$, and $V'V$ and VS the PV coordinates for the low-pressure diagram in the reverse quadrant $V'VS$.

Lay off AB to represent the high-pressure admission at a height XA representing absolute initial pressure; lay off LM at a height TL representing

low-pressure exhaust at a constant absolute back pressure to the same scale.

Locate point B at the cut-off point $AB = .50XY$ on the initial pressure line, and drop a vertical BB^2 and draw similar verticals JJ^2 , GG^2 , MM^2 , at

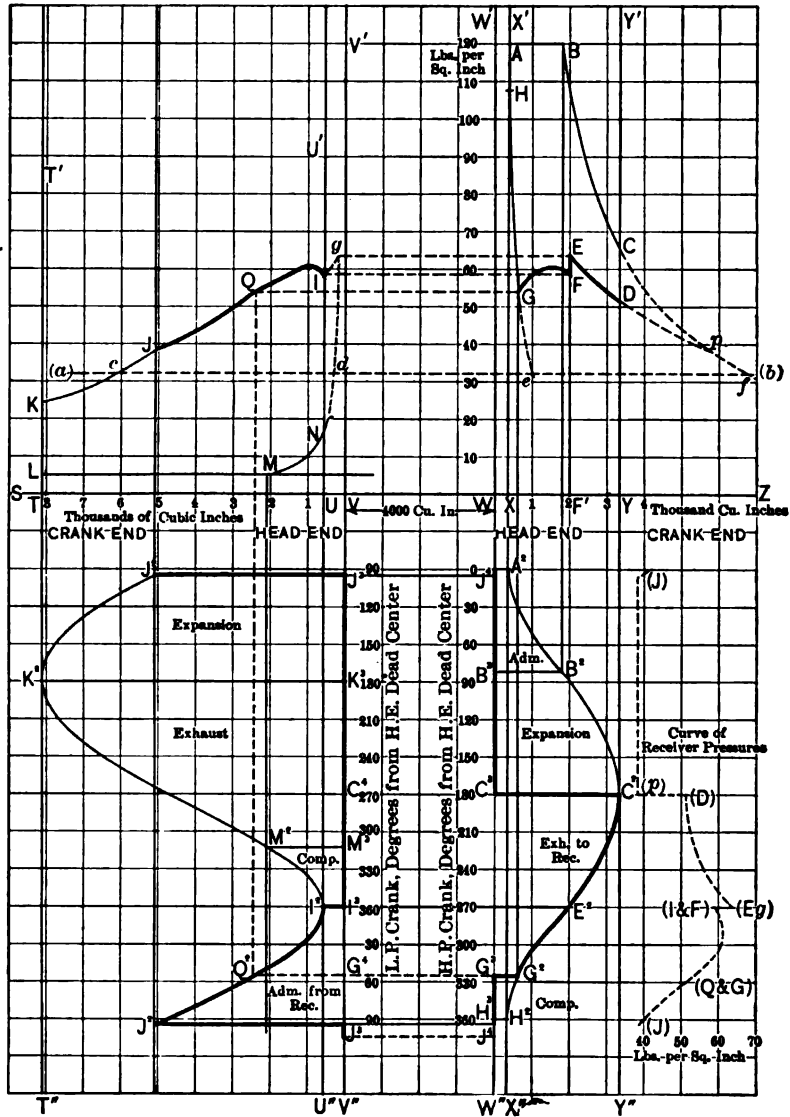


FIG. 83.—Graphical Solution of Compound Engine with Finite Receiver and with Clearance Illustrating General Method of Procedure for any Multiple-expansion Engine.

suitable fractional displacements to represent L.P. cut-off, H.P. and L.P. compression volumes respectively.

This operation will fix two other points besides the points A and L , B the H.P. cut-off at the initial pressure and M the low-pressure compression at the

back pressure. Through the former draw an expansion line BC and through the latter a compression line MN , locating two more points, C and N , at the end of the outstroke of the high and instroke of the low.

At point C the H.P. cylinder steam releases to the receiver of unknown pressure, and at N , the L.P. cylinder steam is opened to both the receiver and high-pressure cylinder at unknown pressure and volume.

To properly locate these pressures and volumes from the previously known pressures and volumes in a simple manner, the construction below the line SZ is used.

Lay off on $W'W''$ the high-pressure crank angles $0-360^\circ$, and to the right of each lay off from the clearance line XX'' the displacement of the piston at the various crank angles for the proper rod to crank ratio, locating the curve $A^2B^2C^2E^2F^2G^2H^2$. This is facilitated by the use of Table (XII) in the Handbook of Tables, but may be laid out graphically by drawing the crank circle and sweeping arcs with the connecting rod as radius.

Opposite H.P. crank angle 270° locate L.P. crank angle $0^\circ=360^\circ$ and draw to left of the low-pressure clearance line UU'' the crank angle displacement curve for that piston.

It will be noted that steam volumes are given in the lower diagram by the distances from either of these curves toward the other as far as circumstances call for open valves. Thus H.P. cylinder volumes are distances from the H.P. displacement curve to WW'' , but when H.P. cylinder is in communication with receiver, the volume of fluid is the distance from H.P. displacement curve to VV'' , and when H.P. cylinder, receiver and L.P. cylinder are all three in communication the volume is given by the distance from H.P. displacement curve to L.P. displacement curve. This pair of displacement curves, located one with respect to the other as called for by the crank angle relations (which may be made to correspond to any other angular relation by sliding the low up or down with respect to high-pressure curve), serve as an easy means of finding and indicating the volumes of fluid occupying any of the spaces that it may fill at any point of either stroke.

On each curve locate the points corresponding to valve periods by the intersection of the curve with verticals to the upper diagram, such as BB^2 . These points being located, the whole operation can be easily traced.

At H.P. cut-off (B) the volume of steam is B^3B^2 . During H.P. expansion (B to C) steam in the high increases in volume from B^3B^2 to C^3C^2 .

During H.P. release (C to D) the volume of steam in the high C^3C^2 is added to the receiver volume C^4C^3 , making the total volume C^4C^2 . During H.P. exhaust (D to E) the steam volume C^4C^2 in H.P. and receiver is compressed to volume I^3E^2 .

At L.P. admission (N) in low and (E) in high, the volume I^2I^3 is added, making the total volume I^2E^2 in high, low, and receiver.

During (I to Q) in low and (F to G) in high the volume I^2E^2 in high, low, and receiver, changes volume until it becomes Q^2G^2 in high, low, and receiver.

At H.P. compression, G in the high, the steam divides to Q^2G^3 in low

and receiver, while G^3G^2 remains in high and is compressed to H^3H^2 , at the beginning of admission in the high. The former volume Q^2G^3 , in low and receiver, expands to J^2J^4 , at the moment cut-off occurs in the low, which divides the volume into, J^3J^4 in receiver, which remains at constant volume till high-pressure release, and the second part, J^2J^3 in the low, which expands in that cylinder to K^2K^3 .

After low-pressure release the volume in low decreases from K^2K^3 to M^2M^3 , when the exhaust valve closes and low-pressure compression begins.

During compression in low, the volume decreases from M^2M^3 to I^2I^3 , which is the volume first spoken of above, that combines with I^3E^2 , causing the drop in the high-pressure diagram from E to F .

The effects upon pressures, of the various mixings at constant volume between high, low, and receiver steam and the intermediate common expansions and compressions may be set down as follows:

At C , steam in high, at pressure P_c , mixes with steam in receiver at pressure P_r , resulting in high and receiver volume at pressure P_d .

From D to E there is compression in high and receiver resulting in pressure P_e .

At E steam in high and receiver at pressure P_e mixes with steam in low, at P_n , locating points I in low and F in high at same pressure.

From (F to G in high) and (I to Q in low) there is a common compression-expansion in high, low, and receiver, the pressures varying inversely as the total volume measured between the two displacement curves. At G in high there begins compression in high alone to H .

In the low and receiver from Q to J there is an expansion and consequent fall in pressure from P_e to P_j .

After low-pressure cut-off at J the expansion takes place in low-pressure cylinder alone, to pressure P_k , when release allows pressure to fall (or rise) to exhaust pressure P_l .

When cut-off in low occurs at J the volume J^3J^4 is separated off in the receiver, where it remains at constant pressure P_j until high-pressure release at point C .

At the point M compression in low begins, increasing the pressure in low alone from P_n to P_m .

There are, it appears, plenty of relations between the various intermediate and common points, but not enough to fix them unless one be first established. One way of securing a starting point is to assume a compression pressure P_r for the beginning of H.P. compression and draw a compression line HG through it, produced to some pressure line af , cutting low-pressure compression line at d . Then the H.P. intercept ($e-f$) must be equal to the low-pressure intercept ($d-c$); this fixes (c) through which a $PV = \text{const.}$ line intersects the L.P. cut-off volume at J .

Now knowing by this approximation the pressure at J , the pressure may be found at D , E , F , and at G . The pressure now found at G may differ considerably from that assumed for the point. If so, a new assumption for

the pressure at G may be made, based upon the last figure obtained and working around the circuit of pressures, J , D , E , F , and back to G should give a result fairly consistent with the assumption. If necessary, a third approximation may be made.

It might be noted that this is much the process that goes on in the receiver when the engine is being started, the receiver pressure rising upon each release from the high, closer and closer to the limiting pressure that is completely reached only after running some time.

These approximations may be avoided by the following computation, representing point pressures by P with subscript and volumes by reference to the lower diagram. P_j is the unknown pressure in receiver before high-pressure release and after low-pressure cut-off.

Pressure after mixing at D is then
$$\frac{P_j(C^4C^3) + P_c(C^3C^2)}{(C^4C^2)} = P_d.$$

The pressure at F , after mixing is

$$\frac{P_d \frac{(C^4C^2)}{(I^3E^2)} + P_n(I^2I^3)}{(I^2E^2)} = \frac{P_j(C^4C^3) + P_c(C^3C^2) + P_n(I^2I^3)}{I^2E^2} = P_f.$$

This pressure multiplied by $\frac{(I^2E^2)}{(Q^2G^2)}$ gives P_g , and this in turn multiplied by $\frac{(Q^2G^3)}{J^2J^4}$ will give P_j . Writing this in full,

$$[P_j(C^4C^3) + P_c(C^3C^2) + P_n(I^2I^3)] \frac{(Q^2G^3)}{(Q^2G^2)(J^2J^4)} = P_j. \text{ Solving for } P_j,$$

$$P_j = \frac{[P_c(C^3C^2) + P_n(I^2I^3)](Q^2G^3)}{(Q^2G^2)(J^2J^4) - (C^4C^3)(Q^2G^3)}, \quad \dots \quad (464)$$

which is in terms of quantities all of which are measurable from the diagram. While this formula applies to this particular case only, the manner of obtaining it is indicative of the process to be followed for other cases.

When there are three successive cylinders the same constructions can be used, the intermediate diagrams taking the position of the low for the compound case, while the low for the triple may be placed under the high and off-set from the intermediate by the volume of the second receiver. In this case it is well to repeat the intermediate diagram. Exactly similar constructions apply to quadruple expansion with any crank-angle relations.

Prob. 1. By means of graphical construction find the horse-power of a 12- and 18×24-in. single-acting cross-compound engine with 6 per cent clearance in each cylinder, if the receiver volume is 5 cu.ft., initial pressure 150 lbs. per sq.in. gage, back pressure 10 lbs. per sq.in. absolute, speed 125 R.P.M., high-pressure compression 30 per cent, low-pressure 20 per cent, high-pressure cut-off 50 per cent, low-pressure 40 per cent, high-pressure crank ahead 70°, logarithmic expansion, and ratio of rod to crank, 4.

Prob. 2. Consider the above engine to be a tandem rather than a cross-compound and draw the new diagrams for solution.

Prob. 3. A double-acting, 15- and 22×24-in. compound engine has the high-pressure crank ahead by 60°, and has 5 per cent clearance in the low-pressure cylinder, 10 per cent in the high, and a receiver 4 times as large as the high-pressure cylinder. What will be the horse-power when the speed is 125 R.P.M., initial pressure 150 lbs. per square inch absolute, back pressure 5 lbs. per square inch absolute, high-pressure cut-off $\frac{1}{2}$, low-pressure $\frac{1}{2}$, high-pressure compression 20 per cent, low-pressure 30 per cent, and ratio of rod to crank, 5? Determine graphically the horse-power in each cylinder.

Prob. 4. Consider the engine of Prob. 3 to be a tandem compound and repeat the solution.

59. Mean Effective Pressure, Engine Power, and Work Distribution and their Variation, with Valve Movement and Initial Pressure. Diagram Distortion and Diagram Factors. Mechanical Efficiency. The indicated power developed by a steam engine is dependent upon three principal factors—piston displacement, speed, and mean effective pressure. The first, *piston displacement*, is dimensional in character, and fixed for a given engine. *Speed* is limited by steam and inertia stresses, with which the present treatment is not concerned, or by losses due to fluid friction in steam passages, a subject that will be further considered under steam flow. *Mean effective pressure* is a third factor which is to be investigated, most conveniently by the methods laid down in the foregoing sections.

In these formulas for mean effective pressure, it will be observed that the terms entering are (a) initial pressure, (b) back pressure, (c) cut-off or ratio of expansion, (d) clearance, and (e) compression, for the single-cylinder engine. It is desirable to learn in what way the mean effective pressure varies upon changing any one of these factors.

Referring to Section 41, Eq. (184) for logarithmic expansion it is seen that the mean effective pressure is the difference between a mean forward pressure and a mean back pressure. The former depends on initial pressure, cut-off, and clearance, and the latter on back pressure, compression, and clearance. To study the effect of varying these terms it is most convenient to draw curves such as are shown in Fig. 84, and examine mean forward pressure and mean backward pressure separately.

Mean forward pressure is seen by inspection to vary in direct proportion to initial pressure. Cut-off, when short, gives a low mean forward pressure, but it is to be noted that zero cut-off will not give zero mean effective pressure so long as there is clearance, due to expansion of steam in the clearance space. Increasing the length of cut-off, or period of admission, increases mean forward pressure, but not in direct proportion, the (m.f.p.) approaching initial pressure as a limit as complete admission is approached. Clearance has the tendency as it increases, to increase the mean forward pressure, though not to a great extent, as indicated by the curve Fig. 84.

Mean back pressure is usually small as compared to initial pressure, though a great loss of power may be caused by an increase of back pressure or compression. Back pressure enters as a direct factor, hence the straight line through the origin in the figure. So long as compression is zero, back pressure and

mean back pressure are equal. When compression is not zero, there must be some clearance, and the ratio of (mean bk.pr.) to (bk.pr.) depends on both clearance and compression, being greater for greater compressions and for smaller clearances.

The mean effective pressures obtained by subtracting mean back from mean forward pressures in Fig. 84 are shown in curve form in Fig. 85.

The *multiple expansion engine* can not be so simply regarded. In a general way each cylinder may be said to be a simple engine, and subject to variations of mean effective pressure due to change in its own initial pressure and back pressure, clearance, cut-off and compression, which is true. At the same time

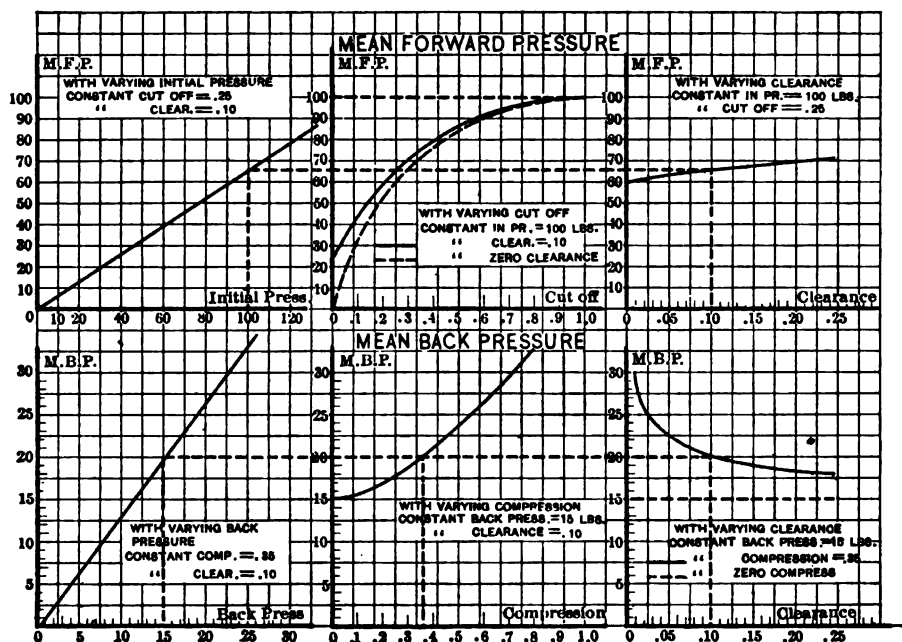


FIG. 84.—Curve to Show Variation of Mean Forward and Mean Back Pressure for Logarithmic Expansion and Compression in a Single-cylinder Engine with Clearance.

these factors are interrelated in a way that does not exist in the simple engine. Consider, for instance, the high-pressure cylinder of a compound engine with infinite receiver, with clearance. An increase of high-pressure compression tends first to raise the mean back pressure according to the reasoning on simple engine, but at the same time the change has decreased the volume of steam passing to receiver. No change having been made in the low-pressure cylinder, the volume admitted to it will remain the same as before, and the receiver pressure will fall, decreasing mean back pressure by a greater amount than compression increased it, and mean forward being the same as before, the increase of high-pressure compression has increased the mean effective pressure of the high-pressure cylinder. The only effect

upon the low-pressure cylinder is that resulting from lowering its initial pressure, i.e., the receiver pressure. This results in a decrease of low-pressure mean effective pressure. Computation will show that the assumed increase of high-pressure compression decreases low-pressure work more than it increases high-pressure work, or in other words, decreased mean effective pressure referred to the low.

It is impracticable to describe all results of changing each of the variables for the multiple-expansion engine. Initial pressure and cut-off in the respective stages have, however, a marked influence upon receiver pressures and work distribution which should be noted. Power regulation

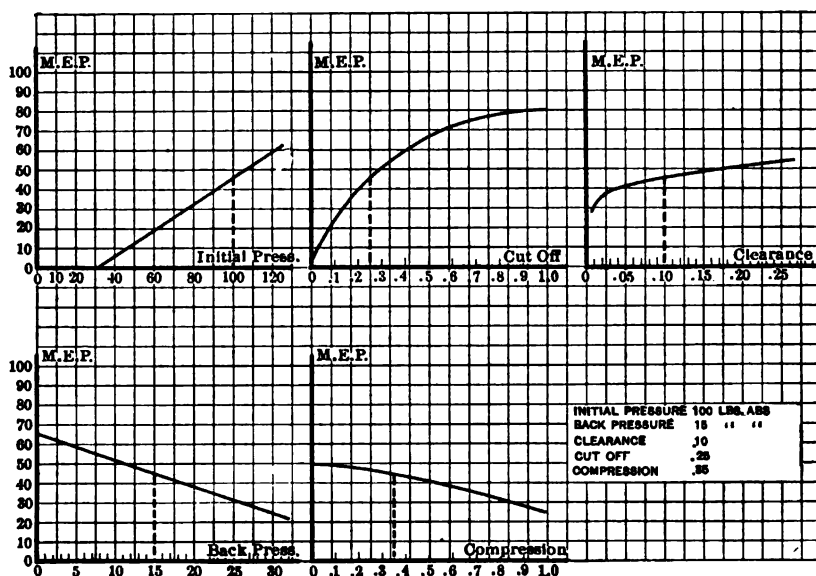


FIG. 85.—Curves to Show Variation of Mean Effective Pressure for Logarithmic Expansion and Compression in a Single-cylinder Engine with Clearance.

is nearly always accomplished by varying initial pressure, i.e., throttling, or by changing cut-off in one or more cylinders.

The effect of decreasing initial pressure is to decrease the pressures on the entire expansion line and in all no-clearance cycles, to decrease absolute receiver pressures in direct proportion with the initial pressure. Since back pressure remains constant, the result is, for these no-clearance cycles, that the mean effective pressures of all but the low-pressure cylinder are decreased in direct proportion to the initial pressure, while that of the low-pressure is decreased in a greater proportion. The same is true only approximately with cycles having clearance and compression.

The conditions giving equal work division have been treated in connection with the individual cycles; it may here be noted in a more general way that if high-pressure cut-off is shortened, the supply capacity of that cylinder is

decreased, while that of the next cylinder remains unchanged. The result is that the decreased supply volume of steam will be allowed to expand to a lower pressure before it can fill the demand of the next cylinder than it did previously, i.e., the receiver pressure is lowered. Similarly shortening cut-off in the second cylinder will tend to increase receiver pressure. To maintain constant work division, there must be a certain relation between cut-offs of the successive cylinders, which relation can only be determined after all conditions are known, but then can be definitely computed and plotted for reference in operation.

So far, in discussing the steam engine, cycles only have been treated. These cycles are of such a nature that they can be only approached in practice, but since all conclusions have been arrived at through reasoning based on assumed laws or hypotheses, the term hypothetical may be applied to all these cycles. It is desirable to compare the actual pressure-volume diagram, taken from the indicator card of a steam engine, and the hypothetical diagram most nearly corresponding with the conditions.

In Fig. 86 is shown in full lines a pressure-volume diagram which has been produced from an actual indicator card taken from a simple non-condensing, four-valve engine having five per cent clearance.

Finding the highest pressure on the admission line $A'B'$ and the lowest pressure on the exhaust line DE' , these pressures are regarded as (initial pressure) and (back pressure) and a hypothetical diagram constructed corresponding to Cycle III, with cut-off and compression at the same fraction of stroke as in the actual engine.

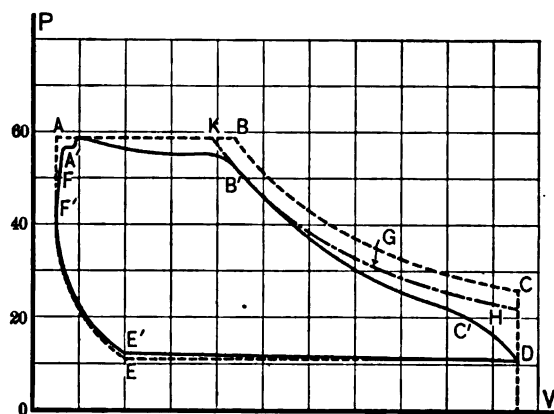


FIG. 86.—Illustrating Diagram Factors.

The first difference between the hypothetical and actual PV diagrams is that the point of release C' is not at the end of stroke, as was assumed for the hypothetical release, C , a difference which is intentional, since it requires time for pressure to fall after release to the exhaust pressure. This same fact may cause the corner of the diagram to be rounded instead of sharp as at D . Similarly, the point of admission F' may occur before the end of the return stroke has been reached, and also the corner A' may be rounded, though if admission be made sufficiently early the corners at A' will be sharp, approaching the hypothetical, A .

These differences, however, have little effect upon the area of the actual diagram, which is seen to be much smaller than the hypothetical. This deficiency of area is the net result of a large number of influences, only a few

of which can be fully explained in connection with the pressure volume discussion.

Beginning with the point of admission, F' , the line $F'A'B'$ represents the *period of admission*. The rounding at A' has been explained; the inclination of the line from A' toward B' is due in part to the frictional loss of pressure as the fluid passes at high velocity through ports and passages from steam chest to cylinder. As the stroke progresses, the linear velocity of the piston increases toward mid-stroke, requiring higher velocities in steam passages. The greater consequent friction causes pressure to fall in the cylinder. The resistance of pipes and valves leading to the engine have an effect on the slope of this line. As cut-off is approached, this pressure fall becomes more rapid, due to the partial closure of the admission valve.

From B' , the point of cut-off, to C' , the point of release, is the *period of expansion*, during which the pressures are much lower than during the hypothetical expansion line BC , due principally to the lower pressure at the point of cut-off B' than at B . Hence, the frictional fall in pressure during admission has a marked effect upon the work done during expansion. The curve $B'C'$ rarely follows the law $PV = \text{const.}$ exactly, though it commonly gives approximately the same work area. During the first part of expansion, the actual pressure commonly falls below that indicated by this curve, but rises to or above it before the expansion is complete. This is largely due to condensation of steam on the cylinder walls at high pressures, and its re-evaporation at lower pressure, to be studied in connection with a thermal analysis of the cycle. The curve of expansion may also depart from this very considerably, due to leakage, either inwardly, through the admission valve, or by piston from a region of higher pressure, or outwardly, through exhaust valve, or by piston into a region of lower pressure, or by drain, indicator, or relief valves, or by stuffing-boxes.

From the opening of the exhaust valve at the point of release, C' , till its closure at compression, E' , is the *period of exhaust*. Pressures during this period, as during admission, are affected by frictional losses in the passages for steam—in this case the pressure in the cylinder being greater than that in exhaust pipe due to friction, by an increasing amount, since the velocity of the piston increases toward mid-stroke. Thus the line DE' rises above the line DE until the partial closure of the exhaust near the point of compression causes it to rise more rapidly.

At the point of compression E' the exhaust valve is completely closed and the *period of compression* continues up to admission at F' . Leakage, condensation, and re-evaporation affect this line in much the same way as they do the expansion, and often to a more marked degree, due to the fact that the volume in cylinder is smaller during compression than during expansion, and a given weight condensed, re-evaporated, or added or removed by leakage will cause a greater change in pressure in the small weight present than if the change in weight had occurred to a large body of steam.

In the *compound engine* all these effects are present in each cylinder in greater or less degree. In addition, there are losses of pressure or of volume in the receivers themselves between cylinders, due to friction or condensation, and where especially provided for, re-evaporation by means of reheating receivers. The effect of these changes in receivers is to cause a loss of work between cylinders, and to make the discharge volume of one cylinder greater or less than the supply volume of the next, while these were assumed to be equal in the hypothetical cases.

The effect of all of these differences between the actual and hypothetical diagrams is to make the actual indicated work of the cylinder something less than that represented by the hypothetical diagram. Since these effects are not subject to numerical calculations from data ordinarily obtainable, they are commonly represented by a single coefficient or *diagram factor* which is a ratio, derived from experiment, between the actual work and that indicated by hypothesis.

It is at once evident that there may be more than one hypothetical diagram to which a certain engine performance may be referred as a standard of comparison. When the heat analysis of the steam engine is taken up, a standard for comparison will be found there which is of great use. For determination of probable mean effective pressure, however, no method of calculation has been devised which gives better results than the computation of the hypothetical mean effective pressure from one of the standard hypothetical diagrams, and multiplying this by a diagram factor obtained by experiment from a similar engine, under as nearly the same conditions as can be obtained.

Such diagram factors are frequently tabulated in reference books on the steam engine, giving values for the factor for various types and sizes, under various conditions of running. Unfortunately, however, the exact standard to which these are referred is not stated. In this text it will be assumed, unless otherwise stated, that the diagram factor for an actual engine is the ratio of the mean effective pressure of the actual engine to that computed for Cycle I, without clearance or compression, logarithmic law, with cut-off at the same fraction of stroke, initial pressure equal to maximum during admission in actual, and back pressure equal to minimum during exhaust of the actual engine.

This is selected as the most convenient standard of comparison for mean effective pressures, as it is frequently impossible to ascertain the clearance in cases where data are supplied. When it is possible to do so, however, closer approximation may be made to the probable performance by comparing the actual with that hypothetical diagram most nearly approaching the cycle, using same clearance, cut-off, and compression as are found in the actual.

Commercial cut-off is a term frequently used to refer to the ratio of the volume AK to the displacement, Fig. 86, in which the point K is found on the initial pressure line AB , by extending upward from the true point of cut-off B' a curve $PV = \text{const.}$

While the diagram factor represents the ratio of indicated horse-power to hypothetical, the output of power at the shaft or pulley of engine is less than that indicated in the cylinders, by that amount necessary to overcome mechanical friction among engine parts. If this power output at shaft or pulley of engine is termed *brake horse-power* (B.H.P.) then the ratio of this to indicated horse-power is called the *mechanical efficiency*, E_m , of the engine

$$E_m = \frac{(\text{B.H.P.})}{(\text{I.H.P.})} \quad (465)$$

The difference between indicated and shaft horse-power is the *power consumed by friction* (F.H.P.). Friction under running conditions consists of two parts, one proportional to load, and the other constant and independent of load, or $(\text{F.H.P.}) = N[(\text{const.}) \times (\text{m.e.p.}) + (\text{const.})_2]$, where N is speed, revolutions per minute. But $N \times (\text{const.})(\text{m.e.p.}) = (\text{I.H.P.})K_1$ and

$$(\text{F.H.P.}) = (\text{I.H.P.})K_1 + N(\text{const.})_2, \quad (466)$$

where K_1 and $(\text{const.})_2$ are constants to be determined for the engine, whose values will change as the conditions of the engine bearing-surfaces or lubrication alters. This value for (F.H.P.) may be used to evaluate E_m ,

$$E_m = \frac{(\text{I.H.P.}) - (\text{F.H.P.})}{(\text{I.H.P.})} = 1 - K_1 - \frac{N(\text{const.})_2}{(\text{I.H.P.})}, \quad (467)$$

but indicated horse-power divided by speed is proportional to mean effective pressure, so that

$$E_m = 1 - K_1 - \frac{K_2}{(\text{m.e.p.})} \quad (468)$$

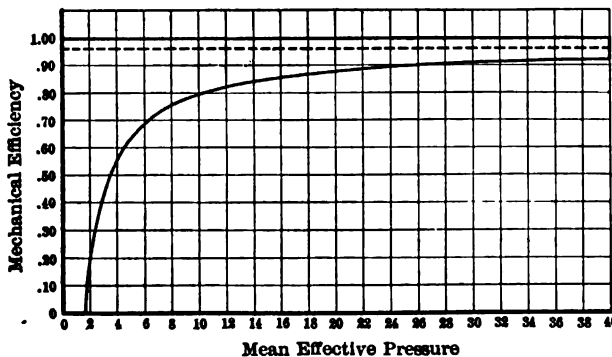


FIG. 87.—Diagram to Show Relation of Mechanical Efficiency and Mean Effective Pressure.

From this expression, speed has been eliminated, which agrees with general observation, that mechanical efficiency does not vary materially with speed. Values of the constants K_1 and K_2 may be ascertained if (m.e.p.) and E_m are known for two reliable tests covering a sufficient range, by inserting their values forming two simultaneous equations.

The numerical values of K_1 found in common practice are between .02 and .05, and for K_2 between 1.3 and 2, in some cases passing out of this range. In Fig. 87 is shown the form of mechanical efficiency curve when plotted on (m.e.p.) as abscissas, using $K_1 = .04$, $K_2 = 1.6$. It may be noted that at higher (m.e.p.) the curve does not approach unity, but the value $(1 - K_1)$

as a limit. The mechanical efficiency becomes zero for this case, at a mean effective pressure of about 1.67 pounds per square inch, which is just sufficient to keep the engine running under no load. For a given speed and size of cylinders, the abscissas may be converted into a scale of indicated horse-power.

Prob. 1. Assuming a back pressure of 10 lbs. per square inch absolute, a clearance of 8 per cent, a cut-off of 40 per cent, and compression of 20 per cent, show how (m.e.p.) varies with initial pressure over a range of 200 lbs., starting at 25 lbs.

Prob. 2. For an initial pressure of 150 lbs. per square inch absolute, show how (m.e.p.) varies with back pressure over a range of 30 lbs., starting at $\frac{1}{2}$ lb. per square inch absolute, keeping other quantities as in Prob. 1.

Prob. 3. For values of initial pressure, back pressure, etc., as given in Probs. 1 and 2, show how (m.e.p.) varies with clearance from 1 per cent to 15 per cent.

Prob. 4. For values of initial pressure, etc., as given in Probs. 1 and 2, show how (m.e.p.) will vary with cut-off from 0 to 1.

Prob. 5. For values of initial pressure, etc., as given in Probs. 1 and 2, show how (m.e.p.) will vary with compression for values from 0 to 50 per cent.

Prob. 6. A certain engine developing 675 I.H.P. at a speed of 151 R.P.M., delivered at the shaft 606 H.P. measured by an absorption dynamometer. A second test at 100 R.P.M. gave 150 I.H.P., and 114 shaft H.P. If this engine is to deliver 500 H.P. at the shaft at a speed of 150 R.P.M., what will be the I.H.P. and the mechanical efficiency?

Prob. 7. A compound Corliss engine, 25 and 52 ins. diameters, 60 ins. stroke, double-acting, was designed for 650 I.H.P. at 63 R.P.M. It was found that at this speed and I.H.P. the mechanical efficiency was 91 per cent. When running with no load, the cylinders indicated 38.1 I.H.P. at 65 R.P.M. Find the probable mechanical efficiency when developing 300 I.H.P. at a speed of 64 R.P.M.

60. Consumption of the Steam Engine and its Variation with Valve Movement and Initial Pressure. Best Cut-off as Affected by Condensation and Leakage. The weight of steam used by a steam engine per hour divided by the indicated horse-power is said to be the water rate or steam consumption of that engine. It is almost needless to say that this is not a constant for a given engine, since it will change with any change of initial pressure, back pressure, or valve setting, leakage, or temperature conditions. Since there are at least two other uses of terms, water rate or consumption, this may be termed the actual water rate, or *actual consumption*, the latter being a more general term which may refer to the weight of *fluid* used per hour per indicated horse-power, whatever the fluid may be, steam, air, carbon dioxide, or any other expansive fluid. The present discussion has special reference to steam.

From the hypothetical diagram, by computations such as are described for the various foregoing cycles, may be obtained a quantity representing the weight of fluid required to develop one horse-power for one hour, by the performance of the hypothetical cycle. This may be termed the *hypothetical consumption* or for steam cycles the hypothetical water rate.

By the use of the actual indicator card, the weight of fluid accounted for

by volumes and pressures known to exist in the cylinder may be obtained, by methods still to be described, this being called the *indicated consumption* of the engine or indicated water rate if the fluid be steam.

The heat analysis of the steam-engine cycle will lead to another standard of comparison which is of the greatest importance as a basis of determining how nearly the actual performance approaches the best that could be obtained if the engine were to use all available energy possessed by the steam. At present the object is to compare the actual and indicated performance with that hypothetically possible with cylinders of the known size. Accordingly attention will be confined first to *hypothetical consumption*, and the quantities upon which it is dependent.

For Cycle III, which is the most general for the single-expansion engine, logarithmic law, the expression for consumption in pounds fluid per hour per indicated horse-power, found in Section 41, Eq. (189), is as follows:

Hypothetical consumption, lbs. per hr. per I. H. P.

$$= \frac{13,750}{(\text{m.e.p.})} \left[(Z+c) - (X+c) \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right) \right] \delta_1,$$

in which the value of mean effective pressure itself depends upon (in.pr.), (bk.pr.), c , Z , and X . The density of the fluid at initial pressure, δ_1 , is to be ascertained from tables of the properties of steam or of whatever fluid is used.

In Fig. 88 are the results of computations on the hypothetical steam consumption, using mean effective pressures as plotted in Fig. 85. For each curve, conditions are assumed to be as stated on the face of the diagram, varying only one of the factors at a time.

Other conditions remaining unchanged, it may be noted that consumption decreases for an increase of initial pressure, though not rapidly in the higher pressure range.

Cut-off has a marked effect upon consumption, the minimum occurring when cut-off is such as to give complete expansion. This occurs when

$$\frac{1+c}{Z'+c} = \frac{(\text{in.pr.})}{(\text{bk.pr.})}, \quad \text{or} \quad Z' = (1+c) \frac{(\text{bk.pr.})}{(\text{in.pr.})} - c, \quad \dots \quad (469)$$

which may be termed *hypothetically best cut-off*. In the case assumed in the

diagram, $Z' = (1+.1) \frac{15}{100} - .1 = .065$.

If clearance be varied, maintaining constant compression and cut-off, large clearance will give high consumption due to an excessive quantity of fluid required to fill the clearance space. Extremely small clearance leads to a high pressure at the end of compression, causing a loss of mean effective pressure, and consequent high consumption. Between, the consumption has a minimum point, which is dependent for its location on both cut-off and compression.

Decreasing back pressure has a beneficial effect upon mean effective pressure and consumption. This would be still more marked in the figure if a case had been selected with a very short cut-off.

Compression, throughout the ordinary range of practice, has but slight effect upon consumption, indicated by the flat middle portion of the curve in Fig. 88. Very small or zero compression permits too much high-pressure steam to be admitted to the clearance space without doing work, and excessively large clearance causes pressures during compression to rise very high, thereby decreasing mean effective pressure; hence this curve of consumption rises at both ends.

Hypothetically, the best attainable consumption for given initial and back pressures is obtained when both expansion and compression are complete.

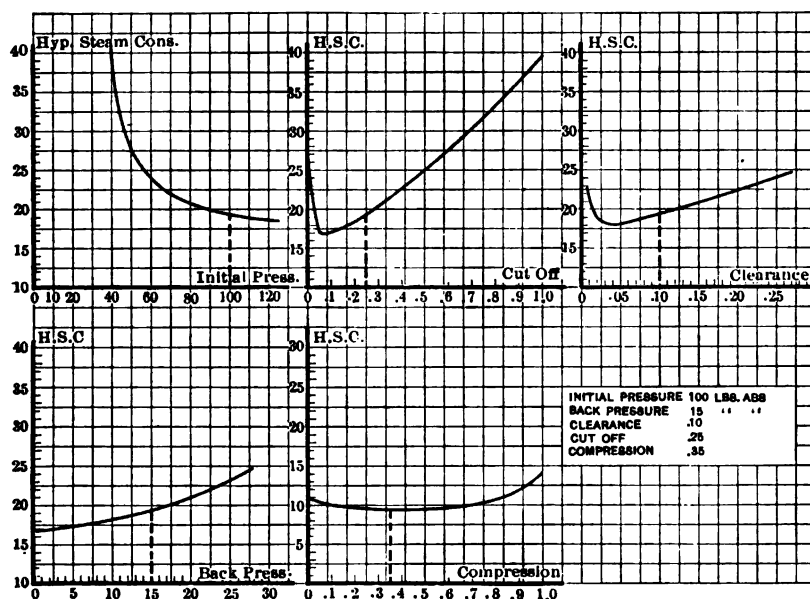


FIG. 88.—Curves to Show the Variation of Hypothetical Steam Consumption of Simple Engines, Logarithmic Expansion and Compression.

The *indicated consumption*, or, as it is frequently called for the steam engine, "steam accounted for by the indicator card" or "indicated water rate," is determined from the indicator card as follows. Let Fig. 89 represent an indicator diagram. The points of cut-off and compression are located from the form of the line, at the highest point on the expansion line and the lowest point on the compression line respectively. The fraction of the card lengths completed at

cut-off, $Z = \frac{\overline{AB}}{\overline{AD}}$, and the fraction of card length from point of compression to

end of stroke, $X = \frac{\overline{AC}}{\overline{AD}}$, are determined, the pressure at cut-off and compression

measured by the proper vertical scale, and the corresponding densities, δ_1 , and δ_2 respectively, are ascertained from steam tables for dry saturated

steam. Clearance, Cl , is known or ascertained by the form of the compression curve (Chap. I, Section 12). D equals displacement of piston in cubic feet.

At the point of cut-off, the weight of dry saturated steam present in the cylinder is $D(Z+c)\delta_1$, and at compression the weight present is $D(X+c)\delta_2$, on the assumption that the steam in the cylinder is of density δ_1 and δ_2 at these two instants. Accepting this assumption, the weight of steam used per cycle is

$$\text{Wt. steam per cycle} = w = [(Z+c)\delta_1 - (X+c)\delta_2]D. \quad (470)$$

The work per cycle $W = 144D(\text{m.e.p.}),$

and for n cycles per minute, $\text{I.H.P.} = \frac{144nD(\text{m.e.p.})}{33,000}.$

The *indicated consumption* is then, in pounds per hour per I.H.P.

$$\frac{60wn}{\text{I. H. P.}} = \frac{60 \times 33,000 \times D[(Z+c)\delta_1 - (X+c)\delta_2]n}{144nD(\text{m.e.p.})}, \quad \text{or,}$$

$$\text{Ind. consumption, lbs. per hr. per I.H.P.} = \frac{13,750}{(\text{m.e.p.})}[(Z+c)\delta_1 - (X+c)\delta_2], \quad (471)$$

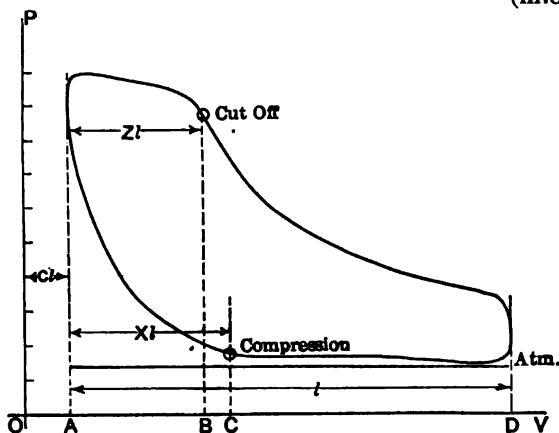


FIG. 89.—Diagram to Illustrate Method of Determining Indicated Water Rate of Steam Engine.

which is the expression used to find indicated consumption for either simple or multiple-expansion engines. In applying this to the *multiple-expansion engine* the terms Z , X and c are found for any one cylinder, and the mean effective pressure is referred to that cylinder. There may be, therefore, as many computations as there are expansion stages. For a compound engine, for instance, indicated consumption according to high-pressure card is found

$$(\text{m.e.p. ref. to H.P.}) = (\text{m.e.p.})_H + (\text{m.e.p.})_L \frac{D_L}{D_H}. \quad (472)$$

If the computation is done by means of events on the low-pressure card, the (m.e.p.) must be referred to the low.

$$(\text{m.e.p. ref. to L.P.}) = (\text{m.e.p.})_H \frac{D_H}{D_L} + (\text{m.e.p.})_L. \quad (473)$$

In general for a multiple-expansion engine

$$(\text{m.e.p. ref. to cyl. A}) = \Sigma(\text{m.e.p.}) \frac{D}{D_A}. \quad (474)$$

It is often difficult and sometimes impossible to determine the point of cut-off and of compression on the indicator card. The expansion and compression lines are very nearly hyperbolic in form and are usually recognizable. The highest point on the hyperbolic portion of the expansion line is regarded as cut-off, and the lowest point on the hyperbolic portion of the compression line, as the point of compression. Since the point of cut-off is thus more or less indeterminate the A. S. M. E. Committee on Standard Rules recommend that the cut-off be defined in exact terms for commercial purposes as follows: Unless otherwise specified the *commercial cut-off* is to be ascertained by drawing a line through the point of maximum pressure parallel to the atmospheric line; through the point on the expansion line near the actual cut-off draw a hyperbolic curve: the point where these two lines intersect is to be considered the commercial cut-off point. The commercial cut-off as thus determined is usually at an earlier point in the stroke than the beginning of the real expansion line, as shown in Fig. 86 and as described on page 257. It must be understood that by reason of the condensation and re-evaporation of steam in cylinders the weight of steam proper is not constant throughout the stroke, so that calculations like the above will give different values for every different pair of points chosen. The most nearly correct results are obtained when steam is just dry and these points are at release and compression.

When under test of actual engines the steam used is condensed and weighed and the indicated horse-power determined, then the *actual steam consumption* or water rate can be found by dividing the weight of water used per hour in the form of steam by the indicated horse-power. This actual water rate is always greater than the water rate computed from the equation for indicated consumption. The reasons for the difference have been traced to (a) leakage in the engine, whereby steam weighed has not performed its share of work, to (b) initial condensation, whereby steam supplied became water before it could do any work, (c) variations in the water content of the steam by evaporation or condensation during the cycle, whereby the expansion and compression laws vary in unpredictable ways, affecting the work.

Estimation of probable water rate or steam consumption of engines cannot, therefore, be made with precision except for engines similar to those which have been tested, in all the essential factors, including, of course, their condition, and for which the deficiencies between actual and indicated consumptions have been determined. *This difference is termed the missing water*, and endless values for it have been found by experiments, but no value is of any use except when it is found as a function of the essential variable conditions that cause it. No one has as yet found these variables which fix the form for an empiric formula for missing water nor the constants which would make such a formula useful, though some earnest attempts have been made. This is no criticism of the students of the problem, but proof of its elusive nature, and the reason is probably to be found in the utter impossibility of expressing by a formula the leakage of an engine in unknown condition, or the effect of its condition and local situation on involuntary steam condensation and evapora-

tion. It is well, however, to review some of these attempts to evaluate missing water so that steam consumption of engines may be estimated. After studying the many tests, especially those of Willans, Perry announced the following for non-condensing engines, in which the expansion is but little,

$$\frac{\text{Missing water}}{\text{Indicated steam}} = m \frac{1 + \frac{1}{Z}}{d\sqrt{N}}, \dots \dots \dots (475)$$

where d is the diameter of the cylinder in inches and N the number of revolutions per minute. This indicates that the missing steam or missing water has been found to increase with the amount of expansion and decrease with diameter of cylinder and the speed. Thermal and leakage conditions are met by the use of different values of m , for which are given,

- $m=5$ for well-jacketed, well-drained cylinders of good construction with four poppet valves, that is, with minimum leakage and condensation.
- $m=30$ or more for badly drained unjacketed engines with slide valves, that is, with high leakage and condensation possibilities.
- $m=15$ in average cases.

For condensing engines Perry introduces another variable—the initial pressure pounds per square inch absolute, p , giving

$$\frac{\text{Missing water}}{\text{Indicated steam}} = \frac{120 \left(1 + \frac{1}{Z}\right)}{d\sqrt{np_1}} \dots \dots \dots (476)$$

It might seem as if such rules as these were useless, but they are not, especially when a given engine or line of engines is being studied or two different engines compared; in such cases actual conditions are being analyzed rather than predictions made, and the analysis will always permit later prediction of considerable exactness, if the constants are fixed in a formula of the right empiric form. Similar study by Heck has resulted in a different formula involving different variables and constants, but all on the assumption that the discrepancies are due to initial condensation. He proposes an expression equivalent to

$$\frac{\text{Missing steam}}{\text{Indicated steam}} = \frac{.27}{\sqrt[3]{N}} \sqrt{\frac{S(x_2 - x_1)}{p_1 Z}}, \dots \dots \dots (477)$$

in which S = the ratio of cylinder-displacement surface in square feet to displacement in cubic feet.

$$= \frac{2 \times \frac{\pi \left(\frac{d}{12}\right)^2}{4} + \pi \frac{d}{12} L}{\frac{\pi \left(\frac{d}{12}\right)^2 \times L}{4}} = \frac{2}{L} + \frac{d}{48}$$

N = R.P.M. of the engine; d = diameter in inches; L = stroke in feet;

The term $(x_2 - x_1)$ is a constant supposed to take into account the amount of initial condensation dependable on the difference between cylinder wall and live-steam temperature and is to be taken from a table found by trial as the difference between the x for the high pressure and x for the low pressure, both absolute, see Table XIX, Handbook of Tables.

In discussing the hypothetical diagrams, it was found that best economy was obtained with a cut-off which gives complete expansion. For other than hypothetical diagrams this is not true, which may be explained most easily by reference to the curves of indicated, and actual consumption, and missing steam, Fig. 90.

The curve ABC is the hypothetical consumption or water rate for a certain steam engine: its point of best economy occurs at such a cut-off, B , that expansion is complete. The curve GHI is computed by Heck's formula for missing water; this shows a falling off for greater cut-offs. Adding ordinates of these two curves, the curve DEF for probable consumption is found. The minimum point in this curve, E , corresponds to a longer cut-off than that of ABC . Since cut-off B gave complete expansion, cut-off E must give incomplete expansion. In other words, due to missing steam, the condition which really gives least steam consumption per hour per indicated horse-power corresponds to a release pressure, which is greater than the back pressure.

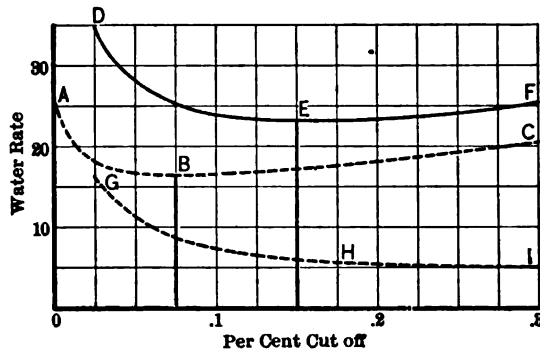


FIG. 90.—Diagram to Show Displacement of Best Cut-off Due to Effect of Missing Water from Point B for the Hypothetical Cycle to Some Greater Value E .

It should be noted that the minimum point mentioned above will not be *best* cut-off, for the output of the engine is not *indicated*, but *brake* horse-power.

In Fig. 91 on cut-off as abscissa are plotted (EFG) consumption pounds per hour per I.H.P., and for the case assumed, (OD) the curve of mechanical efficiency, based on cut-off,

$$\frac{(\text{lbs. steam per hr.})}{\text{I.H.P.}} \div \frac{\text{B.H.P.}}{\text{I.H.P.}} = \frac{(\text{lbs. steam per hr.})}{\text{B.H.P.}}, \text{ or, in other words,}$$

$$\frac{\text{Consumption, lbs. per hr. per I.H.P.}}{E_m} = \text{Consumption, lbs. per hr. per B.H.P.} \quad (478)$$

Owing to the increasing value of E_m for greater cut-offs, the minimum point B corresponds to a cut-off still longer than for the curve EFG , which itself was found in Fig. 90 to give a longer cut-off than that of the hypothetical curve.

Hence the best cut-off for economy of steam, where the net power at the shaft is regarded as the output, will be such as to give incomplete expansion,

or a release pressure above back pressure, this effect being caused by both missing steam and by frictional losses.

Prediction of actual consumption of steam engines as a general proposition is almost hopeless if any degree of accuracy worth while is desired, though the effect on steam consumption of changing the value of any one variable can be fairly well determined by the previous discussion *qualitatively*, that is, in kind, though not quantitatively in amount. Probably the best attempt is that of Hrabak in German, which takes the form of a large number of tables developed from actual tests though not for engines of every class. These tables

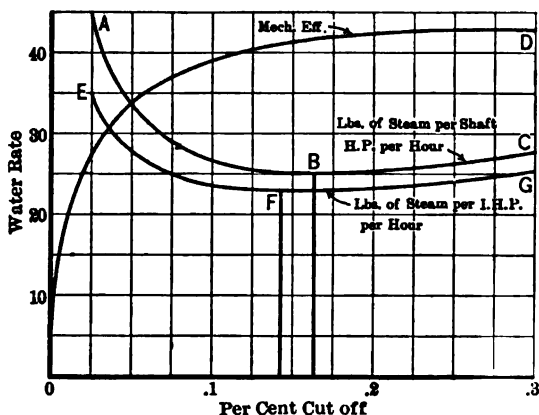


FIG. 91.—Diagram to Show Relation of Steam per Hour per I.H.P. and per B.H.P. to Cut-off.

are too extensive to permit any abstraction of value.

There is, however, a sort of case of steam consumption prediction that can be carried out with surprising precision, and that is for the series of sizes or line of engines manufactured by one establishment all of one class, each with about the same grade of workmanship and degree of fit, and hence having *leakage* and *cylinder condensation* characteristics that vary consistently throughout

the whole range. For such as these, tables and curves of missing water are determined by the best builders, for making guarantees of steam consumption for any service conditions that their engines are able to meet.

The practice of one firm making what is probably the best line of stationary engines in this country is of sufficient interest to warrant description. The primary data are curves of indicated water rate plotted to mean effective pressure for clearances of three or four per cent, and that mean effective pressure is chosen in any one specific case that will give the horse-power desired at the fixed speed for some one set of cylinder sizes available. To this indicated water rate a quantity is added constituting the missing water which is made up of several parts as follows: The first is an addition representing condensation which is plotted in curve form as a function of (a) boiler pressure, (b) superheat in the steam, (c) piston speed, (d) the class of engine—simple, compound or triple, with jacketed or unjacketed cylinders, and for cylinder ratios from 4 to 1, to 6 to 1 in the case of compounds. It is therefore a most complex quantity, the nature of the variations in which can only be indicated here.

For example, increase of piston speed decreases the condensation loss as does multiple expansion, and also jacketing, while increase of superheat in the steam also decreases it, but superheat has less effect in triple than in compounds and less in compound than in simple engines.

The next factor of correction is that covering leakage losses, also additive to indicated water rate, which with it and the condensation loss make up the probable steam consumption. The leakage decreases regularly with increase of piston speed; it is less for large than for small engines, is quite marked from 50 to 200 horse-power, and much less for larger engines, being scarcely appreciable for engines over 2000 horse-power.

Example 1. What cut-off will give the lowest indicated water rate for 9×12-in. engine, with 5 per cent clearance and no compression when running non-condensing on an initial pressure of 100 lbs. per square inch gage, and what will be the value of this water rate? What steam will be used per hour per brake horse-power hypothetically? From Eq. (469)

$$Z' = (1+c) \frac{\text{bk.pr.}}{\text{in.pr.}} - c = (1+.05) \frac{15}{115} - .05 = 8.7 \text{ per cent,}$$

$$\text{and } (\text{m.e.p.}) = 115 \left[.087 \times (.087 + .05) \log_e \frac{1.05}{.137} \right] - 15 = 27.2 \text{ lbs. sq.in.}$$

$$\text{Hence, steam per hour per I.H.P.} = \frac{13,750}{27.2} \left[.137 - .05 \times \frac{15}{115} \right] \times .262 = 17.2 \text{ lbs.}$$

From the curve of Fig. 87, assuming it to apply to the engine, for this value of (m.e.p.) mechanical efficiency is 90 per cent, hence from Eq. (478) the weight of steam per shaft horse-power per hour will be 19.1 pounds.

Prob. 1. Draw diagram similar to Fig. 88 for following case:

Initial pressure, 135 lbs. per square inch gage, back pressure 10 lbs. per square inch absolute, clearance 5 per cent, cut-off 30 per cent, compression 25 per cent.

Prob. 2. From indicator diagram shown in Fig. 86, find the indicated water rate to the engine from which it was taken.

Prob. 3. The indicated water rate of a 9×12-in. jacketed engine when running non-condensing at a speed of 250 R.P.M. with an initial pressure of 100 lbs. per square inch gage and $\frac{1}{2}$ cut-off is 50 lbs. Using Perry's formula, what will be the probable actual steam used by engine per horse-power hour?

Prob. 4. A 24×48-in. engine in good condition is found to have an indicated water rate of 25 lbs. when cut-off is $\frac{1}{2}$, initial pressure 100 lbs. per square inch gage, back pressure 10 lbs. per square inch absolute, and speed of 125 R.P.M. What will be the missing water, and the rate as found by Perry's formula and by Heck's?

Prob. 5. What will be the probable amount of steam used per hour by a 36×48-in. engine with 5 per cent clearance running at 100 R.P.M. on an initial pressure of 150 lbs. per square inch gage, a back pressure of 5 lbs. per square inch absolute, $\frac{1}{2}$ cut-off and 10 per cent compression?

Prob. 6. How will the amount of steam of Prob. 5 compare with that used by a 15×22×36-in. engine with 5 per cent clearance in each cylinder, running at 100 R.P.M. on same pressure range with $\frac{1}{2}$ cut-off in high-pressure cylinder, $\frac{1}{2}$ cut-off in low, and 10 per cent compression in each cylinder?

61. Variation of Steam Consumption with Engine Load. The Willans Line. Most Economical Load for More than One Engine and Best Load Division. However valuable it may be to the user of steam engines to have an engine that is extremely economical at its best load which, it should be noted, may have any relation to its *rated* horse-power, it is more important

usually that the *form* of the economy load curve should be as flat as possible, and this is always true when the engine must operate under a wide range of load. This being the case it is important to examine the real performance curves of some typical engines all of which have certain characteristic similarities as well as differences.

From the discussion of hypothetical and indicated water rates it appears that the curve of steam consumption (vertical) to engine load (horizontal) is always concave upward and always has a minimum point, not at the maximum load. Actual consumption curves are similar in general form, but as has been pointed out, the load at which the water rate is least corresponds to some greater mean effective pressure than that for the hypothetical, so

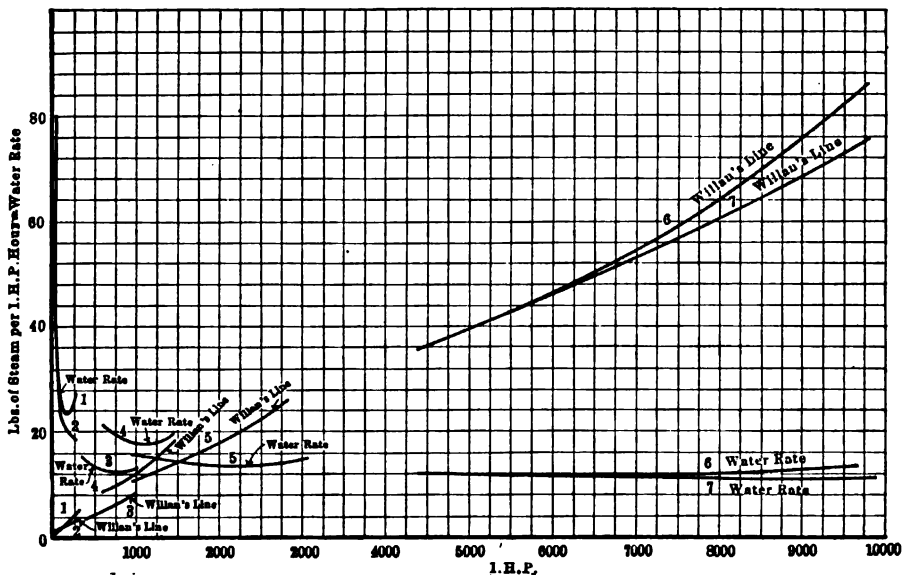


FIG. 92.— Typical Water Rate—Load Curves and Willans Lines for Steam Engines.

the whole curve is displaced upwards and to the right by reason of cylinder condensation and leakage losses. This displacement may be so great as to prevent the curve rising again beyond the minimum point, in which case the least steam consumption corresponds to the greatest load. Just what form the actual water rate-load curve will take depends largely on the form of valve gear and type of governing method in use: by throttling initial pressure with a fixed cut-off or, by varying cut-off without changing initial pressure, with or without corresponding changes in the other valve periods.

Whenever the control of power is by throttling of the supply steam the curve is found to be almost exactly an hyperbola, so that (water rate \times horse-power) plotted to horse-power is a straight line which being characteristic is much used in practical work and is known as the *Willans line*. All other engines, that is, those that govern on the cut-off, have Willans lines that

are *nearly* straight, such curvature as exists being expressed by a second degree equation instead of one of the first degree.

Equations for Willans lines can always be found for the working range of load, that is, from about half to full load, though not for the entire range, except in unusual cases, and these equations are of very great value in predicting the best division of load between units, which is a fundamental step in deciding how many and what sizes of engine to use in carrying a given load in industrial power plants.

Before taking up the derivation of equations some actual test curves will be examined and a number of these are grouped in Fig. 92 for engines of various sizes, simple and compound, up to 10,000 H.P., on which vertical

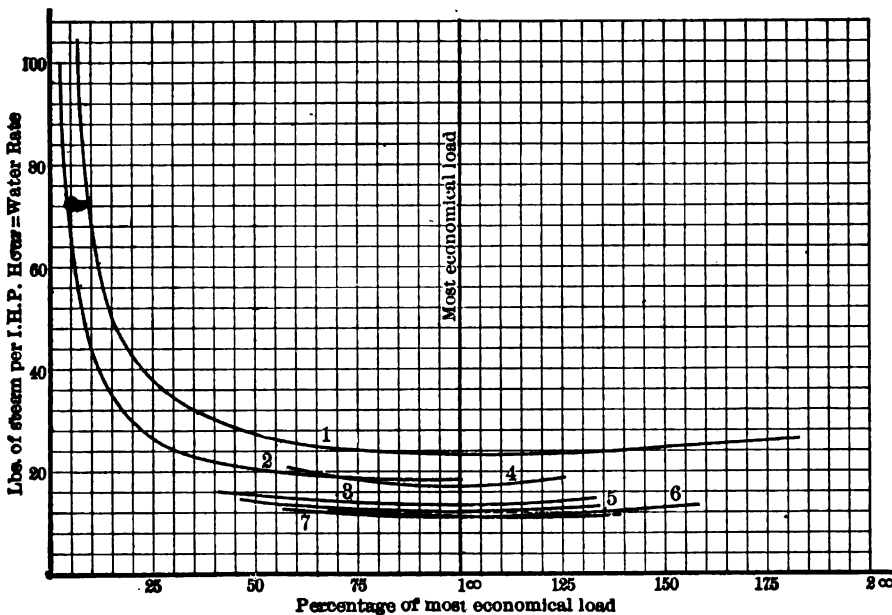


FIG. 93.—Typical Water Rate—Load Curves for Steam Engines Plotted to Fractional Loads.

distances represent pounds of steam per hour per I.H.P., and horizontal, I.H.P. To show the essential similarity of the curves for engines of different size more clearly, these are re-plotted in Fig. 93 to a new load scale based on best load of *each*, which is taken as unity. *This is evidently a function of mean effective pressure*, just what sort of function does not matter here. In every case the Willans line is also plotted in Fig. 92, each line being numbered to correspond to its water rate curve.

As there is a corresponding similarity of form for the water rate and Willans line for steam turbines, though the reasons for it will be developed later, it must be understood that the mathematical analysis that follows applies to both turbine and piston steam engines, and further it makes no difference what units are used for load, whether I.H.P. or B.H.P. or K.W. of a direct-connected electric generator.

In Fig. 94 is shown the water-rate curve to a K.W. base for the 10,000 K.W. Curtis steam turbine at the Chicago Edison Fiske Street Station, for which the following equation fits exactly: $\frac{Y}{P} = \frac{17.02}{P} + 10.54 + .156P$,

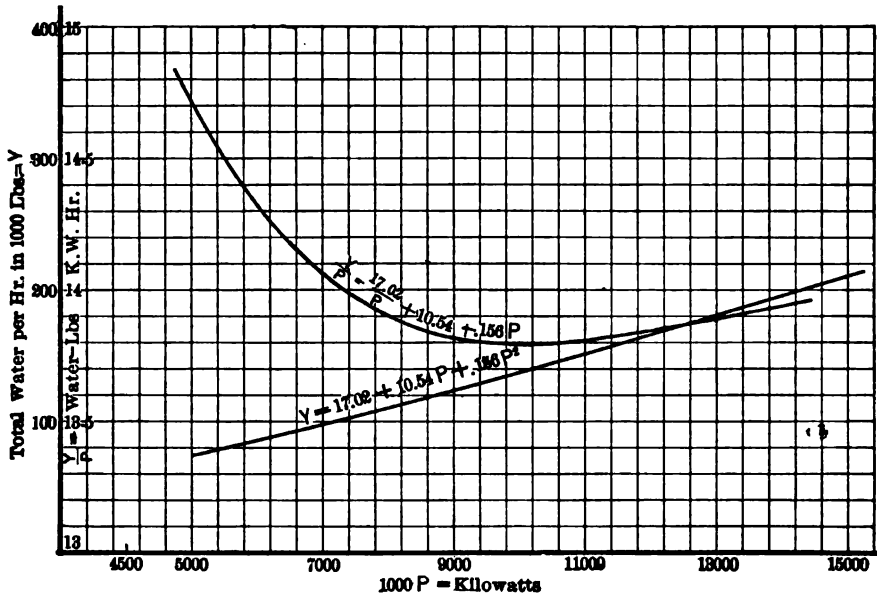


FIG. 94.—Performance of a 10,000 K.W. Steam Turbine.

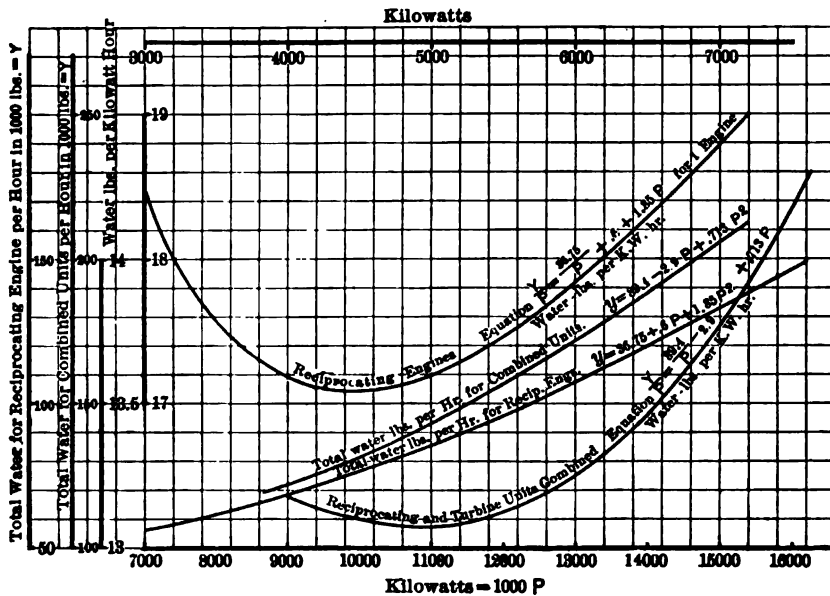


FIG. 95.—Performance of a 7000 H.P. Piston Engine alone and with a Low-pressure Steam Turbine.

where Y = pounds of steam per hour $\div 1000$;
 P = load (in this case in K.W.) $\div 1000$;
 $\frac{Y}{P}$ = pounds of steam per K.W. hr.

A similar equation fits fairly well the curve of Fig. 95, representing the 7000 H.P. piston engines of the Interborough Railway, Fifty-ninth Street station, as well as the combined piston engine and low-pressure steam turbine taking its exhaust steam, in the same station, but with different numerical

constants, as below: Piston engine, $\frac{Y}{P} = \frac{36.75}{P} + .6 + 1.85P$,

Combined piston engine and turbine, $\frac{Y}{P} = \frac{89.4}{P} - 2.90 + .713P$.

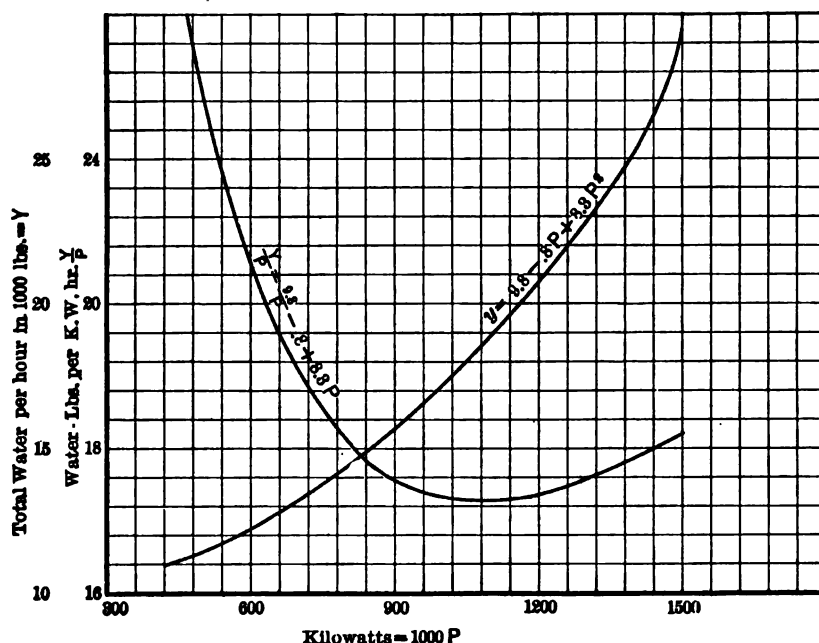


FIG. 96.—Performance of a 1000 K.W. Corliss Engine.

A third case of smaller size is shown in Fig. 96, representing the performance of a 1000 K.W. Corliss piston engine driving a generator for which the equation is

$$\frac{Y}{P} = \frac{9.8}{P} - .8 + 8.3P.$$

These illustrations could be multiplied indefinitely, but those given will suffice to establish the fact that the two following equations are fundamental *over the working range* of any steam engine of whatever type:

Water rate line,
$$\frac{Y}{P} = \frac{A}{P} + B + CP. \quad (479)$$

Water per hour, Willans line, $Y = A + BP + CP^2$, (480)

in which Y is the weight of steam per hour and P the engine load whether expressed in indicated or brake horse-power, or in kilowatts.

At the most economical load the water rate is a minimum, so that

$$\frac{d}{dP}\left(\frac{Y}{P}\right) = 0 = \frac{d}{dP}\left(\frac{A}{P} + B + CP\right)$$

whence the most economical load is $P' = \sqrt{\frac{A}{C}}$ (481)

Where the Willans line is straight, $C=0$, and the most economical load is the greatest load.

Two engines carrying the same load must divide it and some one proportion may be best.

It can be shown that for *similar engines, the best division of load is an equal division.*

When the engines are dissimilar it is convenient to first consider the case of straight Willans lines for which $C=0$. Then for *two dissimilar engines the best division of load is that which puts the greatest possible share on the one with the smaller value of B , in its equation, provided each has a straight Willans line.*

Two dissimilar engines of whatever characteristics yield the equation,

$$\begin{aligned} Y &= A_1 + A_2 + B_1P_1 + B_2P_2 + C_1P_1^2 + C_2P_2^2 \\ &= (A_1 + A_2 + B_1P + C_1P^2) \\ &\quad + (B_2 - 2PC_1 - B_1)P_2 \\ &\quad + (C_1 + C_2)P_2^2. \end{aligned}$$

Differentiating with respect to P_2 , and solving for P_2 , the load for the second engine that makes the whole steam consumption least, gives,

$$P_2 = \frac{B_1 - B_2}{2(C_1 + C_2)} + \frac{C_1P}{(C_1 + C_2)} = \text{constant} + \text{constant} \times P. \quad . . . (482)$$

Therefore, *the load division must be linear* and Eq. (482) gives the numerical value, *when any two engines share a given load.*

This sort of analysis can be carried much further by those interested, but space forbids any extension here. It is proper to point out, however, that by means of it the proper *switch-in* points for each unit in a large power station can be accurately found, to give most economical operation on an increasing station load.

62. Graphical Solution of Problems on Horse-power and Cylinder Sizes.

The solution of problems involving horse-power and cylinder sizes may be greatly facilitated by means of graphical charts or diagrams which will be found in the Handbook of Tables. The diagram of mean effective pressures in terms of initial and back pressures, clearance, compression and cut-off, may be satisfactorily used in the solution of Eq. 184, Section 41, in determining the mean effective pressure for single cylinder engines with clearance and logarithmic expansion and compression.

The mean effective pressure is the difference between mean forward and mean back pressure. The former is dependent upon clearance, cut-off and initial pressure. To illustrate the use of this diagram, Chart No. (9), assume the following:

Example 1. Given clearance 5 per cent, compression 25 per cent, back pressure 2.5 lbs. per square inch absolute, cut-off 12 per cent, initial pressure 115 lbs. absolute. On the right of the diagram the clearance corresponding to 5 per cent is shown at point *A*. Project horizontally to the point *F*, on the contour line for the assumed cut-off, 12 per cent. Project downward to the logarithmic scale for "mean forward pressure in terms of initial pressure" to the point *G*. On the scale for "initial pressure" find the point *H*, representing the assumed initial pressure, 115 lbs. absolute. Through *G* and *H* a straight line is passed to the point *K* on the scale for "mean forward pressure," where the value is read, m.f.p. = 49.5 lbs. absolute.

Mean back pressure is similarly dependent upon clearance, compression and back pressure, and the same process is followed out by the points *A*, *B*, *C*, *D* and *E*, reading the mean back pressure, 3.2 lbs. absolute at the point *E*. Then by subtraction,

$$(\text{m.e.p.}) = (\text{m.f.p.}) - (\text{m.b.p.}) = 49.5 - 3.2 = 46.3 \text{ lbs.}$$

Chart No. (10) is arranged to show what conditions must be fulfilled in order to obtain equal work with *complete expansion in both cylinders* in a compound engine, finite receiver, logarithmic law, no clearance, Cycle VII, when low-pressure admission and high-pressure exhaust are not simultaneous. This is discussed in Section 47, and the diagram represents graphically the conditions expressed in Eqs. (283) to (286).

Example 2. To illustrate its use, assume that in an engine operating on such a cycle, the volume of receiver is 1.5 times the high-pressure displacement, $1.5 = y$, then $\frac{1}{y} = .667$. Locate the point *A* on the scale at bottom of diagram, corresponding to this value. Project upward to the curve marked "ratio of cut-offs" and at the side, *C*, read ratio of cut-offs, $\frac{Z_H}{Z_L} = .572$. Next, extending the line *AB* to its intersection, *D*, with the curve *GH*, the point *D* is found. From *D* project horizontally to the contour line representing the given ratio of initial to back pressure. In this case, initial pressure is assumed ten times the back pressure. Thus the point *E* is located. Directly above *E* at the top of the sheet is read the cylinder ratio, at *F*, $R_C = \frac{D_L}{D_H} = 2.4$.

If cylinder ratio and initial and final pressures are the fundamental data of the problem, the ratio of cut-offs and ratio of high-pressure displacements to receiver volume may be found by reversing the order.

Prob. 1. By graphical means find the (m.e.p.), of a $15 \times 22 \times 30$ -in. cross-compound engine, with 5 per cent clearance in each cylinder, if the receiver volume is 8 cu.ft., initial pressure 125 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute, high-pressure cut-off $\frac{1}{4}$, low-pressure $\frac{1}{8}$, high-pressure compression 40 per cent, low-pressure 20 per cent, high-pressure crank following 90° , logarithmic expansion.

Prob. 2. Show by a series of curves, assuming necessary data, the effect on (m.e.p.) of cut-off, back pressure, clearance, and compression.

Prob. 3. Show by curves, how the indicated, and actual water rate of an 18×24 -in. engine with 5 per cent clearance, running at 125 R.P.M. on an initial pressure of 125 lbs. per square inch gage, and a back pressure of 10 lbs. per square inch absolute may be expected to vary with cut-off from $\frac{1}{10}$ to $\frac{3}{4}$.

PART II

EFFECT OF CHANGE IN HEAT CONTENT OF SUBSTANCE.

CHAPTER VII

EFFECTS OF CHANGES IN HEAT CONTENT OF SUBSTANCES WITHOUT CHANGE OF STATE. TEMPERATURE, PRESSURE, DENSITY.

63. Substances and Heat Effects Important in Engineering. It has been shown in preceding chapters concerned with work in general and with the determination of quantity of work that may be done in the cylinders by, or on expansive fluids that

(a) Fluids originally at low may be put in a high-pressure condition by the expenditure of work;

(b) Fluids under high pressure may do work in losing that pressure.

That work may be done, fluids under pressure are necessary and that the greatest amount of work may be done per unit of fluid the fluid itself must be expansive, that is, it must be a gas or a vapor. Gases or vapors under pressure are, therefore, prerequisites to the economical use of fluids for the doing of work, and that this work may be done at the expense of heat or derived from heat, it is only necessary that the heat be used to create the necessary primary condition of high pressure in vapors and gases. There are two general ways of accomplishing this purpose—first, to apply the heat to a boiler supplied with liquid and discharging its vapor at any pressure as high as desired or as high as may be convenient to manage; second, to apply the heat to a gas confined in a chamber, raising its pressure if the chamber be kept at a fixed volume, which is an intermittent process, or increasing the fluid volume if the size of the chamber be allowed to increase, the fluid pressure being kept constant or not; this latter process may be intermittently or continuously carried out.

These two processes are fundamental to the steam and gas engines that are the characteristic prime movers or power generators of engineering practice, utilizing heat energy, and with the exception of water-wheels, the sole commercially useful sources of power of the industrial world. Thus, the heating of gases and the evaporation of liquids are two most important thermal processes to be examined together with their inverse, cooling and condensation, and necessarily associated in practical apparatus with the heating and cooling of solid containers or associated liquids. From the power standpoint, the effects of heat on solids, liquids, gases and vapors, both without change of state and with change of state, are fundamental, and the substances to be studied as heat carriers

do not include the whole known chemical world, but only those that are cheap enough to be used in engineering practice or otherwise essential thereto. The substances of supreme importance are air and water, with all their physical and chemical variations; next the fuels, coal, wood, oil, alcohol and combustible gases, together with the chemical elements entering into them, and the chemical compounds which mixed together may constitute them.

Probably next in importance from the standpoint of engineering practice, are the substances and thermal processes entering into mechanical refrigeration and ice making. There are but three substances of commercial importance here—ammonia, pure and in dilute aqueous solution, carbonic acid and air. The process of heating or cooling solids, liquids, gases and vapors, together with solidification of water into ice, evaporation and condensation, fundamental to power problems, are also of equal importance here, but there is added an additional process of absorption of ammonia vapor in water and its discharge from the aqueous solution.

Many are the practical applications of heat transfer or transmission, some of which call into play other substances than those named. In the heating of buildings there is first combustion with transfer of heat to water in boilers, flow of the hot water or steam produced to radiators, and then a transfer of heat to the air of the room; in feed-water heaters, heat of exhaust steam warms water on its way to the boilers; in economizers, heat of hot flue gases is transferred to boiler feed water; in steam superheaters, heat of hot flue gases is transferred to steam previously made, to raise its temperature; steam pipes, boiler surfaces and engine cylinders transfer heat of steam to the air which is opposed by covering and lagging; in steam engine condensers heat of exhaust steam is transferred to circulation water; in cooling cold storage rooms and making ice, a solution of calcium or sodium chloride in water is circulated through pipes and tanks and is itself kept cool in brine coolers in which the brine transfers the heat absorbed in the rooms and tanks, to the primary substance, ammonia or carbonic acid, and evaporates it.

While evaporation and condensation as processes are fundamental to the machinery and apparatus of both power and refrigeration, they also are of importance in certain other industrial fields. In the concentration of solutions to promote crystallization such, for example, as sugar, evaporation of the solution and condensation of the distillate, are primary processes; this also is the case in making gasoline and kerosene from crude oil, in making alcohol from a mash, and many other cases found principally in chemical manufacture. These are examples of evaporation and condensation in which little or no gases are present with the vapor but there are other cases in which a gas is present in large proportion, the thermal characteristics of which are different, as will be seen later. Among these processes are: the humidification or moistening of air with water in houses and factories to prevent excessive skin evaporation of persons breathing the air, and excessive shrinkage of wood-work, to facilitate manufacturing processes such as tobacco working and thread spinning. Conversely, air may be too moist for the purpose, in which case it

is dried by cooling and precipitating its moisture as rain or freezing it out as ice; this is practiced in the Gayley process of operating blast furnaces, where excess of moisture will on dissociating absorb heat of coke combustion and reduce the iron output per ton of coke; and in the factories where, for example, collodion is worked, as in the manufacture of photographic films, with which moisture seriously interferes. Humidification of air by water is accomplished only by evaporation of water, and evaporation of water is only to be accomplished by the absorption of heat, so that humidification of air by blowing it over water or spraying water into it must of necessity cool the water; this is the principle of the cooling tower or cooling pond for keeping down the temperature of condenser circulating water, and likewise the principle of the evaporative condenser, in which water cooler and steam condenser are combined in one. The same process, then, may serve to cool water, if that is what is wanted, or to moisten air, when dry air is harmful, and may also serve to remove moisture from solids such as sand, crystals, fabrics, vegetable or animal matter to be reduced to a dryer or a pulverized state.

There are some important examples of humidification in which the substances are not air and water, and one of these is the humidification of air by gasolene or alcohol vapor to secure explosive mixtures for operating gas engines. Here the air vaporizes enough of the fuel, humidifying or carbureting itself to serve the purpose, sometimes without heat being specifically added and sometimes with assistance from the hot exhaust. A somewhat similar action takes place in the manufacture of carbureted water gas when the water gas having no illuminating value is led to a hot brick checkerwork chamber supplied with a hydrocarbon oil, the vapor of which humidifies the gas, the heat of vaporization being supplied by the hot walls and regularly renewed as the process is intermittent. In this case some of the vapors may really decompose into fixed gases, a peculiar property of the hydrocarbon fuels, both liquid and gaseous, which frequently leave residues of tar, or soot, or both.

Finally, among the important processes there is to be noted that of gasification of solid and liquid fuels in gas producers and vaporizers, a process also carried on in blast furnaces in which it is only an accidental accompaniment and not the primary process. Some of the actions taking place in gas producers are also common to the manufacture of coal gas, and coke, in retorts, beehive and by-product ovens.

From what has been said it should be apparent that engineers are concerned not with any speculations concerning the nature of heat, but only with the kind and quantity of effect that heat addition to, or abstraction from, certain substances may be able to produce. While this interest is more or less closely related to philosophic inquiry, having for its object the development of all embracing generalizations or laws of nature, and to the relation of heat to the chemical and physical constitution of matter, the differences are marked, and a clearly defined field of application of laws to the solution of numerical problems dealing with identical processes, constitutes the field of engineering thermodynamics.

It is not possible or desirable to take up and separately treat every single engineering problem that may arise, but on the contrary, scientific methods of grouping thermal processes or substance effects into types should be employed, from which the solution of individual cases may be determined.

Prob. 1. Water is forced by a pump through a feed-water heater and economizer to a boiler where it is changed to steam, which in turn passes through a superheater to a cylinder from which it is exhausted to a condenser. Which pieces of apparatus have to do with heat effects and which with work? Point out similarities and differences of process.

Prob. 2. Air is passed over gasoline in a carbureter; the mixture is compressed, burned and allowed to expand in a gas engine cylinder. Which of the above steps have to do with heat effects and which with work effects?

Prob. 3. In certain types of ice machines liquid ammonia is allowed to evaporate, the vapor which is formed being compressed and condensed again to liquid. Which of these steps is a work phase and which a heat phase? Compare with Prob. 1.

Prob. 4. When a gun is fired what is the heat phase and what is the work phase? Are they separate or coincident?

Prob. 5. Air is compressed in one cylinder, then it is cooled and compressed to higher pressure and forced into a tank. The air in the tank cools down by giving up heat to the atmosphere. From the tank it passes through a pipe line to a heater and then to an engine from which it is exhausted to the atmosphere. Which steps in the cycle may be regarded as heat and which as work phases? Compare with Prob. 2.

64. Classification of Heating Processes. Heat Addition and Abstraction with or without Temperature Change. Qualitative Relations. That heat will pass from a hot to a less hot body if it gets a chance is axiomatic, so that a body acquiring heat may be within range of a hotter one, the connection between them being either immediate, that is they touch each other, or another body may connect them acting as a heat carrier; or they may be remote with no more provable connection than the hypothetical ether as is the case with the sun and earth. A body may gain heat in other ways than by transfer from a hotter body, for example, the passage of electrical current through a conductor will heat it, the rubbing of two solids together will heat both or perhaps melt one, the churning of a liquid will heat it, the mixing of water and sulphuric acid will produce a hotter liquid than either of the components before mixture, the absorption by water of ammonia gas will heat the liquid. All these and many other similar examples that might be cited have been proved by careful investigation, partly experimental and partly by calculation based on various hypotheses, to be examples of transformation of energy, mechanical, electrical, or chemical, into the heat form. While, therefore, bodies may acquire heat in a great many different concrete ways they all fall under two useful divisions:

(a) By transfer from a hotter body;

(b) By transformation into heat of some other energy manifestation.

One body may be said to be hotter than another when it feels so to the sense of touch, provided neither is too hot or too cold for injury to the tissues,

or more generally, when by contact one takes heat from the other. Thus, ideas of heat can scarcely be divorced from conceptions of temperature and the definition of one will involve the other. As a matter of fact temperature as indicated by any instrument is merely an arbitrary number located by somebody on a scale, which is attached to a substance on which heat has some *visible* effect. Temperature is then a purely arbitrary, though generally accepted, number indicating some heat content condition on a scale, two points of which have been fixed at some other conditions of heat content, and the scale space between, divided as convenient. Examination of heat effects qualitatively will show how thermometers might be made or heat measured in terms of any handy effect, and will also indicate what is likely to happen to any substance when it receives or loses heat. Some of the more common heat effects of various degrees of importance in engineering work are given below:

Expansion of Free Solids. Addition of heat to free solids will cause them to expand, increasing lengths and volumes. Railroad rails and bridges are longer in summer than winter and the sunny side of a building becomes a little higher than the shady side. Steam pipes are longer and boilers larger when hot, than cold, and the inner shell of brick chimneys must be free from the outer to permit it to expand when hot without cracking the outer or main supporting stack body. Shafts running hot through lack of lubrication or by overloading in comparatively cool bearing boxes may be gripped tight enough to twist off the shaft, or they may merely score the bearing.

Stressing of Restrained Solids. A solid being heated may be restrained in its tendency to expand, in which case there will be set up stresses in the material which may cause rupture. Just as with mechanically applied loads, bodies deform in proportion to stress up to elastic limit, as stated by Hooke's law, so if when being heated the tendency to expand be restrained the amount of deformation that has been prevented determines the stress. A steam pipe rigidly fixed at two points when cold will act as a long column in compression and buckle when hot, the buckling probably causing a leak or rupture. If fixed hot, it will tend to shorten on cooling and being restrained will break something. Cylinders of gas engines and air compressors are generally jacketed with water, and becoming hot inside, remaining cold outside, the inner skin of the metal tends to expand, while the outer skin does not. One part is, therefore, in tension and the other in compression, often causing cracks when care in designing is not taken, and sometimes even with care, in large gas engines.

Expansion of Free Liquids. Heating of liquids will cause them to expand just as do solids, increasing their volume. Thus, alcohol or mercury in glass tubes will expand, and as these liquids expand more than the glass, a tube which was originally full will overflow when hot, or a tube of very small bore attached to a bulb of cold liquid will on heating receive some liquid; the movement of liquid in the tube if proportional to the heat received will serve as a thermometer. If the solid containing the liquid expanded to the same degree as the liquid there would be no movement. Two parts of the same liquid mass may be unequally hot and the hotter having expanded will weigh less per cubic foot,

that is, be of less density. Because of freedom of movement in liquids the lighter hot parts will rise and the cooler heavy parts fall, thus setting up a circulation, the principle of which is used in hot water heating systems, the hot water from the furnace rising to the top of the house through one pipe and cooling on its downward path through radiators and return pipe. In general, then, liquids decrease in density on heating and increase in density on cooling, but a most important exception is water, which has a point of maximum density just above the freezing-point, and if cooled below this becomes not heavier but lighter. Consequently, water to be cooled most rapidly should be cooled at first from the top and after reaching this point of maximum density, from the bottom, if it is to be frozen.

Rise of Pressure in Confined Liquids. When liquids are restrained from expanding under heating they suffer a rise of pressure which may burst the containing vessel. For this reason, hot water heating systems have at their highest point, open tanks, called expansion tanks, which contain more water when the system is hot than when cold, all pipes, radiators and furnaces being constantly full of water. Should this tank be shut off when the water is cold something would burst, or joints leak, before it became very hot.

Expansion of Free Gases. Just as solids and liquids when free expand under heating, so also do gases, and on this principle chimneys and house ventilation systems are designed. The hot gases in a chimney weigh less per cubic foot than cooler atmospheric air; they, therefore, float as does a ship on water, the superior density of the water or cold gas causing it to flow under and lift the ship or hot gas, respectively. Similarly, hot-air house furnaces and ventilating systems having vertical flues supplied with hot air can send it upward by simply allowing cold air to flow in below, which in turn being heated flows upward and is replaced.

Rise of Pressure in Confined Gases. Gases when restrained from expanding under heat reception will increase in pressure just as liquids do, only over greater ranges, and as the internal stresses in solids increase when a body is heated under restraint. It is just this principle which lies at the root of the operation of guns and gas engines. Confined gases are rapidly heated by explosive combustion and the pressure is thus raised sufficiently to drive projectiles or pistons in their cylinders.

Melting of Solids. It has been stated that solids on being heated expand, but it should be noted that this action cannot proceed indefinitely. Continued heating at proper temperatures will cause any solid to melt or fuse, and the previously-rising temperature will become constant during this change of state. Thus, melting or fusion is a process involving a change of state from solid to liquid and takes place at constant temperature. The tanks or cans of ice-making plants containing ice and water in all proportions retain the same temperature until all the water becomes ice, provided there is a stirring or circulation so that one part communicates freely with the rest and provided also the water is pure and contains no salt in solution. Impure substances, such as liquid solutions, may suffer a change of temperature at fusion or solidification.

For pure substances, melting and freezing, or fusion and solidification, are constant-temperature heat effects, involving changes from solid to liquid, or liquid to solid states.

Boiling of Liquids; Ebullition. Continued heating of solids causes fusion, and similarly continued heating of liquids causes boiling, or change of state from liquid to vapor, another constant-temperature process—just what temperature, will depend on the pressure at the time. So constant and convenient is this temperature-pressure relation, that the altitude of high mountains can be found from the temperature at which water boils. The abstraction of heat from a vapor will not cool it, but on the contrary cause condensation. Steam boilers and ammonia refrigerating coils and coolers are examples of evaporating apparatus, and house heating radiators and steam and ammonia condensers of condensing apparatus.

Evaporation of Liquids; Humidification of Gases. When dry winds blow over water they take up moisture in the vapor form by evaporation at any temperature. This sort of evaporation then must be distinguished from ebullition and is really a heat effect, for without heat being added, liquid cannot change into vapor; some of the necessary heat may be supplied by the water and some by the air. This process is general between gases and liquids and is the active principle of cooling towers, carbureters, and driers of solids like wood kilns. The chilling of gases that carry vapors causes these to condense in part. As a matter of fact it is not necessary for a gas to come into contact to produce this sort of evaporation from a liquid, for if the liquid be placed in a vacuum some will evaporate, and the pressure finally attained which depends on the temperature, is the vapor pressure or vapor tension of the substance, and the amount that will so evaporate is measured by this pressure and by the rate of removal of that which formed previously.

Evaporation of Solids; Sublimation. Evaporation, it has been shown, may take place from a liquid at any temperature, but it may also take place directly from the solid, as ice will evaporate directly to vapor either in the presence of a gas or alone. Ice placed in a vacuum will evaporate until the vapor tension is reached, and it is interesting to note that the pressure of vapors above their solids is not necessarily the same as above their liquids at the same temperature, though they merge at the freezing-point. This is the case with ice-water vapor.

Change of Viscosity. Heating of liquids may have another effect measured by their tendency to flow, or their viscosity. Thus, a thick oil will flow easier when heated, and so also will any liquid. If, therefore, the time for a given quantity to flow through a standard orifice under a given head or pressure be measured, this time, which is the measure of viscosity, will be less for any liquid hot than cold for the same liquid. Viscosity then decreases with heat addition and temperature rise.

Dissociation of Gases. When gases not simple are heated and the heating continued to very high temperatures, they will split up into their elements or perhaps into other compound gases. This may be called decomposition or, better, dissociation, and is another heat effect. Thus, the hydrocarbon C_2H_4 will

split up with solid carbon soot C, and the other hydrocarbon CH_4 ; and steam H_2O , into hydrogen and oxygen. This is not a constant-temperature process, since the per cent dissociated increases as the temperature rises.

Dissociation of Liquids. Similar to the dissociation of gases receiving heat at high temperature is the decomposition of some liquids in the liquid state, notably the fuel and lubricating oils, or hydrocarbons which are compounds of H and C in various proportions, each having different properties. Sometimes these changes of H and C groupings from the old to the new compounds under the influence of heating will be at constant and at other times at varying temperatures; sometimes the resulting substances remain liquid and sometimes soot or C separates out, and this is one of the causes for the dark color of some cylinder oils.

Absorption of Gases in Liquids. Liquids will absorb some gases quite freely; thus, water will absorb very large quantities of ammonia, forming aqua ammonia. Addition of heat will drive off this gas so that another heat effect is the expulsion of gases in solution. Use is made of this industrially in the absorption system of ammonia refrigeration.

Solubility of Solids in Liquids. The heating of liquids will also affect their solubility for solid salts; thus, a saturated solution of brine will deposit crystals on heat abstraction and take them back into solution on heat addition. Certain scale-forming compounds are thrown down on heating the water intended for boilers, a fact that is made use of in feed-water heating purifiers; for these salts increase of temperature reduces solubility. In general then heat addition affects the solubility of liquids for solid salts.

Chemical Reaction; Combustion. If oxygen and hydrogen, or oxygen and carbon, be heated in contact, they will in time attain an ignition temperature at which a chemical reaction will take place with heat liberation called combustion, and which is an *exothermic* or heat-freeing reaction. Another and different sort of reaction will take place if CO_2 and carbon be heated together, for these will together form a combustible gas, CO, under a continuation of heat reception. This is an *endothermic* or heat-absorbing reaction. Neither of these will take place until by heat addition the reaction temperature, called ignition temperature for combustion, has been reached.

Electrical and Magnetic Effects. Two metals joined together at two separate points, one of which is kept cool and the other heated, will be found to carry an electric current or constitute a thermo-electric couple. Any conductor carrying an electric current will on changing temperature suffer a change of resistance, so that with constant voltage more or less current will flow; this is a second electrical heat effect and like the former is useful only in instruments indicating temperature condition. A fixed magnet will suffer a change of magnetism on heating, so that heat may cause magnetic as well as electric effects.

These heat effects on substances as well as some others of not so great engineering importance may be classified or grouped for further study in a variety of ways, each serving some more or less useful purpose.

Reversible and Non-reversible Processes. There may be reversible and non-reversible thermal processes, when the process may or may not be considered constantly in a state of equilibrium. For example, as heat is applied to boiling water there is a continuous generation of vapor in proportion to the heat received; if at any instant the heat application be stopped the evaporation will cease and if the flow of heat be reversed by abstraction, condensation will take place, indicating a state of thermal equilibrium in which the effect of the process follows constantly the direction of heat flow and is constantly proportional to the amount of heat numerically, and in sign, of direction. As an example of non-reversible processes none is better than combustion, in which the chemical substances receive heat with proportional temperature rise until chemical reaction sets in, at which time the reception of heat has no further relation to the temperatures, because of the liberation of heat by combustion which proceeds of itself and which cannot be reversed by heat abstraction. Even though a vigorous heat abstraction at a rate greater than that at which it is freed by combustion may stop combustion or put the fire out, no amount of heat abstraction or cooling will cause the combined substances to change back into the original ones as they existed before combustion. The effect of heat in such cases as this is, therefore, non-reversible.

Constant and Variable Volume or Density. When gases, liquids or solids are heated they expand, except when prevented forcibly from so doing, and as a consequence they suffer a reduction of density with the increase of volume; this is also true of changing liquids to their vapors. It should be noted that all such changes of volume against any resistance whatever occur with corresponding performance of some work, so that some thermal processes may directly result in the doing of work. Heating accompanied by no volume change and during which restraints are applied to keep the volume invariable, cannot do any work or suffer any change of density, but always results in change of pressure in liquids, gases and vapors and in a corresponding change of internal stress in solids.

Constant and Variable Temperature Processes. Another useful division, and that most valuable in the calculation of relations between heat effect and heat quantity, recognizes that some of the heating processes and, of course, cooling, occur at constant temperature, and others with changing temperature. For example, the changes of state from liquid to solid, and solid to liquid, or freezing and fusion, are constant temperature processes in which, no matter how much heat is supplied or abstracted, the temperature of the substance changing state is not affected; the same is true of ebullition and condensation, or the changing of state from liquid to vapor, and vapor to liquid. These latter constant-temperature processes must not be confused with evaporation, which may proceed from either the solid or liquid state at any temperature whether constant or not.

Prob. 1. From the time a fire is lighted under a cold boiler to the time steam first comes off, what heat effects take place?

Prob. 2. What heat effects take place when a piece of ice, the temperature of which is 20°F. , is thrown onto a piece of red-hot iron?

Prob. 3. What heat effects must occur before a drop of water may be evaporated from the ocean, and fed back into it as snow?

Prob. 4. What heat changes take place when soot is formed from coal or oil?

Prob. 5. In a gas producer, coal is burned to CO_2 , which is then reduced to CO . Steam is also fed to the producer, and H and O formed from it. Give all the heat effects which occur.

Prob. 6. By means of what heat effects have you measured temperature changes, or have known them to be measured?

Prob. 7. When the temperature changes from 40°F. to 20°F. , give a list of all heat effects you know that ordinarily occur for several common substances. Do the same for a change in the reverse direction.

Prob. 8. If a closed cylinder be filled with water it will burst if the temperature be lowered or raised sufficiently. What thermal steps occur in each case?

Prob. 9. If salt water be lowered sufficiently in temperature, a cake of fresh ice and a rich salt solution will be formed. State the steps or heat effects which occur during the process.

65. Thermometry Based on Temperature Change Heat Effects. Thermometer and Absolute Temperature Scales. Those thermal processes in which heat addition or abstraction is followed as a result by a corresponding and more or less proportional temperature change, are quite numerous and important both in engineering practice and as furnishing a means for thermometer making, and temperature definition and measurement. According to Sir William Thomson, "every kind of thermoscope must be founded on some property of matter continuously varying with the temperature," and he gives the following:

- (a) Density of fluid under constant pressure.
- (b) Pressure of a fluid under a constant volume envelope.
- (c) Volume of the liquid contained in a solid holder (ordinary mercury or spirit thermometer).
- (d) Vapor pressure of a solid or liquid.
- (e) Shape or size of an elastic solid under constant stress.
- (f) Stress of an elastic solid restrained to constant size.
- (g) Density of an elastic solid under constant stress.
- (h) Viscosity of a fluid.
- (i) Electric current in a thermo-couple.
- (j) Electric resistance of a conductor.
- (k) Magnetic moment of a fixed magnet.

Any, or all of these—pressure, volume, shape, size, density, rate of flow, magnetic or electrical effects, may be measured, and their measure constitutes a measure of temperature indirectly, so that instruments incorporating these temperature effects to be measured, are also thermometers.

Any temperature-indicating device may be called a thermometer, though those in use for high temperatures are generally called pyrometers, which

incidentally indicates the important fact that no thermometer is equally useful for all ranges of temperature. Practically all thermometers in use for temperatures short of a red heat, depend on certain essential relations between the density or volume, or the pressure and temperature of a fluid, though metals are used in some little-used forms in which change of size is measured, or change of shape of a double metallic bar, often brass and iron, consisting of a piece of each fastened to the other to form a continuous strip. The two metals are expanded by the temperature different amounts causing the strip to bend under heating. There are also in use electric forms for all temperatures, and these are the only reliable ones for high temperatures, both of the couple and resistance types, except one, dependent on the color of a high temperature body, black when cold. That most useful and common class involving the interdependence of pressure and temperature, or volume and temperature, of a fluid is generally found in the form of a glass bulb or its equivalent, to which is attached a long, narrow glass tube or stem which may be open or closed at the end; open when the changes of fluid volume at constant pressure are to be observed, and closed when changes of contained fluid pressure at constant restrained volume are to be measured as the effect of temperature changes. For the fluid there is used most commonly a liquid alone such as mercury, or a gas alone such as air; though a gas may be introduced above mercury and there may be used a liquid with its vapor above. When the fluid is a liquid, such as mercury, in the common thermometer, the stem is closed at the end so that the mercury is enclosed in a constant-volume container or as nearly so as the expansion or deformation of the glass, which is not filled with mercury, will permit, this space in the stem is left at a vacuum or filled with a gas under pressure, such as nitrogen, to resist evaporation of the mercury at high-temperatures. Gas-filled mercury thermometers, as the last form is called, are so designed that for the whole range of mercury expansion the pressure of the gas opposing it does not rise enough to offer material resistance to the expansion of mercury or to unduly stress the glass container. It should be noted that mercury thermometers do not measure the expansion of mercury alone, but the difference between the volume of mercury and the glass envelope, but this is of no consequence so long as this difference is in proportion to the expansion of the mercury itself, which it is substantially, with proper glass composition, when the range is not too great. Such thermometers indicate temperature changes by the rise and fall of mercury in the stem, and any numerical value that may be convenient can be given to any position of the mercury or any change of position. Common acceptance of certain locations of the scale number, however, must be recognized as rendering other possible ones unnecessary and so undesirable. Two such scales are recognized, one in use with metric units, the centigrade, and the other with measurements in English units, the Fahrenheit, both of which must be known and familiar, because of the frequent necessity of transformation of numerical values and heat data from one system to the other. To permit the construction of a scale, at least two points must be fixed: These are, *first*, the position of the mercury when the thermometer

is in the vapor of boiling *pure* water at sea level, or under the standard atmospheric pressure of 29.92"=760 mm. of mercury absolute pressure; and *second*, the position of the mercury when the thermometer is surrounded by melting ice at the same pressure. These are equivalent to the boiling or condensation, and melting- or freezing-points, of pure water at one atmosphere pressure; the equal divisions between these points are called *degrees*. The two accepted thermometer scales have the following characteristics with respect to these fixed points and divisions between them:

	Pure Water Freezing-point. at one atm. pr.	Pure Water Boiling-point. at one atm. pr.	Number of Equal Divisions Between Freezing and Boiling.
Centigrade scale.....	0	100	100
Fahrenheit scale.....	32	212	180

From this it appears that a degree of *temperature change* is on the centigrade scale, $\frac{1}{100}$ of the linear distance between the position of the mercury surface at the freezing- and boiling-points of water, and on the Fahrenheit scale, $\frac{1}{180}$ of the same distance.

The relation between scale readings is readily seen, for when the temperature is 0° C., it is 32° F., and when it is 100° C. it is (180+32)=212° F., so that

$$\text{Temperature Fahrenheit} = 32 + \frac{9}{5} (\text{Temperature centigrade}),$$

$$\text{or Temperature centigrade} = \frac{5}{9} (\text{Temperature Fahrenheit} - 32).$$

By reason of the lack of absolute proportionality between temperature and effect, other fixed points are necessary, especially at high temperatures. Tables giving these values have been adopted by the U. S. Bureau of Standards and are considered correct to within 5° C., at 1200° C. See Tables XVII and XVIII, Handbook of Tables.

Thermometers in which a liquid and its vapor exist together, depend on a property to be noted in detail later—the relation of vapor pressure to temperature and its independence of the volume of vapor. So long as any vapor exists above the liquid the temperature will depend only on the pressure of that vapor, so that such thermometers will indicate temperature by the pressure measurement, after experimental determination of this pressure-temperature relation of vapors. Conversely, temperature measurements of vapors by mercury thermometers will lead to pressure values; at the present time some steam plants are introducing mercury thermometers on the boilers and pipe lines, in place of the proverbially inaccurate pressure gages.

Gas thermometer is the name generally applied to the class in which the fluid is a gas, whether air, hydrogen, nitrogen or any other, and whether the pressure is measured for a fixed contained volume, or the volume measured when acted on by a constant pressure.

Using the centigrade scale, fixing freezing point at 0° C., and making the corresponding pressure, p_0 , atmospheric at this point, and reading at 100° C.

another pressure, p_{100} , Regnault found experimentally a relation between these two pressures and the temperature corresponding to any other pressure, p , as given by the empiric formula,

$$t = 100 \frac{p - p_0}{p_{100} - p_0} \quad \dots \dots \dots (483)$$

He also determined the pressure at the boiling-point to be related to the pressure at the freezing-point, by $p_{100} = 1.3665 p_0$, which on substitution gives

$$t = 100 \frac{p - p_0}{.3665 p_0} = 272.85 \left(\frac{p}{p_0} - 1 \right) \quad \dots \dots \dots (484)$$

This constant, 272.85, the reciprocal of which is .003665, is, of course, the pressure-increase factor per degree C. rise of temperature for a gas held at constant volume, but it was found that it had about the same value applied to the other type of thermometer in which gas volumes are measured at constant pressure.

Examination of Eq. (484), giving the relation between two temperatures and the corresponding gas pressures, will show a most important relation. If the pressure be supposed to drop to zero and it is assumed that the relations between pressure and temperature hold, then when $p = 0$, $t = -272.85$. This temperature has received the name of the absolute zero, and may be defined as the temperature at which pressure disappears or becomes zero at constant volume, and correspondingly, at which the volume also disappears, since it was found that similar relations existed between volume and temperature at constant pressure. Calling temperature on a new scale beginning 272.85° below the centigrade zero by the name *absolute temperatures*,

$$\text{then} \quad \left\{ \begin{array}{c} \text{Absolute temperature} \\ \text{centigrade} \end{array} \right\} = 272.85 + \left\{ \begin{array}{c} \text{Scale temperature} \\ \text{centigrade} \end{array} \right\}.$$

As this constant or absolute temperature of the centigrade scale zero is an experimental value, it is quite natural to find other values presented by different investigators, some of them using totally different methods; but as yet there is no absolute agreement as to the exact value. In engineering problems, however, it is seldom desirable or possible to work to such degrees of accuracy as to make the uncertainty of the absolute zero a matter of material importance, and for practical purposes the following values may be used with sufficient confidence for all but exceptional cases.

$$\text{Absolute Temperature } (T) \left\{ \begin{array}{l} \text{Centigrade} = 273 \\ \text{Fahrenheit} = 460 \end{array} \right\} + \text{Scale Temperature } (t).$$

When great accuracy is important it is not possible at present to get a better Fahrenheit value than 459.65, the mean of the two known limits of 459.6 and 459.7, though Marks and Davis in their Steam Tables have adopted 459.64, which is very close to the value of 459.63, corresponding to 273.13 on the centigrade scale, adopted by Buckingham in his excellent Bulletin of U. S. Bureau of Standards.

These experiments with the gas thermometers, leading to a determination of temperature as a function of the pressure-change of the gas held at constant

volume, or its volume-change when held at constant pressure, *really supply a definition of temperature*, which before meant no more than an arbitrary number, and furnish a most valuable addition to the generalization of relations between heat content of a body and its temperature or physical state.

A lack of proportionality between thermometer indication and temperature has already been pointed out, and it is by reason of this that two identical thermometers, or as nearly so as can be made, with absolute agreement between water boiling- and freezing-points, will not agree at all points between, nor will the best constructed and calibrated mercury thermometers agree with a similarly good gas thermometer.

The temperature scale now almost universally adopted as standard is that of the constant volume hydrogen gas thermometer, on which the degree F. is one one-hundred-and-eightieth part of the change in pressure of a fixed volume of hydrogen between melting pure ice and steam above boiling pure water, the initial pressure of the gas at 32° being 100 cm. = 39.37 ins. Hg. An indication of mercury in a glass thermometer is, of course, a measure of the properties of the mercury and glass used, and its F. degree of temperature is defined as one one-hundred-and-eightieth part of the volume of the stem between its indications at the same two fixed points.

One sort of correction that is often necessary in mercury thermometer work is that for stem immersion. Thermometers are calibrated as a rule with the whole stem immersed in the melting ice or the steam, but are ordinarily used with part of the stem exposed and not touching the substance whose temperature is indicated. For this condition the following correction is recommended by the Bureau of Standards, V. 2, No. 3.

$$\text{Stem correction} = .000088 \, n \, (t - t_1)^\circ \text{ F.}$$

When n = number of degrees exposed;

t = temperature indicated Fahrenheit degrees;

t_1 = mean temperature of emergent stem itself, which must necessarily be estimated and most simply by another thermometer next to it, and entirely free from the bath.

Prob. 1. What will be the centigrade scale and absolute temperatures, for the following Fahrenheit readings? -25° , 25° , 110° , 140° , 220° , 283° scale, and 300° , 460° , 540° , 710° , 2000° absolute.

Prob. 2. What will be the Fahrenheit scale and absolute temperatures for the following centigrade readings? -20° , 10° , 45° , 80° , 400° , 610° scale, and 200° , 410° , 650° , 810° , 2500° absolute.

Prob. 3. By the addition of a certain amount of heat the temperature of a quantity of water was raised 160° F. How many degrees C. was it raised?

Prob. 4. To bring water from 0° C. to its boiling-point under a certain pressure required a temperature rise of 150° C. What was the rise in Fahrenheit degrees?

Prob. 5. For each degree rise Fahrenheit, an iron bar will increase .00000648 of its length. How much longer will a bar be at 150° C. than at 0° C. ? At 910° C. absolute than at 250° C. absolute?

Prob. 6. The increase in pressure for SO_2 for a rise of 100°C. is given as .3845 at constant volume. What would have been absolute zero found by Regnault had he used SO_2 rather than air?

Prob. 7. A thermometer with a scale from 40°F. to 700°F. is placed in a thermometer well so that the 200° mark is just visible. The temperature as given by the thermometer is 450° . If the surrounding temperature is 100°F. , what is true temperature in the well?

66. Calorimetry Based on Proportionality of Heat Effects to Heat Quantity. Units of Heat and Mechanical Equivalent. Though it is generally recognized from philosophic investigations extending over many years, that heat is one manifestation of energy capable of being transformed into other forms such as mechanical work, electricity or molecular arrangement, and derivable from them through transformations, measurements of quantities of heat can be made without such knowledge, and were made even when heat was regarded as a substance. It was early recognized that equivalence of heat effects proved effects proportional to quantity; thus the melting of one pound of ice can cool a pound of hot water through a definite range of temperature, and can cool two pounds through half as many degrees, and so on. The condensation of a pound of steam can warm a definite weight of water a definite number of degrees, or perform a certain number of pound-degrees heating effect in water. So that taking the pound-degree of water as a basis the ratio of the heat liberated by steam condensation to that absorbed by ice melting can be found. Other substances such as iron or oil may suffer a certain number of pound-degree changes and affect water by another number of pound-degrees. The unit of heat quantity might be taken as that which is liberated by the condensation of a pound of steam, that absorbed by the freezing of a pound of water, that to raise a pound of iron any number of degrees or any other quantity of heat effect. The heat unit generally accepted is, in metric measure, the calorie, or the amount to raise one kilogramme of pure water one degree centigrade, or in English units, the British thermal unit, that necessary to raise one pound of water one degree Fahrenheit. Thus, the calorie is the kilogramme-degree-centigrade, and the British thermal unit the pound-degree-Fahrenheit; the latter is used in engineering, usually abbreviated to B.T.U. There is also occasionally used a sort of cross unit called the centigrade heat unit, which is the pound-degree centigrade. The relation between these is given quantitatively in Table X in the book of tables.

All the heat measurements are, therefore, made in terms of equivalent water-heating effects in pound-degrees, but it must be understood that a water-pound degree is not quite constant. Careful observation will show that the melting of a pound of ice will not cool the same weight of water from 200°F. to 180°F. , as it will from 60°F. to 40°F. , which indicates that the heat capacity of water or the B.T.U. per pound-degree is not constant. It is, therefore, necessary to further limit the definition of the heat unit, by fixing on some water temperature and temperature change, as the standard, in addition to the selection of water as the substance, and the pound and degree as units of capacity.

Here there has not been as good an agreement as is desirable, some using $4^{\circ}\text{C.}=39.4^{\circ}\text{F.}$ as the standard temperature and the range one-half degree both sides; this is the point of maximum water density. Others have used $15^{\circ}\text{C.}=59^{\circ}\text{F.}$ as the temperature and the range one-half degree both sides; still others, one degree rise from freezing point 0°C. or 32°F. There are good reasons, however, for the most common present-day practice which will probably become universal, for taking as the range and temperatures, freezing-point to boiling-point, and dividing by the number of degrees. The heat unit so defined is properly named the mean calorie or mean British thermal unit; therefore,

Mean calorie $= \frac{1}{100}$ (amount of heat to raise 1 Kg. water from 0°C. to 100°C.).

Mean B.T.U. $= \frac{1}{180}$ (amount of heat to raise 1 lb. water from 32°F. to 212°F.).

In terms of the heat unit thus defined, the amount of heat per degree temperature change is variable over the scale, but only in work of the most accurate character is this difference observed in engineering calculations; in accurate work this difference must not be neglected and care must be exercised in using other physical constants in heat units reported by different observers, to be sure of the unit they used in reporting them. However the great bulk of engineering work involves uncertainties greater than these differences and they may, therefore, be ignored generally.

By various experimental methods, all scientifically carried out and extending over sixty years, a measured amount of work has been done and entirely converted into heat, originally by friction of solids and of liquids, for the determination of the foot-pounds of work equivalent to one B.T.U., when the conversion is complete, that is, when all the work energy has been converted into heat. This thermo-physical constant is the *mechanical equivalent of heat*. Later, indirect methods have been employed for its determination by calculation from other constants to which it is related. All of these experiments have led to a large number of values, so that it is not surprising to find doubt as to the correct value.

From the results of experiments made by himself and many other investigators, Rowland in 1880 concluded that the mechanical equivalent of heat was nearly

778.6 ft.-lbs. = 1 B.T.U., at latitude of Baltimore,

or 774.5 ft.-lbs. = 1 B.T.U., at latitude of Manchester.

with the following corrections to be added for other latitudes.

Latitude.	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
Ft.-lbs.	1.62	1.50	1.15	.62	.15	-.75	-1.41	-1.93	-2.30	-2.43

Since that time other determinations have been made by Reynolds and Morby, using mechanical, and Griffiths, Schuste and Gannon, Callendar and Barnes, using electrical transformation into heat. Giving these latter determinations equal weight with those of Joule and Rowland, the average is

1 mean (small) calorie at 20°C. (nitrogen thermometer) $= 4.1834 \times 10^7$ ergs.

On the discussion of these results by Smith, Marks and Davis accept and use the mean of the results of Reynolds, and Morby and Barnes, which is

$$\begin{aligned} 1 \text{ mean (small) calorie} &= 4.1834 \times 10^7 \text{ ergs,} \\ &= 0.0039683 \text{ B.T.U.} \end{aligned}$$

$$\begin{aligned} 1 \text{ mean (large) calorie} &= 1000 \times 1 \text{ mean (small) calorie,} \\ &= 3.9683 \text{ B.T.U.} \end{aligned}$$

$$1 \text{ mean B.T.U.} = 777.52 \text{ ft.-lbs.,}$$

when the gravitational constant is 980.665 cm. sec², which corresponds to 32.174 lbs., and is the value for latitude between 45° and 46°.

For many years it has been most common to use in engineering calculations the round number 778, and for most problems this round number is still the best available figure, but where special accuracy is needed it is likely that no closer value can be relied upon than anything between 777.5 and 777.6 for the above latitude.

Example. To heat a gallon of water from 60° F. to 200° F. requires the heat equivalent of how many foot-pounds? 1 gallon = 8.33 lbs.,

$$\begin{aligned} 200^\circ \text{ F.} - 60^\circ \text{ F.} &= 140^\circ \text{ F. rise.} \quad 8.33 \times 140 = 1166 \text{ pound-degrees,} \\ &= 1166 \text{ B.T.U.} = 778 \times 1166, \text{ ft.-lbs.} = 90,700 \text{ ft.-lbs.} \end{aligned}$$

Prob. 1. A feed-water heater is heating 5000 gallons of water per hour from 40° F. to 200° F. What would be the equivalent energy in horse-power units?

Prob. 2. A pound of each of the following fuels has the heating values as given. Change them to foot-pounds. Average bituminous coal, 14,000 B.T.U. per lb., Average kerosene, 18,000 B.T.U. per lb. Average small anthracite, 12,000 B.T.U. per lb., Average alcohol, 10,000 B.T.U. per lb. Average gasoline, 20,000 B.T.U. per lb.

Prob. 3. A cubic foot of each of the following gases yields on combustion the number of heat units shown. Change them to foot-pounds. Natural gas (average), 880 B.T.U. per cu.ft., Carbureted water gas, 700 B.T.U. per cu.ft. Coal gas, 730 B.T.U. per cu.ft., Mond gas, 150 B.T.U. per cu.ft. Blast furnace gas, 100 B.T.U. per cu.ft.

Prob. 4. A pool contains 20,000 cu.ft. of water and must be warmed from 40° F. to 70° F. How much work might be done with the equivalent energy?

Prob. 5. How many calories and how many centigrade heat units would be required in Prob. 4?

Prob. 6. In the course of a test a man weighing 200 lbs. goes up a ladder 25 ft. high every 15 minutes. If the test lasted 12 hours how many B.T.U. did he expend?

Prob. 7. A reservoir contains 300 billion gallons of water which are heated each year from 39° F. to 70° F. What is the equivalent number of foot-pounds of work?

Prob. 8. A pound of water moving at the rate of 450 ft. per second is brought to rest so that all of its energy is turned into heat. What will be the temperature rise?

Prob. 9. For driving an automobile 30 horse-power is used. How long will a gallon of average gasoline, sp.gr. = .7, last, if 10 per cent of its energy is converted into work?

Prob. 10. Power is being absorbed by a brake on the flywheel of an engine. If the engine is developing 50 horse-power, how many B.T.U. per minute must be carried off to prevent burning of the brake?

67. Temperature Change Relation to Amount of Heat, for Solids, Liquids, Gases and Vapors, not Changing State. Specific Heats. Provided gases do not decompose, vapors condense, liquids freeze or evaporate, and solids melt, under addition or abstraction of heat, there will always be the same sort of relation between the quantity of heat gained or lost and the temperature change for all, differing only in degree. As the reception of heat in each case causes a temperature rise proportional to it and to the weight of the substances, this constant of proportionality once determined will give numerical relations between any temperature change and the corresponding amount of heat. Making the weight of the substance unity, which is equivalent to the consideration of one pound of substance, the constant of proportionality may be defined as the quantity of heat per degree rise, and as thus defined is the *specific heat* of the substance. Accordingly, the quantity of heat for these cases is equal to the product of specific heat, temperature rise and weight of substance heated.

The heat, as already explained, may be added in two characteristic ways: (a) at constant volume or density, or (b) at constant pressure. It might be expected that by reason of the increase of volume and performance of work under constant-pressure heating, more heat must be added to raise the temperature of one pound, one degree, than in the other case where no such work is done, and both experimental and thermodynamic investigations confirm this view. There are, therefore, two specific heats for all substances capable of definition:

- (a) The specific heat at constant volume, and
- (b) The specific heat at constant pressure.

These two specific heats are quite different both for gases and for vapors, which suffer considerable expansion under constant-pressure heating, but for solids and liquids, which expand very little, the difference is quite small and is to be neglected here. As a matter of fact, there are no cases of common engineering practice involving the specific heat of liquids and solids under constant volume, and values for the specific heats of liquids and solids are always, without further definition, to be understood as the constant-pressure values.

Let C , be the specific heat of solids and liquids suffering no change of state.

C_p , be the specific heat of gases and vapors at constant pressure and suffering no change of state.

C_v , be the specific heat of gases and vapors at constant volume and suffering no change of state.

t_2 and t_1 , be the maximum and minimum temperatures for the process.

w , be the weight in pounds.

Then will the heat added be given by the following equation, if the temperature rise is exactly proportional to the quantity of heat, or in other words, *if the specific heat is constant*:

$$Q = Cw(t_2 - t_1), \text{ for solids and liquids; } \dots \dots \dots (485)$$

$$Q = C_v w(t_2 - t_1), \text{ for gases and vapors (not near condensation)} \\ \text{when volume is constant; (486)}$$

$$Q = C_p w(t_2 - t_1), \text{ for gases and vapors (not near condensation)} \\ \text{when pressure is constant. (487)}$$

When, however, the specific heat is variable, which is the case for many substances, probably for all, the above equation cannot be used except when the specific heat *average* value, or mean specific heat is used. If the variation is irregular this can be found only graphically, but for some substances the variation is regular and integration will give the mean value. It has been the custom to relate the specific heat to the temperature above the freezing-point of water, expressing it as the sum of the value at 32° F., and some fraction of the temperature above this point to the first and second powers, as in Eq. (488).

$$\text{Specific heat at temperature } (t) = a + b(t - 32) + c(t - 32)^2 \quad . \quad (488)$$

In this equation a is the specific heat at 32°, while b and c are constants, different for different substances, c being generally zero for liquids.

When this is true, the heat added is related to the temperature above 32° by a differential expression which can be integrated between limits

$$Q = \int_{t_1-32}^{t_2-32} [a + b(t - 32) + c(t - 32)^2] dt \\ = a[(t_2 - 32) - (t_1 - 32)] + \frac{b}{2}[(t_2 - 32)^2 - (t_1 - 32)^2] + \frac{c}{3}[(t_2 - 32)^3 - (t_1 - 32)^3]. \quad (489)$$

Usually the heats are calculated above 32° so that the heats between any two temperatures will be the difference between the heats from 32° to those two temperatures. In this case $t_1 = 32^\circ$, and $t_2 = t$, whence

$$\text{B.T.U. per lb., from } 32^\circ \text{ to } t, = \left[a + \frac{b}{2}(t - 32) + \frac{c}{3}(t - 32)^2 \right] (t - 32). \quad . \quad (490)$$

For this range of temperature 32° to t , the quantity of heat may be expressed as the product of a mean specific heat and the temperature range

$$\text{or} \quad \text{Heat from } 32^\circ \text{ to } t = (\text{mean sp. heat from } 32^\circ \text{ to } t^\circ) \times (t - 32). \quad . \quad (491)$$

Comparing Eq. (490) with Eq. (491) it follows that

$$\left\{ \begin{array}{l} \text{Mean specific heat} \\ \text{from } 32^\circ \text{ F. to } t^\circ \text{ F.} \end{array} \right\} = a + \frac{b}{2}(t - 32) + \frac{c}{3}(t - 32)^2. \quad . \quad (492)$$

The coefficient of $(t - 32)$ in the mean specific heat expression, is half that in the expression for specific heat *at* t , and the coefficient of $(t - 32)^2$, is one-third. This makes it easy to change from specific heat *at* a given temperature above 32°, to the mean specific heat from 32° to the temperature in question.

The specific heats of some substances are directly measured, but for some others, notably the gases, this is too difficult or rather more difficult than calculation of values from other physical constants to which they are related.

It often happens that in engineering work the solution of a practical problem requires a specific heat for which no value is available, in this case the general law of specific heats, known as the law of Dulong and Petit, for definite compounds may be used as given in Eq. (493).

$$(\text{Specific heat of solids}) \times (\text{atomic weight}) = 6.4. \quad (493)$$

This is equivalent to saying that all atoms have the same capacity for heat, and while it is known to be not strictly true, it is a useful relation in the absence of direct determinations. Some values, experimentally determined for the specific heats of solids, are given in the Handbook of Tables, No. XXII, with values calculated from the atomic weights to show the degree of agreement.

It is extremely probable that the specific heats of liquids all vary irregularly with temperature, so that the constant values given in the Tables No. XXIV must be used with caution. This is certainly the case for water, and is the cause of the difficulty in fixing the unit of heat, which is best solved by the method of means. In Fig. 97 are shown in curve form the values for the

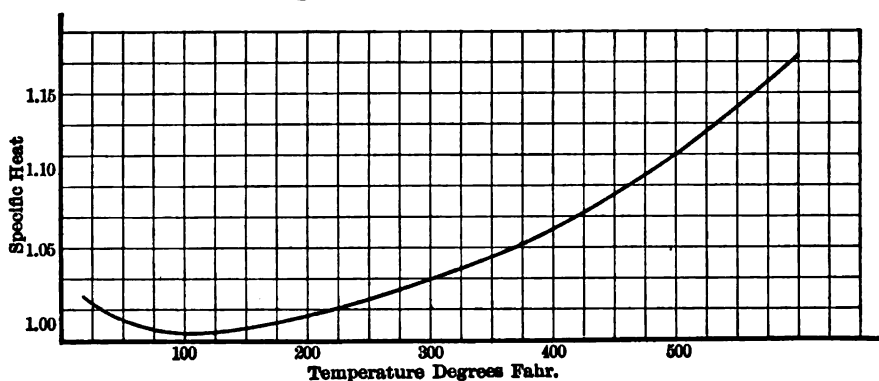


FIG. 97.—Specific Heat of Water at Various Temperatures.

specific heats of water at temperatures from 20° F. to 600° F., as accepted by Marks and Davis after a critical study of the experimental results of Barnes and Dieterici and adjustment of the differences. The integral curve is plotted in Fig. 98, which therefore gives the heat of water from 32° F. to any temperature up to the highest used in steam practice and which is designated in steam tables, summarizing all the properties of water and steam, as the *heat of the liquid*. For the purpose of comparison, the mean specific heat of water is given in Fig. 99 from 32° F. to any temperature which is obtained from the heat of the liquid above 32° F. by dividing it by the temperature above 32° F.

Variability of specific heat is especially noticeable in liquids that are solutions with different amounts of dissolved substance, in which case the specific heat varies with the density and temperature. Problems of refrigeration involve four cases of this kind: (a), calcium, and (b), sodium chloride, brines, the densities of which vary considerably but which are used with

little temperature range, seldom over 20° F. and often not over 5° F., (c), anhydrous ammonia, and (d), carbonic acid.

As the density of brines is often reported on the Baumé scale and liquid fuels always so, a comparison of this with specific gravities is given in connection with the specific heat tables in the Handbook of Tables, in which also will be found values for density of various brine solutions. See Tables XX to XXV.

One of the best-known solutions, so far as accuracy of direct experimental data is concerned, is calcium brine, results for which, from 35° C. to 20° C. are given in the U. S. Bureau of Standards Bulletin by Dickinson, Mueller and

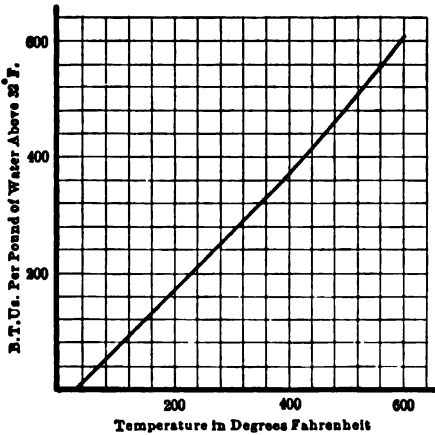


FIG. 98.—Total Heat of Water from 32° F., to any Temperature, *the Heat of the Liquid at that Temperature above 32° F.*

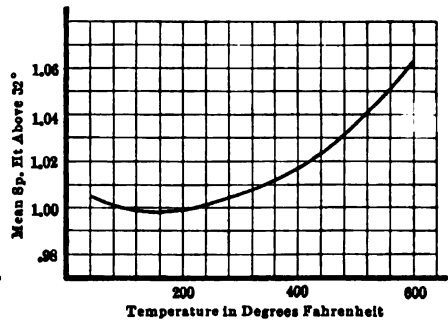


FIG. 99.—Mean Specific Heat of Water from 32° to any Temperature.

George, for densities from 1.175 to 1.250. For chemically-pure calcium chloride in water, the following relation was found between density D , and specific heat C , at 0° C.

$$D = 2.8821 - 3.6272C + 1.7794C^2, \quad (494)$$

and these results when plotted show the specific heat variation with temperature to follow the straight line law very nearly. This being the case the mean specific heat for a given temperature range is closely enough the arithmetical mean of the specific heat *at* the two limiting temperatures.

Anhydrous ammonia liquid has a variable specific heat with temperature, but the experimental values are too few to make its value and law quite certain. Several formulas have been proposed, however, that tend to give an impression of accuracy not warranted by the facts though quite convenient in preparing tables.

Authority.	Specific heat of NH_3 liquid at $t^\circ \text{F.}$			
Zuener	$1.0135 + .00468 (t-32)$	(a)	} . . (495)	
Dieterici	$1.118 + .001156 (t-32)$	(b)		
Wood	$1.1352 + .00438 (t-32)$	(c)		
Ledoux	$1.0057 + .00203 (t-32)$	(d)		

From these expressions the mean specific heat follows by halving the coefficient

of $(t-32)$ F., and these were determined and plotted to scale, together with some direct experimental values of Drewes. From these curves a line is located as the best probability of the value for liquid anhydrous which has the following equation:

$$\left\{ \begin{array}{l} \text{Mean specific heat of anhydrous} \\ \text{liquid NH}_3 \text{ from } 32^\circ \text{ F. to } t^\circ \text{ F.} \end{array} \right\} = 1.07 + .00056(t-32). \quad (496)$$

From this value the heat of liquid ammonia above 32° F. has been calculated and is given in tabular form in the Tables.

Ammonia dissolved in water, giving an aqueous solution as used in the absorption refrigerating system, has a nearly constant specific heat so closely approximating unity as shown by Thomsen, that it is customary in these calculations to ignore any departure from unity, the value for water. These values are 3 per cent NH_3 in water solution, sp.ht., = .997, at 66° F.; 1.8 per cent NH_3 in water solution, sp.ht., = .999, at 66° F.; .9 per cent NH_3 in water solution, sp.ht., = .999, at 66° F.

Liquid carbonic acid, another important substance in engineering, especially in mechanical refrigeration, is less known as to its specific heat than is ammonia, and that is much too uncertain. There is probably nothing better available at present for the necessary range than the results of Amagat and Mollier, reported by Zuener for the heat of the liquid, which are also given in the Tables.

It is, however, with gases that the most complex situation exists with respect to specific heats. As previously pointed out, gases may be heated at constant volume, doing no external work while being heated, or at constant pressure, in which latter case work is done by expansion of the gas against the resisting constant pressure. Therefore, there must be two different specific heats for each gas, one C_p at constant pressure and the other C_v at constant volume, the difference between them representing the heat equivalent of the work of expansion done during the rise of temperature. Most experimental determinations of the specific heats of gases have been made at constant pressure and the constant volume value found from established relations between it and other physical constants. These relations most commonly used are two, Eq. (497) connecting the difference with a constant R and the other Eq. (498) connecting their ratio to a constant γ . These constants have each

$$777.52(C_p - C_v) = R, \quad (497)$$

$$\frac{C_p}{C_v} = \gamma \quad (498)$$

a special significance that may be noted here and proved later, thus R is the ratio of the PV product of a pound of gas to the absolute temperature, and γ the particular value taken by the general exponent s in $PV^s = c$, when the expansion represented takes place with no heat addition or abstraction, i.e., adiabatic, it is also a function of the velocity of sound in gases. Table XXIII in the Handbook of Tables gives some authentic values, with those adopted here, designated by heavy type.

Variability of specific heats of gases and vapors is most marked and of some

engineering importance, because so many problems of practice involve highly heated gases and vapors, the most common being superheated steam and the active gases of combustion in furnaces, gas producers and explosive gas engines. In fact, with regard to the latter it may be regarded as quite impossible with even a fair degree of accuracy to predict the temperature that will result in the gaseous products from the liberation of a given amount of heat of combustion. The first fairly creditable results on the variability of the specific heats of gases of combustion at high temperatures were announced by Mallard and LeChatelier, Vieille and Berthelot, all of whom agree that the specific heat rises, but they do not agree as to the amount. A general law was proposed by LeChatelier, giving the specific heat as a function of temperature by an equation of the following form:

$$\text{Specific heat at } t^{\circ} \text{ F., } (V=C), = C_v = a + b(t-32), \quad \dots \quad (499)$$

in which a = specific heat at constant volume at 32° F. This yields,

$$\left\{ \begin{array}{l} \text{B.T.U. per lb. from } 32^{\circ} \\ \text{F., to } t^{\circ} \text{ F., } (V=C), \end{array} \right\} = Q_{(32 \text{ to } t)} = \left[a + \frac{b}{2}(t-32) \right] (t-32). \quad \dots \quad (500)$$

$$\left\{ \begin{array}{l} \text{Mean specific heat from} \\ 32^{\circ} \text{ F., to } t^{\circ} \text{ F., } (V=C), \end{array} \right\} = C'_v = a + \frac{b}{2}(t-32). \quad \dots \quad (501)$$

The specific heat at constant pressure is obtained by adding a constant to the value for constant volume according to

$$C_p = C_v + \frac{R}{777.52}, \quad \dots \quad (502)$$

$$\text{whence: Specific heat at } t^{\circ} \text{ F., } (P=C), = C_p = a + \frac{R}{777.52} + b(t-32), \quad \dots \quad (503)$$

$$\left\{ \begin{array}{l} \text{B. T. U. per lb. from} \\ 32^{\circ} \text{ F., to } t^{\circ} \text{ F., } (P=C), \end{array} \right\} = Q_{p \text{ to } t} = \left[a + \frac{R}{777.52} + \frac{b}{2}(t-32) \right] (t-32). \quad (504)$$

$$\left\{ \begin{array}{l} \text{Mean specific heat from} \\ 32^{\circ} \text{ F., to } t^{\circ} \text{ F., } (P=C), \end{array} \right\} = C'_p = a + \frac{R}{777.52} + \frac{b}{2}(t-32). \quad \dots \quad (505)$$

The values of these constants have been determined by LeChatelier, Clerk, Callender, and Holborn and Austin, from which the following values are selected:

TABLE II
SPECIFIC HEAT CONSTANTS, GASES.

Gas.	a	$a + \frac{R}{777.52}$	b	$\frac{b}{2}$	Authority.
CO ₂	.1477	.1944	.000097	.0000484	LeChatelier
CO ₂2010	.0000824	.0000412	Holborn and Austin
N ₂	.170	.2404	.0000484	.0000242	LeChatelier
N ₂2350	.000021	.0000105	Holborn and Austin to 2606° F.
N ₂2350	.0000208	.0000104	Callender (1544° F. to 2440° F.)
O ₂	.1488	.2125	.0000424	.0000212	LeChatelier
H ₂ O	.3211000122	.000061	LeChatelier
Air2431	.000135	.0000675	Callender (1544° F. to 2440° F.)

For purposes of comparison various curves have been plotted which are given in the Handbook of Tables, Chart No. 11. These show results of specific heat at constant volume, at temperature $t^\circ \text{F.}$, the total heat above 32°F. per pound of gas, and the mean specific heat from 32°F. to $t^\circ \text{F.}$

Probably there is now more known of the specific heat of superheated steam than of any common gaseous substance, and it is likely that other substances will be found in time to have somewhat similar characteristics. Pure computation from the laws of perfect gases indicates that the specific heat of gases or superheated vapors must be either a constant, or a function of temperature only, and this is what prompted the form of the LeChatelier formula. Bold experimentation on steam, disregarding the law, or rather appreciating that superheated steam is far from a perfect gas, principally by Knobloch and Jacob and by Thomas, showed its specific heat to be a function of both pressure and temperature. Results were obtained that permitted the direct solution of problems of heat of superheat, or the heat per pound of vapor at any temperature above that at which it was produced, or could exist in contact with the liquid from which it came. Critical study of various results by Marks and Davis led them to adopt the values of Knobloch and Jacob with slight modifications, for which evidence was in existence, raising the specific heats at low pressures and temperatures, and their conclusions are adopted in this work. In Fig. 100 is shown (A) the Marks and Davis modification of the C_p curve of Knobloch and Jacobs, the integral of which (C) gives the heat of superheat from any temperature of steam generation to actual steam temperature, while (B) shows the values for the mean specific heat above the temperature of saturation for the particular pressure in question.

When substances of the same class are mixed so that w_1, w_2, w_3 , etc., lbs. of the different substances having specific heats C_1, C_2, C_3 , etc., or C_{p1}, C_{p2}, C_{p3} , etc., or C_{v1}, C_{v2}, C_{v3} , etc., then the specific heat of the mixture is given by

$$C = \frac{C_1 w_1 + C_2 w_2 + C_3 w_3 + \text{etc.}}{w_1 + w_2 + w_3 + \text{etc.}}, \quad \dots \quad (506)$$

$$C_s = \frac{C_{s1} w_1 + C_{s2} w_2 + C_{s3} w_3 + \text{etc.}}{w_1 + w_2 + w_3 + \text{etc.}}, \quad \dots \quad (507)$$

$$C_p = \frac{C_{p1} w_1 + C_{p2} w_2 + C_{p3} w_3 + \text{etc.}}{w_1 + w_2 + w_3 + \text{etc.}}, \quad \dots \quad (508)$$

Example. If 5 lbs. of olive oil at a temperature of 100°F. , 10 lbs. of petroleum at a temperature of 150°F. , and 50 lbs. of water at 50°F. are mixed together, what will be the resultant temperature and how much heat will be required to heat the mixture 100° above this temperature?

Sp.ht. of olive oil = .4, Sp.ht. petroleum = .511, Sp.ht. water = 1.000.

Let x = the final temp. The heat given up by the substances falling in temperature is equal to that gained by those rising, hence

$$50(x - 50) \times 1 = 5(100 - x) \times .4 + 10(150 - x) \times .511, \quad \therefore 50x - 2500 = 200 - 2x + 766 - 5.11x, \\ 57.11x = 3466, \quad \text{or, } x = 60.7^\circ \text{F.}$$

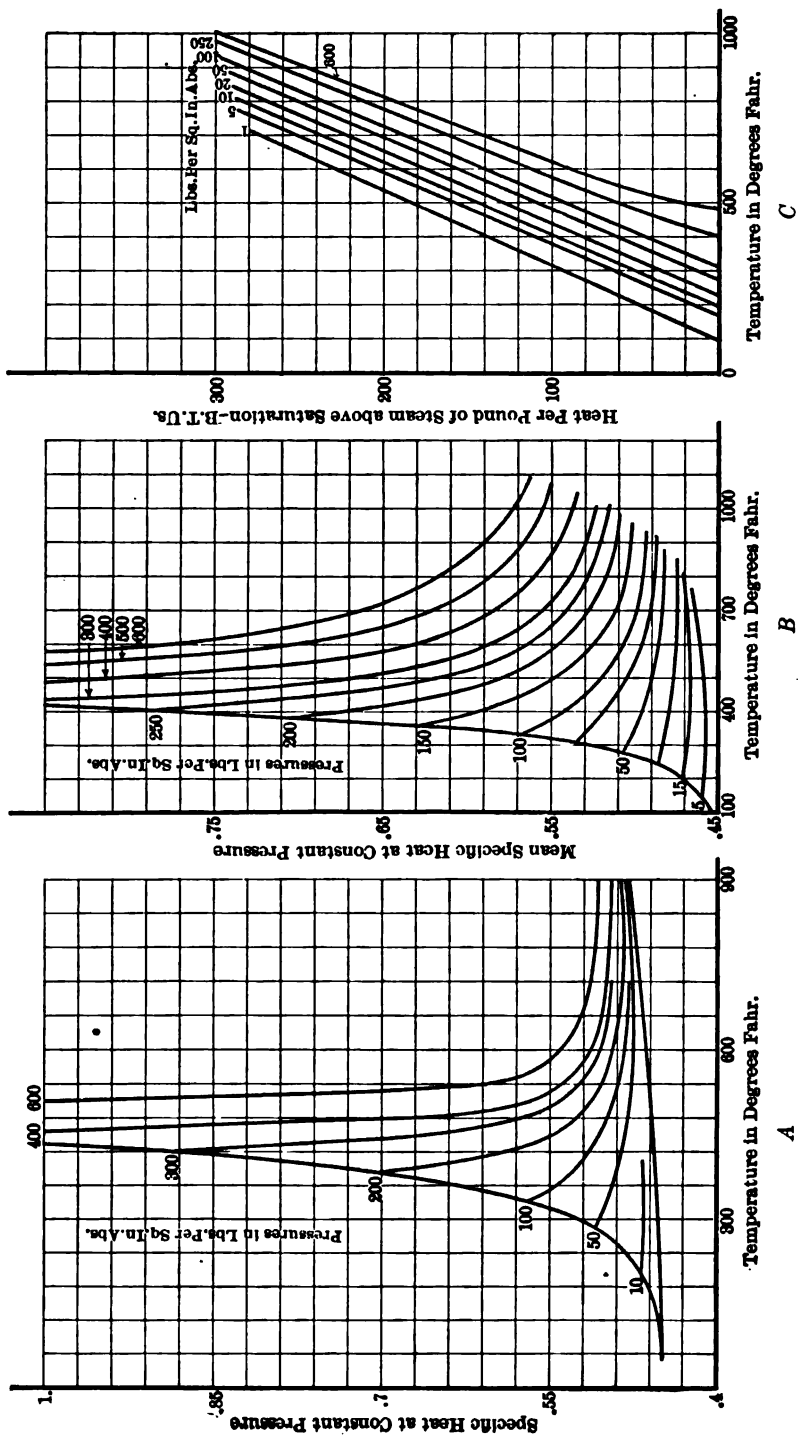


FIG. 100.—Specific Heat of Superheated Steam at, Mean Specific Heat from Saturation Temperature to, and Heat of Superheat per pound from Saturation Temperature to, Various Temperatures.

$$\begin{aligned}\text{Sp.ht. of mixture} &= \frac{C_1w_1 + C_2w_2 + C_3w_3}{w_1 + w_2 + w_3}, \text{ from Eq. (506),} \\ &= \frac{5 \times .4 + 10 \times .511 + 50 \times 1}{5 + 10 + 50} = \frac{57.11}{65} = .8786,\end{aligned}$$

whence the heat required will be $100 \times 65 \times .8786 = 5711$ B.T.U.

Prob. 1. To change a pound of water at 32° F. to steam at 212° F. requires 1150.4 B.T.U.'s. If the same amount of heat be given to a cubic foot of each of the following substances at 32° F., what will be final temperature in each case? (a) copper; (b) iron; (c) mercury; (d) clay; (e) stone.

Prob. 2. How many pounds of the following substances could be warmed 10° F. by the heat required to raise 100 lbs. of water from 40° F. to 200° F.?

(a) Ethyl alcohol from 100° F.; (b) Sea water from 60° F., (density = 1.045); (c) Glycerine from 60° F.; (d) Tin from 480° F.

Prob. 3. If 150 lbs. of water at 200° F. are added to a tank containing 200 lbs. of petroleum at 70° F., what will be the resultant temperature, neglecting any heat absorbed or given up by the tank itself?

Prob. 4. To melt 1 lb. of ice requires 144 B.T.U. How much would this lower the temperature of 1 lb. of the following substances: (1) at constant pressure; (2) at constant volume; (a) air; (b) oxygen; (c) ammonia; (d) hydrogen; (e) nitrogen?

Prob. 5. What would be the specific heats of the following mixture: Hydrogen 3 lbs., oxygen 1 lb., nitrogen 7 lbs., carbon dioxide 20 lbs., carbon monoxide 10 lbs.?

Prob. 6. Air is approximately 77 per cent N_2 , and 23 per cent O_2 by weight. By means of the specific heats of the components, find its specific heats at constant pressure, and at constant volume.

Prob. 7. By means of the specific heats, find the values of R and γ most nearly correct at atmospheric temperature (60° F.) for, hydrogen, air, carbon dioxide, carbon monoxide and nitrogen.

Prob. 8. How much water could be heated from 40° F. to 60° F. by the heat needed to superheat 10 lbs. of steam at 200 lbs. per square inch absolute to 700° F.?

Prob. 9. A building containing 250,000 cu. ft. of space is heated by a hot-water system. Considering the air to change eight times per hour, how many pounds of water per hour must be circulated if the drop in temperature of the water is from 200° to 160° and the temperature of the outside air is 30° F. while that of the room is 70° F. neglecting wall conductor heat?

Prob. 10. How much heat would be required to warm a pound of liquid CO_2 from zero to 80° F.? Compare with water and ammonia.

68. Volume or Density Variation with Temperature of Solids, Liquids, Gases and Vapors, not changing State. Coefficients of Expansion. Coefficients of Pressure Change for Gases and Vapors. Solids increase in length, or in any linear dimension, a certain fraction of their original length for each degree temperature rise and the expansion is usually assumed to be in proportion to temperature rise. The relation between original and final length can be set down in an equation involving the coefficient of expansion.

Let α = coefficient of linear expansion = fractional increase in length per degree.

l_1 and t_1 = original length or any other linear dimension and the corresponding temperature;

l_2 and t_2 = length which l_1 becomes after heating, and the corresponding temperature.

Then increase in length $= l_2 - l_1 = al_1(t_2 - t_1)$, (509)

New length $l_2 = l_1 + al_1(t_2 - t_1) = l_1[1 + a(t_2 - t_1)]$. . . (510)

Solids, expand cubically and the new volume will be to the old as the cubes of the linear dimension.

Let α = coefficient of volumetric expansion;

v_1 = original volume;

v_2 = final volume after heating.

Then when the temperature rises one degree,

$$\frac{v_2}{v_1} = \left(\frac{l_2}{l_1}\right)^3 = (1 + \alpha)^3 = 1 + 3\alpha + 3\alpha^2 + \alpha^3 = 1 + \alpha. \quad (511)$$

If α is small, and it is generally less than $\frac{1}{10000}$, then the square and cube can be neglected in comparison with the first power, whence

$$1 + \alpha = 1 + 3\alpha \quad \text{and} \quad \alpha = 3\alpha,$$

so that the coefficient of volumetric expansion may be taken as sensibly equal to three times the coefficient of linear expansion, and similarly, the coefficient of surface expansion as twice the coefficient of linear expansion.

Liquids, by reason of the fact that they must always be held in solid containers, may be said to have no linear expansion, and therefore, although the expansion may be one direction only, the amount is due to the total change of volume rather than the change of length along the direction of freedom to expand. The same is true of gases, so that for gases and liquids only coefficients of volumetric expansion are of value. Tables of coefficients for various solids, liquids and gases are given in the Handbook of Tables, Nos. XXVI to XXVIII. With liquids and gases it is usual to take the volume at 0°C . or 32°F . and 29.92 ins. Hg pressure as a standard, and the coefficient gives the increase as a fraction of this, per degree departure from the freezing-point. This is the universal practice with gases.

It appears that the coefficients of expansion for solids are quite different from one another, ranging from over 15×10^{-4} for wax, to $.058 \times 10^{-4}$ for Jena normal glass, a range of over two hundred and sixty times. Determinations of the value at various temperatures for any one substance indicate a variation with temperature, which proves that proportionality of increase of dimensions to temperature rise, does not hold true, a fact which has led to formulas of the form

$$l_2 = l_1[1 + x(t_2 - t_1) + y(t_2 - t_1)^2],$$

the value of which is dependent on the determination of the constant and verification of correctness of form, which has not by any means been conclusively

done. For most engineering work the constant values nearest the temperature range will suffice except for certain liquids, vapors, and gases. A more marked tendency to follow such a law of variation with temperature is found with liquids, coefficients for some of which are given in the standard physical tables.

The two important liquids, mercury and water, have been separately studied in greater detail and the latter exhibits a most notable exception to the rule.

Water, as already mentioned, has its maximum density at 39.1° F. and expands with both fall and rise of temperature. Its expansion is given by Scheel, as follows:

$$v_2 = v_1(1 - .03655 \times 10^{-3}t + 2.625 \times 10^{-6}t^2 - 1.161 \times 10^{-8}t^3 + 3822 \times 10^{-10}t^4). \quad (512)$$

Most commonly the expansion of water is not considered in this way, but by comparing densities at varying temperatures; all sets of physical tables contain values which in this work are significant only as affecting the change of volume in turning water to steam *and such values as are needed are incorporated in the steam tables.*

The study of the expansion of gases and vapors at constant pressure, and rise of pressure at constant volume per degree, has perhaps been fairly complete and is of greatest significance, because from it most of the important laws of thermodynamics have been derived. This work may be said to have started with the Regnault air and gas thermometer work, previously referred to. Some of the authentic values collected in the Landolt, Börnstein, Myerhoffer, and Smithsonian Physical Tables, are given in Tables XXVIII and XXIX as previously indicated in the Handbook of Tables.

The remarkable thing about the coefficients for these gases and vapors is the approach to constancy for most of the gases, not only of the coefficients of expansion for $P=c$, nor the similar constancy of the coefficients of pressure rise for $V=c$, but more remarkable than either of these is the similarity of the two constant coefficients. These facts permit the generalization of effect when $P=c$, and when $V=c$, and of the announcement of a law by means of which all such problems can be solved instead of applying separate coefficients for every substance and every different temperature necessary for solids and liquids where, for example, the maximum coefficient was over 260 times as great as the least. The average coefficient for all gases, applying both to pressures and volumes, is the same as that which enters into the gas thermometer work, and its best value is found to be

$$\left. \begin{aligned} \alpha &= \frac{1}{491.52} = .002034, \text{ per degree F.} \\ \alpha &= \frac{1}{273.13} = .003661, \text{ per degree C.} \end{aligned} \right\}, \quad \dots \quad (513)$$

These are the same as the reciprocals of the absolute temperature of the ice-melting point, and are but expressions of conditions for reduction of the volume and pressure at the ice-melting temperature to zero by constant pressure and constant volume abstraction of heat respectively, and, by

stating the amount of reduction per degree, give by implication the number of degrees for complete reduction.

Example. The rails on a stretch of railroad are laid so that they just touch when the temperature is 120°F . How much total space will there be between the rails per mile of track at 0°F ?

For wrought iron α will be nearly the same as for Bessemer steel = .00000648,

Hence the linear reduction in 5280 ft. for a change of 120°F . will be

$$5280 \times 120 \times .00000648 = 4.1 \text{ ft.}$$

Prob. 1. A steam pipe is 700 ft. long when cold (60°F), and is anchored at one end. How much will the other end move, if steam at a temperature of 560°F . is turned into the pipe?

Prob. 2. A copper sphere is one foot in diameter at 50°F . What must be the diameter of a ring through which it will pass at a temperature of 1000°F ?

Prob. 3. A hollow glass sphere is completely filled with mercury at 0°F . What per cent of the mercury will be forced out if the temperature rises to 300°F ?

Prob. 4. A room 100 ft. \times 50 ft. \times 10 ft. is at a temperature of 40°F . The temperature rises to 70°F . How many cubic feet of air have been forced from the room?

Prob. 5. The air in a pneumatic tire is at a pressure of 90 lbs. per square inch gage and at a temperature of 50°F . Owing to friction of the tire on the ground in running, the temperature rises to 110°F . What will be the pressure?

Prob. 6. A brick lighthouse is approximately 200 ft. high. If it should be exactly this at 0°F ., what would it be at 100°F ?

Prob. 7. Show that if a glass tube is rigidly held at each end by brackets attached to an iron tank it will break if the tank is warmed.

Prob. 8. From Eq. (512) find the density of water at 60°F ., 100°F ., 212°F ., and compare with the values in the steam tables.

Prob. 9. A drum containing CO_2 gas at a pressure of 250 lbs. per square inch gage is raised 100°F . above its original temperature. What will be the new pressure?

69. Pressure, Volume and Temperature Relations for Gases. Perfect and Real Gases. Formulating the relations between the pressure change at constant volume and the volume change at constant pressure,

Let P and V be the simultaneous pressure and volume of gas;

t be its scale temperature at the same time, F ;

T be its absolute temperature at the same time, F .

Then at constant volume the pressure reached at condition (a) after heating from 32°F . is given by

$$P_a - P_{32} = \frac{1}{492} P_{32} \times (t_a - 32), \quad \therefore \quad P_a = P_{32} \left[1 + \frac{t_a - 32}{492} \right].$$

Similarly for another temperature t_b , the pressure will be $P_b = P_{32} \left[1 + \frac{t_b - 32}{492} \right]$.

Whence

$$\frac{P_a}{P_b} = \frac{1 + \frac{t_a - 32}{492}}{1 + \frac{t_b - 32}{492}} = \frac{492 - 32 + t_a}{492 - 32 + t_b} = \frac{t_a + 460}{t_b + 460}$$

or
$$\frac{P_a}{P_b} = \frac{T_a}{T_b}, \text{ for } V \text{ constant, } \dots \dots \dots (514)$$

Similarly
$$\frac{V_a}{V_b} = \frac{T_a}{T_b}, \text{ for } P \text{ constant } \dots \dots \dots (515)$$

Both Eqs. (514) and (515) are true for no gas all the time, but very nearly true for all, under any range of change, and a *hypothetical gas is created for which it is exactly true all the time, known as a perfect gas*, about which calculations can be made that would be impossible for real gases, and yet the results of which are so close to what would be the result with real gases as to be good enough for engineering practice. Therefore, with a mental reservation as a guard against too great confidence in the work, all real gases will be assumed perfect, and to follow Eqs. (514) and (515) except when experience shows the results are too far wrong to be useful.

These laws, known by the names both of Charles and Gay-Lussac, are closely associated with another law also doubly named as Boyle's or Mariotte's, and likewise an idealization of experimental observations known to be nearly true for all gases. This is to the effect that so long as temperatures are kept constant the pressures of gases vary inversely as their volume, or that,

$$\frac{P_a}{P_b} = \frac{V_b}{V_a}, \text{ and, } P_a V_a = P_b V_b = \text{constant, for } T \text{ constant. } \dots (516)$$

Study of the PV product, for various gases has revealed a good deal on the general properties of matter, especially as to the transition from one state to another. This is most clearly shown by curves which may be plotted in two ways. To coordinates of pressure and volume a family of equilateral hyperbolas one for each temperature, would represent the true $PV = C$ or *isothermal* relation and any variation in the constancy of the product would be shown by its departure from the hyperbola. Still more clearly, however, will the departure appear when the product PV is plotted against pressures, constancy of product would require all lines to be straight and inconstancy appear by departures from straight lines. To illustrate, the data from Young for carbon dioxide are plotted both ways in Fig. 101, from 32° F. to 496° F., the values of PV at 32° and 1 atm. are taken as unity on one scale. It appears that up to the temperature of 88° F., known as the critical temperature, each isothermal plotted to P and PV coordinates consists of three distinct parts:

- (a) a curved line sloping to the right and upwards;
- (b) a straight line nearly or exactly horizontal;
- (c) a nearly straight line sloping upward rapidly and to the left.

In this region then the isothermals are discontinuous, and this is caused by the liquefaction or condensation of the gas, during which increase of pressure produces no change of volume, provided the temperature is low enough. It also appears that each PV line has a minimum point and these minima joined result in a parabola. Tabular values of PV at three different temperatures and various pressures for oxygen, hydrogen, carbon dioxide, and ammonia, in

terms of the values at 32° and 1 atm., will be found in the Handbook (Table XXX), for further comparison and use. Except near the point of condensation or liquefaction, gases or vapors, which are the same thing save as to nearness to the critical state, follow the Boyle law closely enough for engineering purposes.

None of these approximate laws, Eqs. (514), (515) and (516) can be considered as general, because each assumes one of the variables to be constant, but a general law inclusive of both of these follows from further investigation of a fixed mass of gas undergoing various pressure-volume and temperature changes, such as occur in the cylinders of compressors and gas engines. A table of simultaneous experimental values of pressure, volume, and temperature, for any

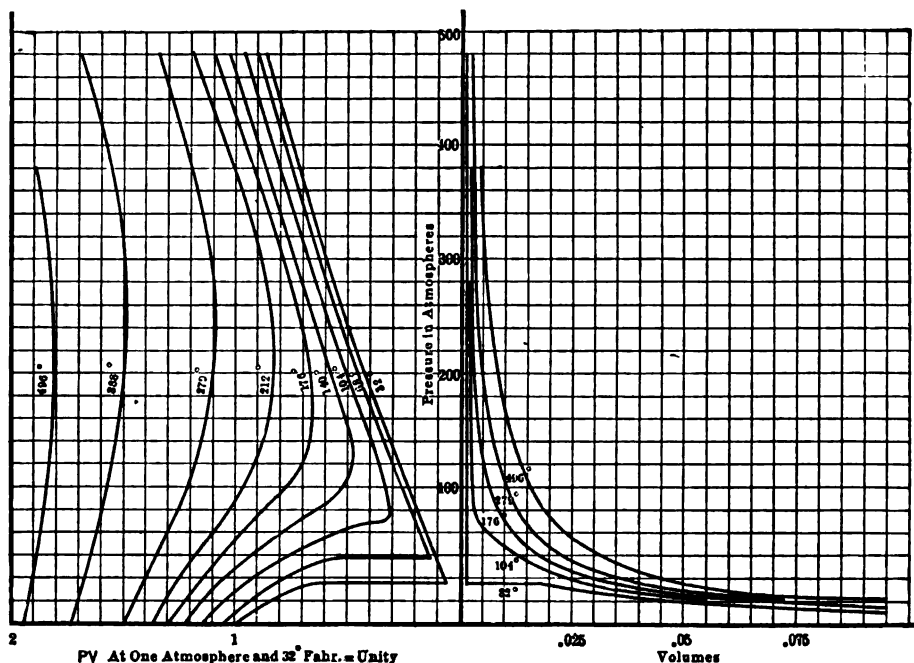


FIG. 101.—Isothermals or PV Products for Carbon Dioxide.

gas will reveal the still more general relation inclusive of the preceding three as follows:

$$\frac{P_a V_a}{T_a} = \frac{P_b V_b}{T_b} = \frac{PV}{T} = C, \quad (517)$$

in which C , is approximately constant for any one gas and assumed constant for perfect gases in all calculations. For twice the weight of gas at the same pressures and temperatures C , would be twice as large, so that taking a constant R for one pound, generally known as the "gas constant," and introducing a weight factor w , the general characteristic equation for the perfect gas is,

$$PV = wRT. \quad (518)$$

This general law may be derived from the three primary laws by imagining in Fig. 102, two points, *A* and *B*, in any position and representing any two states of the gas. Such points can always be joined by three lines, one constant pressure *A* to *X*, one constant temperature *X* to *Y*, and the other constant volume *Y* to *B*. For these the following relations hold, passing from *A* to *B*:

$$V_a = V_x \frac{T_a}{T_x}. \text{ But } V_x = V_y \frac{P_y}{P_x}, \text{ and } T_x = T_y, \text{ whence } V_a = V_y \frac{P_y}{P_x} \frac{T_a}{T_y}, \text{ or } \frac{P_a V_a}{T_a} = \frac{P_y V_y}{T_y}.$$

$$\text{Passing to } B, P_y = P_b \frac{T_y}{T_b}. \text{ But } V_y = V_b, \therefore \frac{P_a V_a}{T_a} = \frac{P_y V_y}{T_y} = \frac{P_b \frac{T_y}{T_b} V_b}{T_y} = \frac{P_b V_b}{T_b},$$

or in general $\frac{PV}{T} = \text{constant} = wR$, when the weight of gas is *w* lbs.

For numerical work, the values of *R* must be fixed experimentally by direct measurement of simultaneous pressure, volume, and temperature, of a known

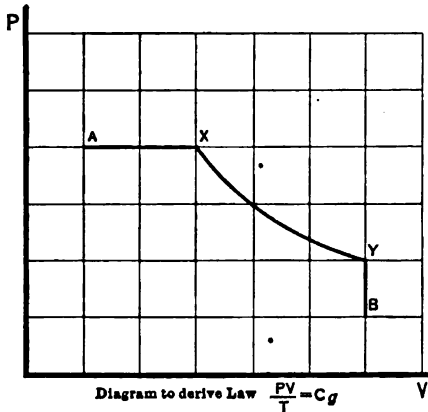


FIG. 102.—Curve of Continuous Relation between *P*, *V*, and *T* for Gases.

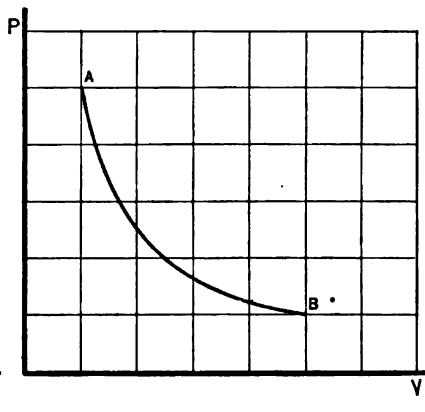


FIG. 103.—Expansion or Compression of Gas between *A* and *B*, Causing a Change of Temperature.

weight of gas, or computed from other constants through established relations. One such relation already mentioned but not proved is

$$R = 777.52(C_p - C_v). \quad (519)$$

It is extremely unlikely that the values of *R* found in both ways by a multitude of observers under all sorts of conditions should agree, and they do not, but it is necessary for computation work that a reasonable consistency be attained and that judgment in use be cultivated in applying inconsistent data. In the latter connection the general rule is to use that value which was determined by measurement of quantities most closely related to the one being dealt with. Thus, if *R* is to be used to find the state of a gas as to pressure, volume, and temperature, that value of *R* determined from the first method should be selected, but the second when specific heats or Joule's equivalent are involved. Of course, a

consistency could be incorporated for a perfect gas, but engineers deal with real gases and must be on guard against false results obtained by too many hypotheses or generalizations contrary to the facts. Accordingly, two values of R will be found in the Tables, one obtained from measurements of specific heats at constant pressure and determinations of the ratio of special heats, unfortunately not always at the same temperature and generally by different people; and the other by direct measures of gas volume at standard 32° F. temperature and 1 atm. pressure. These measurements are separately reported in Sections (67) and (70), respectively. See Table XXXI in Handbook.

If a gas in condition A , Fig. 103, expand in any way to condition B , then it has been shown that $P_a V_a^s = P_b V_b^s$, in which s has any value, which becomes numerically fixed only when the process and substance are more specifically defined. Comparing the temperatures at any two points A and B , it follows that

$$T_a = \frac{P_a V_a}{wR}, \text{ and } T_b = \frac{P_b V_b}{wR}, \text{ whence } \frac{T_b}{T_a} = \frac{P_b V_b}{P_a V_a}.$$

$$\text{But } P_a V_a V^{s-1} = P_b V_b V^{s-1}, \text{ and } \frac{P_b V_b}{P_a V_a} = \left(\frac{V_a}{V_b}\right)^{s-1}, \text{ whence } \frac{T_b}{T_a} = \left(\frac{V_a}{V_b}\right)^{s-1}. \quad (520)$$

$$\text{and} \quad \frac{V_b}{V_a} = \left(\frac{T_a}{T_b}\right)^{\frac{1}{s-1}}, \quad \dots \dots \dots (521)$$

Eqs. (520) and (521) give the relation between temperatures and volumes

$$\text{but} \quad \frac{V_a}{V_b} = \frac{T_a P_b}{P_a T_b} = \frac{T_b P_a}{T_a P_b}, \text{ which, substituted in above, gives}$$

$$\frac{T_b}{T_a} = \left(\frac{T_a}{T_b}\right)^{s-1} \left(\frac{P_b}{P_a}\right)^{s-1}, \text{ or } \left(\frac{T_b}{T_a}\right)^s = \left(\frac{P_b}{P_a}\right)^{s-1}, \text{ and } \frac{T_b}{T_a} = \left(\frac{P_b}{P_a}\right)^{\frac{s-1}{s}}. \quad (522)$$

$$\text{or} \quad \frac{P_b}{P_a} = \left(\frac{T_b}{T_a}\right)^{\frac{1}{s-1}}, \quad \dots \dots \dots (523)$$

Eqs. (522) and (523), give the relation between pressures and temperatures. It is convenient to set down the volume and pressure relations again to complete the set of three pairs of most important gas equations.

$$\frac{V_b}{V_a} = \left(\frac{P_a}{P_b}\right)^{\frac{1}{s}} \quad (524); \quad \frac{P_b}{P_a} = \left(\frac{V_a}{V_b}\right)^s \quad (525) \quad \dots \dots \dots (524)$$

These are perfectly general for any expansion or compression of any gas, but are of value in calculations only when s is fixed either by the gas itself or by the thermal process, as will be seen later.

Example. A pound of air has a volume of 7.064 cu.ft. at a pressure of two atmospheres and a temperature of 100° F. Find the value of R for air from the data; also the final volume and temperature if expansion occurs so that $s = 1.4$ until the pressure becomes $\frac{1}{2}$ an atmosphere.

$$PV = wRT, \text{ or } 2116 \times 2 \times 7.064 = 1 \times R \times 560, \text{ or } R = 53.38,$$

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{s-1}{s}} = (4)^{\frac{1.4}{1.4}} = 1.49, \quad \therefore T_2 = T_1 + 1.49 = \frac{560}{1.49} = 352 \text{ abs.} = -108^\circ \text{ F.}$$

$$\frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{s}} = 2.7, \quad \text{or} \quad V_2 = 2.7V_1 = 19.1 \text{ cu.ft.}$$

Prob. 1. A perfect gas is heated in such a way that the pressure is held constant. If the original volume was 10 cu.ft. and the temperature rose from 100° F. to 400° F. , what was the new volume?

Prob. 2. The above gas was under a pressure of 100 lbs. per square inch gage at the beginning of the heating. If the volume had been held constant what would have been the pressure rise?

Prob. 3. A quantity of air, 5 lbs. in weight, was found to have a volume of 50 cu.ft. and a temperature of 60° F. What was the pressure?

Prob. 4. A cylinder holding 12 cu.ft. has a pressure of 250 lbs. per square inch gage, and the temperature is 50° F. What would be the weight of its contents were it filled with (a) CO_2 ; (b) NH_3 ; (c) Oxygen; (d) Hydrogen?

Prob. 5. At a pressure of 14.696 lbs. per square inch and a temperature of melting ice, one pound of air has a volume of 12.387 cu.ft. From the data find the value of R for air. The specific heats of air are given by one authority as $C_p = .2375$ and $C_v = .1685$. Find R from the data and see how the two values obtained compare.

Prob. 6. At the same pressure and temperature as in Prob. 5, a pound of the following substances has a volume as shown. From the data and the values of specific heats, find R by the two methods.

Substance.	Cu.ft. per lb.	C_p .	C_v .
Hydrogen.....	178.93	3.409	2.412
Carbon dioxide....	8.15	.217	.1535
Oxygen.....	11.21	.2175	.1551
Nitrogen.....	12.77	.2438	.1727

Prob. 7. Five cu.ft. of gas at a pressure of 3 atmospheres absolute and a temperature of 50° F. expand to atmospheric pressure. What will be the final volume and temperature, if $s = 1.35$?

Prob. 8. 1000 cu.ft. of gas at atmospheric pressure and 60° F. are compressed into a tank of 100 cu.ft. capacity. What will be the pressure in the tank and the temperature of the gas at the end of the process, if the gas is CO_2 and the compression adiabatic?

Prob. 9. What will be the final volume, pressure and temperature, if a pound of air at atmospheric pressure (14.7 lbs. per square inch) and a temperature of 60° F. be compressed adiabatically until its absolute temperature is six times its original value?

70. Gas Density and Specific Volume and its Relation to Molecular Weight and Gas Constant. The density of a gas is best stated for engineering purposes as the weight of a cubic foot, but as this becomes less on rise of temperature or decrease of pressure it is necessary to fix a standard condition for reporting this important physical constant. It is best to take one atmosphere 760 mm. or 29.92 ins. of mercury as the pressure, and $0^\circ \text{ C.} = 32^\circ \text{ F.}$ as the standard temperature, though it is in some places customary in dealing with

commercial gases, such for example as those used for illumination, to take the temperature at 60° F., since illuminating gas at this condition is often known among gas engineers as standard gas. In this work, however, the freezing-point and standard atmosphere will be understood where not specifically mentioned, as the conditions for reporting gas density and its reciprocal, the specific volume of gases or the cubic feet per pound. The relation of volume and density at any pressure and temperature to the volume and density under standard conditions is graphically shown in the book of tables, Chart No. 13.

These constants have been fairly accurately determined by many investigators, whose figures do not agree absolutely, but the disagreement is found only in the last significant figures. Some selected values of reliable origin for the important gases are given in the Handbook of Tables, No. XXXII, for use in computation work.

It often happens in dealing with gases and especially superheated vapors that a value is needed for which no determination is available, so that general laws of density or specific volumes of substances are necessary to permit the needed constant to be estimated. These relations may be applied to vapors as well as to gases even though the standard conditions are those for the liquid state, on the assumption that all gases and vapors will expand under temperature, or contract under pressure rise, to the same degree, retaining the same relations between all substances which exist at the standard atmosphere and freezing-point. A vapor thus reported below its point of condensation and assumed to have reached that condition from one of higher temperature at which it exists as vapor, is often called steam gas, or alcohol gas, for example in the case of water and alcohol.

Such general relations between the densities of gases, so desirable and useful in practical work, have been found by studying the manner in which gases chemically combine with respect to the volume relations before and after the reaction. Following several experimenters, who reported observed relations, Gay-Lussac stated a general law, as follows:

When two or more gaseous substances combine to form a compound, the volumes of the combining gases bear a simple ratio to each other and to that of the compound when it is also a gas.

He also attempted to derive some relation between this law and Dalton's atomic combining law, which states that, in combining chemically, a simple numerical relation exists between the number of atoms of different elements which unite to form a compound. This was not successful, but Avagadro later found the expected relation by assuming that it is a particle, or a number of atoms, or a molecule, that is important in combining, and the law stated is as follows:

Equal volumes of different gases measured at the same pressure and temperature contain the same number of molecules.

It is possible by analysis of these two laws to obtain a relation between the volumes of gases and the weights of their molecules because the molecular relation of

Avagadro, unites with the combining law of Gay-Lussac to define the relation between the number of combining molecules. At the same time the weight relations in chemical reactions, based on atomic weights, may be put into a similar molecular form, since the weight of any one substance entering is the product of the number of its molecules present and the weight of the molecule. Applying the relation between the number of molecules derived previously, there is fixed a significance for the weight of the molecule which for simple gases, like hydrogen and oxygen, is twice the atomic weight and for compound gases, like methane and carbon dioxide, is equal to the atomic weight. Applying this to the Avagadro law, the weights of equal volumes of different gases must be proportional to their molecular weights, as equal volumes of all contain the same number of molecules.

Putting this in symbolic form and comparing the density of any gas with that of hydrogen, because this is the lightest of gases and has well determined characteristics, requires the following symbols, denoting hydrogen values by the subscript H . Let m = molecular weight of a gas,

$$\delta = \text{density in lbs. per cu.ft.} = \frac{1}{V},$$

$$\text{then } \frac{\delta_1}{\delta_2} = \frac{m_1}{m_2} \quad (526); \quad \text{and } \frac{\delta_1}{\delta_H} = \frac{m_1}{m_H} \quad (527) \quad$$

But as the molecular weight of hydrogen is for engineering purposes equal to 2 closely enough, and as hydrogen weighs .00559 lb. per cu.ft. = δ_H , at 32° F. and 29.92 ins. Hg, we have,

$$\text{Lbs. per cu.ft.} = \delta_1 = .00279 m_1. \quad (528)$$

To permit evaluation of Eq. (528) it is necessary that there be available a table of molecular weights of gases and the atomic weights of elements from which they are derived. As atomic weights are purely relative they may be worked out on the basis of any one as unity; originally chemists used hydrogen as unity, but the custom has changed to $\frac{1}{16}$ the value for oxygen as unity. These atomic weights are not whole numbers but nearly so, therefore for convenience and sufficient accuracy the nearest whole number will be used in this work and hydrogen be taken as unity except where experience shows it to be undesirable.

The reciprocal expression to Eq. (528) can be set down, giving the specific volume of a gas or its cubic feet per pound at 32° F. and 29.92 ins. Hg, as follows:

$$\text{Cu.ft. per lb.} = \frac{1}{\delta_1} = \frac{1}{.00279} m_1 = \frac{358}{m_1}. \quad (529)$$

This is a most important and useful conclusion as applied to gases and vapors for which no better values are available; in words it may be stated as follows:

The cubic feet per pound of any gas or vapor at 32° and 29.92 ins. Hg, is equal to 355.87 divided by its molecular weight, or

The molecular weight of any gas or vapor in pounds, will occupy a volume of 358 cu.ft. at 32° and 29.92 ins. Hg.

The approach to truth of these general laws is measured by the values given in the tables for specific volume and density (a) experimentally derived, and (b) as derived from the hydrogen value by the law.

Another and very useful relation of a similar nature is derivable from what has been established, connecting the gas constant R with molecular weights.

The general law $PV = wRT$ when put in the density form by making $\delta = \frac{w}{V}$ becomes

$$\frac{P}{\delta} = RT. \quad (530)$$

Whence, comparing gases with each other and with hydrogen at the same pressure and temperature

$$\frac{\frac{P_1}{\delta_1}}{\frac{P_2}{\delta_2}} = \frac{\delta_2}{\delta_1} = \frac{R_1}{R_2}, \quad \frac{\delta_1}{\delta_H} = \frac{R_H}{R_1}, \quad \text{and} \quad \delta_1 = \frac{\delta_H R_H}{R_1}, \quad (531)$$

which indicate that the *densities of gases are inversely proportional to the gas constants*, or the density of any gas is equal to the density of hydrogen times the gas constant for hydrogen divided by its own.

Inserting the values of density at 32° and 29.92 ins. Hg and of the gas constant for hydrogen, it follows that for any gas

$$\text{Lbs. per cu.ft.} = \delta_1 = \frac{4.2812}{R}, \quad (532)$$

the reciprocal of which gives the specific volume at 32° F. and 29.92 ins. Hg, or

$$\text{Cu.ft. per lb.} = V_1 = \frac{R}{4.2812}. \quad (533)$$

Example. 1. To find the pounds per cubic foot of gas at 32° F. and 29.92 ins. of mercury when its value is known for 90° and 13 lbs. per sq.in. On the temperature scale of Chart No. 13, pass vertically until the temperature line is reached, then horizontally until the curve for 13 lbs. absolute is reached. The value on the scale below is found to be 1.265, so that the density under the standard conditions is 1.265 of the value under known conditions. Had it been required to find the cubic feet per pound the process would be precisely the same, the value being taken from the lower scale, which for the example reads .79, or the cubic feet per pound under standard conditions is 79 per cent of the value under conditions assumed.

Example 2. By means of the molecular weight find the density of nitrogen at 32° F. and 29.92 ins. Hg, and the cubic feet per pound for these conditions.

$$\text{From Eq. (526)} \quad \frac{\delta_1}{\delta_H} = \frac{m_1}{m_H}, \quad \text{or} \quad \delta_1 = \frac{28 \times .00559}{2}.$$

Hence δ for nitrogen = .07826 pounds per cu.ft., and $\frac{1}{\delta} = \frac{1}{.07826} = 12.709$ cu.ft. per lb.

Prob. 1. Taking the density of air from the table, find the value of R for air, by means of Eq. (532) and compare its value with that found in Section 69.

Prob. 2. Compare the density of carbon monoxide when referred to 32° F. and 60° F. as the standard temperature, as found both ways.

Prob. 3. By means of their molecular weights find the density of oxygen, nitrogen and carbon dioxide at 32° F. and 29.92 ins. Hg.

Prob. 4. What are the cubic feet per pound of acetylene, methane and ammonia at 32° F. and 29.92 ins. Hg?

Prob. 5. An authority gives the following values for R . Compare the densities found by this means with the densities for the same substance found by the use of the molecular weights. Oxygen 48.1; Hydrogen 764.0; Carbon monoxide 55.0.

Prob. 6. What will be the volume and density under standard conditions of a gas which contains 12 cu.ft. per pound at a temperature of 70° F. and a pressure of 16 lbs. per square inch absolute?

Prob. 7. What will be the difference in volume and density of a gas when considered at 60° and 29.92 ins. of Hg, and at 32° F. and 29.92 ins. of Hg?

71. Pressure and Temperature Relations for Vapor of Liquids or Solids. Vaporization, Sublimation and Fusion Curves. Boiling- and Freezing-points for Pure Liquids and Dilute Solutions. Saturated and Superheated Vapors. Substances may exist in one of three states, solid, liquid or gas, the latter being generally called vapor when, at ordinary temperatures, the common state is that of liquid or solid, or when the substance examined is near the point of liquefaction or condensation—just which state shall prevail at any time depends on thermal conditions. Within the same space the substance may exist in two of these three states or even all three at the same time under certain special conditions. These conditions may be such as to gradually or rapidly make that part in one state, turn into another state, or may be such as to maintain the relative amounts of the substance in each state constant; conditions of the latter sort are known as conditions of equilibrium. These are experimental conclusions, but as in other cases they have been concentrated into general laws of which they are but special cases. The study of the conditions of equilibrium, whether of physical state or chemical constitution, is the principal function of physical chemistry, in the pursuit of which the Gibbs phase rule is a controlling principle. According to this rule each possible state is called a phase, and the number of variables that determine which phase shall prevail or how many phases may exist at the same time in equilibrium for one chemical substance like water, is given by the following relation, which is but one of the conclusions of this general principle of equilibrium:

Number of undefined variables = 3 — number of phases.

Now it is experimentally known that if water be introduced into a vacuum chamber some of it will evaporate to vapor and that, therefore, water and its vapor may coexist or the number of phases is two, but this does not state how or when equilibrium will be attained. The rule above, however, indicates that for this case there can be but one undefined or independent variable and, of course, since the pressure rises more when the temperature is high than when

low, the two variables are pressure and temperature, of which accordingly only one is free or independent, so that fixing one fixes the other. In other words when a vapor and its liquid are together the former will condense or the latter evaporate until either pressure or temperature is fixed, and fixing the one the other cannot change, so that the conditions of equilibrium are indicated by a curve to coordinates P and T , on one side of which is the vapor state and on the other that of liquid. Such a curve is the *vapor pressure-temperature curve* of the substance, sometimes called its *vapor tension curve*. Much experimental information exists on this physical property of substances, all obtained by direct measurement of simultaneous pressures and temperatures of a vapor above its liquid, carefully controlled so that the pressure or the temperature is at any time uniform throughout.

The conditions of equilibrium between vapor and liquid, defined by the vapor tension curve extend for each substance over a considerable range of pressure and temperature, but not indefinitely, nor is the range the same for each. At the high-pressure and temperature end a peculiar interruption takes place due to the expansive effect of the temperature on the liquid and the compressive effect of the pressure on the vapor, the former making liquid less dense and the latter making vapor more dense, the two densities become equal at some pressure and temperature. The point at which this occurs is the "*critical point*" at which the equilibrium between liquid and vapor that previously existed ends, and there is no longer any difference between vapor and liquid. This point is a most important one in any discussion of the properties of matter; while difficult to exactly locate, it has received much experimental attention, and some of the best values are given in the Tables (No. XXXIV) for the pressure, density, and temperature defining it, for the substances important in engineering.

There are also presented in the Tables the vapor tension curves of water, to a large scale (Charts Nos. 16 to 21), derived from the tabular data. Similar curves will be found in Dr. Lucke's Engineering Thermodynamics for ammonia and carbon dioxide. The data used in obtaining the charts are partly direct experimental determinations and partly corrections obtained by passing a smooth curve representing an empiric equation of relation between pressure and temperature, through the major part of the more reliable experimental points.

In calculations for this work the values of Marks and Davis, and Marks have been accepted and used. These indicate the critical point as located at 706.1° F. and 3200 lbs. per square inch.

Just as the vapor-liquid curves indicate the conditions of equilibrium between vapor and its liquid, dividing the two states and fixing the transition pressure or temperature from one to the other, so also does a similar situation exist with respect to the vapor-solid relations. In this case the curve is that of "*sublimation*," and indicates the pressure that will be developed above the solid by direct vaporization at a given temperature in a closed chamber.

By plotting a curve of sublimation of vapor-ice, it can be shown that the line divides the state of ice from that of vapor so that at a constant pressure,

decrease of temperature will cause vapor to pass directly to ice, and at constant temperature a lowering of pressure will cause ice to pass directly to vapor.

Likewise the liquid, water, may pass to the solid, ice, by lowering temperature at a fixed pressure; a curve plotted to these values will then become the curve of "*fusion*."

If curves be plotted for vapor tension, sublimation, and fusion to the same scale they will meet at a point located at a pressure of 4.6 mm. Hg = .18 ins. Hg (= 0.088 lbs. per sq.in.), and temperature $+ .0076^{\circ}\text{C.} = 32.01^{\circ}\text{F.}$, ordinarily taken at 32°F. , which point is named the triple point, as indicated in Fig. 104. The fact that the vapor pressure for water extends below freezing-point and parallels more or less that of ice indicates the condition of supercooled water, one of unstable equilibrium instantly dispelled by the introduction of a little ice at the proper stable state for this temperature.

Ordinary engineering work is not concerned with the entire range indicated in Fig. 104 for any substance, but with the higher temperature ranges for some and the low for others; with transition from solid to liquid state for metals and similar solids, and the transition from liquid to vapor for a great many, of which water comes first in importance; then the refrigerating fluids, ammonia and carbon dioxide; and last certain fuels like alcohol and the petroleum oils with their distillates and derivatives.

Melting-points, or the fusion temperature of such solids as are important, are usually given for only one pressure, the standard atmosphere, as in ordinary practice these substances are melted only at atmosphere pressure, and some such values are given in Table No. XXXIX.

This is not the case, however, for *boiling-points*, which must be defined a little more closely before discussion. The vapor-pressure curves indicate that as the temperature of a liquid rises, the pressure rises also if the substance is enclosed, but if the pressure were relieved by opening the chamber to a region of lower pressure and kept constant, then the temperature would no longer rise and boiling or ebullition would take place. The boiling-point then is the highest temperature to which the liquid and its vapor could rise under the existing pressure. When not otherwise defined the term boiling-point must be taken to mean the temperature of ebullition for atmospheric pressure of 29.92 ins. Hg. Values for several substances are given in the Handbook of Tables (Table XXXVII).

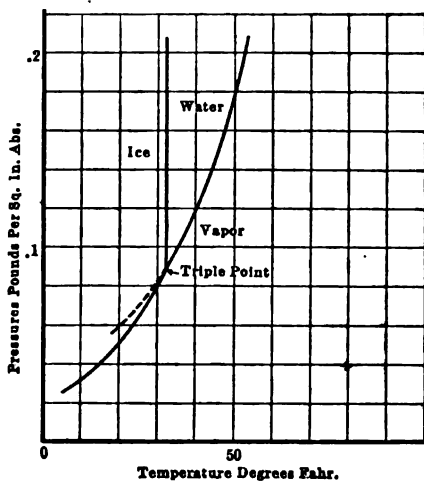


FIG. 104.—Water Vapor—Water—Ice, Combined Curves of Pressure-temperature Relation. The Triple Point.

Vapor having the temperature required by the pressure of the pressure-temperature curve is known as *saturated vapor*, and this may be defined as vapor having the lowest temperature at which it could exist as vapor, under the given pressure. Vapors may, however, be superheated, that is, have higher temperatures than saturated vapors at the same pressure, but cannot so exist for long in the presence of liquid. Superheating of vapors, therefore, implies isolation from the liquid, and the *amount of superheat* is the number of degrees excess of temperature possessed by the vapor over the saturation temperature for the pressure. In steam power plant work, especially with turbines, it is now customary to use steam with from 75° F. to 150° F. of superheat, and it might be noted that all so-called gases like oxygen and nitrogen are but superheated vapors with a great amount of superheat.

It has already been mentioned that the saturated vapor pressure-temperature curve of direct experiment is seldom accurate as found, but must be corrected by empiric equations or smooth average curves, and many investigators have sought algebraic expressions for them. These equations are quite useful also in another way, since they permit a more exact evaluation of the rate of change of pressure with temperature, which in the form of a differential coefficient is found to be a factor in other physical constants. One of these formulas for steam as adopted by Marks and Davis in the calculation of their tables is given in Eq. (534), the form of which was suggested by Thiessen:

$$(t+459.6) \log \frac{p}{14.7} = 5.409(t-212^\circ) - 3.71 \times 10^{-10}[(689-t)^4 - 477^4], \quad (534)$$

in which t = temperature F.; and p = pressure lbs. sq.in.

This represents the truth to within a small fraction of one per cent up to 400° F., but having been found inaccurate above that point Professor Marks has very recently developed a new one, based on Holborn and Baumann's high temperature measurements, which fits the entire range, its agreement with the new data being one-tenth of 1 per cent, and with the old below 400° F., about one-fifth of 1 per cent, maximum mean error. It appears to be the best ever found, and in developing it the methods of the physical chemists have been followed, according to which a pressure is expressed as a fraction of the critical pressure, and a temperature, a fraction of the critical temperature. This gives a relation between *reduced* pressures and temperatures and makes use of the principle of *corresponding states* according to which bodies having the same reduced pressure and temperature, or existing at the same fraction of their critical are said to be in equivalent states. The new Marks formula is given in Eqs. (535) and (536), the former containing symbols for the critical { pressure p_c
temperature T_c abs. } and the latter giving to them their numerical values, in pressure pounds per square inch, and temperature absolute F.

$$\log \frac{p_c}{p} = 3.006854 \left(\frac{T_c}{T} - 1 \right) \left[1 + .0505476 \frac{T}{T_c} + .629547 \left(\frac{T}{T_c} - .7875 \right)^2 \right], \quad (535)$$

$$\log p = 10.515354 - 4873.71 T^{-1} - .00405096 T + .000001392964 T^2. \quad (536)$$

As the method used in arriving at this formula is so rational and scientific, it has been adopted for a new determination of the relations between p and T for ammonia and carbon dioxide, so important as substances in refrigeration, especially the former. According to this method if p_c and T_c are the critical pressures and temperatures, both absolute, and p and T those corresponding to any other point, then according to Van der Waals,

$$\log \frac{p_c}{p} = f\left(\frac{T_c}{T} - 1\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (537)$$

Accordingly, the logarithm of the critical divided by any other pressure, is to be plotted against the quantity [(critical temperature divided by the temperature corresponding to the pressure) - 1], and the form of curve permits the determination of the function, after which the values of the critical point are inserted. This has been done for NH_3 and CO_2 with the result for NH_3 ,

$$\log \frac{p_c}{p} = .045 + 2.75\left(\frac{T_c}{T} - 1\right) + .325\left(\frac{T_c}{T} - 1\right)^2. \quad . \quad . \quad . \quad . \quad (538)$$

Inserting the critical constants,

$$\left. \begin{array}{l} p_c = 114 \text{ atm.} = 1675.8 \text{ lbs. per square inch} \\ T_c = 727.4^\circ \text{ F. absolute} \end{array} \right\} \begin{array}{l} \text{which are the Vincent and Chap-} \\ \text{puis' values,} \end{array}$$

$$\text{this becomes,} \quad \log p = 5.60422 - 1527.54T^{-1} - 171961T^{-2} \quad . \quad . \quad . \quad (539)$$

$$\text{For } \text{CO}_2 \text{ it was found that } \log \frac{p_c}{p} = .038 + 2.65\left(\frac{T_c}{T} - 1\right) + 1.8\left(\frac{T_c}{T} - 1\right)^3. \quad . \quad (540)$$

Inserting the critical constants,

$$\left. \begin{array}{l} p_c = 77 \text{ atm.} = 1131.9 \text{ lbs. per square inch} \\ T_c = 547.27^\circ \text{ F. abs.} \end{array} \right\} \begin{array}{l} \text{which are Andrews' values,} \end{array}$$

the equation becomes,

$$\log p = 7.46581 - 4405.765T^{-1} + 1617501.366T^{-2} - 257086165.8706T^{-3}. \quad (541)$$

Curves showing the relation of reduced and actual temperatures and pressures are given in the Handbook of Tables, Chart 14, for ammonia and in Chart 15, for carbon dioxide.

Equations of physical relations can be no better than the data on which they are based, and for the substances, ammonia and carbonic acid, the charts or formulas must be used with a good deal of suspicion.

In all engineering calculations requiring one of these constants even for steam no one is justified in using a formula; the nearest tabular or chart value must be employed and it will be as accurate as the work requires. Time is at least as important as accuracy, if not more so, for if too much time is required to make a calculation in commercial work, it will not be made because of the cost, therefore indirect and approximate methods are substituted.

It is sometimes useful in checking the boiling-point of some substance little known, to employ a relation between boiling-points of different substances at the same pressure applied to a substance well-known.

Let T_a and T_b be absolute temperatures of boiling for substances A and B under same pressure;

T_a' and T_b' be absolute temperature of boiling for substances A and B under some other pressure.

Then,
$$\frac{T_a'}{T_b'} = \frac{T_a}{T_b} + c(T_b' - T_b). \quad (542)$$

Such equations as this are useful in finding the saturation curve of other substances from that for water, which is now so well established, when enough points are known for the other substances to establish the constant c . Also

the ratio $\frac{T_a'}{T_b'}$ plotted against the temperature difference $T_b' - T_b$ should give

a straight line, and if the line is not straight the experimental values may be wrong or the law untrue.

All of the preceding refers, of course, to pure substances, but in practical work there are frequently encountered problems on solutions where large differences may exist compared to the pure liquids. Thus, for salts in water, it is well known that addition of a salt lowers the freezing-point, that more salt lowers it more, and it was first thought that the depression was in proportion to the amount dissolved. This being found to be untrue, recourse was had again to molecular relations by Raoult, who announced the general law that *the molecular depression of the freezing-point is a constant*.

$$\text{Molecular lowering of freezing-point } E' = \frac{\Delta t m}{w} = \text{const.}, \quad (543)$$

in which Δt = depression of freezing-point in degrees F.;
 w = weight dissolved in 100 parts of solvent;
 m = molecular weight of substance dissolved.

From Eq. (543) the freezing-point for brines may be found as follows:

$$\text{Freezing-point degrees F. of aqueous solutions} = 32^\circ - (\text{const.}) \times \frac{w}{m}. \quad (544)$$

Just as the pressure of dissolved substances in liquids lowers the freezing-point, so also does it lower the vapor pressure at a given temperature or raise the boiling-point at a given pressure. Investigation shows that a similar formula expresses the general relation:

$$\text{Molecular rise of boiling-point degrees F.} = E = \frac{\Delta t m}{w} = \text{constant} = 9.4, \quad (545)$$

when water is the solvent.

From Eq. (545) the rise of the boiling-point is found to be

$$\text{Rise of boiling-point} = 9.4 \frac{w}{m}. \quad (546)$$

When liquids are mixed, which is the case with all fuel oils and with denatured alcohol, the situation is different than with salts in solution, and these cases fall into two separate classes: (a) liquids infinitely miscible like alcohol and water or like the various distillates of petroleum with each other, and (b) those not miscible, like gasolene and water.

The vapor pressure for miscible liquid mixtures is a function of the pressure of each separately and of the molecular per cent of one in the other when there are two. This rule, which can be symbolized, is no use in engineering work, because in those cases where such mixtures must be dealt with there will be generally more than two liquids, the vapor pressure characteristic and molecular per cent of each, or at least some of which, will be unknown.

When, however, the two liquids in contact or rather any number, are non-miscible they behave in a very simple manner with respect to each other, in fact are quite independent in action. Each liquid will evaporate until its own vapor pressure is established for the temperature, as if the other were not there, and the vapor pressure for the mixture will be the sum of all the separate ones. On the other hand the boiling-point will be the temperature at which all the vapor pressures together make up the pressure of say the atmosphere, and this is necessarily lower than the highest and may be lower than the lowest value for a single constituent. This action plays a part in vaporizers and carburetors using alcohol and petroleum products, which will be discussed later.

Example 1. Through how many degrees has ammonia vapor at a pressure of 50 lbs. per square inch absolute been superheated, when it is at the temperature at which steam is formed under a pressure of 100 lbs. per square inch absolute?

From the curve of pressure and temperature of steam, see Chart No. 16 in the Tables, the temperature is 328°F . for the pressure of 100 lbs. From the table for ammonia (Table XLII), vaporization occurs under a pressure of 50 lbs. at a temperature of 22°F . Hence, $\text{superheat} = 328 - 22 = 306^{\circ}\text{F}$.

Prob. 1. Three tanks contain the following liquids: water, ammonia, and carbon dioxide respectively, at a temperature of 30°F . What pressure exists in each tank? If the temperature rises to 70°F . how much will the pressure rise in each?

Prob. 2. The pressure exerted by water vapor in the atmosphere when saturated, is that due to the temperature and is independent of the pressure of the air. The total pressure read by a barometer is the sum of the air pressure and the water-vapor pressure. What is the pressure due to each under a saturated condition for temperatures of 50°F ., 100°F ., 150°F ., and 200°F ., the barometer in each case being 29.92 inches of Hg?

Prob. 3. In order to secure a sufficiently high rate of heat transfer the steam in a radiator must be at a much higher temperature than the room to be warmed. If it is to be 150° above room temperature what must be its pressure for room temperatures of 50°F ., 60°F ., 70°F ., 80°F ., and 125°F ?

Prob. 4. In one type of ice machine ammonia gas is condensed at a high pressure and evaporated at a low pressure. What is the least pressure at which gas may be condensed with cooling water of 70°F ., and what is the highest pressure which may be carried in the evaporating coils to maintain a temperature in them of 0°F ?

Prob. 5. Should carbon dioxide be substituted in the above machine what pressures would there be in the condensing coils, and in the evaporating coils?

Prob. 6. How many degrees of superheat have the vapors of water, ammonia, and carbon dioxide at a pressure of 15 atmospheres and a temperature of 500° F.?

Prob. 7. At the temperature of melting ice what will be the vapor pressure of ammonia and carbon dioxide? At the temperature of melting tin what will be the pressure of water vapor? At this same temperature how many degrees of superheat would ammonia vapor under 100 lbs. pressure have, and how many degrees superheat would carbon dioxide vapor have under 1000 lbs. pressure?

Prob. 8. If 10 lbs. of common salt, NaCl, be dissolved in 100 lbs. of water, what will be the boiling point of the solution at atmospheric pressure? What the freezing-point?

CHAPTER VIII

CHANGES OF PHYSICAL STATE WITH HEAT CONTENT. LATENT HEAT. GAS AND VAPOR MIXTURES. ABSORPTION OF GASES.

72. Change of State with Amount of Heat at Constant Temperature. Latent Heats of Fusion and Vaporization. Total Heats of Vapors. Relation of Specific Volume of Liquid and of Vapor to the Latent Heat. As previously explained, a liquid boils or is converted into a vapor at constant temperature when the pressure on the surface is constant. Then *during the change of state the amount of heat added is in direct proportion to the amount of vapor formed.* The amount of vapor to convert a pound of liquid into vapor at any one steady temperature, is the *latent heat of vaporization* some values for which are given in the Tables (No. XXXV). It must be understood that this latent heat is also the amount given up by the condensation of a pound of vapor. Latent heat is not the same for different pressures or temperatures of vaporization but is intimately associated with the volume change in the transition from the liquid to the vapor state. That this should be so, is clear on purely rational grounds because there is necessarily external mechanical work done in converting the liquid to the vapor since this is accompanied by a change of volume against the resisting pressure at which the conversion takes place.

Thus, if V_v = specific volume of the vapor in cubic feet per pound;

V_L = specific volume of the liquid in cubic feet per pound;

P = pressure of vaporization lbs. per sq.ft. absolute;

$$\text{then } \left\{ \begin{array}{l} \text{Mechanical external work done dur-} \\ \text{ing vaporization of 1 lb.} \end{array} \right\} = P(V_v - V_L) \text{ ft.lbs.} \quad . \quad . \quad (547)$$

At high temperatures the volume of a pound of liquid is greater than at low because of its expansion with temperature rise, and under the corresponding higher pressures the volume of a pound of vapor is less, because of the compressional effect of the pressure, than at low pressures, so that as pressures and temperatures rise the difference $V_v - V_L$ becomes less and disappears at the critical point where it is zero. The latent heat being thus associated with a factor that becomes less in the higher ranges of temperatures and pressure may be expected likewise to become less, unless some other factor tends to increase. All the energy of vaporization making up the latent heat may be said to be used up in (a) doing external work as above, or (b) overcoming attraction of the molecules for each other. As at the critical point there is no molecular change and no external work, the latent heat becomes zero at this point.

This relation between latent heat and volume change was formulated by Clausius and Clapeyron, but Eq. (548) is generally known as Clapeyron's equation:

Let L = latent heat;

J = mechanical equivalent of heat = 778, or better 777.52, in such cases as this;

T = absolute temperature of vaporization;

$\frac{dP}{dT}$ = rate of increase of vapor pressure per degree change of corresponding temperature;

Then
$$L = \frac{T}{777.52} \frac{dP}{dT} (V_v - V_L). \quad (548)$$

This formula is used to calculate latent heat from the specific volumes of vapor and liquid and from the curvature of the saturation curve when they are known, but as these volumes are especially difficult to measure, direct experimental determination of the latent heat should be depended upon to get numerical values wherever possible. The formula will then be useful for the inverse process of calculating specific volumes from latent heats or as a means of checking experimental values of both, one against the other. It is, however, just as useful to calculate latent heats from the specific volumes, and $\frac{dP}{dT}$ of the vapor curve, when the latent heats are less positively determined than the volumes or densities.

Experimental data are quite numerous for water, but as generally determined they include the *heat of liquid* water from some lower temperature to the boiling-point. The amount of heat necessary to warm a pound of liquid from temperature 32° F. to some boiling-point, and to there convert it entirely into vapor is designated as the *total heat* of the dry saturated vapor above the original temperature. This is also equal to the heat given up by the condensation of a pound of dry saturated vapor, at its temperature of existence, by the subsequent cooling of the water to some base temperature taken universally now as 32° F. in engineering calculations.

From observations by Regnault and formulated by him in 1863 the present knowledge of the total heat of water may be said to date. He gave the expression, Eq. (549), in which the first term is the latent heat at 32° and one atmospheric pressure:

Total heat per pound dry saturated steam = $H = 1091.7 + .305(t - 32). \quad (549)$

This was long used as the basis of steam calculations, but is now to be discarded in the light of more recent experimental data, the best of it based on indirect measurements by Grindley, Griessmann and Peake, who observed the behavior of steam issuing from an orifice, together with the results of Knobloch and Jacob and Thomas on specific heats of superheated steam, and in addition on direct measurements by Dieterici, Smith, Griffiths, Henning, Joly. All this work

has been recently reviewed and analyzed by Davis, who accepts 1150.3 B.T.U. as the most probable value of the total heat under the standard atmosphere, and the following formula as representing total heats from 212° up to 400° F.

$$H = 1150.3 + .3745(t - 212) - .000550(t - 212)^2. \quad (550)$$

The Davis curve containing all the important experimental points and the accepted line, extended dotted from 212° to 32°, is presented in Fig. 105.

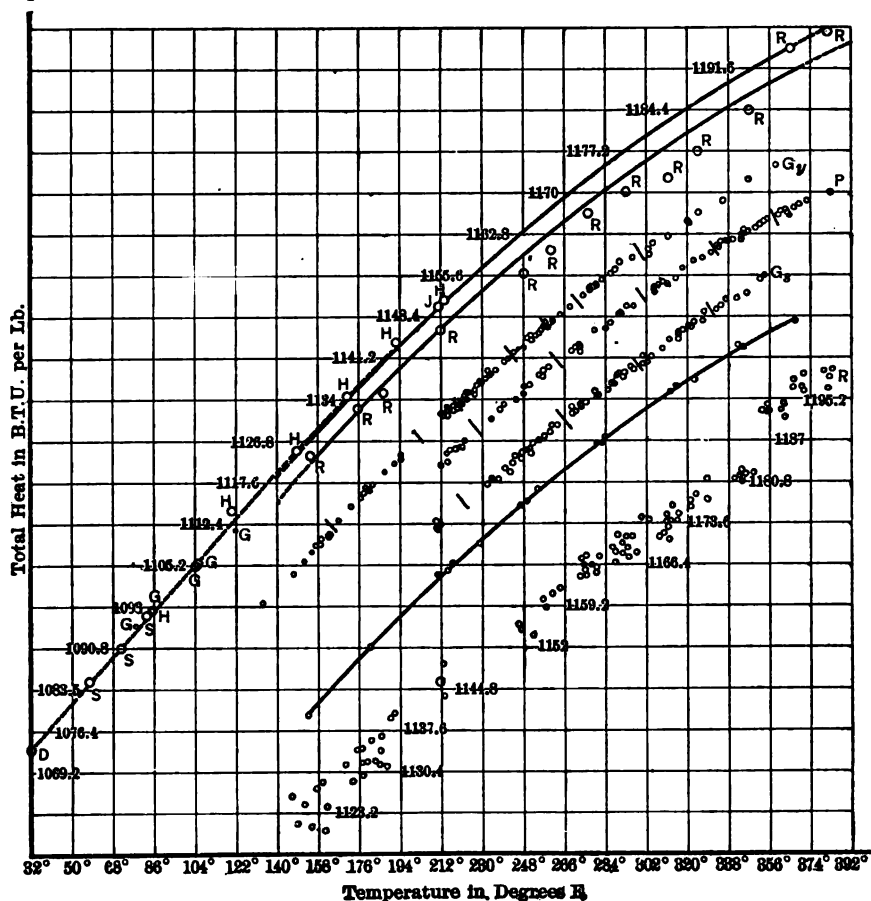


FIG. 105.—Total Heat of Dry Saturated Steam above 32° F. (Davis).

From the total heats given by this formula the latent heat is obtainable by subtraction, according to the relation,

$$\text{Latent heat } (L) = \text{total heat of vapor above } 32^\circ \text{ F. } (H) - \text{heat of liquid from } 32^\circ \text{ F. to boiling point } (h), \quad (551)$$

in which the heat of the liquid is computed from a mean curve between Dieterici's and Regnault's values, having the equation $h = .9983 - .0000288(t - 32) + .0002133(t - 32)^2$. This is the basis of the values for latent and total heats in the

Marks and Davis steam tables referred to, and accepted as the best obtainable to-day. From these tables charts have been prepared for latent heat and total heat of dry saturated steam, which will be found in the Handbook of Tables. The specific volume and density of dry saturated steam, given in the charts and table are calculated, as this seems to promise more exact results than direct experiment.

For ammonia and carbonic acid there are no data available on total heats by either direct measure or by the orifice expansion properties, and very few determinations of the latent heat itself, so that the process that has proved so satisfactory with steam cannot be directly followed with these substances. Accordingly, a process of adjustment has been used, working from both ends, beginning with the pressure temperature relations on the one hand and specific volumes of liquid and vapor on the other, the latent heat is determined by Clapeyron's equation, and where this does not agree with authentic values an adjustment of both latent heat and specific volume is made.

This process is materially assisted by the so-called Cailletet and Mathias' law of mean diameter of the curves of density of liquid and vapor one of which, that for water, is given in Fig. 106.

The line BD is the line of mean density which passes through the critical volume B ; its abscissa is found from the following general equation:

$$s = \frac{1}{2} \left(\frac{1}{V_v} + \frac{1}{V_L} \right) = a + bt + ct^2. \quad (552)$$

This expression has the form given in Eq. (553) for water, ammonia, and carbon dioxide:

$$\left. \begin{array}{l} \text{For water.} \dots\dots s = 28.7 - .015(t-300) - .000015(t-300)^2. \quad (a) \\ \text{For ammonia.} \dots\dots s = 20 - .022(t-30). \quad (b) \\ \text{For carbonic acid.} \dots\dots s = 33.1 - .0219(t-20) - .00016(t-20)^2. \quad (c) \end{array} \right\} . \quad (553)$$

A more nearly exact equation for water has been determined by Marks and Davis and is given in their steam tables as follows:

$$s = 28.424 - .01650(t-320) - .0000132(t-320)^2. \quad . . . (554)$$

From the smooth curve, which has the above equation, the volumes and densities of liquid and vapor that are accepted have been derived, and are given in the Handbook of Tables; the values for water being those of Marks and Davis.

From these volume differences and the $\frac{dp}{dT}$ relation the latent heats have been calculated.

The total heats are obtained by adding to the latent heat the heat of liquid above 32° from -50° F. up to the critical point for CO_2 , and to 150° F. for NH_3 , which include the working range for refrigeration. These liquid heats have been determined in Section 67 in discussing specific heats.

Additional tables give the final values of total heat, heat of liquid, latent

heat, specific volume and density of dry saturated vapor based on large-scale plottings, without equations beyond that for the pressure-temperature relations for saturated vapor, and the results are believed to be as reliable as it is possible to obtain without more experimental data.

The properties of dry-saturated steam, superheated steam, dry-saturated ammonia vapor, and dry-saturated carbon dioxide vapor thus determined will be found in the Tables; those for steam are also given in diagrammatic form (Charts Nos. 16 to 21).

The volumes of dry-saturated steam determined from the tables when compared with their pressures show that there exists an approximate relation of the form for steam,

$$p(V_v - V_L)^{1.0646} = \text{constant} = 497, \dots \dots \dots (555)$$

when pressures are in pounds per square inch and volumes are in cubic feet. This curve plotted to PV coordinates is called the saturation curve for the vapor. It is useful in approximate calculations of the work that would be done by steam expanding so that it remains dry and saturated, or the work required to compress vapor such as ammonia under the same conditions. But as the specific volume of liquid is generally negligible it may be written as one of the general class $PV^s = \text{constant}$, for which $s = 1.0646$ and $\text{constant} = 497$.

This curve supplies a means for computing the work for wet vapors (not too wet) as well as dry, provided only that they at no time become superheated or change their quality, by using for V some fraction of the true specific volume representing the *dryness*. The very fact that a great volume of vapor may be formed from an insignificant volume of liquid makes the saturation curve a useful standard of comparison with actual expansion and compression lines for wet vapors.

Plotting the saturation curve beside an actual cylinder expansion or

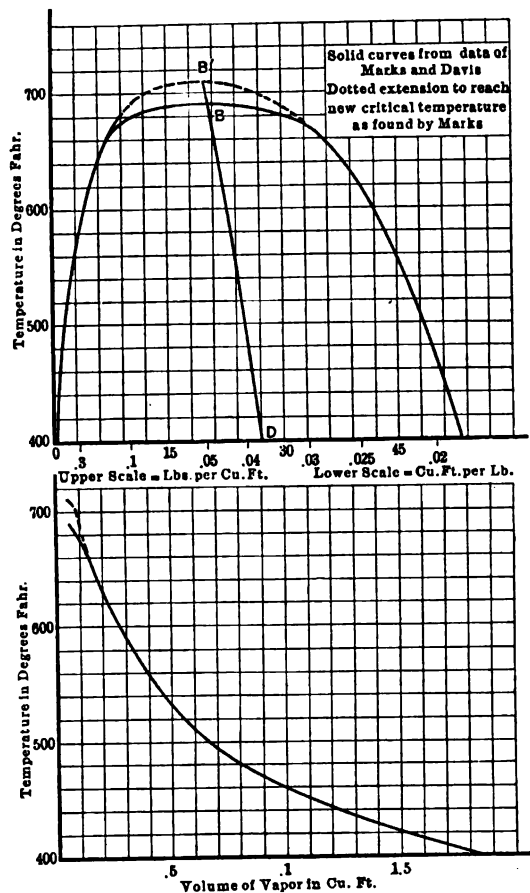


FIG. 106.—Specific Volume and Density of Liquid and Dry Saturated Vapor of Water.

compression curve will show the quality of vapor at all times and also give a measure of evaporation and condensation taking place during the process. In Fig. 107 is shown a set of diagrams taken from a simple Corliss engine, 18×24

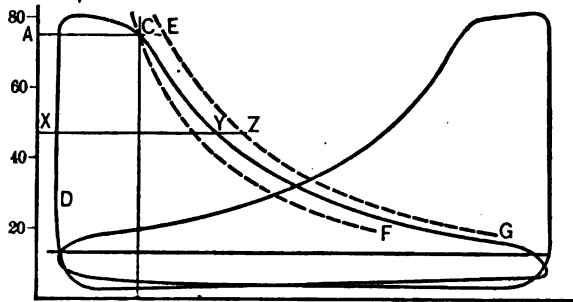


FIG. 107.—Comparison of Steam Expansion Line of an Indicator Card with the Saturation Line for both Dry Saturated Vapor and for Vapor Constantly Wet at the Initial Value.

ins. with 4 per cent clearance, a 2½-inch piston rod and tail rod, running at 150 R.P.M., to which have been added lines of zero pressure and volume by the method explained in Chapter I. The discharge from the condenser per hour for a constant load of the value to give the above cards was 2600 lbs. Allowing for the rods, the displacement volumes of each end of the cylinder

will be 5990 cu.ins., and since the clearance volume is 4 per cent, the steam volume will be 239.6 cu.ins. From the left-hand card it will be seen that the cut-off was at point C, 16.5 per cent of stroke, hence the volume at C is $(.165 \times 5990) + 239.6 = 1228$ cu.ins. It will also be seen from the card, that the pressure at C was 73.5 lbs. per square inch absolute. From the curves or the tables it will be found that 1 cu.ft. of dry steam at this pressure weighs

.1688 lb. and hence the weight of steam in this end of cylinder was $\frac{1228}{1728} \times .1668$

or .1185 lb. at cut-off. From the card it will also be seen that at the end of the return stroke, denoted by the point D, the pressure was 30 lbs., at which the weight of 1 cu.ft. of dry steam is .0728 lb., hence the weight of steam in the

cylinder was $\frac{239.6}{1728} \times .0728 = .01010$ lb., and the amount admitted was .1185—
.0101 = .1084 lb.

Inasmuch as the two ends of cylinder are identical and as the cards from both ends are practically the same, it may be assumed that the same weight of steam was in each end, or that $.1084 \times 2 = .2168$ lb. is accounted for by the card per revolution, or $.2168 \times 150 \times 60 = 1950$ lbs. per hour. There is then the difference to account for otherwise, of $2600 - 1950 = 650$ lbs. per hour, which can only have been lost by condensation. 2600 lbs. per hour is $2600 \div (150 \times 60 \times 2) = .1442$ lb. per stroke, which with the .0101 lb. left from previous stroke would make .1543 lb. in the cylinder at cut-off, and if it were all steam its volume would be 1581 cu.ins., denoted by point E on diagram. The ratio of \overline{AC} to \overline{AE} gives the amount of actual steam present in the cylinder at cut-off, to the amount of steam and water. The saturation curves CF and EG are drawn through C and

E from tabular values and represent in the case of CF the volumes which would have been present in the cylinder at any point of stroke had the steam and water originally present expanded in such a way as to keep the ratio or dryness constant, and in case of EG , volumes at any point of the stroke if all the steam and water originally present had been in form of steam and had remained so throughout the stroke. Just as the ratio of \overline{AC} to \overline{AE} shows per cent of steam present at cut-off, so does the ratio of distances of any points Y and Z , from the volume axis denote the per cent of steam present at that particular point of the stroke. By taking a series of points along the expansion curve it is possible to tell whether evaporation or condensation is occurring during expansion. In this case the ratio,

$$\frac{\overline{AC}}{\overline{AE}} = .795, \text{ and } \frac{\overline{XY}}{\overline{XZ}} = .86.$$

Hence, it is evident that evaporation is occurring since the percentage of steam is greater in the second case.

For some classes of problems it is desirable that the external mechanical work be separated from the latent heat, and for this reason latent heat is given in three ways:

(a) External latent heat, (b) internal latent heat, (c) latent heat total.

The external latent heat in foot-pounds is the product of pressure and volume change, or expressing pressures in pounds per square inch,

$$\text{External latent heat} = \frac{144}{J} p(V_v - V_L) \dots \dots \dots (556)$$

This is sometimes reduced by neglecting V_L as insignificantly small, as it really is for most problems which are limited to temperatures below 400° for saturated vapor, in which case,

$$\text{External latent heat} = \frac{144}{J} pV_v \dots \dots \dots (557)$$

In all cases: Internal latent heat = $L - (\text{Ext. Lat. Ht.})$ (a);

$$= L - \frac{144}{J} p(V_v - V_L) \text{ (b); } = L - \frac{144}{J} pV_v \text{ (c).} \dots \dots \dots (558)$$

Fusion and freezing are quite similar to vaporization and condensation in that they are constant temperature processes with proportionality between the amount of substance changing state and the amount of heat exchanged. They are different inasmuch as little or no volume change occurs. As there is so little external work done it may be expected that there is little change in their latent heats with temperature and pressure, but as a matter of fact it makes very little difference in most engineering work just how this may be, because practically all freezing and melting takes place under atmospheric pressure. There does not appear to be any relation established between heats of fusion like those for vaporization that permit of estimates of value from other constants, so direct experimental data must be resorted to. As a matter of fact such laws would be of little use.

Example 1. Pigs of iron having a total weight of 5 tons and a temperature of 2000° F. are cooled by immersing them in open water at a temperature of 60° F. If one-half of the water is evaporated by boiling, how much must there have been originally?

The iron was evidently cooled to the final temperature of the water, which must have been 212° F. Also the heat given up by the iron will be the product of its weight, specific heat and temperature difference, or, considering the mean specific heat to be .15, $10,000 \times (2000 - 212) \times .15 = 2,682,000$ B.T.U. The heat absorbed by the water in being heated, assuming its specific heat as unity, will be its weight times its temperature change and, since one-half evaporates, the heat absorbed in evaporating it will be half its weight times the latent heat, or

$$W[(212 - 60) + \frac{1}{2} \times 970] = 637W \text{ B.T.U.}$$

These expressions for heat must be equal, hence $W = \frac{2,682,000}{637} = 4210$ lbs.

Example 2. A tank of pure water holding 1000 gallons is to be frozen by means of evaporating ammonia. The water is originally at a temperature of 60° F. and the ice is finally at a temperature of 20° F. The ammonia evaporates at a pressure of one atmosphere and the vapor leaves the coils in a saturated condition. How many pounds of ammonia liquid will be needed, how many cubic feet of dry saturated vapor will be formed, and how much work will be done in forming the vapor?

The heat to be removed is the sum of that to cool the water, the latent heat of fusion of ice, and that to cool the ice, or for this case,

$$[(60 - 32) + 144 + .5(32 - 20)] \times 8333,$$

8333 being the weight of 1000 gallons of water. Hence the B.T.U. abstracted amount to 1,466,608.

Each pound of ammonia in evaporating at atmospheric pressure absorbs 594 B.T.U.'s as latent heat, and therefore 2470 lbs. are needed. At this pressure each pound of vapor occupies 17.5 cu.ft., hence there will be 43,200 cu.ft. of vapor. At this same pressure the volume of a pound of liquid is .024 cu.ft., so that the work done per pound in evaporating the ammonia is 37,000 ft.-lbs. and the total work is 915×10^6 ft.-lbs.

Prob. 1. How much ice would be melted at 32° F. with the heat necessary to boil away 5 lbs. of water at atmospheric pressure, the water being initially at the temperature corresponding to the boiling-point at this pressure?

Prob. 2. What is the work done during the vaporization of 1 lb. of liquid anhydrous ammonia at the pressure of the atmosphere?

Prob. 3. As steam travels through a pipe some of it is condensed on account of the radiation of heat from the pipe. If 5 per cent of the steam condenses how much heat per hour will be given off by the pipe when 30,000 lbs. of steam per hour at a pressure of 150 lbs. per square inch absolute is passing through it?

Prob. 4. Brine having the specific heat of .8 is cooled by the evaporation of ammonia in coils. If the brine is lowered 5° F. by ammonia evaporating at a pressure of 20 lbs. per square inch gage, the vapor escaping at NH_3 temperature, how many pounds of brine could be cooled per pound of ammonia?

Prob. 5. Steam from an engine is condensed and the water cooled down to a temperature of 80° F. in a condenser in which the vacuum is 28 ins. of Hg. How many pounds of cooling water will be required per pound of steam if the steam be initially 10 per cent wet?

Prob. 6. A pound of water at a temperature of 60° F. is made into steam at 100

lbs. per square inch gage pressure. How much heat will be required for this, and what will be the volumes at (a) original condition; (b) just before any steam is made; (c) after all the water has been changed to steam?

Prob. 7. A sand mold weighs 1000 lbs. and 100 lbs. of melted cast iron are poured into it. Neglecting any radiation losses and assuming the iron to be practically at its freezing temperature, how much of the iron will solidify before the mold becomes of the same temperature as the iron?

Prob. 8. How many pounds of ice could be melted by heat given up by freezing 50 lbs. of lead?

73. Gas and Vapor Mixtures. Partial and Total Gas and Vapor Pressures. Volume, Weight, and Gas Constant Relations. Saturated Mixtures. Humidity. One of the characteristic properties of gases distinguishing them from liquids, which also extends to vapors with certain limitations, is that of *infinite expansion*, according to which no matter how the containing envelope or volume of the expansive fluid may vary, the space will be filled with it at some pressure and the weight remain unchanged except when a vapor is brought to condensation conditions, or the pressure lowered on the surface of a liquid which will, of course, make more vapor. A given weight of gas or vapor (within limits) will fill any volume at some pressure peculiar to itself, and two gases, two vapors, or a vapor and gas, existing together in a given volume, will fill it at some new pressure which is the sum of the pressures each would exert separately at the same temperature (if *non-miscible*). This fact, sometimes designated as Dalton's Law, permits of the derivation of equations for the relation of any one pressure, partial or total, to any other total or partial, in terms of the weights of gas or vapor present, and the gas constants R . It also leads to equations for the various constituent and total weights in terms of partial and total pressures and gas constants. Such equations supply a basis for the solution of problems in humidification and drying of air, in carburetion of air for gasolene and alcohol engines, or of water gas for illumination, and are likewise useful as check relations in certain cases of gas mixtures such as the atmospheric mixture of nitrogen and oxygen, producer gas or gaseous combustibles in general.

Let w_1, w_2 and w_x be the respective weights of the constituents of a mixture;

$w_m = \Sigma w$ be the weight of the mixture;

P_1, P_2, P_x be the respective partial pressures of the constituents;

$P_m = \Sigma P$ be the pressure of the mixture;

R_1, R_2, R_x be the respective gas constants;

R_m be the gas constant for the mixture.

Then if w_1 lbs. of one, and w_2 lbs. of another gas or vapor at temperature T_m occupy the volume V_m cubic feet together,

$$V_m P_1 = w_1 R_1 T_m \text{ (a), and } V_m P_2 = w_2 R_2 T_m \text{ (b) . . . (559)}$$

$$\text{whence } V_m(P_1 + P_2) = (w_1 R_1 + w_2 R_2) T_m \text{ or, in general, } V_m \Sigma P = \Sigma (w R) T_m. \text{ (560)}$$

$$\text{Or putting } \Sigma P = P_m, \text{ and } \Sigma (w R) = R_m w_m, \text{ or } R_m = \frac{\Sigma (w R)}{w_m}, \text{ . . . (561)}$$

$$\text{then } P_m V_m = w_m R_m T_m, \text{ (562)}$$

so that the *mixture will behave thermally quite the same as any one gas with such exceptions as may be due to a different gas constant R_m .*

Dividing Eq. (559) by Eq. (560) or (561) gives

$$\frac{P_1}{P_m} = \frac{w_1 R_1}{w_1 R_1 + w_2 R_2} = \frac{w_1 R_1}{\Sigma(wR)}, \quad \dots \dots \dots (563)$$

which gives the ratio of any partial pressure to that for the mixture in terms of the individual weights and gas constants. Hence

$$\frac{P_1}{P_m} = \frac{w_1 R_1}{w_m R_m}, \quad \dots \dots \dots (564)$$

which gives the *ratio of any partial pressure to that for the mixture in terms of its own weight and gas constant and those for the mixture.*

It is possible to express the ratio of weights as a function of gas constants alone which will permit of a third expression for the partial pressures in terms of gas constants without involving any weights. For two gases

$$w_1 = w_m - w_2. \quad \text{Whence} \quad \frac{w_1}{w_m} = 1 - \frac{w_2}{w_m}, \quad \dots \dots \dots (565)$$

But from Eq. (561) $w_1 R_1 + w_2 R_2 = w_m R_m$, or $\frac{w_2}{w_m} = \frac{1}{R_2} \left(R_m - \frac{w_1}{w_m} R_1 \right)$, so that

$$\frac{w_1}{w_m} = 1 - \frac{w_2}{w_m} = 1 - \frac{1}{R_2} \left(R_m - \frac{w_1}{w_m} R_1 \right), \text{ or } \frac{w_1}{w_m} \left(1 - \frac{R_1}{R_2} \right) = 1 - \frac{R_m}{R_2}, \text{ and } \frac{w_1}{w_m} = \frac{R_2 - R_m}{R_2 - R_1}, \quad (566)$$

which is the ratio of partial to total weights in terms of gas constants. On substitution in Eq. (564),

$$\frac{P_1}{P_m} = \frac{R_1}{R_m} \left(\frac{R_2 - R_m}{R_2 - R_1} \right), \quad \dots \dots \dots (567),$$

which gives the ratio of partial pressures of two gases or vapors to that for the mixture in terms of the individual gas constants and that for the mixture, and a similar expression can be found for more than two gases. The ratio of any one partial to the total weight can also be found from Eq. (564) in terms of its gas constant and partial pressure, and the mixture-gas constant and pressure, from Eq. (566) in terms of the gas constants for the constituents and for the mixture. This ratio of partial to total mixture weight gives the fractional composition by weight.

It is sometimes necessary to know the volume relations in a mixture of two gases existing at the same pressure or two vapors or a vapor and gas, such, for example, as air and water vapor. In this case two different volumes existing together at a common temperature and pressure together make up a mixture volume equal to their sum. Using similar symbols

$$P_m V_1 = w_1 R_1 T_m \quad \text{and} \quad P_m V_2 = w_2 R_2 T_m, \quad \dots \dots \dots (568)$$

where V_1 and V_2 are the volumes occupied by the two constituents respectively when at a mixture pressure P_m and temperature T_m , whence for the mixture

$$P_m (V_1 + V_2) = (w_1 R_1 + w_2 R_2) T_m \quad \text{or} \quad P_m \Sigma(V) = \Sigma(wR) T_m, \quad \dots \dots \dots (569)$$

These Eqs. (569) are identical in form with those in (560) except that V and P replace each other so that all equations just derived also apply to volumes, as the volume proportion will be identical with pressure proportions. For convenience of reference these may be set down.

From Eq. (563),
$$\frac{V_1}{V_m} = \frac{w_1 R_1}{\Sigma w R} \quad (570)$$

which gives the ratio of any partial volume, to that for the mixture in terms of the individual weights and gas constants.

From Eq. (564)
$$\frac{V_1}{V_m} = \frac{w_1 R_1}{w_m R_m} \quad (571)$$

which gives the ratio of any partial volume to that of the mixture in terms of its own weight and gas constant and those for the mixture.

From Eq. (567)
$$\frac{V_1}{V_m} = \frac{R_1 (R_2 - R_m)}{R_m (R_2 - R_1)} \quad (572)$$

which gives the ratio of any partial volume to that of the mixture in terms of the individual gas constants and that for the mixture.

The volumetric composition of air is given by Eq. (571) or its equal numerically, Eq. (567), and since the partial pressure of oxygen and nitrogen in air are 78.69 per cent and 21.31 per cent, these are its volumetric per cents.

When one of the constituents is a vapor, all the preceding applies, provided the condition of the vapor is such that at the temperatures assumed it is not near the condition of condensation, but then the relations become more definite since the *partial pressure of the vapor is fixed by the temperature*. In practical work with gas and vapor mixtures the failure of the perfect gas laws near the condensation condition is ignored and they are assumed to be true.

All liquids, and many if not all solids, will, if placed in a vacuum chamber, evaporate until the pressure has reached a certain value depending on the temperature, at which time the liquid and its vapor are in equilibrium, and evaporation may be said either to cease or proceed at a rate exactly equal to the rate at which vapor condenses, or more precisely, at equilibrium the weight of vapor in the vapor form remains constant. The weight of vapor that will rise over a liquid in a given space depends on the temperature and pressure which are related in the so-called vapor tension or vapor pressure tables and curves, such as shown in Fig. 108, for some liquid fuels, or as given in the previous section for water. Additional curves for hydrocarbons and light petroleum distillates, as well as curves for the vapor pressure of the alcohols will be found in the book of tables, Charts Nos. 22 and 24. At any fixed temperature the vapor will continue to rise until it exerts its own vapor pressure for the temperature, the process being often described as evaporation without ebullition. If the liquid or solid be introduced into a chamber containing dry gas the evaporation will proceed precisely the same as in the vacuum until the pressure has risen by an amount corresponding to the vapor pressure for the temperature, because each substance exerts

the pressure it would if alone occupying the volume; when they both occupy the same volume, the pressure will be their sum and equal to the pressure of the gas alone added to the vapor pressure for the same temperature. There is one important practical condition, and that is, time enough for the completion of the process of evaporation which proceeds very slowly toward the end. If time enough is allowed the vapor pressure will establish itself and the gas is

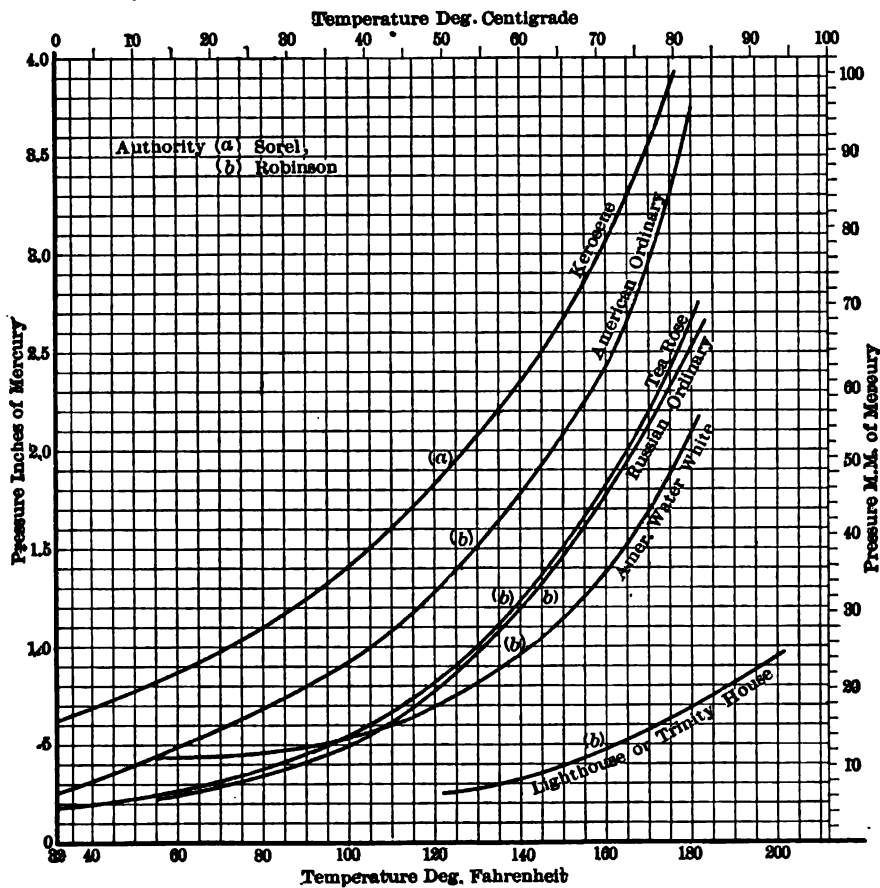


FIG. 108.—Vapor Pressure of Heavy Petroleum Distillates of the Kerosene Class.

said to be saturated; this is an important special case of gas-vapor mixtures. It is the condition in which the gas may be said to carry the maximum weight of vapor possible for the total pressure and temperature. The gas in contact with the liquid may carry less vapor if it has not been in contact long enough at the given temperature, and a gas no longer in contact with the liquid may carry less, because, (a) of insufficient time of original contact; (b) of condensation of some it originally carried; (c) of a rise of temperature after leaving the liquid. To all such general cases the equations above apply without change, but for the special case of saturated mixtures they have a simpler form.

Let P_v be the vapor pressure of the liquid for temperature T , which is the partial pressure of the vapor in a saturated vapor gas mixture;

P_g be the partial pressure of the gas at same temperature;

then for a gas saturated with vapor at temperature T , Eq. (564),

$$\left(\frac{\text{Weight vapor}}{\text{Weight mixt.}}\right) = \left(\frac{\text{Vapor pressure}}{\text{Mixture press.}}\right) \times \left(\frac{R \text{ for mixture}}{R \text{ for vapor}}\right). \quad (573)$$

But according to Eq. (526) and following

$$\left(\frac{R \text{ for vapor}}{R \text{ for mixt.}}\right) = \left(\frac{\text{Density of mixture}}{\text{Density of vapor}}\right) = \left(\frac{\text{Mol. wt. of mixt.}}{\text{Mol. wt. of vapor}}\right). \quad (574)$$

whence $\frac{w_v}{w_m} = \frac{P_v}{P_m} \frac{m_v}{m_m}$. Also $\frac{w_g}{w_m} = \frac{P_g}{P_m} \frac{m_g}{m_m}$, and $\frac{w_v}{w_g} = \frac{P_v}{P_g} \frac{m_v}{m_g}$. . . (575)

The presence of water vapor in the atmosphere, and problems connected with it, constitute a specific case of vapor-gas mixture, subject to the foregoing laws. This subject has been given most attention by the United States Weather Bureau; tables have been prepared for ready computation and for certain problems for which only experimental data or empiric formulas afford solution.

Air is said to be "*saturated with moisture*" when it contains the saturated vapor of water. It might be better to say that the *space* is saturated since the presence or absence of the air has no effect upon the water vapor other than imposing its temperature or imparting heat to the water vapor, and also that the air retards the diffusion of water particles. The weight of saturated aqueous vapor per cubic foot depends only on the temperature, and not on the presence of air.

If the space contains only a certain fraction of the weight of aqueous vapor corresponding to saturation, that fraction is called the "*relative humidity*" or degree of saturation, and the corresponding percentage, the per cent of saturation. If air containing saturated water vapor be cooled ever so little, some of the vapor will be condensed and appear in the liquid form. If air is cooled at constant pressure, from a given initial condition, the degree saturation approaches unity, and finally reaches that value at a temperature called the "*dew point*" corresponding to the initial condition. At this temperature the condition of saturation has been reached and any further cooling will cause the precipitation of liquid water, as occurs in the formation of dew, rainclouds or fog.

A space or body of air carrying water vapor in smaller quantity than that of saturation, in reality contains superheated steam. If the vapor density and the temperature of the mixture be known, the degree of superheat may be ascertained from the temperature of the mixture, and the temperature corresponding to saturated water vapor having a pressure equal to the partial pressure of the vapor in the mixture.

Humidity of atmospheric air is ordinarily determined by an instrument called the psychrometer, which consists merely of two thermometers, one with a bulb exposed directly to the air and the other covered with a piece of wick

which is kept moist with water. The two are mounted together so that they can be whirled or swung about in the air until a stable condition has been reached. The dry-bulb thermometer should record the temperature of the air. The wet-bulb thermometer will record something lower than the air temperature, dependent upon the rate at which evaporation takes place, since the process of evaporation abstracts heat. Were there no other influence, this process of evaporation would continue till the temperature of the wet bulb became that of the dew point. The temperature of the wet-bulb thermometer never falls to the dew point, however, because of conduction of heat between the cold bulb and the warmer surrounding air. From extensive experiments conducted by the U. S. Weather Bureau, Professor Ferrel has devised the following formula for the vapor pressure, h in ins. of mercury corresponding to given readings of the wet- and dry-bulb thermometers, t_d and t_w degrees F.:

$$h = h' - .000367 h_b (t_d - t_w) \left(1 - \frac{t_w - 32}{1571} \right), \quad (576)$$

where h_b is barometric height in inches, after applying all corrections, and h' is pressure of saturated water vapor, in inches of mercury, corresponding to the temperature t_w .

The relations shown by this formula are expressed graphically in much more convenient form in the curves devised by Prof. H. L. Parr, which are given in the Handbook of Tables, Charts Nos. 25 and 26.

Example 1. By means of the relation of gas constants find the proportion of nitrogen and oxygen in the air.

R for nitrogen is 54.92 and for oxygen 48.25 and for air 53.35. From Eq. (572)

$$\frac{V_o}{V_m} = \frac{R_o (R_N - R_M)}{R_m (R_N - R_o)}, \text{ which, on substituting the above values for } R_N, R_o, \text{ and } R_m,$$

gives $\frac{V_o}{V_m} = \frac{48.25 (54.92 - 53.35)}{53.35 (54.92 - 48.25)} = .213$, or air is 21.3 per cent oxygen by volume.

Example 2. At what temperature will air containing $\frac{1}{2}$ lb. of water per pound of dry air at atmospheric pressure be saturated?

If the vapor pressure be known, the temperature may be found from tables.

$$\text{From Eq. (575)} \quad \frac{w_s}{w_g} = \frac{P_s m_g}{P_g m_g}, \quad \text{or} \quad P_s = \frac{P_g m_g w_s}{m_s w_g}, \quad \text{and} \quad P_s + P_g = 760 \text{ mm. of Hg.}$$

For air $m_g = 28.88$ and for water $m_s = 18$, hence substituting those values

$$P_s = \frac{(760 - P_s)(28.88 \times .5)}{18 \times 1}, \quad \text{or} \quad P_s = \frac{760 \times 7.22}{16.22} = 337 \text{ mm. Hg,}$$

which corresponds to a temperature of 172° F.

Example 3. A pound of alcohol requires 9.06 lbs. of air for a proper combustible mixture for gas engines. At what temperature will these proportions constitute a saturated mixture?

$$\text{From Eq. (575)} \quad P_s = \frac{P_g m_g W_o}{m_o W_g}.$$

For alcohol $m_a=46$, for air $m_g=28.88$, and $P_a+P_g=760$ mm. of Hg for atmospheric pressure. Substituting these values in the above equation

$$P_g = \frac{(760 - P_a)28.88}{46 \times 9.06} = 49 \text{ mm. Hg.}$$

From the curve of vapor tension of alcohol, the temperature corresponding to 49 mm. of Hg is about 72° F. See Chart 24 in the book of tables

Prob. 1. Air at 80 per cent humidity, atmospheric pressure and 70° F., is cooled to 40° F. How much water will be thrown down per 1000 cu.ft. of moist air?

Prob. 2. The same air is compressed adiabatically to five atmospheres, and again cooled to 40° F. at this pressure. How much moisture per 1000 cu.ft. of moist air will be separated out when the temperature becomes 70° F, and how much at 40° F.?

Prob. 3. What will be the weight of water in a pound of air and water vapor if the value of R for the mixture is taken as 55.25, for air as 53.35 and for water vapor as 91?

Prob. 4. At what temperature will air containing its own weight of water vapor be saturated at atmospheric pressure?

Prob. 5. An internal combustion engine uses a saturated mixture of air and gasolene vapor in which ratio of air to gasolene is 15.3. Considering the gasolene to be hexane, at what temperature will the mixture be?

Prob. 6. Should kerosene regarded as decane, $C_{10}H_{22}$, be substituted for gasolene in the above problem, what would be the change in temperature of mixture, assuming it still to be saturated?

Prob. 7. Air containing moisture equal to one per cent of the weight of the air alone is at a temperature of 150° F. How much is the water vapor superheated? What is the humidity?

Prob. 8. The reading of a dry bulb of a psychrometer is 90° F. and of the wet bulb 70° F. By means of Parr's charts, find the dew point, relative humidity, and grains of water per cubic foot of air.

74. Absorption of Gases by Liquids and Adsorption or Occlusion by Solids. Relative Volumes and Weights with Pressure and Temperature. Heats of Absorption and of Dilution. Properties of Aqua Ammonia. In the attainment of high vacuua in steam condensers, separate removal of considerable quantities of non-condensable gases is found necessary by means of dry vacuum pumps, a fact that proves in a practical way the freedom with which the boiler water had absorbed gases. These gases, for otherwise pure water are carbon dioxide and air, probably air mainly, but may include many others, notably the products of organic decomposition, especially when condensing water is taken from the neighborhood of sewers, as is generally the case when power plants are located on city water fronts. To a very much greater extent, however, is ammonia soluble in water, the latter being capable of taking up about 1000 volumes of ammonia at 0° and one atmosphere, against about 30 volumes of air, and one-fiftieth of a volume of hydrogen. It is the freedom of solution of ammonia in water that makes the process useful as a means of removing anhydrous ammonia from the cooling coils in mechanical refrigerating plants, as a substitute for the mechanical removal by piston compressors.

In all cases the solubility of gases in liquids decreases with rise of tem-

perature, a fact associated with the separation of gases from boiler feed-water during their heating in feed-water heaters, economizers and the boiler itself. This property is also depended upon to free the aqua ammonia that has absorbed its ammonia charge from the cooling coils, of the amount so taken up, by heating the solution in a separate chamber from which the ammonia vapor escapes to the ammonia condenser to become liquid anhydrous, while the weak liquor returns to the absorber for a new charge. To permit the calculation of the quantity of liquor to be circulated, in order that a given amount of anhydrous ammonia may be absorbed from the cooling coils and delivered later to a condenser by heating to high temperature, requires accurate data on the maximum possible ammonia content of solutions at various temperatures and pressures. Rise of temperature will always reduce the gas content of the solution if originally saturated, but the volume dissolved is independent of pressure for slightly soluble gases like nitrogen or hydrogen, the weight dissolved, of course, being greater and directly proportional to pressure at a given temperature by reason of the increased density.

This law of independence of volume and pressure or proportionality of weight to pressure, is known as Henry's Law; it is hardly true for gases as freely soluble as ammonia, probably due to some action between water and the gas, equivalent to that studied by Thomsen for solids, which tend to form hydrates of various kinds. For such gases as are soluble by weight in proportion to pressure, it is not the total pressure of the solution that is significant, but the partial pressure of the gas alone, so that the amount of mixed gases like air dissolved in water will depend, for the oxygen part, on the specific solubility of oxygen and its partial pressure in the air, which is approximately one-fifth that of the air, and the same is true for the nitrogen. Thus, in examining the solubility conditions for ammonia in water, while in practice the total pressure only is known, it is to the separate pressure of the ammonia that the amount dissolved must be referred in any attempt to establish a law of relation.

Just as gases dissolve in liquids, so are they absorbed by solids, though in this case the process is described as one of *adsorption* or occlusion. This phenomenon is now being studied in connection with coal, which it is found more or less freely absorbs air, the oxygen of which under comparatively low temperatures unites with the coal causing spontaneous combustion if the heat is *conserved* as in a pile or its liberation in any way accelerated by heating or otherwise. Most investigations of adsorption or occlusion have been made with charcoals, the more dense varieties of which have greater adsorptive power than others. The quantity of different gases adsorbed is believed by Dewar to be the same in volume per unit of charcoal when each is held at its own condensation temperature. The quantity increases with rise of pressure but not in proportion, and decreases rapidly with rise of temperature; a curve showing the temperature and pressure at which equal volumes are adsorbed is similar in form to a vapor tension curve.

In the establishment of the properties of aqueous solutions of ammonia,

progress was for many years very slow. Early experimental data have been more recently supplemented, so that it has been possible to graphically fill in the data of the unexplored region with reasonable, though not yet satisfactory, accuracy. The results in the form of charts are given in the Handbook of Tables, in which the three variables of pressure, temperature and concentration are grouped in pairs. Chart No. 27 is arranged for pressure-temperature; No. 28 for pressure-concentration, and No. 29 for temperature-concentration, from which can be read with reasonable accuracy any quantity needed in calculations. Table XLIV, also given in the Handbook, has been prepared from these diagrams; in this table the lower numbers are new, and the upper those as given by Starr several years ago and since used by engineers engaged in refrigeration, as standards.

These data refer to the equilibrium conditions of the solution, and in using them for practical problems care must be taken to avoid applying them to other conditions, for example to solutions that are not homogeneous, or in which there has not been sufficient time for the establishment of equilibrium.

Any change in the ammonia content of a solution is thermal in character and is, therefore, accompanied by heat changes. When water absorbs ammonia heat is liberated, which is also the case when ammonia in solution is diluted with more water, the latter being really a further absorption in the fresh water of the ammonia already contained in its solution being diluted by it. Likewise, when ammonia is absorbed by an ammonia solution heat is also liberated, but heat is absorbed by a solution from which ammonia is escaping, as in evaporation of liquids. There are three cases of the exothermic process, each with an endothermic inverse, and these are: (a) Absorption of ammonia by pure water; (b) Absorption of ammonia by ammonia solution; (c) Dilution of water of an ammonia solution.

Data on the amounts of heat liberated in these cases are not sufficient to establish firmly any general law of change, but are sufficient to give an approximation. The first important fact in this connection is that the heat liberated per pound of ammonia when pure water absorbs ammonia, depends on the amount of water. One pound of ammonia absorbed in a little water gives out a little heat, more is liberated when more water is present, but when the amount of water is large, 200 times the weight of ammonia, according to Thomsen, the *heat of absorption is constant* and does not increase beyond this point. It may easily be, however, that the point is reached with fifty water weights, or that some heat continues to be generated for any amount of water to infinity, but so small in quantity as to be impossible to measure in the great weight of liquid present. For example, if one B.T.U. were liberated in 100 lbs. of solution, the temperature rise would be somewhere near $1/100^{\circ}$ F., and to detect this in the presence of radiation and conduction influences, and make allowance for the heat of stirring, would be difficult.

For engineering purposes, however, it may be accepted that the heat of absorption of a pound of ammonia is constant if the weight of water is large; its value as determined by Thomsen is 8430 calories per gram molecule of NH_3 ,

which is equivalent to $\left(\frac{9}{5} \times \frac{8430}{17}\right) = 893$ B.T.U. per pound. This value is accepted and defined as the heat of *complete absorption* for the want of a better term, and in view of the desirability of distinguishing it from the heat of absorption in *small* amounts of water or in solutions already containing appreciable amounts of ammonia, which will be designated as heat of *partial absorption*.

Experiments have further established another important relation between the heats of dilution of solutions and their original strength. According to this, solutions behave like ammonia itself with respect to pure water and liberate a little heat when a little water is added, more with larger amounts attaining a constancy for very large amounts of water. Thus a solution of a given ammonia strength, it may be assumed, will always liberate the same amount of heat when diluted with water, if the total amount of water after dilution exceeds about two hundred times the weight of ammonia present. This heat per pound of NH_3 contained will be designated as the heat of *complete dilution*, and is defined as the heat liberated when a solution containing 1 lb. of ammonia in a given amount of water is completely diluted with water, or brought to the condition of 200 lbs. or more of water per pound of ammonia contained.

There is a rational relation between these three heats, that of *complete absorption*, which is a constant, that of *complete dilution*, which depends only on the original ratio of ammonia to water, and that of *partial absorption*, which is a function of the character of solution receiving it, or if pure water, the amount. This relation is

$$\left\{ \begin{array}{l} \text{Heat of} \\ \text{complete} \\ \text{absorption} \end{array} \right\} = \left\{ \begin{array}{l} (\text{Heat of partial absorption}) \\ (+ \text{Heat of complete dilution}) \end{array} \right\} = 893 \text{ B.T.U. per lb. NH}_3. \quad (577)$$

Numerical values for heats of partial absorption are entirely lacking, but Berthelot has given ten values for the heats of complete dilution for solutions containing from 1 lb. of water per pound of ammonia, to a little over one hundred, but at only one temperature, 57°F .

BERTHELOT'S DATA ON HEATS OF COMPLETE DILUTION OF AMMONIA SOLUTIONS

Original solution, 1 lb. Ammonia in (w) lbs. Water.	When Completely Diluted gives	Original Solution, 1 lb. Ammonia in (w) lbs. Water.	When Completely Diluted gives
1.04	136	3.76	34
1.06	134	6.11	22
1.13	124	10.06	2
1.98	51	57.39	0
3.18	41	116.47	0
B.T.U. per lb. ammonia		B.T.U. per lb. ammonia	

Berthelot's results are given in the preceding table, from which he derives a general law of relation given by the following formula Eq. (578):

$$\text{Heat of complete dilution, B.T.U. per lb. NH}_3 = \frac{142.5}{w} \text{ (Berthelot).} \quad (578)$$

The agreement, is not very good for larger values of w than 4 or 5, which is unfortunate, as commercial ammonia lies between one part ammonia to nine parts water, or $w=9$, and one part ammonia to 39 parts water, or $w=39$. Nevertheless, engineers have been using this formula in these doubtful ranges for some years.

By means of the few but probably accurate figures given by Thomsen and experimentally determined by him it is possible to check this practice. He measured not the heat of complete dilution, as did Berthelot, but the heats of partial dilution, and the manner in which his data merge into those of Berthelot, make the combined results of increased value because of the difference in method. Thomsen added a limited amount of water to a solution of ammonia containing 3.39 lbs. water per pound of ammonia and measured the heat, which was not the heat of complete dilution, with the following results:

THOMSEN'S DATA ON HEATS OF PARTIAL DILUTION OF AMMONIA SOLUTIONS

Original Solution. 1 lb. Ammonia in (w) lbs. Water.	Final Solution. 1 lb. Ammonia in (w) lbs. Water.	B.T.U. per lb. Ammonia Absorbed by the Ammonia Solution.
$w = \begin{cases} 3.39 \\ 3.39 \\ 3.39 \end{cases}$	$w = \begin{cases} 19.27 \\ 29.86 \\ 56.33 \end{cases}$	$\begin{matrix} +34 \\ +37 \\ +40 \end{matrix}$

These results have been fitted into those of complete dilution by the relations of Eqs. (579) and (580), and by the introduction of one assumption.

$$\left\{ \begin{array}{l} \text{Heat of complete dilution of original ammonia solution per lb. NH}_3 \end{array} \right\} = \left\{ \begin{array}{l} \text{Heat of partial dilution from original to some greater water content per lb. NH}_3 \end{array} \right\} + \left\{ \begin{array}{l} \text{Heat of complete dilution of the new solution per lb. NH}_3 \end{array} \right\} \quad (579)$$

or

$$\left\{ \begin{array}{l} \text{Heat of complete dilution of new solution per lb. NH}_3 \end{array} \right\} = \left\{ \begin{array}{l} \text{Heat of complete dilution of original solution per lb. NH}_3 \end{array} \right\} - \left\{ \begin{array}{l} \text{Heat of partial dilution from original to final solution per lb. NH}_3 \end{array} \right\} \quad (580)$$

If the heat of complete dilution of the original solution containing 3.39 lbs. water per pound ammonia be taken from the Berthelot equation, its value is 42; assuming this to be correct it may be introduced in Eq. (580) as a constant, giving with the Thomsen figures the following:

$$\left\{ \begin{array}{l} \text{Heat of complete dilution of new solution, per lb. NH}_3 \end{array} \right\} = 42 - \left\{ \begin{array}{l} \text{Heat of partial dilution from original to final solution per lb. NH}_3 \end{array} \right\}$$

Heat of complete dilution per pound NH_3 with

$$(w=19.27) = 42 - 34 \text{ (Thomsen)} = 8;$$

$$(w=29.86) = 42 - 37 \text{ (Thomsen)} = 5;$$

$$(w=56.33) = 42 - 40 \text{ (Thomsen)} = 2.$$

These three new points have been plotted and agree well with curve obtained from the Berthelot equation, better even than the original Berthelot points themselves, so that Thomsen's partial dilution figures seem to confirm Berthelot's complete dilution data, and Eq. (578) may be taken as truly representative of the heat of complete dilution of ammonia solutions and indirectly, of course, heats of partial dilutions as well.

Heats of absorption are more often needed in practical problems than heats of complete or partial dilution, but these heats follow on the assumption that the *heat of complete absorption must be equal to the sum of the heat of partial absorption, and the heat of complete dilution of the solution so formed.* Hence

$$\left\{ \begin{array}{l} \text{Heat of partial absorption} \\ \text{in } w \text{ lbs. water B.T.U.} \\ \text{per lb. NH}_3. \end{array} \right\} = \left\{ \begin{array}{l} \text{Heat of complete} \\ \text{absorption in ex-} \\ \text{cess water B.T.U.} \\ \text{per lb. NH}_3. \end{array} \right\} - \left\{ \begin{array}{l} \text{Heat of complete dilution by} \\ \text{excess water of solution} \\ \text{with } w \text{ lbs. water per lb.} \\ \text{NH}_3, \text{ B.T.U. per lb. NH}_3. \end{array} \right\}$$

or $\left\{ \begin{array}{l} \text{Heat of partial absorption in water} \\ \text{B.T.U. per lb. NH}_3 \text{ absorbed} \end{array} \right\} = 893 - \frac{142.5}{w} \quad \dots \quad (581)$

Most important are the heats liberated when ammonia is absorbed not by water but by weak ammonia solutions themselves, and these heats of partial solution of ammonia in ammonia solutions are obtainable from the data already established by a comparatively simple relation. In this case there are two different solutions in question, an original one which becomes the second one on receiving more ammonia. If the water received all the ammonia contained in the second solution a certain quantity of heat would be liberated, and it must be equal to the total amount liberated by absorption of the first ammonia in the water, and by the absorption of the second ammonia in the resulting solution, whence this last quantity is obtainable by differences between the heats of partial solution of ammonia in water alone.

Let w = lbs. water per lb. ammonia in original solution which, therefore, consists of $w+1$ lbs. solution, 1 lb. of ammonia and w lbs. of water.

A = lbs. ammonia added per lb. ammonia already present, making new solution containing w lbs. of water and $A+1$ lbs. of ammonia or $\frac{w}{A+1}$ lbs. water per lb. ammonia in $w+A+1$ lbs. of solution.

$$\text{Then } \left\{ \begin{array}{l} \text{Heat of partial absorption of 1 lb.} \\ \text{NH}_3 \text{ by solution containing 1 lb.} \\ \text{NH}_3 \text{ in } w \text{ lbs. water, B.T.U. per} \\ \text{lb. NH}_3 \end{array} \right\} = 893 - \frac{142.5}{w} (A+2) \quad \dots \quad (582)$$

As ammonia solution strengths are often given in terms of per cent of ammonia present by weight, and the heat of absorption in terms of changes in the per cent strength, the following conversion factors are useful:

$$\text{Per cent ammonia in original solution} = S_1 = \frac{100}{w+1},$$

$$\text{Per cent ammonia in final solution} = S_2 = \frac{100(A+1)}{w+A+1}.$$

Whence $w = \frac{100 - S_1}{S_1}$ (a); $A = \frac{100}{S_1} \left(\frac{S_2 - S_1}{100 - S_2} \right)$ (b) (583)

These on substitution in Eq. (582) give the heat of absorption per pound of ammonia absorbed to change the per cent NH_3 from S_1 to S_2 .

$$\left\{ \begin{array}{l} \text{Heat of partial absorption of 1 lb.} \\ \text{of } \text{NH}_3 \text{ in a solution containing} \\ \text{1 lb. } \text{NH}_3 \text{ in } w \text{ lbs. water, B.T.U.} \end{array} \right\} = 893 - 142.5 \left(\frac{S_1}{100 - S_1} + \frac{S_2}{100 - S_2} \right). \quad (584)$$

When, however, the absorbed ammonia is to be driven off from the solution by heating it, the discharge will consist partly of ammonia vapor and partly water vapor, so the heat of liberation of a given amount of ammonia from solution will be in practice that for the ammonia and equal to what would be liberated by its absorption, and also in addition, the heat of vaporization of the water vapor. When, as in absorption refrigerating machine generators, the discharged vapors meet incoming solution and are thereby partly condensed, practically all except perhaps 2 per cent of the heat of vaporization of water vapor is returned, and the net heat of ammonia liberation is not materially different from the value for absorption. If this is not done a correction must be introduced for the water vapor.

To assist in the solution of problems on the amount of non-condensable gases to be handled by dry vacuum pumps serving steam condensers, and on the change in the composition of gases when scrubbed by water in the course of cooling and cleaning after manufacture, a table of solubilities of various gases in water is given in the Handbook of Tables, as compiled from various sources and reported in the Smithsonian physical tables. The numbers in the tables are volumes of *standard* gas, that is, gas measured at 32°F. and 1 atm. pressure per unit volume of water, though they are at a different volume as absorbed or when absorbed at the temperature given. The pressure of the solution is in every case 29.92 ins. Hg absolute, and this is the combined pressure of both the gas and the water vapor. See Tables XLVI and XLVII.

Example. In the absorber of an ice machine of the absorption type, a weak solution of ammonia in water takes up the ammonia vapor coming from the refrigerating coils, the heat found being removed by water. The weak liquor, as it is called, is continuously supplied and the rich liquor continuously pumped away to the generator where, by heating, some of the ammonia vapor is driven off to the condenser. Assuming the action in the absorber to be merely one of ammonia dissolving in a weak solution and that no water vapor leaves the generator, what will be the heat produced in the absorber and needed in the generator per pound of ammonia for the following assumed conditions: Weak solution, 15 per cent NH_3 ; strong solution, 30 per cent NH_3 ; temperature in absorber, 80°F.

From Eq. (584), the heat of absorption per lb. of ammonia absorbed will be

$$893 - 142.5 \left(\frac{S_1}{100 - S_1} + \frac{S_2}{100 - S_2} \right),$$

where S_1 and S_2 are the per cents of ammonia in weak and rich solutions, respectively,

or $Q = 893 - 142.5 \left(\frac{15}{85} + \frac{30}{70} \right) = 807 \text{ B.T.U. per lb. } \text{NH}_3 \text{ absorbed.}$

Prob. 1. Ammonia is being absorbed by water at a temperature of 100°F . The solution contains 10 per cent of ammonia. If the total pressure is 15 ins. of Hg, what part of this is due to foreign gases, what part to ammonia and what part to water vapor?

Prob. 2. How many cubic feet and how many pounds of the following gases can be separately dissolved in 1000 gallons of pure water at atmospheric pressure and a temperature of 50°F .—air, carbon dioxide, and hydrogen? How would the results be changed if the pressure were doubled? If the temperature were doubled?

Prob. 3. When either water or ammonia is added to an ammonia solution, heat is evolved. Explain why this is so.

Prob. 4. Ammonia is being absorbed by a stream of running water, there being 5 lbs. of water per pound of ammonia. What will be the heat developed per pound of ammonia liquor formed?

Prob. 5. Ten pounds of the above solution receives an additional pound of ammonia. How much heat is generated by this action?

Prob. 6. A solution containing 10 per cent of ammonia receives an addition of another 10 per cent. What was the amount of heat developed per pound of ammonia and per pound of solution when the second portion of the ammonia was added?

Prob. 7. The pressure in the absorber of an ammonia absorption machine is one atmosphere and the temperature is 80°F . What is the maximum per cent of ammonia which can be absorbed by the water?

Prob. 8. The generator is working under a pressure of 125 lbs. per square inch gage and the heat is supplied by a steam coil in which the pressure is 30 lbs. per square inch gage. What per cent of ammonia will be left in solution after passing through the generator and about how much steam must condense per lb. NH_3 discharged?

CHAPTER IX

CHANGE OF CHEMICAL STATE WITH HEAT. COMBUSTION AND HEAT TRANSMISSION.

75. Combustion and Related Reactions. Relative Weights and Volumes of Substances and Elements, before and after Reaction. Not only may matter assume the three states of solid, liquid and vapor separately, in pairs simultaneously, or even all three together with various accompanying or causal temperature, pressure, or heat content, conditions, without chemical change of the matter itself, but matter may change in kind. As Mellor puts it—"matter appears to be endowed with properties in virtue of which two or more dissimilar substances, when brought into close contact, give rise to other forms of matter possessing properties quite distinct from the original substance. The process of change is called a chemical reaction." Chemical changes are assumed to be characterized by molecular rearrangement according to which molecules of elements may divide into atoms and the separated atoms of one combined with those of another element, to form a molecule of a new substance to be called a compound. Similarly, the molecules of compounds may split and re-associate, part of one with part of another, to make a new compound, or a single compound may split up into its elements which remain separated, the last case being generally termed dissociation. All three classes of change of substance are classifiable as chemical reactions, and there is really no very rigid line to be drawn between the sub-classes of reaction known as combination and dissociation except when applied to the same substances, in which case one is the reverse of the other. The complete or partial destruction of a substance as such is commonly termed decomposition as, for example, when hydrocarbon constituents of coal volatile, or liquid fuel, are changed by excessively high temperature into free carbon, and other hydrocarbons or even free hydrogen.

Every chemical change whether of combination or dissociation is accompanied by a heat change of the system or group of substances. When the reaction is such that heat is set free, tending to raise the temperature of the reacting mass unless it be carried away as liberated, the reaction is classed as *exothermic*. On the contrary, those reactions that are accompanied by heat absorption, tending to lower the temperature, unless heat be added to supply the absorption, are classed as *endothermic*.

It appears then that every reaction *tends* to change the temperature of the system, causing it to rise or fall according as the reaction is exothermic or endothermic, except in the one case where several reactions occur simulta-

neously, in which all the exothermic set free just enough heat to supply what is required for the endothermic.

The most important reaction in engineering is combustion, defined as the chemical reaction between fuels and the oxygen of the air, which is exothermic or heat liberating, and the source of practically all the heat used in engines for conversion into work. Combustion is often classed as an oxidation process, and is thus distinguished from another important engineering class of related reactions—reduction or the reverse of oxidation, the most prominent case of which is the change from carbon dioxide to carbon monoxide in gas producers, and in a lesser degree in furnaces, according to which there is a reduction of oxygen content per molecule; this process is endothermic. The formation of carbon monoxide directly from carbon by its oxidation, is sometimes defined as partial combustion of carbon or incomplete oxidation because the product, carbon monoxide, may further oxidize or burn to carbon dioxide by taking up more oxygen. The substances produced by the partial combustion or partial oxidation of one fuel element may also be considered as the result of the dissociation or reduction or deoxidation of the substances produced by its complete combustion. Ordinarily, partial combustion and reduction are considered as reverse processes producing the same substances, by (a) exothermic reaction of the primary substance in partial combustion, and (b) the endothermic deoxidizing reaction of the products of complete exothermic reactions of the same primary substances. It is also common to consider only the reaction of a substance or so-called fuel element with oxygen, as combustion and other processes, whether involving oxygen or not, as related reactions. Thus, carbon combining with oxygen to form carbonic acid is complete, and carbon combining with oxygen to form carbon monoxide incomplete combustion of carbon, while carbon monoxide and steam reacting can hardly be considered as combustion, and is best classified as a related reaction.

The number of elements entering into combustion and related reactions is small, but the number of possible substances and reactions between them is very large. The prediction of just what reactions will take place between various groupings or mixtures of these substances is extremely difficult and in some cases quite impossible. The study of possible reactions has become the province of physical chemistry, especially when the conditions controlling the result are also subjects of study. These conditions include the temperature, pressure, electrical state and the mutual relation of the elementary compounds present, and the relation of these various conditions to the primary and resultant substances; the intermediate, successive, simultaneous or parallel reactions constitute the subject matter of the study of chemical equilibrium. If chemical equilibrium were better understood, it would be possible to predict the resultant from primary substances for specified conditions, but at the present time this is quite impossible. Even though the exact prediction of the direction in which reaction would proceed in an ordinary complex system and the extent to which it would go in that direction, were made possible by a more complete thermochemistry than now exists, it would not be of much use in engineering,

because it is seldom possible to define the conditions that are present or to be met. For example, in gas producers, solid coal, air and steam are the primary materials and the product or result of their mutual reaction is a combustible gas; but it is not possible to predict and control the exact composition of this gas, nor will it in all probability ever be possible, because it would first be necessary to fix the chemical and thermal character of the coal, air and steam, to fix their relative quantities, to maintain an absolutely uniform fuel bed as to size, porosity and quality, with some other conditions equally elusive.

It must be understood, therefore, that while the possibility or even probability of certain reactions taking place may be known, it is quite impossible to predict just what will happen, or what products will result, when a given group of primary substances mutually react, so that many important problems of combustion in boiler furnaces and gas producers cannot be solved except by approximation.

The approximation takes the form of a calculation which is exact, based on an hypothesis which does not represent the facts of the case. That is, engineering calculations about combustion are always to be prefaced by a statement that certain substances are going to change completely or within a given degree to certain others, whether they will or not. Furthermore, the substances must be defined chemically by their symbols, and then will it be possible to calculate the relative weights and volumes of the various substances that can so react, the corresponding relative weights and volumes of the products, and the heat liberated or absorbed, but not otherwise. It is quite important that too much confidence be not put in these results, which are no more nearly correct than is the assumption of what substances are to be formed.

From the study of chemical equilibrium a few principles for guidance have been developed that help a little but not very much. For example, van't Hoff's "*law of movable equilibrium*" says that when the temperature of a system is raised reaction takes place which is accompanied by absorption of heat, and conversely. Another similar law is to the effect that a rise of pressure in a system in equilibrium causes that reaction which is accompanied by a reduction of volume. There are more of this character but they are entirely too general to make it possible to avoid the procedure adopted by engineers of assuming the kind of reaction, and then calculating quantities and heats that should and would accompany it, if it did take place; however crude this may look it is very useful, and in most cases sufficiently satisfactory considering the knowledge of other conditions to be met.

Carbon and hydrogen are the only chemical elements of fuel value, and all commercial fuels, including wood, peat, lignite, bituminous and anthracite coals, charcoal, coke, crude petroleum oils with their distillates, gasoline, kerosene, and their residues, tars, heavy oils, alcohols, benzole, a bituminous coal product, natural gas as well as blast furnace, carbureted and uncarbureted water gas, coal gas, producer gas, and oil gas, are *compounds*, *mixtures* and *mixtures of compounds* of these fuel elements, with oxygen in some cases. The one exception is sulphur, which exists in many solid,

liquid and gaseous fuels as an impurity, but which also has some small fuel value.

This being the case, the number of products to be formed by the *complete* combustion of fuel is also small, and includes only carbon dioxide and water vapor, with the nitrogen carried by the air, and a small amount of sulphur compounds, often ignored.

The process of reaction, whether combustion or one of the related ones, is to be described by the usual chemical equation which has the additional significance of showing the relative weights involved directly, because,

- (a) The total number of atoms of each chemical element on each side of the equation must be the same.
- (b) The sum of the products of the atomic weight of each atom and the number present must be the same on each side of the equation.

This is the same as saying, (a) that the total weight of hydrogen in the products must be the same as the total weight of hydrogen in the original mixture and so also for the other elements, and (b) the total weight of the original mixture of reacting substances must be the same as that of the products.

Natural fuels, while sometimes consisting of simple elements like carbon or hydrogen alone, or simple compounds like carbon monoxide or methane alone, are more often mixtures. Their reaction equations are then to be derived from combinations of the equations representing the reactions of elements and of compounds with oxygen or with each other.

These and any other reactions are characterized by definite weight relations which are given directly by the reaction equation by introducing the weight of each element from a table of atomic weights, and for this purpose the nearest whole number is close enough. For example, the complete combustion of hexane, C_6H_{14} , the main constituent of gasolene, to carbon dioxide and water, is given by,



This may be interpreted as follows, taking the atomic weight of C=12, of H=1, and of O=16.

$$\left. \begin{array}{l} (6 \times 12 = 72) \text{ lbs. carbon} \\ + (14 \times 1 = 14) \text{ lbs. hydrogen} \\ + (9\frac{1}{2} \times 2 \times 16 = 304) \text{ lbs. oxygen} \end{array} \right\} \text{ make } \left\{ \begin{array}{l} (6 \times 12 = 72) \text{ lbs. carbon} \\ + (6 \times 2 \times 16 = 192) \text{ lbs. oxygen} \\ + (7 \times 2 \times 1 = 14) \text{ lbs. hydrogen} \\ + (7 \times 16 = 112) \text{ lbs. oxygen} \end{array} \right.$$

$$\left. \begin{array}{l} \text{or } (72 + 14 = 86) \text{ lbs. hexane} \\ + (304) \text{ lbs. oxygen} \end{array} \right\} \text{ make } \left\{ \begin{array}{l} (72 + 192 = 264) \text{ lbs. carbon dioxide} \\ + (14 + 112 = 126) \text{ lbs. water vapor} \end{array} \right.$$

It might also be said from the same data that,

1 lb. hexane completely burned with oxygen yields $\frac{390}{86}$ lbs. products.

1 lb. oxygen will completely burn $\frac{86}{304}$ lbs. hexane and yield $\frac{390}{304}$ lbs. products.

This particular example can be analyzed in another way, yielding a general expression for the combustion of an analyzed fuel.

Fuel analyses are reported in two ways: (a) Proximate, giving per cent of

each independent compound, or separately determined constituent substance; (b) ultimate, giving the per cent of each chemical element. Applying this distinction to the original mixture of hexane and oxygen and its products, the proximate analyses are:

Original mixture	Hexane	$= \frac{86}{390}$	= 22.1 % by weight.
	Oxygen	$= \frac{304}{390}$	= 77.9 % by weight.
	Total		<u>100.00%</u>
Products	Carbon dioxide	$= \frac{264}{390}$	= 67.6 % by weight.
	Water vapor or liquid	$= \frac{126}{390}$	= 23.4 % by weight.
	Total		<u>100.00%</u>

Using the ultimate analysis method of designation for mixtures and products

Original mixture	Carbon (in hexane)	$= \frac{72}{390}$	= 18.4 % by weight.
	Hydro. (in hexane)	$= \frac{14}{390}$	= 3.6 % by weight.
	Oxygen (free)	$= \frac{304}{390}$	= 78.0 % by weight.
	Total		<u>100.00%</u>
Products	Carbon (in CO ₂)	$= \frac{72}{390}$	= 18.4 % by weight.
	Hydrogen (water)	$= \frac{14}{390}$	= 3.6 % by weight.
	Oxygen (in CO ₂)	$= \frac{192}{390}$	$= \frac{304}{390} = 78.0 \%$ by weight.
	Oxygen (water)	$= \frac{112}{390}$	
	Total		<u>100.00%</u>

Neither combustion nor its related reactions take place with oxygen alone, but with air containing oxygen, 23.2 per cent, and nitrogen, 76.8 per cent by weight, each pound of oxygen carrying with it $\frac{76.8}{23.2} = 3.31$ lbs. of nitrogen or existing in 4.31 lbs. of air. The nitrogen is generally considered neutral, though it may form compounds with hydrogen, such as ammonia directly, or with oxygen, such as nitrous oxide if conditions are right. If neutral, it

has the effect of changing the weight of the mixture by an amount depending on the proportion of oxygen that came from air.

$$\left\{ \begin{array}{l} \text{(Weight of mixture with oxygen alone)} \\ + 3.31 \text{ (wt. oxygen present)} \end{array} \right\} = \text{(weight of mixture with air),}$$

or $\left\{ \begin{array}{l} \text{(weight of nitrogen to be added when} \\ \text{air is used instead of oxygen)} \end{array} \right\} = 3.31 \text{ (wt. of oxygen present).}$

This will add in the present case $304 \times 3.31 = 1006$ lbs. of nitrogen in the combustion of one molecule or 86 lbs. of hexane, the sum of 304 lbs. of oxygen and 1006 lbs. of nitrogen, giving 1310 lbs., the weight of air required, and raising the total weight of the mixture to 1396 lbs. The per cent of the various constituents of the mixture and products is now changed, but the amount of change is quickly computed. All these relative numbers can be conveniently stated in tables of conversion factors, such a table for hexane being given below as a type form, useful in every-day work when it is necessary to make repeated calculations with some one fuel.

TABLE III
CONVERSION FACTORS FOR WEIGHTS IN THE COMPLETE COMBUSTION OF
HEXANE WITH AIR TO WATER AND CARBON DIOXIDE

Original Mixture.	Final Mixture.	Hexane C_6H_{14} .	Oxygen O_2 .	Nitrogen N_2 .	Air.	Carbon Dioxide CO_2 .	Water H_2O .
1396.	1396.	86.	304.	1006.	1310.	264.	126.
1.	1.	.0616	.218	.720	.938	.189	.091
16.22	16.22	1.	3.54	11.68	15.22	3.07	1.46
4.59	4.59	.283	1.	3.31	4.31	.87	.41
1.387	1.387	.085	.302	1.	1.302	.262	.125
1.066	1.066	.066	.232	.768	1.	.202	.096
5.29	5.29	.326	1.15	3.81	4.96	1.	.48
11.08	11.08	.683	2.42	7.98	10.40	2.10	1.

The average analysis of pure air is given by Parkes as being $O_2 = 20.96$ per cent; $CO_2 = .04$ per cent; $N_2 = 79.00$ per cent by volume, giving $O_2 = 23.20$ per cent; $CO_2 = .06$ per cent; $N_2 = 76.74$ per cent by weight. Regarding the CO_2 as negligible, the relation may be used for the purpose of computations of this sort, $O_2 = 23.2$ per cent and $N_2 = 76.8$ per cent. On the assumption that any nitrogen present came from air and not from any other compound, such as ammonia, it must have been associated with oxygen in the proportion 3.31 to 1 of O_2 . The weight of oxygen on both sides must be equal and the weight of air on one side must be 4.31 times the weight of oxygen. Of the nitrogen that is present $\frac{1}{3.31} \times$ (the weight of nitrogen present) must have come from air and the rest from the other substances.

A much-used weight relation is, for the weight of air products per pound of fuel, and on the assumption that H = part of hydrogen by weight per pound fuel, and C = part of carbon by weight per pound fuel:

$$(\text{Weight of air per lb. of fuel}) = C \times \frac{32}{12} \times 4.31 + H \times 8 \times 4.31 = 11.49C + 34.46H, \quad (585)$$

$$\begin{aligned} \text{Weight of products per lb. fuel} &= 3\frac{1}{3}C \text{ lbs. CO}_2 + 9H \text{ lbs. H}_2\text{O} + \frac{32}{12} \\ &\quad \times 3.31C \text{ lbs. N} + 8 \times 3.31H \text{ lbs. N} \\ &= 3.67C \text{ in form of CO}_2 + 9H \text{ in form of H}_2\text{O} + 8.82C + 26.46H \text{ in form of N} \\ &= 12.49C + 35.46H, \quad \dots \dots \dots (586) \end{aligned}$$

for complete combustion in air with no more air supplied than enters into the reaction.

Most of the practical problems concerning the relative amounts of substances involved in combustion and reduction processes are concerned with gases, at least on one side of the equation and sometimes on both. Direct combustion and the gasification of fuels in producers and complete combustion in boiler furnaces yields gas mixtures, the composition of which is always volumetrically determined by analysis, while the explosive mixtures or primary working substances of gas engines are gaseous initially and remain so after combustion. It is quite necessary, therefore, to transform the weight relations of the reaction equation into another form yielding volumes. There are three ways of doing this, all equivalent and all yielding the same result if the constants used are consistent, and if the gases and vapors follow the Avagadro law on which the most useful of the methods depends.

1. The volume at standard conditions of any substance can be found from the weight present by multiplying that weight by the specific volume of the substances, in cubic feet per pound, at standard condition of 1 atm. at 32° F.

2. The molecular weight in pounds of any gaseous or vapor substance assumed to follow Avagadro's law occupies 358 cu.ft.

3. The volumetric relations are given directly by the coefficients of the substance in the chemical equation when the substances are gaseous and all enter into the reaction, and when each is expressed in terms of molecules present.

The first method needs no explanation or further development and the second and third are really one. *If in the reaction equations there be written for any substance the number of pound molecules $\times 358$, the product will be the volume of that substance in the reaction in cubic feet at 32° F. and standard atmospheric pressure of 29.92 ins. Hg, if the substance could so exist and even if not, the relation between the volumes will hold at any other simultaneous pressure and temperature for all.* When the substances all are gaseous the coefficient of the symbol, representing the number of molecules, indicates the relative volumes at once. Thus, for complete combustion of carbon with oxygen alone, $C + O_2 = CO_2$, or (one molecule of carbon) + (one molecule) of oxygen makes (one molecule of CO_2) or 12 lbs. carbon + 358 cu.ft. oxygen makes 358 cu.ft. of CO_2 . One pound of carbon requires 29.8 cu.ft. of oxygen and will make 29.8 cu.ft. of CO_2 at 32° F. and 29.92 ins. Hg. Also for hydrogen $2H_2 + O_2 = 2H_2O$, or (two molecules of hydrogen) + (one molecule of oxygen) makes (two molecules of water); or 2 cu.ft. of hydrogen + 1 cu.ft.

oxygen, makes 2 cu.ft. gaseous water; or 4 lbs. hydrogen, requires 358 cu.ft. of oxygen, hence 1 lb. of hydrogen requires 89.5 cu.ft. of oxygen. For *hexane* $C_6H_{14} + 9\frac{1}{2}O_2 = 7H_2O + 6CO_2$, or 1 cu.ft. of hexane requires $9\frac{1}{2}$ cu.ft. of oxygen and makes 7 cu.ft. of gaseous water and 6 cu.ft. of carbon dioxide. It thus appears that the volumetric analysis of gaseous mixtures or products is given simply by the ratio of the number of molecules of each substance to the total number of molecules present.

When the oxygen is to be derived from the air each cubic foot of oxygen will carry with it $\frac{79.04}{20.96} = 3.771$ cu.ft. of nitrogen which is to be added, or is to be found in 4.771 cu.ft. of air.

Prob. 1. Complete the following equations for combustion with oxygen and with air: $CH_4 + 2H_2 + -O_2 =$; $C_6H_6 + -O_2 =$; $C + 3H_2 + 4S + -O_2 =$.

Prob. 2. What will be the weight of the air needed in each of the cases of Prob. 1 and what will be the weight of the products? The volumetric composition?

Prob. 3. When 1 lb. of propylene is burned how much air is needed and what is formed? What is the proportion of each product by weight? What would be the approximate volume and volumetric composition of the products at a temperature of 60° F.?

Prob. 4. After combustion the products of a combustible mixture were found to be 17 cu.ft. of nitrogen, 3 cu.ft. of carbon dioxide and water, equivalent to 6 cu.ft. of water vapor. What was the composition prior to combustion and what was the volume at 60° F.?

Prob. 5. Five lbs. of carbon dioxide, 4 lbs. of oxygen, and 2 lbs. of water would result from the combustion of how much carbon and hydrogen? In what amount of oxygen? What would be the complete weight of the products if the combustion had taken place in air?

Prob. 6. When steam is passed through a bed of hot coke the coke takes the oxygen from the water to form CO and the resulting gas is called water gas. For 10 lbs. of carbon how much gas by weight could be made and how much steam would be required?

Prob. 7. A sample of producer gas gave upon analysis the following per cubic foot:

$H_2 = .18$ cu.ft. $CO = .25$ cu.ft. $CH_4 = .03$ cu.ft. $CO_2 = .07$ cu.ft. $N_2 = .47$ cu.ft.

What will be the cubic feet of air needed to burn a cubic foot of the gas and what would be the composition of the products?

76. Heats of Reaction. Calorific Power of Combustible Elements and Simple Chemical Compounds. B.T.U. per Pound and per Cubic Foot. Combustion of the so-called fuels is always exothermic, and the heat set free per pound of fuel is its calorific power. This heat of reaction may also be expressed per unit weight or per unit volume of any other element or substance entering into the reaction, but all are derivable from the calorific power by the weight and volume relations. Each is more directly useful in some particular calculation, for example, the B.T.U. per cubic foot of combustible gaseous mixture is an important factor in the mean effective pressure of gas engines;

also the temperature rise during combustion is proportional to the B.T.U. per pound of products. To facilitate boiler calculations and reduce the large number of B.T.U. per pound of fuel, its equivalent evaporative power from water at 212° F. to dry saturated steam at the same temperature is often given and is the B.T.U. per pound divided by 970.4. This value is described as equivalent evaporative power, the number of evaporation units, or as evaporative power from and at 212° F.

All calorific powers can be determined exactly only by direct experimental observations of burning the fuel in water-cooled calorimeters, and the heat of combustion is directly given by the pound-degrees of the water with a correction for radiation during test, and also for the heat absorbed by the metal and other parts simultaneously heated, called the water equivalent of the instrument. It is assumed in calorimeter tests that all the substances are brought to their original temperature after combustion, if not, another correction must be made for the residual heat remaining in the hot products, ash or instrument parts, which has not been imparted to the calorimeter.

It should be noted that whenever hydrogen burns to superheated water vapor in a calorimeter, the products in cooling will somewhere reach the saturation temperature for the pressure existing. When this point is reached water vapor will condense and the calorimeter water will receive the latent heat of condensation of the water produced by combustion. If, for example, the combustion proceeded at atmospheric pressure, as in all gas calorimeters, this point would be 212° F., and as the condensation proceeds at this temperature there may be two calorific powers reported for products of combustion cooled to 212° F., the first for products condensed and the second for the products not condensed. The former will be greater than the latter by 970.4 B.T.U. per pound of water condensed and not per pound formed because all will not condense at any temperature. Of course this same difference exists when the products are cooled below 212°, but there is in this case another difference, inasmuch as the substances giving up heat to the calorimeter water are not all gaseous, but a liquid, some fixed gases and some water vapor, a little of which continues to condense for each degree drop, and this is in contrast to the conditions above 212°, where nothing but gases and superheated vapor, which behaves the same as gases, are being cooled.

It is thus apparent that not only may there be two calorific powers for fuels containing hydrogen, one *high* and the other *low*, but that the exact measure of the difference between them per pound of hydrogen burned or per pound of water formed is a question of how far or to what temperature the cooling be carried. In view of this possibility of more than one interpretation, it is sufficiently close for engineering work of the usual routine to accept as the difference, $970.4 \times \text{weight of water vapor formed}$, and this has the advantage of being definite and possible to calculate, even though it may not be absolutely in accordance with actual conditions. Since 1 lb. of hydrogen makes 9 lbs. of water in combustion, the weight of water is nine times that of the hydrogen, therefore it should be accepted that

$$\left\{ \begin{array}{l} \text{Difference between high} \\ \text{and low calorific power} \end{array} \right\} = 970.4 \times 9 \times (\text{weight of hydrogen per pound fuel}).$$

$$= 8734 \times \left\{ \begin{array}{l} (\text{weight of hydrogen}) \\ \text{per pound fuel} \end{array} \right\}. \quad \dots \quad (587)$$

Engineers have been forced to take this difference into consideration owing to the greatly increased use of hydrocarbon gaseous and liquid fuels and the alcohols, the former for burning under boilers and in internal combustion engines and the latter in gas engines alone. In both of these classes of apparatus the products of combustion leave quite hot, always above 212° F., so that it may be said with some propriety that they did not receive the full heat of combustion, as reported by the calorimeter, for products reduced to initial temperature. On the other hand, it may with equal propriety be contended that they have received even less than the low value because the products are discharged hotter than 212° F., but that they should receive no credit for the difference which is to be charged against them because they are unable to utilize low temperature heat. In this book the most commonly accepted practice will be adopted, and this is to charge against boilers and all open or atmospheric pressure fires or furnaces the high calorific power, and against explosive combustion or that taking place in closed chambers at pressures considerably in excess of atmosphere the low value as previously defined.

Unless otherwise defined *calorific power* will be understood to mean the heat liberated by combustion obtained by burning 1 lb. of fuel at constant pressure in free oxygen, both fuel and oxygen being at equal temperature before, and products reduced to that same temperature after, combustion. This is the high value, but not so named unless for special reasons. Constant volume combustion would yield a different value, which some authorities insist on applying to gas engine explosions, but this is hardly necessary, in engineering, where conditions of service are always somewhat undefined and practical factors of performance must be introduced to allow for inaccuracies.

Since the mean specific heat of the original fuel and oxygen may not be the same as that of the products, it will make some difference what the base temperature of the experiment is. This effect will be greatest in the case of hydrogen burning to water, liquid or vapor, and the fact is illustrated best by a specific computation. Julius Thomsen gives the experimental heat of combustion of hydrogen as being 61,200 B.T.U. per pound hydrogen, when the original hydrogen and oxygen are at a temperature of 18° C. or 64° F., and the resulting water brought to the same temperature. Since 9 lbs. of water are formed per pound hydrogen the heat obtained from the calorimeter, in case the product, water, were finally at 212° would be less than the above by the difference between heat of the liquid at 212° and at 64° for the 9 lbs. Then per pound of H₂ (H₂ and O₂ at 64° F. burning to liquid water at 64° F.) gives 61,200 B.T.U.; (H₂ and O₂ at 64° F. burning to liquid water at 212° F.) gives 61,200 - 148 × 9 = 59,868 B.T.U. If, however, the original substances H₂ and

O₂ are at 212° instead of 64°, more heat would be removed from the calorimeter, by the amount of heat necessary to raise the temperature of 1 lb. H₂ and 8 lbs. O₂ from 64° to 212° F. Or (H₂ and O₂ at 212° F. burning to liquid water at 212° F.) gives $59,868 + 148(1 \times 3.4 + 8 \times .217) = 60,626$ B.T.U. per lb. H₂. If the final products of combustion be vapor of water at 212° instead of liquid, the (H₂ and O₂ at 212° F. burning to water vapor at 212° F.) gives $60,626 - 9 \times 970.4 = 51,892$ B.T.U.

The temperature 212° F. has no significance when the pressure is other than atmospheric, and in the products of combustion there are nearly always other substances than water vapor present, so that very rarely is the partial pressure of the resulting vapor equal to standard atmosphere. Nevertheless, to conform to practice which considers the difference between high and low value as equal to the product of the weight of water vapor formed by the latent heat at 212°, and at the same time to be consistent with scientific information available, it will be regarded that the expression "high value" for heat of combustion of hydrogen refers to the figure 60,626 B.T.U. per pound hydrogen, as derived above, while the term "low value" refers to the figure 51,892 B.T.U. per pound hydrogen, both being referred to an arbitrary base temperature of 212° F.

Investigations by thermo-chemists have shown that the heat of combustion is determined not only by the chemical elements that burn, but also on their molecular structure, whether a single chemical element is involved or a chemical compound. That is, assuming carbon and hydrogen to be the fuel elements in a fuel, its calorific power is not equal to the sum of the products of these two calorific powers into their respective partial weights. There is a difference and the difference is not the same for different compounds, but follows a simple law for the compounds of one series, like the hydrocarbons of the paraffine series, for example. Furthermore, for carbon alone there are several different calorific powers, that for soft, porous carbon being at one end and that for crystalline diamond at the other. Fortunately, these differences are not to be considered in engineering as two successive samples of the same natural fuel will differ from each other more than would warrant fine corrections like that for the carbon. The molecular structure as a cause of difference is, however, useful in explaining the lack of agreement between a calorimeter test which is always conclusive and a calculation based on chemical composition and calorific power of the elements.

As combination and dissociation are inverse processes the heat liberated by the combustion of hydrogen to form water, which may be regarded as the heat of water formation, must be equal to the heat necessarily absorbed in separating the water into hydrogen and oxygen, whence the heat of dissociation of a compound formed by combustion is equal to the heat of combustion of its fuel element. This is important in high temperature work in which steam is often dissociated as hydrogen is burned, and where in successive steps steam dissociation and hydrogen combustion may follow each other. In high temperature combustion as, for example, gas engine explosions, it is

quite possible that before the fuel is burned entirely the temperature has risen to the value at which dissociation of products takes place, and therefore combustion or combination will be retarded until heat be carried away. When this occurs, all the heat of combustion does not enter into the raising of temperature indefinitely but only partly so, the rest acting as liberated to maintain a high temperature as heat is being abstracted or otherwise being disposed of.

As the calorific power of a commercial fuel is a function of the calorific power of the carbon, hydrogen and sulphur it contains, it is important that their calorific powers be firmly established, and some of the more authoritative values are given in Table LI, Handbook of Tables.

In the case of carbon, which may burn to carbon monoxide or to carbon dioxide, it is important that the corresponding reaction heats be *consistent*. This will be the case when the heat of combustion of 1 lb. of carbon burning to CO, added to the heat of combustion of the amount of CO containing a pound of carbon, be together equal to the heat of combustion of carbon to CO₂ directly. One lb. of C is contained in $\frac{28}{12}=2.33$ lbs. of carbon monoxide according to atomic weights compared to O=16, or $\frac{11.99+15.88}{11.99}=2.324$, according to atomic weights compared to H=1.

Therefore, for atomic weight O=16.

$$\left\{ \begin{array}{l} \text{B.T.U. per lb. C} \\ \text{burning to CO} \end{array} \right\} + \left\{ \begin{array}{l} 2.333 \times \text{B.T.U. per lb.} \\ \text{CO burning to CO}_2 \end{array} \right\} = \left\{ \begin{array}{l} \text{B.T.U. per lb. C} \\ \text{burning to CO}_2 \end{array} \right\}$$

or for atomic weights H=1,

$$\left\{ \begin{array}{l} \text{B.T.U. per lb. C} \\ \text{burning to CO} \end{array} \right\} = \left\{ \begin{array}{l} 2.324 \times \text{B.T.U. per lb.} \\ \text{CO burning to CO}_2 \end{array} \right\} = \left\{ \begin{array}{l} \text{B.T.U. per lb. C} \\ \text{burning to CO}_2 \end{array} \right\}.$$

This is but a special form of the general thermo-chemical law. *The heat of combustion of a compound (CO) together with the heat of formation of that compound from its elements (C and O) must be equal to the heat of combustion of the elements (C and O) direct to the final products CO₂ or the heat of formation of products directly from the elements.*

Taking the Favre and Silberman value for the complete combustion of carbon to carbon dioxide, which is accepted by Julius Thomsen, as 14,544 B.T.U. per pound C, and Thomsen's own value for the heat of combustion of CO to CO₂ as 4369 B.T.U. per pound CO, the heat of combustion of the compound must be (using atomic weights C=12 and O=16 because Thomsen used these values):

$$\text{B.T.U. per pound C burning to CO} = \left\{ \begin{array}{l} (\text{B.T.U. per pound C burning to CO}_2) - 2.333 \times (\text{B.T.U.} \\ \text{per pound CO burning to CO}_2), \\ = 14,544 - 2.333 \times 4369 = 14,544 - 10,193 = 4351. \end{array} \right.$$

From the above it is clear that the heat of combustion of CO burning to CO₂ is 10,193 B.T.U. per pound of *carbon*.

It is evident from the general law that the heat of combustion of compounds, like the hydrocarbons and alcohols, cannot be computed from the heat

of combustion of the carbon and hydrogen they contain, without correcting for the heats of formation of these compounds from their elements. These heats are, however, fairly well determined for some of the definite compounds that are constituents of natural fuels; and among these the most important are the hydrocarbons. Such figures, however, are not of very much use in the practical work of dealing with the volatile of coals or liquid fuels because it is not possible to tell precisely what hydrocarbons are present. For all such practical work approximate methods are used. If the hydrocarbon is represented by the symbol C_xH_y , the heat of formation of the products is (high value)

$$\frac{12 \times 14,544x + 60,626y}{12x + y}$$

This is the heat which would be obtained by the burning of that amount of hydrogen and carbon in the free state which exists in the combined state in the hydrocarbon, which is not the heat of combustion of the hydrocarbon, since such combustion requires the supply of heat of formation of the hydrocarbon itself, either positive or negative.

Calculations dealing with the combustion of gases and vapors are often more conveniently carried through, on a volume rather than on a weight basis. In doing this, it is desirable to have available the volumetric calorific powers of the standard gases and vapors reduced to 32° F., and 29.92 ins. Hg absolute pressure even if the particular compound is a liquid under these conditions, for the proportionality will hold for higher temperatures as if it were a vapor or gas. Gases reported for these conditions of pressure and temperature are termed *standard*, and the reduction to this standard from any other pressure and temperature, or to any desired pressure and temperature, is to be made on the assumption of perfect gas behavior.

Let $P_{29.92}$ = absolute pressure in inches of mercury = (29.92), of standard gas or vapor;

$V_{32/29.92}$ = volume in cubic feet per pound of standard gas;

$d_{32/29.92}$ = density in pounds per cubic foot of standard gas;

$$(\text{B.T.U. per cubic foot})_{32/29.92} = \frac{\text{B.T.U. per lb.}}{V_{32/29.92}} = \text{B.T.U. per cubic foot standard gas};$$

P = any other pressure in inches of mercury at which the gas may be under study;

t = any other Fahrenheit temperature;

V and d = corresponding specific volume and density for P and t ;

(B.T.U. per cubic foot) = corresponding volumetric heating power for P and t .

$$\text{Then } \frac{PV}{(t+460)} = \frac{P_{29.92} \times V_{32/29.92}}{492} = \frac{29.92 \times V_{32/29.92}}{492}.$$

$$\text{Whence } V = \left(\frac{t+460}{492} \right) \left(\frac{29.92}{P} \right) V_{32/29.92}, \text{ and } d = \left(\frac{492}{t+460} \right) \left(\frac{P}{29.92} \right) d_{32/29.92},$$

therefore,
$$\begin{aligned} (\text{B.T.U. per cu.ft.}) &= \frac{\text{B.T.U. per lb.}}{\left(\frac{t+460}{492}\right)\left(\frac{29.92}{P}\right)V_{32/29.92}} \\ &= \frac{(\text{B.T.U. per cu.ft.})_{32/29.92}}{\left(\frac{t+460}{492}\right)\left(\frac{29.92}{P}\right)} \quad \dots (588) \end{aligned}$$

The values for the (B.T.U. per cubic foot)_{32/29.92}, that is, of standard gases given in the Tables, are derived from the B.T.U. values per pound reported, both high and low values, and from the standard volumes of the pound; the direct experimental determination being used wherever it is available, otherwise as calculated from the molecular weights.

A consistency is desirable whenever not seriously in conflict with facts, and the molecular relations of density are necessary in dealing with liquid fuels and the gasification of solid ones; the values per cubic foot for hydrogen and carbon monoxide are often derived in this way from the pound value instead of making use of direct calorimetric values given in the table. This is always done for the vapors.

According to the formulas $2\text{H}_2 + \text{O}_2 = 2\text{H}_2\text{O}$ and $2\text{CO} + \text{O}_2 = 2\text{CO}_2$, it appears that the calorific power of hydrogen per cubic foot standard must be

$$\left. \begin{aligned} \text{B.T.U. per cu.ft. hydrogen} &= \frac{2 \times 60626}{358} = 339 \text{ high} & (a) \\ &= \frac{2 \times 51892}{358} = 290 \text{ low} & (b) \end{aligned} \right\} \dots (589)$$

Then if a molecule of CO has the same volume as a molecule of H_2 ,

$$\text{B.T.U. per cu.ft. carbon monoxide} = \frac{28 \times 4369}{358} = 342. \quad \dots (590)$$

For the hydrocarbons the same procedure can be followed, neglecting the heats of formation of the compounds.

Thus, for the paraffines $\text{C}_n\text{H}_{2n+2}$, taking the molecular weight of C=12, the following formulas may be obtained:

$$\left. \begin{aligned} \text{Wt. C in compound} &= \frac{12n}{12n+2n+2} = \frac{6n}{7n+1} \text{ per lb. fuel} & (a) \\ \text{Wt. H in compound} &= \frac{2n+2}{12n+2n+2} = \frac{n+1}{7n+1} \text{ per lb. fuel} & (b) \end{aligned} \right\} \dots (591)$$

$$\left. \begin{aligned} \text{B.T.U. per lb. fuel} &= \frac{14544 \times 6n + 60626(n+1)}{7n+1} = \frac{147890n + 60626}{7n+1} \text{ (high)} & (a) \\ &= \frac{14544 \times 6n + 51892(n+1)}{7n+1} = \frac{139156n + 51892}{7n+1} \text{ (low)} & (b) \end{aligned} \right\} \dots (592)$$

from which the B.T.U. per cu.ft. vapor may be determined. In the same way formulas for the ethylene series C_nH_{2n} may be obtained. Results calculated from such formulas have been plotted to a base of n in Fig.

109, and illustrate beautifully how the calorific power varies with the position of the hydrocarbon in the series with the paraffine series at the top. These values are the basis of work on coal and oil gas and liquid fuels to be taken up later. To the calculated values on the curve may be added the available direct determination for comparison from the Slaby empiric formula

$$(\text{B.T.U. per cubic foot}) = 112 + 18,880 \times (\text{pounds per cubic foot.}) \cdot (593)$$

The calorific power of a commercial fuel or lighting gas per cubic foot is the sum of the volumetric per cents of its constituents times the calorific

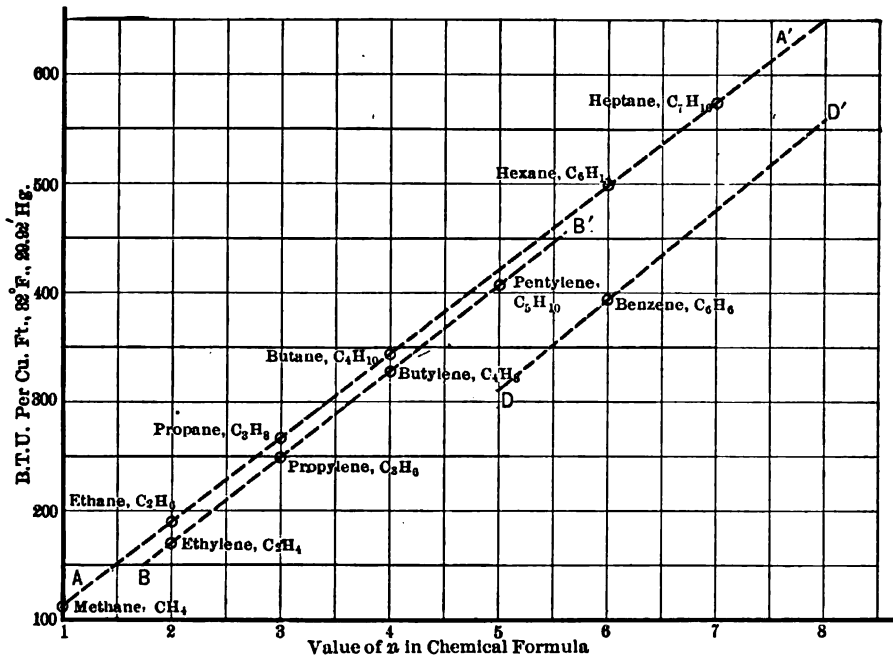


FIG. 109.—Heat of Formation of Products for Combustion of Hydrocarbons, in B.T.U., per cu.ft. at Standard Conditions.

power of those constituent gases which include carbon monoxide, hydrogen, methane, ethylene, ethane and sometimes some higher hydrocarbons, the nature of which is assumed, like benzene. Coals and oils cannot be so treated, but the methods used here with varying degrees of approximation will be taken up later. Of course, in all cases an estimate may be made from the combustible chemical elements if they are known in amount, by assuming their molecular condition.

Example 1. A certain fuel consists of 5 per cent hydrogen, 8 per cent oxygen, 1 per cent sulphur, and 87 per cent carbon, by weight. Find the heat of formation of the products of complete combustion, if all of the oxygen in the fuel is originally combined with hydrogen in form of moisture.

.08 lb. oxygen with $\frac{.08}{8} = .01$ lb. hydrogen constitute .09 lb. water originally in fuel, leaving $.05 - .01 = .04$ lb. hydrogen to be burned.

$$.04 \times 60626 = 2425 \text{ B.T.U.}$$

$$.01 \times 3998 = 40 \text{ B.T.U.}$$

$$.87 \times 14544 = 12653 \text{ B.T.U.}$$

Heat of formation of products = 15118 B.T.U. high value

Prob. 1. A certain fuel contains 20 per cent H_2 and 80 per cent C by weight. If the heat of formation of the compound was 8 per cent of the heat of formation of the products, what are the high and low values of the heat of combustion?

Prob. 2. What would be the heat of combustion for the reactions given in Problem 1, Section 75?

Prob. 3. It is known that a cubic foot of natural gas under standard conditions contains 980 B.T.U.'s. What would be its calorific power at 70°F. and at an altitude of 8000 ft.?

Prob. 4. A sample of producer gas gave the following analysis per cubic foot. What would be its high and low heating value per cubic foot? $H_2 = .18$; $CO = .25$; $CH_4 = .03$; $CO_2 = .07$; $N_2 = .47$.

Prob. 5. What would be the cost per 100,000 cu.ft. of standard gas and per bbl. of oil to give the same cost per B.T.U. as coal at \$4.00 per ton, when analyses are as follows? Analysis of coal and oil by weight, gas by volume.

Coal.	Oil.	Gas.
C = .75	$C_8H_{18} = .10$	$N_2 = .04$
$CH_4 = .07$	$C_6H_{14} = .30$	$H_2 = .02$
Moisture = .08	$C_8H_{20} = .50$	$CH_4 = .94$
Ash = .10	$C_{12}H_{28} = .10$	

Prob. 6. An evaporation of 7.7 lbs. of water from and at 212°F. was obtained with the following coal. What per cent of the heat of formation of products was delivered to the steam? C = .758; $H_2 = .049$; $O_2 = .085$; S = .023; Ash = .085.

Prob. 7. An internal combustion engine gives a horse-power hour with a fuel consumption of .5 lb. of hexane, while another engine uses 8 cu.ft. of methane during the same period and develops the same power. What are the respective efficiencies?

Prob. 8. By burning a pound of oil, the analysis for which by weight is given, 500 lbs. of water were heated 40°F. in a calorimeter. What was the heat of formation of the compounds? $C_{17}H_{32} = 10$ per cent; $C_8H_{18} = 80$ per cent; $C_6H_{14} = 10$ per cent.

77. Heat Transmission Processes. Factors of Internal Conduction, Surface Resistance, Radiation and Convection. Coefficients of Heat Transmission. When air supports combustion, the gaseous products may be said to be heated directly, and to define the process, the term "internal combustion heating" is generally applied; likewise when a liquid cools by evaporation from its surface due to a lesser surface-vapor pressure than corresponds to the temperature, it may be said to be directly or internally cooled. With such exceptions as these it is universally necessary in dealing with heat in commercial apparatus to transmit or transfer it from one place or one substance to another place or substance. The heat for boiling water in steam boilers must come through

tubes and plates from the fire or hot products of combustion, and a similar transfer takes place in closed feed-water heaters, in economizers, steam superheaters; in all heated surfaces of boiler, pipe and engine parts; in surface condensers for steam, ammonia or carbon dioxide; in steam and hot-water radiators; through the walls of buildings outward when heated during the winter and inward when cooled for cold storage below the atmosphere; likewise in the brine coolers, ice cans and other parts of mechanical refrigerating systems; in apparatus for concentrating solutions in chemical manufacture or in distillation; in the cooling of gas engine and compressor cylinders by water jackets, and cooling air between compressor cylinders; or drying and superheating steam between cylinders of multiple expansion engines, or preheating of compressed air for air engines. Thus, the practical application of heat for obtaining a desired result, generally though not always involves transmission; and most often through solid containers of great variety ranging from the thin brass tubes of surface condensers or the steel of boiler shells or ammonia evaporating coils, to steam pipe covering or the brick wall of a dwelling house.

That the movement of heat from one place or substance to another of lower temperature follows certain laws of flow was long ago recognized, even before anything was known of the transformation of heat into work. However, there is no general acceptance of what those laws are except in a few special cases, and the reason for this situation becomes clear after even a brief analysis of what must really happen in the course of an actual case of transmission. Notwithstanding the absence of laws that will permit reasonably accurate calculation of the relations between quantity of flow, the dimensions and kind of surfaces and quantities and kinds of material exchanging heat, with their corresponding changes of temperature or state, engineers must design apparatus that is needed, as best they can, and for this purpose there has been determined a method of approximation with which is used a series of coefficients developed by experience for the more common and recurrent cases. The treatment of this subject will, therefore, be divisible into two parts, first the examination of such laws as are fairly well fixed; and second, the development of an approximate method of treatment for the practical everyday problems that the fundamental laws do not solve.

Heat may pass from one substance or place to another in three essentially different ways and every practical case includes one or all of these and *no others*.

1. Heat may flow along a bar or wire or along any line in a solid body from a high to a low temperature point, by simple molecular communication. Each molecule heats the next without any discontinuity and with a regular continuous fall of temperature from the high to the low temperature point; this is *internal conduction* of heat. The conducting capacity of nearly all the common substances is fairly well determined and is represented by coefficients of thermal conductivity, some selected values for which are given in the Tables. Heat could not flow by internal conductivity alone beyond the limits of one piece of substance, but at the boundary it may be communicated to the next body, and so on pass any number of dividing surfaces or through

any number of bodies. This is a most common mode of *conduction* about which there is no exact information comparable with that for *internal conduction*. See Table LII in the Handbook of Tables.

2. Heat may flow from a hot body through space to a distant colder body entirely without reference to the separating medium, and the best example is the passage of the sun's heat to the earth. This mode of transmission is defined as *radiation* and heat so received by the cold, or discharged by the hot body, as radiant heat. The capacity for radiation of heat for different substances is practically unknown, though there is *fair* though not good agreement as to the relation of heat quantity to the two controlling temperatures. There is practically nothing known of the radiating capacity of any but originally black bodies brought to high temperature conditions or incandescence.

3. Heat may pass from one place or substance to another by being carried as a *charge* on a particle that is moved bodily between the two places. Thus, a particle of water heated at the bottom of a vessel becomes less dense and floating in the colder, or more dense medium, actually carries its heat to the top; this is also the case in chimneys where the flotation tendency produces the necessary draft. This is *convection* communication of heat and takes place only in liquids and gases, where in action it is generally associated most intimately with radiation and conduction. There may be a movement of particles of liquid or gas, not due to differences in density characteristic of convection, caused by pumps or fans, which is quite effective in carrying or transmitting heat, but this is an externally forced circulation of heated particles rather than a self-caused heat transfer.

Transmission of heat by internal conduction takes place according to a law of proportionality to temperature difference, but as the conductivity of substances depends on the temperature itself there must be involved a temperature term. This gives, for solids, a comparatively simple law which is apparently also applicable to liquids and gases, but, for gases variations enter that have been the subject of prolonged study by eminent physicists without acceptable solution.

For solids and liquids the conductivity is given by the following Eq. (594):

$$K_t = K_{32}[1 + x(t - 32)], \quad (594)$$

in which K_t = B.T.U. per hour per square foot of cross-section per degree F. difference in temperature between the points under consideration, per inch of path, when the body has the temperature t° F.;

K_{32} = B.T.U per hour per square foot per degree F. per inch of thick-
at 32° F.;

t = temperature degrees F.;

x = a constant.

This conductivity *at any temperature* is not directly useful except in comparing the conductivity of substances, but in computing the amount of heat flowing between a high temperature and a low temperature point the mean con-

ductivity for this range must be determined. As conductivity is a straight line function of temperature the mean conductivity between the temperature is the arithmetical mean of the conductivities at these temperatures, hence

Let t_h = high temperature degrees F.;
 t_l = low temperature degrees F.;
 $K_{t_h-t_l}$ = mean conductivity between t_h and t_l .

$$\begin{aligned} \text{Then } K_{t_h-t_l} &= \frac{K_{32}[1+x(t_h-32)] + K_{32}[1+x(t_l-32)]}{2} \\ &= K_{32} \left(1 + \frac{x}{2} [(t_h-32) + (t_l-32)] \right). \quad . \quad . \quad . \quad . \quad (595) \end{aligned}$$

From this mean conductivity expression Eq. (595) the amount of heat that will flow through a body by *internal conduction* is given by Eq. (596):

$$\begin{aligned} Q &= \frac{A}{l} (t_h - t_l) K_{t_h-t_l} \\ &= A l K_{32} \left(1 + \frac{x}{2} [(t_h-32) + (t_l-32)] \right) (t_h - t_l), \quad . \quad . \quad . \quad (596) \end{aligned}$$

where Q = B.T.U. per hour;

A = cross-section of conducting path in square feet;

l = length of conducting path in inches.

Applying expressions like this to solids is a direct and safe proceeding if the high and low temperatures are taken *within* the conducting body itself and not in some external substance like a liquid or gas in contact. The same expression and limitation applies to liquids *if there is no convection flow*, in which case the heat flow will be the algebraic sum of the conduction and convection parts. For gases there enters likewise a radiation factor as a third disturber, which suggests a doubt as to the value of the expressions at all for gases except as a means of comparing them with liquids and solids. In no case can these equations be applied to heat flow across a surface boundary or joint in a solid, or from solid to liquid or to gas.

Inspection of the table of conductivities reveals some most interesting and valuable relations—thus, silver is about at the head of the list as a conductor and carbon dioxide at the foot with an enormous difference between them, silver being able to conduct about 35,000 times as much heat as carbon dioxide for equal temperature differences. In fact, the substances group themselves naturally into good and bad conductors, all the gases being in the latter class as well as many solids, like felt and wool. These solids are the natural heat insulators and their insulating value, seemingly dependent on their porosity, at least to some degree, has led to the theory that the gases confined between fibers or granules of solids are strong contributors to the insulating values of otherwise solid bodies. Liquids fall into an intermediate class, being universally better conductors than gases, not so good as the metals, which as a class are the best conductors, but of the same order as the group of solids generally considered as insulators.

As the relative conductivity of solids, liquids and gases, or their reciprocals—the resistances to heat, are most useful in explaining or even predicting the relative characteristics of the common heat transfer paths in commercial apparatus, it is important that a table be developed from existing data. These data are of two sorts, first, the absolute internal conductivities of the table; and second, direct experimental measurement of relative conducting power, which is determined by different methods than the absolute value. These relative conductivities are similarly useful in selecting from conflicting values, like those for iron, that which is most probable, though, of course, commercial irons being alloys merging into the steels must have small differences depending on composition, but so small as to be negligible for most engineering purposes.

It is extremely likely that the conductivity of gases does not follow the linear law with respect to temperature that seems satisfactory for solids, and it is from Clark Maxwell that the best suggestion for a substitute has come. He predicted from the kinetic theory of gases that the conductivity is proportional to the product of the coefficient of gas viscosity and specific heat at constant volume or $K = \mu C_v \times (\text{a constant})$. Dalby from a study of values of μ finds it can be put proportional to the three-quarter power of the absolute temperature T , so that $K = cC_v (T)^{3/4}$, or $\frac{K_T}{K_{93}} = \left(\frac{T^\circ \text{ F.}}{492} \right)^{3/4}$, approximately if spe-

cific heats are assumed constant. This more than doubles the conductivity value for 32° F., at the temperature 930° F., which is an important addition because the gases of furnaces are very hot indeed while the tubes, plates and water are comparatively cool. See Table LIII in the Handbook of Tables.

Conductivity beyond the boundaries of a body is known to suffer a rapid decrease, or heat flow to encounter a resistance through joints in metal bars or between plates at point of contact. Boiler plates offer appreciable resistance and boiler seams are forbidden in contact with fires by certain inspection laws, because experience shows that the plates frequently burn, which they would not do were there not a large joint resistance. It is also known that several layers of thin boards make better insulation than equal thickness of single boards for refrigerated boxes and rooms. Whether this joint resistance is due to a thin layer of poor conducting air or is a strictly separate phenomenon is not known, but most carefully made accurately fitted joints in experimental bars in which there could not be over one ten-thousandth, of an inch air layer, gave measurable joint resistance, smaller, of course, than a bad joint. It is probably true that there is a real joint or surface resistance of unknown character at the boundary of every body, and likewise a fluid film of some thickness as well, though probably not uniform. When heat passing from fluid to plate or plate to fluid, encounters a film of fluid adhering to the plate and for all practical purposes sticking to it, the heat resistance is materially increased. Inspection of the table indicates that, taking the relative conductivity of iron at about 6000, a layer of carbon dioxide of $\frac{1}{6000}$ of an inch thick will offer as much resistance to the passage of heat as a one-inch thickness of metal, and taking water

as 40, relative conductivity, a layer of water $\frac{40}{8000} = \frac{1}{150}$ of an inch thick would offer the same resistance, both in addition to the separate joint resistance.

The existence of such heat-resisting films of gas and liquid in heat transfer plates is conclusively proved, so that the problem of *estimating heat flow from a body on the other side involves an estimate of the thickness of the adhering fluid film, which is a very indefinite quantity*. In addition to the film and joint resistance difficulty, there is another when dealing with boiler, condenser, feed-water heater, economizer, superheater and similar tubes and plate surfaces, and that is the scale, grease, soot, rust or dirt layer that is always present in some condition of density and thickness, and also highly resistant as belonging in the heat insulator group. Furthermore, such a layer on both sides involves another joint resistance, so that the practical everyday problem of deciding on how much surface to allow for a given heat flow or how much heat can flow past a given surface, is not one that can be solved by the laws of conduction even if data on all substances were available; such problems probably never will be solved because they require first, an hypothesis fixing the thickness of fluid film, the joint resistance and the kind and thickness of fouling or layer of dirt. Problems of this character are to be solved in another way by means of coefficients of transmission found by experiment and applied to the same kind of apparatus as that on which the experiment was made, and covering all resistances together, though each cannot be separated from the rest.

There is no doubt now as to the existence and great importance of the surface films, especially on the gas side of plates receiving heat from or delivering it to gases, or involving the presence of gases such as deposit on steam condenser tubes, or intermittent layers of vapor on boiler surfaces when ebullition is taking place; nor can there be longer any doubt about the *reduction of film thickness and resistance by fluid agitation*. This agitation generally is the result of velocity increase of the fluid over the surface, but such data as exist on this effect do not permit a determination of actual film thickness or resistance, but rather of overall coefficients of transmission, to be considered later.

If a series of resistances to heat flow as reciprocals of conductivity are known, the whole heat flow temperature relations, can be set down algebraically; the real practical difficulty arises from their numerical evaluation.

Let l_1, l_2 , etc. = thickness of each portion of plate or film along the path of heat flow, in inches;

K_1, K_2 , etc. = mean or constant coefficients of conductivity of each material;

ρ_1, ρ_2 , etc. = $\frac{1}{K_1}, \frac{1}{K_2}$, etc., thermal resistances of each material;

σ_1, σ_2 , etc. = surface resistances;

t_h and t_l = high and low temperatures at the limits of the path.

Then
$$Q = \frac{A(t_h - t_l)}{\sum \rho l + \sum \sigma} \text{ (a); } = AU(t_h - t_l) \text{ (b). (597)}$$

In (b) a general overall coefficient U has been introduced inclusive of all internal and boundary resistances for the whole complex heat flow path.

In this case U = B.T.U. per hour per square foot, per degree difference of temperature between any two points, usually in practical problems taken in two separated fluids, one receiving heat from the other or

$$U = \frac{1}{\Sigma \rho l + \Sigma \sigma} \quad \dots \dots \dots (598)$$

Direct evaluation of this coefficient of transmission U is the usual practical procedure, inclusive of all separate conductivities or thermal resistances, together with radiation and convection influences without separately evaluating each.

Heat flow by radiation alone has been experimentally studied and reduced to laws by a number of skillful physicists, and while they do not agree, the so-called Stefan and Boltzmann law is now quite generally accepted. According to this law heat will be radiated from an incandescent "black" body as follows:

$$\text{B.T.U. per hour radiated per square foot} = 16 \times 10^{-10} (T_1^4 - T_2^4), \quad (599)$$

where T_1 = absolute temperature F of the "black" radiating body,

and T_2 = absolute temperature F of the receiving body.

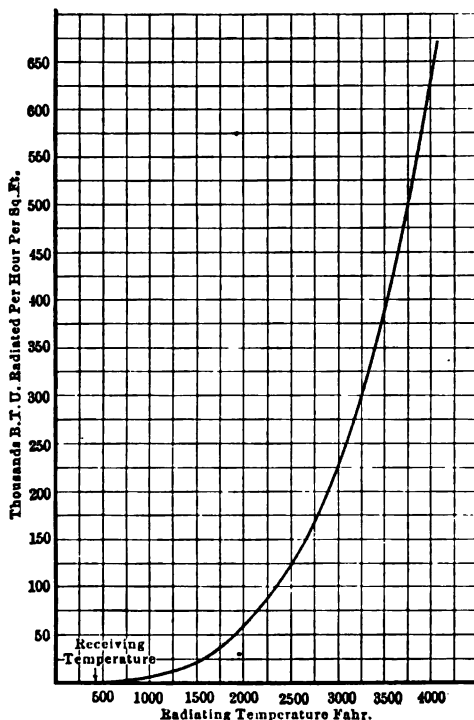


FIG. 110.—Heat Radiation from "Black Body," according to Stefan and Boltzmann Law.

A *black body* is defined as one capable of absorbing all heat rays received, neither reflecting nor transmitting any, and for practical purposes may be assumed to be any porous carbon, that is all carbon except the crystalline. Bodies not "black," like hot gases or fire brick or boiler plates, do reflect some heat received, and fail to radiate to the same extent as a black body does. For them absorption and radiation increase with darkness (when cold) and roughness, which is one reason for nickel plating and polishing surfaces from which radiation is to be resisted.

Numerical values on relative radiation or absorption and reflecting power for various substances are quoted in the Tables (No. XLIX), but these figures are probably wrong; they are given in the absence of any better, which even if available would be of little real use.

For the black body itself the preceding curve, Fig. 110, is calculated from the Stefan and Boltzmann law for various temperatures of the radiator up to 5000° absolute F. to a heat absorber at 500, 600, 700, 800, 900 and 1000° absolute. The very great quantities of heat that may thus pass, independent entirely of conduction, convection and mechanical carrying by gases and equally independent of dead gas films from an incandescent coal fire to boiler plates or furnace interiors is at first surprising. It must not be assumed, however, that all this will be taken up directly by the plates of a firebox, for example, as they will reflect possibly a quarter to a half of what is so received, sending it out again to other walls, the reflection being especially severe when the heat ray strikes at a small angle, when nearly all the heat will be immediately reflected. However, these quantities are significant of the great steam-forming capacity of enclosed fireboxes common to locomotives and to the Scotch marine boiler. Experiments all prove the firebox capacity for evaporating, greater than all the rest of the boiler as ordinarily made.

Example. The wall of a house consists of 6 ins. of brickwork lined with 1 in. of plaster. The junction of the plaster to the brick being equivalent to 10 ins. of plaster. The windows are of glass $\frac{1}{8}$ -in. thick and have a film of moisture on them .003 in. thick, the junction of the water and glass being equivalent to 3.5 ins. of glass. How does the coefficient of heat transfer for the walls compare with that of the windows?

$$\text{From Eq. (598), } U = \frac{1}{\Sigma \rho l + \Sigma \sigma} = \frac{1}{\frac{1}{8} \times 6 + \frac{1}{2} \times 10 + 1} = 0.077 \text{ for the wall.}$$

For the window, $K=5$ for the glass, and $K=3.5$ for the water.

$$\text{Hence } U = \frac{1}{\frac{1}{5} \left(\frac{1}{8} + 3.5 \right) + \frac{1}{3.5} \times .003} = 1.38.$$

Prob. 1. A bar of copper and one of glass are heated by the same source of heat. How much longer will it take for the end of the glass bar furthest from the source of heat to reach a uniform temperature than it will that of copper?

Prob. 2. Neglecting joint effects, how much will the heat lost from a pipe $\frac{1}{2}$ in. thick carrying steam at 500° F. and running through a room at 50° F., be reduced by the application of a plaster of Paris coating 2 ins. thick?

Prob. 3. A standard 2-in. boiler tube has a thickness of .095 in. On the inside of the tube is a layer of scale $\frac{1}{16}$ in. thick which may be considered as plaster of Paris; on the outside of the tube is $\frac{1}{8}$ -in. coating of soot having the same conductivity as wool. Neglecting the resistance due to the junction of these substances, what will be the difference in heat transmitted through this tube as compared with a new clean one?

Prob. 4. The walls of a room are as follows: 6 ins. concrete, 6 ins. sawdust, 3 ins. concrete. What will be the heat lost per square foot per hour for an inside temperature of 40° F. and an outside temperature of 90° F.?

Prob. 5. The copper tubes of a condenser are .05 in. in thickness. On each side is a water film .01 in. thick. How will the heat transferred through this tube compare with that of a dry one in contact with air?

Prob. 6. A tin tube .05 in. thick is silver plated, the plating being .01 in. thick

and the joint resistance being equal to 1 in. of silver. How does the plating affect the value of U ?

Prob. 7. Considering steel to have a radiating effect equal to .2 of carbon, what will be the value of the radiant heat of a steel sphere having 10 sq.ft. of surface at a temperature of 1500°F . when the receiving body is at 50°F .?

Prob. 8. A boiler has 1700 sq.ft. of heating surface of which 100 are exposed to direct radiant heat. The remainder is heated by hot gases of average temperature of 1000°F . The temperature of the radiating fire is 3000°F . and the temperature of the water in the boiler is 350°F . If the surface not subject to radiation consists of iron tubes .1 in. thick with .1-in. scale and the junction of scale to plate is equal to an inch of iron, what is the relative importance of each part of the boiler?

78. Heat Transmission between Separated Fluids. Mean Temperature Differences, Coefficients of Transmission. Heat flow, in terms of B.T.U. per hour transmitted, is usually taken to be proportional to the temperature difference when one fluid is giving up heat to another separated by plates or tubes, as for internal conduction, though experimental determinations have indicated the existence of other relations. For example, Grashof, Rankine and others have announced the square of the mean temperature difference between the two fluids as the correct function, while Orrok, recently experimenting with surface condensers, reports a seven-eighths power instead of the first power. The temperatures that are measurable in engineering work are those in the body of fluid, gas or water entering or leaving a coil or pipe, or the temperature in a condenser or boiling mass at some selected points. It is not possible to determine outside of the laboratory the skin temperature of a boiler tube, the mean temperature of a kettle shell, or even the temperature of the fluid next the cooling or heating surface. Exact scientific analysis of heat flow laws would involve those and many other quantities, hence the practical necessity for approximate calculations and the absence of any generally inclusive law or equation. Each class or case must be studied by itself though in the light of relations to others.

Mathematical analysis and experimental observations both in the laboratory on special apparatus, and in the field on standard working equipment, has led to the following generalizations beside the proportionality of flow to some power of the mean temperature difference between the giving and receiving fluid. *The quantity of heat transmitted is found to be proportional to the velocity of the fluid to some power when one of the fluids is in motion; and to some power of each, not necessarily the same, when both move.* Thus, for the case of water in the tubes of surface condensers and feed-water heaters, the one-third power has been used by Joule and Ser, the one-half power by Hageman, Josse and Orrok, and the first power by Stanton. The heat flow has been related to the steam velocity at the condensing surface by Hausbrand and Ser as a function of its one-half power, but found to be independent for such conditions as exist in surface condensers and feed-water heaters by Orrok, which corresponds to the zero power. *Air or other gas in motion giving up heat does so at a rate proportional, not to velocity according to Jordan and Reynolds, but*

to the weight passing per unit of cross-section of gas stream directly, a conclusion that may reconcile the discrepancies noted previously where the velocities used by different experimenters might be the same, whereas temperature, density and channel cross-section were in all cases different. When steam condenses, the heat flow is strongly controlled by the collection of gaseous matter on the surfaces, and data by Smith and others show that in ordinary apparatus this may affect the heat flow as much as 50 per cent.

For the purposes of calculation the cases are grouped into types, for example, surface condensers and feed-water heaters are typical of heat transmission from condensing steam to water entering at a low and leaving at a higher temperature, but still lower than that of the steam. If the water temperatures are fixed and the amount of heat to be exchanged, likewise, the problem becomes one of finding the necessary surface. On the other hand the problem may be set to find the rise of water-temperature for a given amount of surface.

A different situation exists, for example, in steam superheaters in which steam temperature rises by heat received from hot gases, the temperature of which is falling. In this case, given the surface and the initial temperatures of the gases and the steam, the final temperature of both may be required for a given amount of surface; or inversely the object sought may be the surface necessary to accomplish a given rise of steam or given fall of gas temperature.

In all these practical cases of transfer, what is sought is the relation between amount of surface and the corresponding heat flow or its equivalent in quantity of substance heated or cooled, vapors condensed or liquids evaporated. The temperatures of the substance are not always uniform throughout the mass or over the heating surface and ordinarily change as the substance passes over the surface. Therefore, while the quantity of heat passing is proportional to some power of the temperature difference at any instant, the instantaneous temperature differences being indeterminate, it is necessary to establish a *mean* temperature difference for the whole period of thermal contact of the substance in question.

In addition to the fixing of a *mean* temperature difference for each typical kind of fluid flow to which heat flow is related, it is also necessary in the solution of numerical problems to have the *constant of proportionality* for each class apparatus or typical transfer. This constant of proportionality is known as the coefficient of heat transmission and is defined as the B.T.U. per hour, transmitted through one square foot of surface, or cross-section of heat flow, path, per degree F. difference in temperature between the two substances measured in the body of each and not at the surfaces of contact. Accordingly,

Let Q = B.T.U. per hour transmitted;

U = coefficient of heat transmission = B.T.U. per hour per square foot per degree F. measured in the substances;

t_m = mean temperature difference for whole period of transfer;

A = square feet of surface through which transfer is taking place or the cross-sectional area of the heat flow path in square feet.

Then

$$Q = AUt_m. \quad . \quad . \quad . \quad . \quad . \quad . \quad (600)$$

There is a definite relation between the mean temperature difference and the initial and final temperatures of the fluids exchanging heat. In establishing this relation five different cases must be recognized, as follows:

- CASE I. The substance giving up heat may be at a constant temperature and the substance receiving heat suffer a rise of temperature.
- CASE II. The substance giving up heat may suffer a drop in temperature and the substance receiving heat remain at a constant temperature.
- CASE III. Both substances may remain at a constant temperature.
- CASE IV. Both substances may change temperature, that giving up heat falling and that taking the heat rising by *parallel flow*, the final temperatures of both tending to become equal.
- CASE V. Both substances may change temperature by *counter current flow*, i.e., in opposite directions, the *final* temperature of one substance tending to become equal to the *initial* temperature of the other.

For each of these cases there is a different relation between the initial and final temperatures of both substances and the mean temperature difference, and these algebraic relations will be derived after a further examination of the typical cases to show how these five are inclusive of all ordinary conditions. Constant temperatures characterize the two cases of condensing vapors and evaporating liquids, though not strictly so, because it is well known that in a boiler or condenser the water on the vapor side is not quite constant in temperature, but for the purposes of such calculations as these it must be taken so, or no calculation at all would be possible. Substances changing temperature regularly and indefinitely may be liquid or gaseous. With these distinctions the following classification of heat transfer cases can be made with reference (a) to the kind of thermal change taking place in the substances; (b) type of relation between initial, final temperatures and mean temperature difference; and (c) specific examples of apparatus in which each of the type actions takes place which, therefore, are grouped as similar or dissimilar.

It might be noted in the case of ice cans submerged in brine that before freezing begins the case is one of heat of water cooling to brine warming, and after freezing begins, constant temperature latent heat of freezing to brine warming, but as the brine is not allowed to rise more than a few degrees the process is nearly one of constant temperature on both sides and is not listed above because it is a peculiar case without parallel. A similar complexity exists with respect to reheating receivers of multiple expansion engines in which the heat of condensing high pressure steam is added to steam passing between cylinders, and where if the working steam is wet it is first dried at constant temperature and afterward warmed or superheated to a temperature approaching that of the live steam. Still other cases of possible complexity are the cooling tower and evaporative condensers in which more than one action may take place and in which the specific construction exerts a strong or controlling influence.

Thermal Action in Substances.		Class of Relation between temperatures and mean temperature differences.	Examples of Standard Thermal Apparatus.
Giving up Heat.	Receiving Heat.		
Liquid Cooling	Liquid warming	Case IV or V	Aqua ammonia exchanger. Water coils of ammonia absorber. Brine water cooler.
	Gas warming	Case IV or V	Hot-water house-heating radiator; cooling tower. Automobile radiator.
	Liquid boiling	Case II	Ammonia and carbon dioxide brine cooler. Evaporator with hot liquid coils or jacket.
Gas cooling	Liquid warming	Case IV or V	Brine coils in cold storage rooms. Economizers. Cylinder jackets. Forecooler ammonia condenser. Brine coil air dryer or cooler. Compressor intercooler.
	Gas warming	Case IV or V	Dense air machine coils in cold storage rooms. Steam superheaters. Compressed air engine preheater.
	Liquid boiling	Case II	Steam boiler. Ammonia or carbon dioxide direct expansion coils in cold storage rooms.
Vapor condensing	Liquid warming	Case I	Exhaust steam boiler feed-water heater. Steam, ammonia and carbon dioxide condenser.
	Gas warming	Case I	Steam radiator and tempering coils. Steam piping radiation. Multiple expansion engine reheating receiver. Dry air condenser.
	Liquid boiling	Case III	Exhaust steam vac. evaporator. Multiple effect evaporator. Coil and jacketed evap. kettle. Aqua ammonia generator. Evaporative condenser.

It is structure entirely that determines whether Case IV for parallel flow or Case V for counter-current flow shall apply to the transfer between liquids and gases, one to the other in which both substances suffer temperature change. In some constructions it is quite impossible to fix the flow relations, and this is in general true when a large mass of substance is under treatment, as in a tank, instead of in coils or pipes, or flowing between guiding partitions, and in these cases it is necessary to assume the nearest typical case as representative. Derivations of mean differences will first be based on *proportionality of heat transmitted to the first power of the temperature difference*.

Case I. Mean temperature difference, for constant high temperature source

to rising temperature cold substance. In Fig. 111 the temperature of the two substances is plotted vertically, and the surfaces over which the flow causes temperature change, horizontally for the conditions specified as Case I.

Let t_h = constant temperature of the hot substance degrees F.;

t_c = any momentary temperature of the cold substance degrees F.;

t_{c1} = initial temperature of the cold substance degrees F.;

t_{c2} = final temperature of the cold substance degrees F.;

w_c = pounds of cold substance flowing per hour over surface.

C_c = specific heat of cold substance;

A = surface in square feet;

t_m = mean temperature difference between hot and cold substance;

U = coefficient of heat transfer = B.T.U. per hour per square foot per degree F.

Then will the heat transmitted per hour through the elementary surface dA be given by $U(t_h - t_c)dA$, and as the temperature rise for this period will be dt_c , the heat transmitted per hour is also given by $C_c w_c dt_c$ which is the amount taken up by the cold substance. These two quantities must be equal, whence

$$dA = \frac{C_c w_c}{U} \frac{dt_c}{t_h - t_c}.$$

If, as is here assumed to be the case, the hot substance be at a constant temperature, t_h is independent of t_c and the coefficient of heat transfer U like-

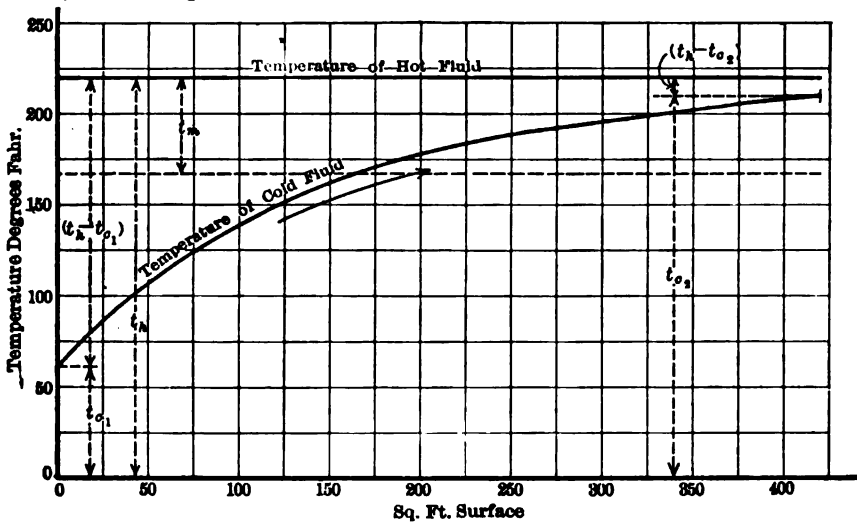


FIG. 111.—Heat Transfer from Constant Temperature Source to Rising Temperature Fluid, Case I.

wise, then will the relation between temperature rise of the cold substance and the surface traversed be given by,

$$A = \frac{C_c w_c}{U} \int_{t_{c1}}^{t_{c2}} \frac{dt_c}{(t_h - t_c)} = \frac{C_c w_c}{U} \log_e \left(\frac{t_h - t_{c1}}{t_h - t_{c2}} \right). \quad \dots (601)$$

For the whole period of transfer if U is independent of the temperature difference

$$t_m UA = C_c w_c (t_{c2} - t_{c1}), \quad \text{or} \quad t_m = \frac{C_c w_c}{UA} (t_{c2} - t_{c1}). \quad (602)$$

Substitution in Eq. (602) of the value of UA from Eq. (601) in terms of temperature differences gives the mean temperature difference,

$$t_m = \frac{(t_{c2} - t_{c1})}{\log_e \left(\frac{t_h - t_{c1}}{t_h - t_{c2}} \right)}, \quad (603)$$

This value is indicated by the distance from the high temperature line t_h to the dotted line on the diagram Fig. 111.

Case II. A constant temperature of the cold substance T_c with a falling temperature of the hot one constitutes the second case, represented in Fig. 112.

With the use of similar symbols and as indicated on the diagram,

$$dA = \frac{C_h w_h}{U} \frac{dt_h}{(t_h - t_c)}, \quad A = \frac{C_h w_h}{U} \int_{t_{h2}}^{t_{h1}} \frac{dt_h}{(t_h - t_c)}, \quad (604)$$

$$t_m UA = C_h w_h (t_{h1} - t_{h2}), \quad t_m = \frac{(t_{h1} - t_{h2})}{\log_e \left(\frac{t_{h1} - t_c}{t_{h2} - t_c} \right)}. \quad (605)$$

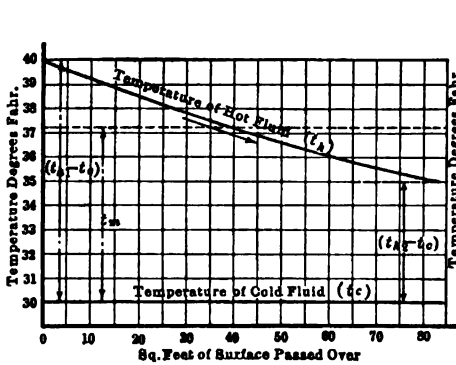


FIG. 112.—Heat Transfer from a Falling Temperature Source to a Constant Low Temperature Fluid, Case II.

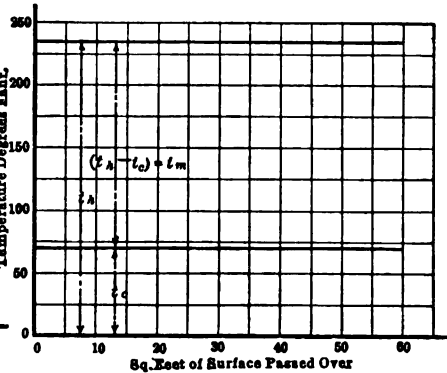


FIG. 113.—Heat Transfer between Two Fluids Each at a Constant Temperature, Case III.

Case III. When both temperatures are constant the relations shown in Fig. 113 are most simple since the mean temperature difference is constant and the heat transmitted per hour is directly proportional to the surface in action.

$$t_m = t_h - t_c. \quad (606)$$

Case IV. Parallel flow conditions of two substances both changing temperature are indicated in Fig. 114 which, therefore, represents diagrammatically the fourth case. After passage over the elementary surface dA , the cold substance will rise in temperature an amount dt_c and the temperature of hot

substance fall correspondingly dt_h ; therefore, the heat transmitted per hour is given by $U(t_h - t_c)dA = C_c w_c dt_c = C_h w_h dt_h$. It appears, therefore, that the relation between surface and temperature change can be given two forms, as follows,

$$A = \frac{C_c w_c}{U} \int_{t_{c1}}^{t_{c2}} \frac{dt_c}{(t_h - t_c)} = \frac{C_h w_h}{U} \int_{t_{h1}}^{t_{h2}} \frac{dt_h}{(t_h - t_c)}.$$

These are really identical because t_h is a function of t_c at any moment since the heat gained by the cold body must be equal to that lost by the hot one, which fact fixes the relation between temperatures.

Hence after any interval of time or surface traversed

$$C_h w_h (t_{h1} - t_h) = C_c w_c (t_c - t_{c1}). \quad \text{Therefore} \quad t_h = t_{h1} + \frac{C_c w_c}{C_h w_h} (t_{c1} - t_c).$$

$$\text{Subtracting } t_c \text{ from each side, } t_h - t_c = t_{h1} + \frac{C_c w_c}{C_h w_h} t_{c1} - \left(1 + \frac{C_c w_c}{C_h w_h}\right) t_c.$$

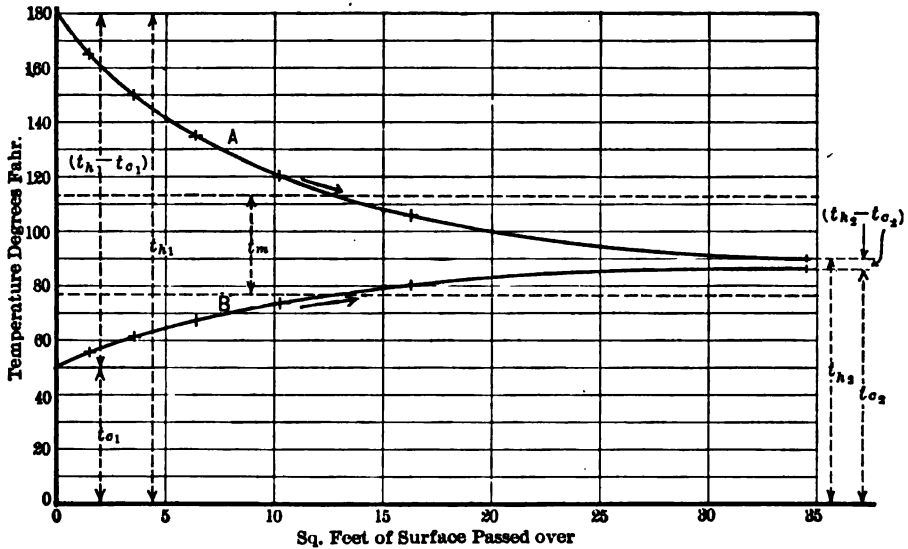


FIG. 114.—Heat Transfer between Fluids, Each Changing Temperature, Parallel Flow, Case IV.

Substituting this in the differential equation where the variable is t_c , gives

$$\begin{aligned} A &= \frac{C_c w_c}{U} \int_{t_{c1}}^{t_{c2}} \frac{dt_c}{\left[t_{h1} + \frac{C_c w_c}{C_h w_h} t_{c1} - \left(1 + \frac{C_c w_c}{C_h w_h}\right) t_c \right]} \\ &= \left[\frac{C_c w_c}{U \left(1 + \frac{C_c w_c}{C_h w_h}\right)} \right] \log_e \left[\frac{t_{h1} + \frac{C_c w_c}{C_h w_h} t_{c1} - \left(1 + \frac{C_c w_c}{C_h w_h}\right) t_{c1}}{t_{h1} + \frac{C_c w_c}{C_h w_h} t_{c1} - \left(1 + \frac{C_c w_c}{C_h w_h}\right) t_{c2}} \right], \end{aligned}$$

Therefore
$$A = \left[\frac{C_c w_c}{U \left(1 + \frac{C_c w_c}{C_h w_h} \right)} \right] \log_e \left(\frac{t_{h1} - t_{c1}}{t_{h2} - t_{c2}} \right), \quad \dots \dots \dots (607)$$

This can be put in a slightly different form by the relation

$$1 + \frac{C_c w_c}{C_h w_h} = 1 + \frac{t_{h1} - t_{h2}}{t_{c2} - t_{c1}} = \frac{(t_{c2} - t_{c1}) + (t_{h1} - t_{h2})}{t_{c2} - t_{c1}},$$

which may be inserted in Eq. (607) when the problem indicates a necessity for it.

For the whole transfer, $t_m UA = C_c w_c (t_{c2} - t_{c1}) = C_h w_h (t_{h1} - t_{h2})$,

whence
$$t_m = \frac{C_c w_c (t_{c2} - t_{c1})}{UA} = \frac{C_c w_c (t_{c2} - t_{c1})}{\left[\frac{C_c w_c}{\frac{(t_{c2} - t_{c1}) + (t_{h1} - t_{h2})}{(t_{c2} - t_{c1})}} \right] \log_e \left(\frac{t_{h1} - t_{c1}}{t_{h2} - t_{c2}} \right)}.$$

Therefore
$$t_m = \frac{(t_{c2} - t_{c1}) + (t_{h1} - t_{h2})}{\log_e \left(\frac{t_{h1} - t_{c1}}{t_{h2} - t_{c2}} \right)} = \frac{(t_{h1} - t_{c1}) - (t_{h2} - t_{c2})}{\log_e \left(\frac{t_{h1} - t_{c1}}{t_{h2} - t_{c2}} \right)}. \quad \dots \dots \dots (608)$$

Case V. Counter-current flow conditions for this case are indicated on Fig. 115, which shows a rising temperature of the cold body to a value approaching that of the initial temperature of the hot one, instead of its final temperature, as in the case of parallel flow. Beginning with the entrance of the cold body into the system its temperature will have risen dt_c degrees from t_{c1} , after passage over the elementary surface dA , and in the same time the hot body will have fallen dt_h degrees to the value t_{h2} . Therefore, as for parallel flow

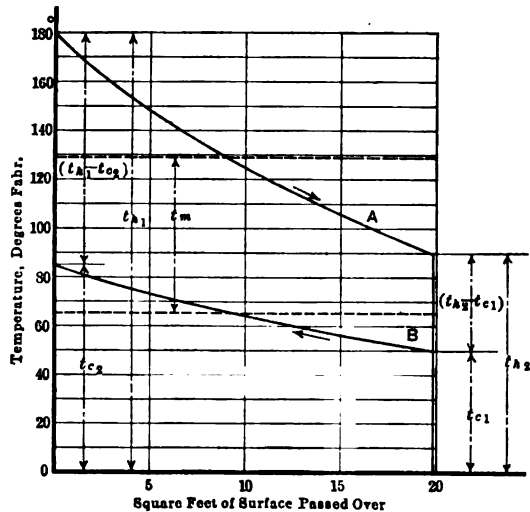


FIG. 115.—Heat Transfer between Fluids, Each Changing Temperature, Counter-current Flow, Case V.

$$\begin{aligned} A &= \frac{C_c w_c}{U} \int_{t_{c1}}^{t_{c2}} \frac{dt_c}{(t_h - t_c)} \\ &= \frac{C_h w_h}{U} \int_{t_{h2}}^{t_{h1}} \frac{dt_h}{(t_h - t_c)}. \end{aligned}$$

Elimination of one variable t_h by introducing its relations to t_c is effected in a similar manner but by terms differently involved. After any appreciable interval $C_h w_h (t_h - t_{h2}) = C_c w_c (t_c - t_{c1})$. Therefore

$$t_h = t_{h2} + \frac{C_c w_c}{C_h w_h} (t_c - t_{c1}), \quad \text{and} \quad t_h - t_c = t_{h2} - \frac{C_c w_c}{C_h w_h} t_{c1} + \left(\frac{C_c w_c}{C_h w_h} - 1 \right) t_c.$$

Substitution in the differential equation where the variable is t_c gives, after reduction,

$$A = \left[\frac{C_c w_c}{U \left(\frac{C_c w_c}{C_h w_h} - 1 \right)} \right] \log_e \left(\frac{t_{h1} - t_{c2}}{t_{h2} - t_{c1}} \right). \quad (609)$$

The weight, specific heat ratio can be eliminated by the relation between it and temperatures which is given by

$$\frac{C_c w_c}{C_h w_h} - 1 = \frac{t_{h1} - t_{h2}}{t_{c2} - t_{c1}} - 1 = \frac{(t_{h1} - t_{h2}) - (t_{c2} - t_{c1})}{t_{c2} - t_{c1}}. \quad \text{For the whole transfer}$$

$$t_m U A = C_c w_c (t_{c2} - t_{c1}) = C_h w_h (t_{h1} - t_{h2}), \quad \text{and} \quad t_m = \frac{(t_{h1} - t_{c2}) - (t_{h2} - t_{c1})}{\log_e \left(\frac{t_{h1} - t_{c2}}{t_{h2} - t_{c1}} \right)}. \quad (610)$$

All of these equations for mean temperature difference are identical when properly interpreted, however different they may look in terms of the specific symbols. Inspection will show that the numerator is in every case the initial temperature difference less the final temperature difference, while the denominator is only the hyperbolic logarithm of the ratio of initial to final temperature differences.

$$(\text{Mean temp. diff.}) = \frac{(\text{Initial temp. diff.}) - (\text{Final temp. diff.})}{\log_e \left(\frac{\text{Initial temp. difference}}{\text{Final temp. difference}} \right)}. \quad (611)$$

By the adoption of coefficients of transfer U , or rather numerical values for U , for the common cases of practice, it becomes possible to calculate relation between quantities of substance heated, or quantity of heat transmitted, and surface; but only approximately. The coefficients used in most engineering work allow a good safe margin in the form of sufficient surface so the apparatus will always be big enough to perform the required service, but economy in the use of material demands care in design to bring to bear as large a coefficient as possible by whatever means are available; these are confined to the use of high velocities of fluid or equivalent means of fluid film reduction, and cleanliness. In the Tables are given some average values of U that have been in common use in designing apparatus. These are presented not as accurately determined values but as average values illustrating the most important fact that in all cases where gases are involved the coefficient is much less than when they are not present. See Table L, Handbook of Tables.

Example 1. Case I, Fig. 111. A feed-water heater, with exhaust steam at 220°F. and a water supply at 60°F. $U = 200$, specific heat = 1, water 30,000 lbs. per hour, which in round numbers corresponds roughly to the feed of a 1000 H.P. boiler. Compute the surface to give the feed water a final temperature of 90° , 120° , 150° , 180° and 210° , also find t_m .

$$A = \frac{C_c w_c}{U} \log_e \left(\frac{t_h - t_{c1}}{t_h - t_{c2}} \right). \quad \text{For } t_c = 90^\circ \text{ F., } A = \frac{30000}{200} \log_e \left(\frac{220 - 60}{220 - 90} \right) \text{ sq.ft.}$$

$$= 150 \log_e \frac{160}{130} = 150 \log_e 1.23 = 150 \times .207 = 31.05 \text{ sq.ft.}$$

In the same way for $t_{c1}=120^{\circ}\text{ F.}$, 150° F. , 180° F. and 210° F. , A will have the following values: 70.5; 124.95; 207.9; 415.5 sq.ft.

$$t_m = \frac{t_{c2} - t_{c1}}{\log_e \left(\frac{t_h - t_{c1}}{t_h - t_{c2}} \right)} = \frac{210 - 60}{\log_e \left(\frac{160}{10} \right)} = \frac{150}{2.77} = 54^{\circ} \text{ mean temperature difference.}$$

Example 2. Case II, Fig. 112. An ammonia brine cooler where the evaporation takes place at 30° F. and the brine enters the coils at 40° and leaves at 35° F. , a total fall in temperature of 5° F. Take the specific heat of the brine at .68 and assume 17,700 lbs. per hour, which about corresponds to one ton of ice per 24 hours. Take $U=100$. Find the surface corresponding to 39° F. , 38° , 37° , 36° and 35° F.

$$A = \frac{C_h w_h}{U} \log_e \frac{(t_{h1} - t_c)}{(t_{h2} - t_c)} = \frac{.68 \times 17700}{100} \log_e \frac{(40 - 30)}{(t_{h2} - 30)} = 120 \log_e \frac{10}{(t_{h2} - 30)}.$$

$$\text{For } t_{h2}=39^{\circ}\text{ F.}, A = 120 \log_e \frac{10}{9} = 12.6 \text{ sq.ft.}$$

In the same way for $t_{h2}=38^{\circ}$, 37° , 36° and 35° F. , the corresponding surfaces, A , in sq.ft. will be 26.8; 42.9; 61.4; and 83.2.

$$t_m = \frac{t_{h1} - t_{h2}}{\log_e \left(\frac{t_{h1} - t_c}{t_{h2} - t_c} \right)} = \frac{40 - 35}{\log_e \left(\frac{10}{5} \right)} = 7.21^{\circ} \text{ mean temperature difference.}$$

Example 3. Case III, Fig 113. Assume that exhaust steam at a temperature of 235° F. is used to supply heat to an evaporator in which alcohol is being evaporated at a temperature of 70° F. So long as the pressures and the quality of the alcohol remain constant, these temperatures will remain the same. What will be the surface required if 3,600,000 B.T.U. are to be supplied to the evaporating fluid per hour? Assume $U=400$.

$$(t_h - t_c) = t_m = 235 - 70 = 165^{\circ} \text{ temperature difference. } \therefore \text{ Heat transfer per sq.ft.}$$

$$= 165 \times 400 = 60,000 \text{ B.T.U. per hour. Hence required surface} = \frac{3600000}{60000} = 60 \text{ sq.ft.}$$

Example 4. Case IV, Fig 114. Water leaving a still at a temperature of 180° F. passes through a cooler before entering ice tanks in which it is to be made into ice. Let this cooler be assumed to have a parallel flow of distilled water and of cooling water. The cooling water enters at a temperature of 50° F. The cooler is to be capable of cooling 10 tons of water per day from a temperature of 180° F. to 90° F. using for cooling $2\frac{1}{2}$ lbs. of cooling water per pound of distilled water. Take $\bar{U}=60$.

$$w_h = 834 \text{ lbs. per hour; } w_c = 2080 \text{ lbs.; } C_h = 1; C_c = 1.$$

Find the required cooling surface for each 15° fall of temperature of the hot water. Heat necessary to cool 20,000 lbs. through 15° will heat 50,000 lbs. of water $\frac{2}{5} \times 15^{\circ} = 6^{\circ}$, which is accordingly the corresponding increment for t_c as shown in tabular form:

Substitution in Eq. (607) gives the following expression for area corresponding to a given temperature difference, $t_{h1} - t_{c1}$,

t_h	t_c	$t_h - t_c$
180°	50°	130°
165	56	109
150	62	88
135	68	67
120	74	46
105	80	25
90	86	4

$$A = \left[\frac{1 \times 2080}{60 \left(1 + \frac{1 \times 2080}{1 \times 834} \right)} \right] \log_e \left(\frac{180 - 50}{t_h - t_c} \right) = 9.9 \times \log_e \left(\frac{130}{t_h - t_c} \right).$$

Substituting in this formula there are obtained the following values of A corresponding to the values t_h as noted: $t_h = 165^\circ \text{ F.}$, $A = 1.74$; for 150° , $A = 3.88$; for 135° , $A = 6.56$; for 120° , $A = 10.30$; for 105° , $A = 16.32$ and for 90° , $A = 34.5 \text{ sq.ft.}$

The mean temperature difference is

$$t_m = \frac{(t_{h1} - t_{c1}) - (t_{h2} - t_{c2})}{\log_e \left(\frac{t_{h1} - t_{c1}}{t_{h2} - t_{c2}} \right)} \frac{130 - 4}{\log_e \left(\frac{130}{4} \right)} = \frac{126}{3.48} = 36 \text{ degrees.}$$

Example 5. Case V, Fig. 115. Assume a problem similar to that for Case IV except that the flow is counter-current instead of parallel, i.e., into a counter-current cooler flows 834 lbs. per hour of distilled water at a temperature of 180° F. , which is to be cooled to 90° upon leaving. Cooling water is supplied at 50° F. at the rate of 2080 lbs. per hour. How much cooling surface is required to cool by intervals of 15° ? What is the mean temperature difference when cooling to 90° ?

$C_h = 1$; $C_c = 1$; $w_h = 834$; $w_c = 2080$. Assume $U = 60$; $t_{c1} = 50^\circ$. Since for any given interval, with counter flow $C_h w_h (t_{h1} - t_{h2}) = C_c w_c (t_{c2} - t_{c1})$,

$$t_2 = t_1 + (t_{h1} - t_{h2}) \frac{C_h w_h}{w_c C_c} = 50 + (180 - 90) \frac{834 \times 1}{2080 \times 1} = 86^\circ \text{ F., as shown in the table.}$$

Next, let $t_{h2} = 165^\circ \text{ F.}$, and $t_{c2} = 86 - (180 - 165) \frac{834}{2080} = 80^\circ \text{ F.}$

t_h	t_c	$(t_h - t_c)$
180	86	94°
165	80	85
150	74	76
135	68	67
120	62	58
105	56	49
90	50	40

For each successive fall for t_h of 15° , there is a fall of 6° for t_c since the cold fluid is flowing in the opposite direction and the change of temperature of the one is due to the change of temperature of the other. See Example 4.

From Eq. (609) the area from the point of entrance of the hot liquid to the point at which temperature has fallen 15° is found to be:

$$\text{For } t_{h2} = 165^\circ \text{ F., } A = \frac{1 \times 2080}{60 \left(\frac{2080}{834} - 1 \right)} \log_e \left(\frac{180 - 86}{165 - 80} \right) = 23.1 \times \log_e \frac{94}{85} = 2.33 \text{ sq.ft}$$

Other values of A may be readily determined by substitution in the above formula.

$$\text{The mean temperature difference is, } t_m = \frac{C_h w_h (t_{h1} - t_{h2})}{UA} = \frac{834 \times 90}{60 \times 19.8} = 63.1 \text{ degrees,}$$

or by Eq. (710)

$$t_m = \frac{94 - 40}{\log_e \left(\frac{94}{40} \right)} = \frac{54}{1.855} = 63.1 \text{ degrees.}$$

Prob. 1. In the manufacture of condensed milk, the water is evaporated at a low pressure by the use of steam. If the pressure in the vacuum kettle is such that the temperature is 90° F. , the latent heat per pound will be about 1100 B.T.U.'s. On the assumption that this condition remains the same during the entire process, that $U = 300$,

and that the heat is derived from steam at a temperature of 215°F ., how much surface will be required to evaporate 2 tons of water per hour?

Prob. 2. The rate of heat transfer in a surface condenser is 400. The vacuum desired calls for a temperature in the condenser of 100°F . The cooling water amounts to 1,000,000 lbs. per hour, enters at 50°F . and leaves at 95°F . How much surface will be needed if the heater is a five-pass heater, and what will be the temperature at the end of each pass?

Prob. 3. The boiler for a steam-heating plant runs at a pressure of 5 lbs. per square inch gage, and evaporates a thousand pounds of steam per hour. The products of combustion have an initial temperature of 1000°F ., a final temperature of 400°F . and a specific heat of .25. Assuming 1000 B.T.U.'s needed per pound of steam and that U is 2.5, how much surface will be needed?

Prob. 4. The oil in an electric transformer is cooled by running it through annular tubes inside of which is circulated cold water in a counter-current direction. 1000 lbs. of oil per hour must be cooled from 150°F . to 80°F . while there is available 5000 lbs. of water per hour at a temperature of 50°F . The specific heat of the oil may be taken as .4, and U as 50. At what temperature will the water leave the end of the cooler, and how much surface will be needed for every 10° of cooling? If it were desired to cool oil down to 70°F . how much additional surface would be required?

Prob. 5. To reduce the quantity of moisture in air it is passed over coils containing cold brine and thereby lowered below the dew point. Disregarding the heat to be removed from the moisture what must be the area of coils to lower 500,000 cubic feet of air per hour from 70°F . to 20°F .? The air and brine pipes enter the cooling duct at the same end. Using the following data, what will be the required surface? Specific heat of air .25, of brine .7, brine enters at -20°F . and leaves at 15°F . What will be the mean temperature difference?

Prob. 6. Solve Prob. 5 for counter-current flow.

79. Variation in Coefficient of Heat Transmission Due to Kind of Substance, Character of Separating Wall and Conditions of Flow. Nothing could well be more striking than the fluctuations in the accepted workable values of U for different conditions of surface, kinds of substance and rate of flow, ranging as they do from about $U=2$, to somewhere near $U=1000$. It is quite natural in view of the uncertainty that must accompany the selection of the correct value for a given practical problem that many investigators have sought to explain by experiment and analysis the variations that exist, with a view to associating all values by a general law. This ultimate aim appears, however, to be quite hopeless of attainment, and even if it were not without hope it would probably be useless, because any general law must include terms to account for the conditions of surface, the gas content of liquids or vapors, something equivalent to the thickness or resistance of fluid films and other things equally indeterminate by a computer engaged in predicting what will happen in an apparatus for which these things must first be evaluated but cannot be. About all that one could reasonably expect in this connection is a relation of U to the *definable* variables in each characteristic class of heat transfer cases with perhaps a few general principles, and some of these principles are fairly well established, though none quite beyond the limits of controversy.

Among the general principles, one already cited appears most important

of them all, that when a gas is either giving or receiving heat the coefficient of heat transfer is very much lower than when a liquid is involved as such, or as changing state with its vapor. *So very high is the resistance and so very low the value of U when a gas is involved, that the main resistance to flow is always on the gas side unless most extraordinary means are used to change it.* As transfer always proceeds from one substance to another and through a third generally, it may be considered as a three-stage operation, of which the first step is the giving up of heat by the hot one, the second its transmission proper and the third its absorption by the cooler body. The rate with which it will pass, while a function of all the heat resistances encountered is practically controlled by the one single highest resistance when that one is much larger than the others. Thus, in a given case the rate of heat exchange may be limited by the ability of the fluid on one side or the other to give or to take, and in the great majority of cases this is so; for a steam boiler the water can take up heat faster than the hot gases can give it, except possibly for surfaces receiving radiant heat, while in a steam condenser the limit is imposed by the ability of the circulating water to take the heat, and this is also the case for steam feed-water heaters as well. Therefore, even if means were available to increase the heat-absorbing capacity of water next to the heating surface of boilers, it would be of no value until the gases had been first caused to give up heat at a very much faster rate than they do.

The usual plan of experimenters seeking to relate the coefficient of heat transfer, or the hourly transfer of heat to some prime variable, has been to vary every condition that might produce a change, one at a time, plot the results to coordinates and seek an equation for the curve. Practically all that is known of the conditions that control the value of U has been derived in this manner with the exception of one result announced by Osborne Reynolds and derived by mathematical analysis based on the kinetic theory of gases and on certain hypotheses with regard to the condition of affairs at the metal surface.

A study of the results of these experiments illustrates the complexity of the relations involved and the almost complete hopelessness of any attempt to generalize much beyond the following principles in addition to those of a single controlling resistance, and the almost universally higher value of gas resistance over liquid.

The transmission per hour is said to be proportional to the temperature difference of the fluids directly according to most experimenters, so that the coefficient in B.T.U. per hour per square foot per degree is independent of temperature difference, however much it may vary with other things.

When the heat transmitted per hour is directly proportional to temperature difference, the value of U , being the B.T.U. per square foot per degree mean temperature difference, is independent of the temperature difference, and the mean temperature difference t_m is a function of the initial and final, as determined in the last section. Should the heat flow per hour be proportional to any but the first power of the temperature difference then the value of U will be itself a function of temperature difference, and the mean temperature

difference a different function of the actual temperatures than those previously derived. This can be shown by analysis for one case as follows, a similar method being applicable to other kinds of flow and cases of transfer though not worked out here because of the uncertainty of the value of the exponent.

Consider the case of flow of heat from condensing vapor at a constant temperature to a liquid or gas with rising temperature, which of course includes surface condensers and feed-water heaters. Then, from an element of surface dA there will pass $U(t_h - t_c)dA$, B.T.U. per hour, which is equal to the product of weight per hour, specific heat and temperature rise of the substance receiving the heat, whence,

$$dA = \frac{Cw}{U} \frac{dt_c}{t_h - t_c}.$$

If now the heat per hour be proportional to the n th power of the temperature difference, then U is proportional to the $(n-1)$ power of the temperature difference or, $U = K(t_h - t_c)^{n-1}$. Therefore

$$A = \frac{Cw}{K} \int_{t_{c1}}^{t_{c2}} (t_h - t_c)^{1-n} dt_c = \frac{Cw}{K(1-n)} [(t_h - t_c)^{1-n} - (t_h - t_{c1})^{1-n}], \quad (612)$$

which is the relation between the surface and the temperatures resulting from the passage over that surface. The mean temperature difference is given by the relation $t_m U_m A = Cw(t_{c2} - t_{c1})$, in which U_m is the average value of U for the whole heat exchange.

$$\begin{aligned} \text{Whence } t_m &= \frac{Cw(t_{c2} - t_{c1})}{U_m A} = \frac{Cw(t_{c2} - t_{c1})}{U_m \frac{Cw}{K(1-n)} [(t_h - t_c)^{1-n} - (t_h - t_{c1})^{1-n}]} \\ &= \frac{K(1-n)}{U_m} \frac{(t_{c2} - t_{c1})}{(t_h - t_{c1})^{1-n} - (t_h - t_{c2})^{1-n}}. \quad (613) \end{aligned}$$

It appears from this that the mean temperature difference is a function of the temperature rise of the water or gas being heated, of the $(1-n)$ power of the initial and final temperature differences and of the mean value of the coefficient of heat transfer which is itself now a variable. This mean value of U can be found by integration, but is not, because, first the resulting expression is very complicated for practical use, and second the completed expression is hardly worth while notwithstanding its apparent accuracy, because of the uncertainty in the value of n and the real value of U , so that the result is not and cannot be accurate.

Both hourly rate and the value of U assumed independent of temperature difference, that is, for $n=1$, are found to be more clearly a function of flow conditions which are defined by different experimenters in different ways. For example, all observers agree in assigning higher values to U when the flow is vigorous than when it is sluggish, that is, *U is found to increase in some manner with rate of flow in all cases but one, and that is when the increased rate of flow takes place in the fluid on that side of the surface where the resistance is but a small fraction of the total.* Thus, for hot gas warming water, increase in flow of the

water does not sensibly increase the value of U , but increase in the rate of gas flow will do so. This is one source of discrepancy in the reports of experiments not conducted in such a way as to control both rates when seeking to relate heat flow to one of them, many experimenters reporting results for variations in one rate of fluid flow without making sure that the rate of flow of the other fluid was either constant or of negligible resistance. This point is clearly demonstrated by the following experimental data, comparing one set of experiments with another.

The first series of experiments was conducted by Mr. W. D. Monks in the mechanical laboratories of Columbia University, under the direction of Professor Lucke, on the transfer of heat from hot gases to water under varying conditions of water flow with the gas flow intentionally neglected, except as was necessary to control gas temperature. The metal walls were those of a 2-in. tube 88½ ins. long, set vertically in a brick flue and heated outside by the products from a gas fire. The water flow was controlled partly by varying velocity in the tube as set and partly by varying the area of the water passage in the tube, while surface conditions of the tube on the gas side were also varied by applying in one case cast-iron rings of the Foster steam superheater elements. The first series was run with a perfectly plain tube supplying varying quantities of water while holding the gas temperature

constant, then the process was repeated with a higher gas temperature; later a plugged 1-in. standard pipe was inserted as a core and the same repeated, and finally a helical strip of metal substituted to give the water a rotary movement. Some of the results are plotted in Fig. 116, showing the relation of U to water velocity up to 160 ft. per minute, and these are representative. They show a small increase in U with water velocity as might be expected, since there is some water resistance, but hardly enough to be worth while; for example, the increase along any one of the curves is less than 1 B.T.U. per hour per square

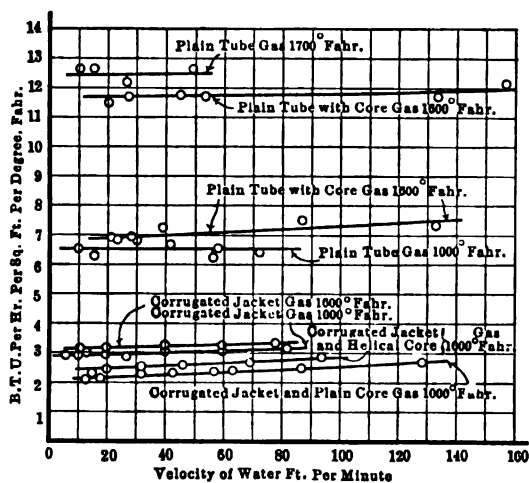


FIG. 116.—Variation of Heat Transfer Coefficient with Velocity of Water when Water Receives Heat from a Gas, Illustrating Substantial Constancy and Proving the Controlling Resistance to be on the Gas Side.

foot per degree, and all values for whatever water velocity or tube condition lie between $U=2$, and $U=13$. There are two influences affecting U more than water velocity, and these are gas temperatures and the outside condition of the tube. For the plain tube increasing the gas temperatures from 1000° F. to

1700° F. had the effect of approximately doubling the value of U , but it is not possible to say whether this effect is due inherently to temperature or to increased gas velocity which accompanies it, or to some other related quantity such as, perhaps, gas density. In every case the jacketed tube gave much lower values than the unjacketed, calculating surface as that exposed to the water and not that exposed to the gas, or *about the same when calculating surface as that exposed to the gas, illustrating well the constancy of gas film resistance for equal gas flow and temperature conditions*. The whole series demonstrates absolutely the controlling character of the resistance on the gas side and leads to the conclusion that whatever variations in U may be found are due to varying film resistance, and may be related to velocity of fluid directly or indirectly. The situation appears to be somewhat as illustrated in Fig. 117.

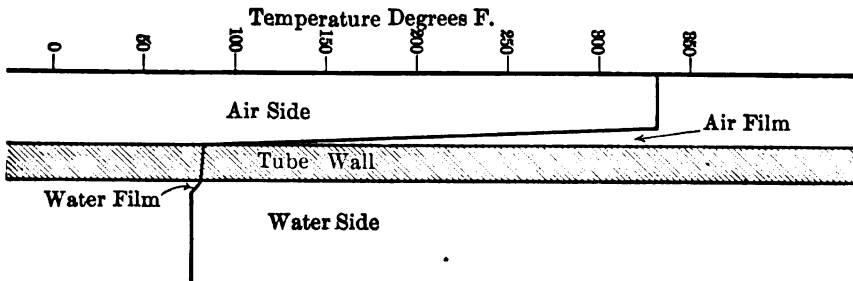


FIG. 117.—Probable Relations between the Heat Resistances or Drops in Temperature when Heat Passes from Hot Gas to Water.

Another series of experiments carried out in great detail by H. P. Jordan, on the transmission of heat from hot air to water confirms the above conclusions but extends them, as the amount and velocity of the air were accurately determined and the transmission phenomenon divided into two parts: first from air to metal, and second from metal to water. The superior resistance on the gas side is shown by the curve of Fig. 118, plotted from one experiment, by the temperature drops, which on the air side was 246° F. with only 6.7° F. on the water side, that through the metal itself being negligible. In every case the rate of transmission from the hot gas to metal is found to increase with increase of flow, and here a most striking relation is demonstrated by an appropriate selection of the prime variable representing rate of flow. *When this variable is taken as pounds of air per square foot of area of cross-section of air passage per second, the B.T.U. per hour per square foot per degree difference between air and metal relation, is linear and the curve a straight line as shown in Fig. 118, for one series.* These lines all have the equation

$$U = A + B \frac{w}{a}, \quad \dots \dots \dots (614)$$

where U = B.T.U. per square foot per degree difference between gas and metal;
 w = pounds air per second;
 a = square foot cross-section area of air passage.

In this equation A and B are factors, not the same numerically for different circumstances, and found by Jordan to depend on the following conditions: A , which is the zero flow value of U , appears to be independent of both cross-section of channel and temperature, but probably fixed by the condition of the surface as to cleanliness or smoothness; while B , which measures the rate of increase in U with flow, seems to be dependent on both the air temperature and on the dimensions of the air passage. Its value is

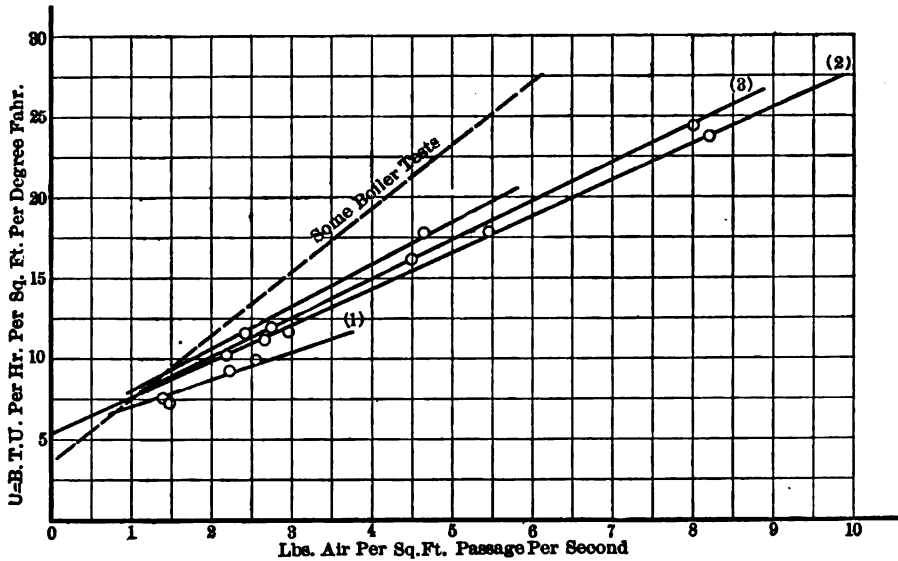


FIG. 118.—Variation of Heat Transfer Coefficient with Rate of Flow per sq.ft. of Stream Cross-section of the Gas (Air), when Water Receives Heat from a Gas, Illustrating Linear Relation between U and Gas Mass Flow.

found from, $B = c + c'q + c''t_m$, where c , c' and c'' are constant, t_m the arithmetical mean of the air and metal temperature, and

$$q = \text{mean hydraulic depth in inches} = \frac{\text{area of flow of channel.}}{\text{perimeter of cooling surface}} \quad (615)$$

Substituting the values of the constants and factors he found that

$$\frac{U}{3600} = .0015 + [.000506 - .00045q + .00000165t_m] \frac{w}{a}$$

This gives the coefficient of heat transfer in terms of all variables investigated, including weight of flow of gas in pounds per second, cross-section of channel, shape of channel or mean hydraulic depth and temperature of air and metal. The laws are summarized by Jordan as follows:

- (a) For a constant mass flow $\left(\frac{w}{a} = c\right)$ the rate of transfer is proportional to temperature difference directly;

- (b) For a given temperature difference ($t_m = c$) the rate of transfer increases with speed by a linear law;
- (c) For a given rate of flow and temperature difference the rate of transfer increases with the value of the temperature;
- (d) The rate of transfer depends on the condition of surface;
- (e) The rate of transfer depends on the size of channel, and the smaller

the ratio, $\frac{\text{area}}{\text{perimeter}} = q$, the greater the rate of transfer.

These experiments, as well as another series by Dr. T. E. Stanton, on the transfer from water to water through metal, may be regarded as confirming the theoretical predictions on the laws of transfer as announced by Prof. Osborne Reynolds forty years ago. *He gave on purely theoretic grounds a linear law of relation between rate of transfer per hour per degree and the product of density and velocity of fluid, but this product is equal to the weight per second per square foot of cross-section or the mass flow, as it may be called.*

The conclusion that when a gas is present on one side or the other the coefficient of heat transmission increases in a linear law manner with a mass flow is a most important one, because the greatest practical use of this sort of transmission is made in the steam boiler, the laws for which have for over a century been a subject of controversy and doubt, from which some order and agreement is beginning to appear. In the case of the boiler the temperatures on the two sides of the surface are unknown, greatest variations and uncertainty are found on the gas or flame side. This being the case it is quite impossible to find, or if found for one case, to use in engineering practice, any particular value of the coefficient per hour per degree per square foot. *It is possible, comparatively easy, and entirely practicable, to discuss and use the rate of transmission per hour per square foot, and this evidently in any case will also bear a linear relation to the quantity of gas per hour being discharged through the flues or tubes.* This is proved by all accurate tests on boilers and will be taken up later as the subject is important enough to warrant a more detailed treatment.

The next important class of heat transfer cases in view of practical applications in standard engineering apparatus is the transfer from condensing steam to moving water, typical of exhaust steam feed-water heaters, surface condensers, steam-heated kettles and similar equipment. This has long been a subject of experimental investigation and there are available many test results, all of which do not, however, agree in detail, though all do agree in assigning increasing values to U with water velocity, usually making it proportional to some power of the water velocity—Ser and Joule the one-third power, Hege-man the one-half power and Stanton the first power. In some of the work attempts have been made to relate U to the steam velocity or mass flow of steam approaching the condensing surface as well, but these are in the main unsuccessful. *One great source of variation in U noticed by some and ignored by others as peculiar to this class, is the collection of non-condensable gases in bubbles and films on the condensing surface, preventing actual vapor-metal*

contact and interposing great heat resistance. In general it may be said that for this class the principal resistance is on the water side and the steam can condense as fast as the water is able to carry heat away, so that increase of water velocity always increases U by decreasing the film thickness and corresponding heat resistance. The value of U is always higher than for cases where gases are present, because liquid film resistances are always less than those for gases of equal thickness. When gases collect on the condensing side the principal resistance may no longer rest in the water film, but may be transferred to the vapor side, in which case increase of water velocity results in little or no gain.

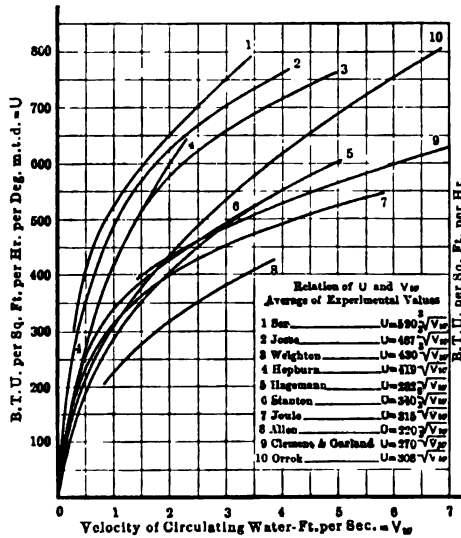


FIG. 119.—Variation in Heat Transfer Coefficient with Water Velocity, when Water Receives Heat from Condensing Steam (Orrok).

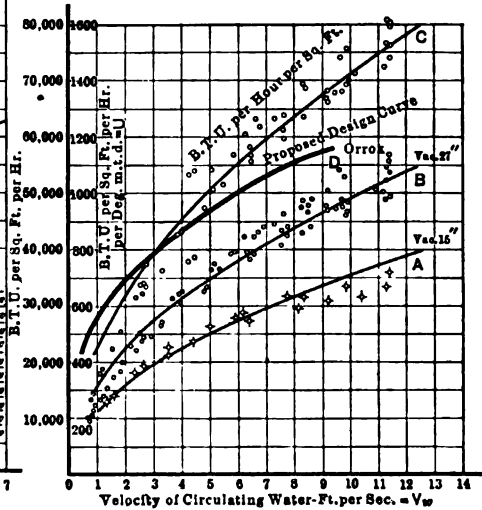


FIG. 120.—Heat Transfer Coefficient as a Function of Water Velocity for Steam Condensers, Showing Curve of Accepted Law and Experimental Points by Orrok.

In no series of experiments has more care been exercised to obtain accurate data, with a full understanding beforehand of the difficulties and interferences to be overcome, than those of Orrok reported in the A.S.M.E. Proceedings. In this series extraordinary precautions were taken to exclude air and other gases, and steam generated from a constant mass of water in a closed system was condensed under vacuum on tubes of different compositions and surface conditions supplied with water of varying velocity. The results reported are consistent and reliable, but are not applicable to commercial steam generated by fresh supplies of water carrying gases in solution, as all water supplies do, without correction or allowance for the increase of heat resistance due to collection of gas films on condensing surfaces.

The curves of Fig. 119, reproduced from Orrok's paper, give a graphic summary of the value of U in terms of water velocity as determined by various

experimenters and includes his own result. These crossing lines of different curvature and slope prove conclusively the complex nature of the case under examination and the fact that U cannot be regarded as a function of water velocity alone. More in detail Orrok's results for the two vacua 15 and 27 ins. Hg are given in Fig. 120, curves (A) and (B), being the values of U and (C), that of B.T.U. per square foot per hour or $U \times t_m$; it should be noted how widely distributed are the points about the curve in spite of great care in testing. All these curves, as located in the bands of points, follow the square root of velocity law having the equations,

$$U = 308\sqrt{V_w} \text{ for 27 ins. vacuum (a); } U = 224\sqrt{V_w} \text{ for 15 ins. vacuum (b). (616)}$$

The effect of steam velocity, amount of air present in the steam as represented by the partial pressure of the air and the nature of the tube were also investigated, the results being summed up in the following equation,

$$U = K \frac{C' \rho^5 \mu \sqrt{V_w}}{t_m^{1/4}} \quad (617)$$

Where C' is a cleanliness coefficient varying from 1.00 to .5;

$$\mu \text{ is material coefficient} = \begin{cases} 1.00 \text{ for copper;} \\ .98 \text{ for admiralty metal;} \\ .97 \text{ for admiralty aluminum lined;} \\ .92 \text{ for admiralty black oxidized;} \\ .87 \text{ for admiralty aluminium bronze;} \\ .80 \text{ for cuprous nickel;} \\ .79 \text{ for tin or admiralty lead lined;} \\ .75 \text{ for zinc;} \\ .74 \text{ for monel metal;} \\ .63 \text{ for Shelby steel;} \\ .55 \text{ for admiralty badly corroded;} \\ .47 \text{ for admiralty inside vulcanized;} \\ .25 \text{ for glass;} \\ .17 \text{ for admiralty vulcanized both sides;} \end{cases}$$

$$\rho = \text{ratio of } \frac{\text{Partial pr. steam}}{\text{Total pressure}}, \text{ varying from 1.00 to 0.}$$

V_w = water velocity feet per second;

t_m = mean temperature difference;

K = a constant = 630 approximately.

For high vacuum condensers the standard vacuum is 28 ins. and guarantees are usually based on this and on a circulating water inlet temperature of 70° F. with 20° F. allowable rise. For these conditions $U = 435C' \left(\frac{P_i}{P_t} \right)^5$.

The effect of air or other gases, deposited on the condensing surface, on the rate of heat transmission was investigated most thoroughly by J. A. Smith, whose results are the best available. He finds that enough gases to be

equivalent to a partial pressure of $\frac{1}{20}$ in. Hg will at 90° F. decrease the transmission 25 per cent, and $\frac{3}{20}$ in. Hg, 50 per cent.

Probably the most complex of the heat transmission cases, so far as concerns the variability of U , is that in which the two fluids are the same in kind and both suffer a change of temperature; these include heat passage from liquid to liquid, or gas to gas. *For all of these it is quite impossible to fix off-hand the location of the controlling resistance, as it may be on either side, and with variations in flow may pass alternately from one side to the other,* as in liquid heat exchangers, steam superheaters and similar apparatus. As there are no available data covering a sufficient range of conditions, about the best that can be done is to analyze such cases by trial with assumed metal temperature, taking Jordan's data for air relating U to mass flow, and Orrok's for water relating U to the square root of the water velocity. These cases are not so common nor so important in practice, which probably accounts for lack of data, but they are sufficiently important to warrant some trouble in establishing experimentally the needed laws.

CHAPTER X

HEATING BY COMBUSTION

FUELS, NATURAL AND MANUFACTURED.

80. Origin of Heat and Transformation to Useful Form. Complexity of Fuels as Sources of Heat. General Classification. Solid, Liquid, Gaseous, Natural and Artificial. All heat for power purposes is derived from the combustion of fuels of some sort, whether natural or artificial, so also is nearly all the heat used for warming for domestic and industrial purposes; but some heat used for these purposes is derived from electricity generated from water power; in other cases heat is derived directly from bodies in their natural state, as for example, when foods are cooled for safe storage or when ice is made. In general, the source of heat with which engineering operations must deal is (a), that of an exothermic chemical reaction, or (b), that of bodies in their natural state, and this heat when taken into some system for the purpose of accomplishing some industrial end must be traced through the whole series of processes necessary to that end and finally disposed of, as no energy can be destroyed. The ultimate disposition of the heat is a matter of just as much importance as its generation or acquirement at a point of origin, and the almost infinite variety of intermediate exchanges and transformations between its reception into the system and its final disposition.

Heat received into a system is ultimately disposed of in one of three ways: first, it is most commonly discharged from the system in the form of hot water, hot gases or vapors, and from them communicated to the surrounding bodies or radiated into space; or second, it may be bound up by endothermic chemical reactions or changes of state as when certain chemical compounds or solutions are formed by heat absorption; or finally it may have been transformed into energy of another form, permanent or transient, as when a mine cage has been lifted or light generated in the electric lamp. Omitting such dispositions as correspond to transformation of energy into another form as but temporary, and to be treated subsequently, the two final dispositions are the inverse of the two original sources, chemical or physical changes of state of bodies or temperature changes of the general surroundings, earth and water and air.

When the heat is received into the system at high temperatures as is the case when its origin is the combustion of fuel, its progress is generally downward as to temperature, each exchange from substance to substance resulting in lower temperature, but the final disposition by transfer to water or atmospheric

air, in steam condensers, for example, must take place at a temperature of the heat carrier, in excess of that of the final heat receiver, that is, the final temperature of the steam, the heat of which must be disposed of by heating water or air, must be higher than that of the available water that is to receive it. Between the reception and final disposition all sorts of things may be caused to happen; power may be generated and transformed into all the various forms of energy; metals be extracted from their ores, or chemical compounds of value made from those of no value.

When, however, the heat is received at low temperatures, as when ice is to be made or a quicksand to be frozen to permit shaft sinking, or food is to be preserved, or natural water cooled for industrial or drinking purposes, or air to be dried of its moisture, then heat can be received into the system only by having in the system something colder than the body whose heat is to be taken. In general, these operations result in heat reception at a lower temperature than the surrounding air and water, to which it must ultimately be returned, and which can receive it only when it has been raised to a higher temperature, so that *at least one step in the series of processes must be a raising of temperature by doing work on the working substance, usually by compression.*

The practical use of heat, therefore, is absolutely limited as to process, methods and quantities by the conditions of reception and those of ultimate disposition, but between these limits all operations or processes may be classified as

- (a) Heat transformation into work or vice versa;
- (b) Change of substance form, chemically or physically, with heat absorption or liberation,
- (c) Heat transfer from regions of high to regions of low temperature.

These may also be grouped into two broader divisions, somewhat more convenient than those above.

I. Thermal processes in which no mechanical work is done but involving heat exchange or substance change with heat addition or abstraction.

II. Thermal processes involving the doing of mechanical work on, or by, a working medium or heat carrier. These are all work-heat transformations.

The majority of fuels must be changed in character before the heat in them is in a form suitable for conversion into work. All work is produced by a change in the volume of a substance with or without a change in pressure. As before stated, those substances giving the greatest volume changes are the gases and vapors. *For this reason then the heat in the fuel must be transferred to a gas or vapor unless the fuel be originally one or the other of these.*

The first type of heat-carrying substance, a gas, is generally prepared under pressure by the so-called explosive burning of a gas with air in a closed vessel. As few of the natural fuels are gases, the first step in the utilization of other forms of fuels for such cases as these, is to gasify them by means of gas producers, retorts or similar apparatus at the expense of some of their own heat or heat from another source.

The second type of heat-carrying substance, used for securing work at the

expense of heat, a vapor, is obtained by evaporating some volatile liquid, usually water, in a boiler by means of the heat produced by the combustion of any natural or artificial fuel.

Therefore, the problem of the fuels, their transformation, one form into another, more especially their gasification and complete combustion, is of the utmost practical importance and of considerable difficulty. Although the only combustible chemical elements of all fuels are carbon, hydrogen and to a small extent sulphur, and the complete combustion reactions few and simple, it must not be understood that fuels are simple in nature, their suitable and proper combustion easy; or that it is possible to predict their heat of combustion or its effects with precision, even when their chemical composition is known. *No problem of equal practical importance is more difficult or elusive than the prediction of fuel effects by calculation, or the determination of the most suitable fuel for a given purpose.*

Fuels may be regarded as mixtures of the chemical elements, carbon, hydrogen and sulphur, so far as their combustible parts are concerned, in the proportions indicated by their ultimate analysis, but it is known that these elements make a great variety of compounds, so that if these compounds are present in a fuel, its heat of combustion cannot be the sum of the heats of combustion of its chemical elements, but this sum less the amount necessary to break the bonds of molecular union. It would be more proper from the heat liberating standpoint, to say that the heat of combustion of a fuel is the sum of the heats of combustion of all those chemical compounds that as a mixture, make up the fuel. Unfortunately this precise statement is of little help because no one knows that a fuel except the gaseous varieties, does consist of a mixture of separate and distinct fuel compounds, or what they are. It is probably nearer the truth to say that solid fuels are themselves chemical compounds of great molecular complexity with other chemical compounds in *solution* or some equivalent state; it is known that all liquid fuels are solutions of a lot of liquids in each other with perhaps a little of the simple mixture condition. If all the compounds that together constitute a fuel were known, if the heats of solution or absorption of such substances so held by the others were known, and their quantities, then the heat of combustion of a fuel could be expressed as an algebraic sum of all these heats exactly. This, however, is quite impossible, and even if it were possible would be of comparatively little use because of its complexity and the fact that no two samples of the same fuel externally judged, give quite the same results, though all will average up within certain reasonably close limits.

There is another possible assumption with regard to a solid fuel composition, that is, while approximate, quite useful; this considers the combustible to consist of fixed carbon or carbon that burns directly from the solid coke, mixed with a gaseous combustible, termed the volatile; and the heat of combustion as the sum of the heat of so much fixed carbon as is present, and the heat of the volatile. This assumption leads to an estimate of the fuel value of one coal, from calorimeter determinations of others of the same class in

terms of the heat of combustion per pound of the volatile, which is different for each class of coals, and that of the fixed carbon which is the same for all. In a precisely similar approximate way, the gaseous and liquid fuels may be treated, which treatment emphasizes the tremendously variable and complex nature of fuels which must be handled for the accomplishment of engineering ends, and the necessity for some study of their nature that will throw light not only on their calorific power, but on the best way in which to burn them to develop it to the fullest extent and in the most efficient manner.

The very chemical complexity that the study of calorific powers reveals, is also responsible for the remarkably different behaviors of the fuels even of the same class, in fires. Furthermore, each different use to which fuels are put, requires within certain limits special properties in the fuel to make it adaptable to the purpose. For example, internal combustion engines require gaseous fuel and where gaseous fuel is not available, it must be made from whatever solid or liquid fuel is most available, availability depending partly on cost. Blast furnaces for the extraction of iron from the ore are best served with coke, and coke must be made for the purpose from coal, or where this is not available, a hard, natural coal may be substituted in some cases. Certain metallurgical and chemical manufacturing operations require a very steady, high temperature such as can be obtained only with gaseous and in some cases liquid fuel. Boilers are fired with various fuels—that which produces the cheapest steam being selected. This may be a gas, an oil, or any one of the great number of different solid fuels that may be had in a given district, each of different quality and price and usually each giving a different boiler horse-power and efficiency; so it may be, that either the most expensive coal per ton, or the lowest price coal per ton, or some intermediate one, may make the most steam per dollar's worth of fuel. Additional complexity arises from the fact that the cheapest steam-making coal may yield so low a boiler horse-power as to make the boiler installation investment charge, per unit of steam, so large that greater horse-power with more expensive fuel might really be more economical. In cities the household use of gas for illumination is now combined with the use of the same gas for domestic heating, and a suitable gas must be manufactured to meet the demand for this double service.

These examples will suffice to show: first, that the use to which a fuel is put more or less dictates the type of fuel; second, that in apparatus capable of using more than one fuel the problem of selection is partly one of properties and partly one of price, factors that may or may not be related; third, that while natural fuels exist in great variety it is good engineering to manufacture from them other fuels to secure desired results. These facts make it necessary to enquire a little more closely into the properties of the natural and artificial fuels, and their differences; and the sort of process by which one class or kind of fuel may be manufactured from another.

The first step in such a study is a comprehensive classification of all fuels as given in the following table as a basis for the treatment:

TABLE IV
GENERAL CLASSIFICATION OF FUELS

Physical State.	Name of Natural Fuel.	Name of Manufactured Fuel.
Solid.....	Wood, peat, lignite, bituminous and anthracite coal	Charcoal, coke, briquettes.
Liquid....	Crude mineral oil.....	Alcohol, mineral oil, distillates and residues, oil, gasoline, kerosene, fuel oil, and oil tar.
Gaseous...	Natural gas.....	Coal gas, coke-oven gas, water gas, carbureted water gas, producer gas, blast-furnace gas, oil gas, oil and alcohol vapors.

The general processes by which the artificial fuels are made from the natural ones are divisible into groups. The first is a simple heating or roasting of natural solid fuels, which leaves a coke and produces a volatile mixture that on cooling leaves some permanent gases more or less saturated with vapors, and some liquid, generally termed tar or tar liquor. The second is analogous but as applied to liquid fuels is termed distillation or fractionation. The third is quite different; it is based on the reactions of fixed carbon with the oxygen of the air, and with steam, to make carbon monoxide and hydrogen. Often these three type processes are mixed and all three take place at once and are associated with various dissociations and inter-reactions. It is, however, convenient to think of all processes being divisible into:

- (a) Mere heating of the natural fuel;
- (b) Chemical reactions between the fuel constituents, air, and steam.

Prob. 1. The simplest power-plant consists of a boiler and non-condensing engine. Trace the heat from its source in the fuel to its final disposition.

Prob. 2. The above power-plant had added to it various auxiliaries for increasing the efficiency. They consist of a feed-water heater in the exhaust of all auxiliaries, and an economizer in the flue, a superheater in the first pass of the boiler, an induced draft steam-driven fan, and a condenser. Trace the heat through this system from the coal to its ultimate disposition.

Prob. 3. Gasoline is burned in the cylinder of a water-jacketed engine, provided with an air-cooled radiator. Show where the original heat of the fuel goes.

Prob. 4. Coke is burned in a blast furnace and some of the gaseous products, which are rich in CO, are washed and used in gas engines driving the blast compressors. Between the blower and furnace the air passes over brickwork previously heated by burning some of the gas. The gas-engine cylinder is, of course, water-jacketed. Through what steps does that portion of the heat of the fuel not used up by the reduction of the iron ore pass and what is its ultimate disposition?

Prob. 5. Heat is removed from a cold-storage room and delivered to the atmospheric air. The process in an ammonia-compression system is to allow ammonia to evaporate in coils at a low pressure, compress the vapor and condense it by allowing water and

air to pass over tubes receiving the high pressure vapor. Through what transfer and transformation processes does the heat pass before reaching the atmosphere? Assume the compressor to be steam driven and trace the heat of the coal as well.

Prob. 6. Air is compressed by a water-jacketed steam-driven compressor and discharged to a large pipe-coil receiver, then through a long pipe line and a coal-fired preheater to an air engine. Show where the air receives heat and what becomes of it and also what becomes of the heat of the coal.

Prob. 7. A coal-fired boiler supplies a steam-driven electric generator, the exhaust from which passes to radiators placed throughout a building lighted by the current, the excess steam passes to the roof exhaust. Trace the heat of coal to its final disposition.

Prob. 8. Air and steam are supplied to a gas producer. What becomes of the heat originally in the coal?

81. Natural Solid Fuels, Wood, Peat, Lignite, Bituminous and Anthracite Coal. Chemical and Physical Properties. Classifications Based on Ultimate and Proximate Analysis and on Behavior on Heating. All solid fuels are of vegetable origin notwithstanding the variations in properties from the hard anthracites to the soft peats, and while the tracing of the process of transformation of each from its origin or one to another is more a problem of chemical geology, or physical and organic chemistry, than of the engineering of fuel manufacture or combustion, yet such an analysis throws so much light on the nature of the fuels and the methods of treatment in combustion or gasification as to render a brief review decidedly worth while.

There are three ways of defining a fuel: First, by a general name, indicating geologic condition, behavior on heating, or some other generally understood but not very definite characteristic, which is responsible for the names lignite, peat, bituminous and anthracite, long flame, short flame, cannel, coking, caking, and gas coal, together with many others; some indicating size like run-of-mine, slack, broken, furnace, egg, nut, pea, buckwheat, and rice, the latter names applying principally to sizes of anthracite. Also the name of the district or mine is used, such as Hocking Valley or Kanawha, or Pocahontas coal or Pennsylvania anthracite or Illinois coal.

The second way of defining or fixing a solid fuel is based on what is termed its *proximate analysis*, which has a meaning dependent solely on the methods employed, the following applying to the laboratory work of the U. S. Geological Survey. One gram of pulverized coal so selected that it represents the average of the lot, is heated in an oven at 221° F. for one hour, and quickly weighed. The loss in weight is termed the *moisture*, and the residue after burning the dried sample in a crucible is termed the *ash*; the difference or *combustible*, being divided by other operations into two parts, the *volatile* and the *fixed carbon*, the former being the loss of weight by seven minutes heating of the dry sample, in closed crucibles excluding the air, in the flame of a Bunsen burner, and the latter the residue after subtracting the ash.

By complete analytical methods the chemical elements of the coal may be found and reported, which define the coal by its *ultimate analysis*, the methods

being those common to quantitative organic chemistry. Special constituents reported by analysts for fuels are each determined by special methods.

Wood is in all cases the primary organic substance from which solid fuels are derived and the term must be used to include not only the trunks of trees, but branches, leaves, and roots as well as small plants and mosses. It is composed chemically of cellulose, $C_6H_{10}O_5$, as the fiber, and of sap, or sap deposits between the fiber. Cellulose consists, according to its formula, of a definite weight proportion of carbon, hydrogen, and oxygen, but the proportions of these same elements will be different in real woods by reason of the sap properties or the materials deposited by sap, which are of both nitrogenous and non-nitrogenous character. Sap is really a very complex substance, consisting of protein, tannin and several vegetable acids, starch, sugar, essential oils and resins among its organic constituents, together with various inorganic salts derived from the soil. These are of some importance in explaining the differences found in the coals derived from the woods and are mentioned here to call attention to the relation. For analysis of cellulose and average wood see Table LIV, Handbook of Tables.

It is from such chemical origin that the solid fossil fuels were derived by a natural process, termed 'carbonification, because it is characterized by an increase of carbon content over the original vegetable substance. These fossil solid fuels are usually divisible by age into younger and older groups, though local conditions may disturb the division, the former including *peat and lignite*, and the latter *bituminous and anthracite* coals, which calls attention to the fact that time is the first element in increase in per cent of carbon. The next factor is the action of oxygen. Wood will absorb oxygen with a resulting slow reaction to CO_2 even at low temperatures, and so also will all the coals of whatever grade. There is a further reaction of organic nature between the hydrogen, whether free or combined and the carbon, to form hydrocarbons, among which is methane CH_4 as the principal one. Mixing of the deposits with sand and clay increases the ash content of the coals over that of the woods, just as sap deposits give to the woods an ash content not existing in cellulose.

It is certain that the processes of carbonification produce certain products consisting of carbon dioxide and hydrocarbons, and perhaps many other complex compounds, and leave a residue which is the coal. Increase of both pressure and temperature promotes the process, the coal residue containing a higher per cent of carbon and ash than the original plants from mixture and reaction with earths, water and air, while oxygen and nitrogen decrease with time and the favorable nature of surrounding conditions. One very active agent in the changes is heat, promoting chemical reaction and expelling gases and the vapors of liquid resultant substances, which are compounds of carbon, hydrogen, and nitrogen, largely hydrocarbons and carbon dioxide. Vapors may condense in other colder places into which they are driven by the pressure, forming oil deposits, while gases may impregnate rocks and sands and remain in place when overlaid by impervious rock. Thus the

origin of natural gas and crude oils is traced with reference to coal formation as products of successive decomposition and reactions beginning with wood, the coal constituting the residue; but there is good reason to believe that hydrocarbons forming natural gases and oils may have been formed from carbides of iron, as the U. S. Geological Survey has recently shown that the regions of great magnetic needle deflection are those of known oil deposits.

The various coals are divided into classes, with names for convenience of discussion, according to their properties relating to the completeness and character of the change they have suffered; but unfortunately there is a general lack of agreement as to the meaning of the names. Coal classification is generally based on chemical properties, Muck making the total carbon content of the ultimate analysis the basis, Frazer, the fixed carbon divided by the volatile combustible matter of the proximate analysis, while Campbell of the U. S. Geological Survey uses the ratio of the total carbon to the total hydrogen of the ultimate analysis. These three classifications with the corresponding names are given in the following table:

TABLE V
CLASSIFICATION OF COALS BY COMPOSITION

Class.	Name.	Campbell. Total C. Total H.	Frazer. Fixed C. Vol. Comb.	Muck. Per Cent. Total C. Dry and Ash Free.	General. Per cent Total C. Dry and Ash Free.
A	Graphite and graphitic coal	∞ to ?	Anthracite		
B	Anthracite (1).....	? to 30	100 to 12	Anthracite 95	Anthracite 97 to 92.5
C	Anthracite (2).....	30 to 26			
D	Semi-anthracite.....	26 (?) to 23	12 to 8		92.5 to 87.5
E	Semi-bituminous.....	23 (?) to 20	8 to 5		87.5 to 75.0
F	Bituminous (1).....	20 to 17	Bitumi- nous 5	Common coal 82	
G	Bituminous (2).....	17.0 to 14.4			Bituminous east- ern U. S. 75.0 to 60
H	Bituminous (3).....	14.4 to 12.5	to		
I	Bituminous (4).....	12.5 to 11.2	0		Bituminous west- ern U. S. 65 to 50
J	Lignite.....	11.2 to 9.3		70	Under 50
K	Peat.....	9.3 (?) to ?		59	
L	Wood or cellulose.....	7.2		50	

It is to be regretted that there were not enough data to positively fix the numbers marked (?), but it is a fact that no sharp line of division exists between

one class and the next, each merging into the other by almost imperceptible gradations.

Another basis of classification of considerable value depends on the behavior of coals on heating with respect to (a) the amount of gas produced, (b) the character of the coke residue, and (c) the changes in the mass before the coke sets. These various conditions have been arranged in tabular form and are given in Table (LVII), in the Handbook of Tables, which indicates, first, that this sort of classification, while common in practice, is still not reduced to any acceptable form; second, that as an indication of the use to which the different coals are well adapted, a classification based on behavior under heating, as to gas yield and character of coke formed may be more valuable than others based on composition; third, gas and coke property classification is not definitely related to composition, though there is some sort of relation.

These classification properties are not of mere laboratory importance, but largely control the actual useful service of the coals. Coals that melt too completely cannot well be burnt under boilers without constantly breaking up the cake that forms, so as to let air pass through, but moderate caking tendencies are good as preventing fine material from passing through the grate, so anthracite rice or dust is often advantageously mixed with some caking bituminous coal.

Peat is the least removed in character from its vegetable origin of all the solid fossil fuels and two general varieties are recognized with reference to origin, sometimes described as (1), high-bog peats, formed from heath and swamp moss and usually located in high altitudes, and (2), low-bog peats, formed largely from grasses about the borders of low bodies of water.

In most peats the fibrous structure is still visible; in color they range from yellow through brown to black; some are soft and others hard, hard especially when the fibrous structure has almost or nearly disappeared, and in all cases there is a very large moisture content even in air-dried peat. In but few cases is peat strong enough to resist crushing when piled high during combustion as in gas producers. Probably its most distinguishing characteristic is the very large percentage of nitrogen and oxygen and the poorly combustible character of its volatile matter.

The average calorific power of peat as given by Scheere in B.T.U. per pound is:

Zero per cent water, zero per cent ash = 9090	25 per cent water and zero per cent ash = 6840
Zero per cent water, 15 per cent ash = 7992	30 per cent water and 10 per cent ash = 1462

Lignite is only one step removed from peat, yet is it very different in character, having been formed largely, if not entirely, from plants rich in resin, including the coniferous trees, cypress and palms, but in some cases others are included. Its appearance is non-fibrous and is either brown or black in color. It readily absorbs oxygen and gives off carbon dioxide at all temperatures. The volatiles of lignites are so largely carbon dioxide that they are chemically in the same general class as the peats, and this distinguishes

both, as much as any property, from all other coals, the volatiles of which are mainly hydrocarbons. This is a most interesting distinction but must not be interpreted as meaning that no hydrocarbons are found in lignites but rather that they are mixed with large amounts of non-combustible diluents.

Many lignites contain water, as water of crystallization, so that, on drying even in the sun, loss of water will sometimes cause a freshly mined, hard, shiny piece to fall into a heap of sand-like particles. This is important in the problem of firing this fuel because varieties that behave like this, may fall through a boiler grate or pack a gas producer bed, so that the air blast cannot be forced through. When lignites contain bituminous matter that melts on heating this will act as a binder and may correct the disintegration evil in boiler fires or perhaps increase the packing difficulty in gas producers. There are among lignites, as among peats, very great ranges of change and variety, but all varieties contain much water and yield much non-combustible matter in the volatile gases. The latter fact is the reason why classification based on the per cent of fixed carbon or its relation to the volatile of coals, often fails in distinguishing lignites from bituminous coals, and why also even the total carbon-hydrogen ratio fails.

Bituminous coal is next to lignite in order of age, but embraces in its own class so great a variety of coals as to lead to the different designations of coking, non-coking, long flame and short flame, and to the four sub-classes based on composition, proposed by Campbell, in addition to a semi-bituminous class. It requires only a brief inspection of the coal table (No. LV), given in the Tables, to show how hopeless must be any attempt to draw a sharp line between bituminous coals of low grade and the lignites. In general practice perhaps nothing distinguishes a bituminous coal more than the considerable quantities of very rich hydrocarbon volatile that characterizes the best varieties. Perhaps the best division of bituminous varieties for practical purposes, however indefinite it may be, is that descriptive of the flame or the coke. According to this, the lowest coal in the bituminous series is the non-caking long flame coal.

Non-caking long-flame bituminous coals, called by the Germans "sand coals," leave coal particles after heating, unchanged as to form, except as to the cracking of large masses, but give off combustible hydrocarbon gases very freely. They are black or brown in color, generally hard, and include the English and Scotch splint coals, which have been used for blast furnaces instead of coke and also for reverberatory furnaces. They occur in America but are most common in England and on the Continent. According to Jüptner the total carbon of these coals is between 75 and 80 per cent, hydrogen 5.5 to 4.5 per cent, oxygen and nitrogen 19.5 to 15.5 per cent, indicating the presence of non-combustible in the volatile. The calorific power is placed by him between 14,400 and 15,300 B.T.U. per pound, which is very high compared to lignites.

Cannel coals belong more nearly to this class than to any other, but are quite different in many ways from the general run of coals, hardly fitting

into such a series as here described. They are dull black, hard but easily broken, give a very large amount of gas on heating and a long flame when burning, crack as they burn but do not melt nor cake. They appear to be related to the shales that yield oil on distillation more than to other coals. Industrially cannel coals are used only for gas making by retort roasting processes.

Bituminous gas coals come next in order and are distinguished from the previous class by the melting, caking or coking tendency during heating, and by the greater volatile content or gas yield, and its hydrocarbon character, though they contain nitrogen diluents and yield ammonia. Lumps will stick together but not puff up, nor melt completely with loss of original form, during heating. They are especially adapted to coal-gas making by the retort roasting process, to reverberatory furnaces because of their long flames, but may also be used to advantage in boiler furnaces. According to Jüptner the total carbon content is greater than for the non-caking variety, being between 80 and 85 per cent, the hydrogen between 5.8 and 5 per cent, so that this is also a little higher, and oxygen and nitrogen 14.2 to 10 per cent, somewhat lower, indicating gases of more distinctly hydrocarbon character with less non-combustible diluents. This also is indicated by a higher calorific power, placed between 15,300 and 15,840 B.T.U. per pound.

Bituminous furnace coals is the term applied by Sexton to the next in the series; these have more strongly developed caking tendencies, softening and swelling during combustion and yielding long, luminous gas flames by the combustion of the hydrocarbon volatile. According to Sexton the volatile matter is between 25 per cent and 35 per cent, materially less in total quantity, than for the long flame non-caking coals. Jüptner places the total carbon between 84 per cent and 89 per cent, total hydrogen 5 to 5.5 per cent, oxygen and nitrogen 11 to 5.5 per cent. The rise of the hydrocarbons is indicated also by the rise of calorific power, which is placed between 15,840 and 16,740 B.T.U. per pound. These coals are used in England for domestic purposes and everywhere for boiler fires, forges, reverberatory furnaces and coal-gas making.

Bituminous coking coals are those next in order, distinguished not only by a strong melting tendency on heating, and by the very hard, large masses of coke they yield from slack coal, but equally important by a lesser gas yield which is responsible for the designation, *short flame*, applied to them. This is the most important industrial class of coals, as it is applied to the making of blast furnace coke and is good for boiler work. The coke yield varies from 65 per cent to 80 per cent, depending as much on the oven as on the coal itself, and the content of oxygen and nitrogen is less than for the preceding varieties.

Semi-bituminous, semi-anthracite, and anthracite coals are not distinguishable on the basis of coke or flame properties but rather on the basis of hardness and composition; they merge very gradually from the last group in the order named to graphite. The volatile becomes of a simple character, being

almost entirely methane in the anthracite and small in quantity, usually less than 4 per cent, whereas the more distinctly bituminous coals contain very large quantities of volatile that are complex hydrocarbons, some of them of the liquid form, termed tar, which may be over 5 per cent by weight, while the total volatile may be quite high. The total carbon content of anthracites may reach 98 per cent and as they are very hard, are not so easy to burn. Some varieties termed graphitic coals cannot be burned at all except by mixing with other varieties and only in the hottest fires. The calorific power of the anthracites approaches very closely to that of pure carbon, the small excess heating value of the volatile balancing the ash, but there is a most important exception found in small sizes. Anthracite generally occurs with slate streaks, which, as the coal is broken into commercial sizes, is separated out by hand, and as sizes become smaller the slate cannot be distinguished from the coal so that the smaller sizes often contain as much as 35 per cent of ash. This relation is shown approximately in the following table, applying to the Wilkesbarre and Scranton, Pa., products.

TABLE VI
WILKESBARRE ANTHRACITE SIZES AND AVERAGE ASH CONTENTS

Name.	Passes through Hole, Inches.	Passes over Hole, Inches.	Ash Per Cent.	
			General Average.	One Mine.
Broken.....	3½	2½	5.00	
Egg.....	(a) 2½	2	5.5	5.66
	(b) 2½	1½		
Stove.....	(a) 2	1½	8	10.17
	(b) 1½	1½		
Chestnut.....	(a) 1½	¾	12	12.67
	(b) 1½	¾		
Pea.....	¾	¾	15	14.66
Buckwheat.....	(a) ¾	¾	19	16.62
	(b) ¾	¾		
Rice or buckwheat, No. 2.....	(a) ¾	¾	25	
	(b) ¾	¾		

Semi-anthracites are usually the border coals of the anthracite fields as the semi-bituminous are border coals of more distinctly bituminous fields, but the latter may more often separately occur. In the United States these semi-bituminous coals include those from Clearfield, Cambria, and Somerset counties, Pa., Cumberland, Md., Pocahontas, Va., and New River, West Va. They are perhaps most definitely fixed by the volatile content of 18 to 22 per cent low ash and sulphur, and are considered the best boiler coals in America.

More than any single thing the character of the volatile characterizes the different coals and serves to group them naturally into classes, the bituminous coals, more especially those of the caking or coking class, having complex hydrocarbon mixtures with much of the liquid variety yielding tar; the non-

coking semi-bituminous varieties yield a simpler set of hydrocarbons with anthracite at one end characterized by almost pure methane volatile, and lignites and peats at the other, characterized by much non-combustible matter in their volatile, diluting the hydrocarbon constituents. It is not possible to say that the volatile gases or vapors that finally appear really existed in the coal as such. It is more likely, as has been pointed out, that they appear as products of decomposition of other oxygenated compounds.

Table LV, previously referred to, presents a collection of proximate and ultimate analyses selected, from thousands available, by reason of the high authority and reputation for accuracy of the analysts reporting them and also because they fairly well represent, when grouped together, the whole range of solid fuels from the peats to the anthracites. For each fuel the calorific power, as determined by the bomb calorimeter, is given which will be used later as a basis for a general study of calorific power.

The impurities in coal, notably the ash and sulphur, have an appreciable influence on the uses to which coals may be put, often not inferior, to the properties of the volatile or the calorific power of the coal. Sulphur generally occurs as iron pyrites, FeS_2 , in nuggets or veins and on combustion usually gives off SO_2 as a gas, leaving iron oxide, Fe_2O_3 , as part of the ash if there is insufficient air, as is usually the case in the interior of the cinder; while in the region of high temperature there will be formed iron sulphide, FeS , and sulphur which escapes as vapor. Sulphur is also present in some coals as calcium sulphate or gypsum, which leaves calcium sulphide when heated in contact with carbon. Sulphur in almost any form will make compounds with other things present, which fact bears on the availability of the fuel for certain metallurgical operations; for example, the making of iron. Sulphides, especially of iron, are apparently important factors in the fusing of the ash and the making of clinker. Both in the solid combined form and as a gas, sulphur is actively corrosive and grate bars as well as boiler plates and the pipes and scrubbers of gas generators will suffer from its presence so that high sulphur coal is considered generally undesirable, though sometimes some sulphur may be removed by washing.

Fusion of ash, and clinker formation is a serious matter, limiting the air supply, sticking to grate bars, and when it contains sulphur actually corroding the bar by chemical reaction, while in gas producers clinker collection is often so severe as to stop the operation entirely. In fact certain coals cannot be used in producers at all because of this tendency, though otherwise desirable. The formation of clinker starts with the fusion of some one of the constituents of the ash or the fluxing of one constituent by another, the liquefied material flowing over and enclosing solid matter, even large pieces of unburned coal, and tending to stick to other pieces, to firebrick linings or to grate bars. The clinkering tendency of a given coal cannot be predicted even when the ash is completely analyzed and the melting-points of its constituents known, because the possible compounds of those constituents are not known. The chemical compounds in ash are usually oxides of aluminum

Al_2O_3 , silica SiO_2 , iron FeO and Fe_2O_3 , calcium CaO , potassium K_2O , magnesium MgO , sodium Na_2O , sulphur SO_2 , with many more all combined with each other in unknown ways. Those occurring in the largest quantities are usually the alumina and silica, the melting-points of which are 3400°F . and 2510°F . respectively, which are within range of the usual furnace temperature of 3000°F . or thereabouts. The melting-point of either of these constituents or a mixture of them is lowered by fluxing with the other compounds, making glasses and slags, some of which melt below 2000°F ., so that in practically every ash some fusion will occur and clinker form if the fire is pushed at all. Just what is to be done in any case to avoid trouble, in view of the certainty of some clinker and the possibility of a prohibitive amount cannot at present be settled on any scientific grounds, but is a matter of trial and judgment, first in the treatment of the most available coal and second in the selection of a substitute.

82. Calorific Power of Coals and the Combustible of Coals. Calculation of Calorific Power from Ultimate and Proximate Analyses. Calorific Power of the Volatile. If the carbon, hydrogen and sulphur in coal were just mixed together, the calorific power would be the sum of the products of each fractional weight into the calorific power of the respective element, but no such simple relation exists, so that calculation of calorific power from the ultimate analysis must be based on certain assumptions as to molecular relations, expressed or implied.

Most of the ordinary fuels are mixtures of chemical compounds of unknown molecular constitution, and usually great complexity of molecular structure, not at all revealed by ultimate analysis and only partly so by proximate analysis, or as in the case of the liquids by fractional distillation. They are all compounds and mixtures of the elements carbon, hydrogen, nitrogen oxygen, and sulphur, with or without neutral ash. This fact lies at the basis of most of the empiric formulas for their calorific power, for if the fuel elements, C, H_2 , and S, shown to be present by an ultimate analysis be assumed to be just mixed together in the weights given, the calorific power is the sum of that of all the parts and can be expressed symbolically:

Let Q_H = B.T.U. per pound of hydrogen;
 Q_C = B.T.U. per pound of carbon;
 Q_S = B.T.U. per pound sulphur;
 Q = B.T.U. per pound fuel;
 w_H = weight of hydrogen per pound fuel;
 w_C = weight of carbon per pound fuel;
 w_S = weight of sulphur per pound fuel;
 w_O = weight of oxygen per pound fuel.

Then

$$Q = Q_C w_C + Q_H w_H + Q_S w_S. \quad \dots \dots \dots (618)$$

This, however, takes no account of the oxygen which may be assumed to be free and so ignored, or assumed to be combined as an incombustible.

By weight, 1 lb. H+8 lbs. O₂, make 9 lbs. water, so that each pound of O₂ that is combined with H₂ as water takes from the total H₂ present $\frac{1}{8}$ of the oxygen weight, leaving as burnable only what is left or $(w_H - \frac{1}{8}w_O)$. Introducing this expression in the formula above

$$Q = Q_C w_C + Q_H (w_H - \frac{1}{8}w_O) + Q_S w_S = Q_C \left[w_C + \frac{Q_H}{Q_C} (w_H - \frac{1}{8}w_O) + \frac{Q_S}{Q_C} w_S \right]. \quad (619)$$

This is the form of one of the first empiric formulas as given by Dulong; the sulphur effect was neglected and 14,500 and 62,100 in round numbers taken as the calorific power of carbon and hydrogen respectively, giving the form

$$Q = 14,500w_C + 62,100(w_H - \frac{1}{8}w_O). \quad (620)$$

In the American Society of Mechanical Engineers Code for Boiler Testing there is recommended a formula which retains the sulphur effect and assigns 4000 as the calorific value, changing the others for carbon and hydrogen to give

$$Q = 14,600w_C + 62,000(w_H - \frac{1}{8}w_O) + 4000w_S \quad (621)$$

Neither of these expressions or others which have been proposed will agree with experimental determinations owing to the very wide complexity of molecular structure of the fuels they are supposed to represent. Most of them have been developed for coals of a single class, thus, for example, the Dulong expression Eq. (620) was intended to nearly represent a class of bituminous coals characterized by much volatile hydrocarbon.

As an example of an attempt to base the formula, at least in part, on the physical character of the coal that of Goutal, Eq. (622), is especially interesting and suggestive:

$$Q = 14,670w_C' + Kw_V, \quad (622)$$

where w_V is the volatile hydrocarbon weight, while w_C' represents the fixed carbon or coke, K is a constant supposed to have values, according to Goutal, varying with the per cent of volatile, and ranging from 23,400 when the volatile is 2-15 per cent to 18,000 when volatile is 15-30 per cent. When the volatile is 35-40 per cent $K = 16,200$. The percentage of volatile is by weight.

It has been pointed out that the solid fuels may be regarded as the sum of a combustible and a non-combustible part; and the combustible as the sum of a fixed carbon and a volatile part. On this basis it is possible to make approximations of calorific power if the nature of the volatile is known or the average calorific power of the volatile, without an ultimate analysis, using instead the simpler and easier proximate analysis which does not require elaborate chemical laboratory equipment for its determination.

The principal combustible gases of the volatile are hydrocarbons and these are of fairly constant calorific power per pound over a very considerable range of character. The highest calorific power is that of methane, 23,646 B.T.U. per pound, and few of those hydrocarbons that enter into coal volatiles, so far as known, have less than 75 per cent of this. If, therefore, it is found that

one group of coals is characterized by the same group of hydrocarbons in its volatile, in even approximately constant proportions, the calorific power of that volatile will be constant, at least as nearly constant as the calorific power of all the combustible taken together. It is also reasonable to expect that each class of coal will have a different group of hydrocarbon combustibles in the volatiles, the calorific power of which will vary accordingly, being highest for anthracites with methane as the principal constituent, lower for the bituminous varieties and least for lignites, the volatile of which is much diluted with non-combustible gases and vapors, similar to those yielded by wood. Thus the character of heating power of the volatile of the coals furnishes a new basis of classification with direct reference to availability as fuels.

To illustrate the peculiarities of the calorific power of coals and the dependence of calorific power on class of coal a new table, Table LVI, has been computed from the data given in the table of coal analyses.

Various charts were also drawn in order to determine these characteristics graphically. There appears to be no very definite relation between calorific power and Campbell's classification of the coal; great confusion exists in the bituminous region and also in the peats, so that any representative curve is impossible. The same is true when plotted according to Frazer's classification, namely, the ratio of fixed carbon to volatile. The points are differently distributed in this case, but with no more hope of a smooth curve being drawn through them.

The explanation of this situation must be found in the nature of the combustible and in its amount and kind, but especially in kind. As combustible is partly fixed carbon and partly volatile, and as fixed carbon has always the same calorific power, the real explanation must be sought in variations in the nature of the volatile itself, and, of course, its amount. In order that this possibility may be traced, the chemical and thermal properties of these coals are redetermined, as ash and moisture free, in Table LVI in the Handbook of Tables, which, therefore, gives the properties of the combustible matter only. In this table the calorific power of the combustible is reported, total and as divided between the fixed carbon and the volatile parts, and finally the calorific power of the volatile itself per pound is found. To get this result, the calorific power of the coal as determined by the bomb calorimeter is divided by the sum of the partial weights of fixed carbon and volatile, the quotient being the calorific power per pound of total combustible. The product of the fractional weight of the fixed carbon and 14,544, its known calorific power, gives the heat due to the combustion of the fixed carbon part of the combustible, and this subtracted from the B.T.U. per pound of combustible gives the heat per pound of combustible derived from its volatile. The heat per pound of combustible derived from its volatile only, when divided by the fractional weight of volatile in the combustible gives the B.T.U. per pound of volatile itself. These values of the calorific power of the volatile matter of the coals were also plotted, Fig. 121, and while the points thus

obtained were more or less scattered it was possible to draw a fair curve through them as shown. It is reasonable to expect that direct determination of the calorific power of volatile alone, will lead to a much smoother curve, but even with the present data it is possible to make a very useful approximation to the calorific power of a coal from its proximate analysis by the formula, Eq. (623), derived from the smooth solid curve of Fig. 121.

Let w_c' = fractional weight of fixed carbon in coal;

w_v = fractional weight of volatile in coal.

Then
$$Q = 14,544 w_c' + 27,000 w_v \left[1 - \frac{1}{\left(\frac{w_c'}{w_v} \right) + .5} \right] \dots \dots (623)$$

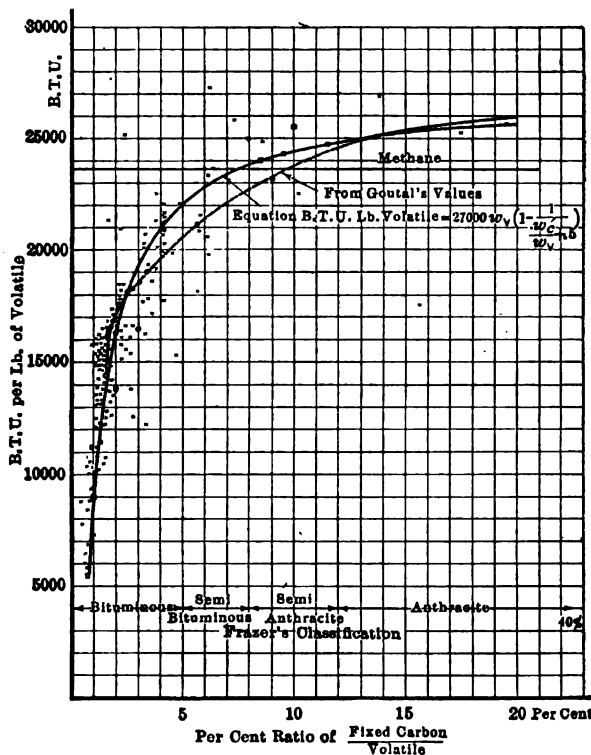


FIG. 121.—Calorific Power of the Volatile of Coals.

This equation is of the same general form as Goutal's Eq. (622), but with his constant C evaluated as a function of the fixed carbon to volatile ratio, instead of assigning it separate values for each class of coal by the volatile content alone. The differences due to this are indicated in Fig. 121 by the relation of the dotted to the solid line.

These relations not only make it possible to calculate the calorific power of a coal with fair accuracy from its easily found proximate analysis, but more important in a broad sense it brings out the relation of the coals as fuels to the

oils and natural gas fuels, as will appear a little more clearly when these latter are examined.

Example. A coal which gives a heating value of 13,961 B.T.U. when tested in the calorimeter, was found to have an ultimate analysis as follows: C=80.03, H₂=4.13, N₂=1.40, O₂=3.20, S=1.90, ash=9.34. How do the heating values derived by means of the Dulong Eq. (620) and A.S.M.E. Eq. (621) compare with the experimental?

Substituting the above values in Eq. (620) and (621) we obtain, respectively,

$$Q = 14,500 \times .8003 + 62,100 \left(.0413 - \frac{.0320}{8} \right) = 13,952 \text{ B.T.U. per lb.}$$

$$\text{and } Q = 14,600 \times .8003 + 62,000 \left(.0413 - \frac{.0320}{8} \right) + 4000 \times .019 = 14,069 \text{ B.T.U. per lb.}$$

Prob. 1. Which of the following coals would be worth most per ton on the basis of the heating value found by the A.S.M.E. formula? By the others?

For the first C=83.75; H₂=4.13; S=.57; O₂=2.65.

For the second C=74.39; H₂=4.98; S=3.44; O₂=6.42.

For the third C=73.5; H₂=5.19; S=2.54; O₂=8.05.

Prob. 2. A boiler horse-power is 33,486 B.T.U. received by the water in the boiler per hour. If 70 per cent of the heat of the coal is available, what will be the horse-power of a boiler under which is being burned one ton per hour of the following coal, the correct calorific power of which is given by the Dulong expression? What is the weight of the coal per hour per boiler horse-power? C=79.20; H₂=4.30; O₂=2.15.

Prob. 3. Natural gas of 1000 B.T.U. per cubic foot may be purchased in a certain locality for 10 cents a thousand cubic feet. What price could coal containing 75 per cent of C, 6 per cent of H₂, 1.5 per cent of O₂, and 1.5 per cent of S, bring in the same locality on the basis of heat content only?

Prob. 4. A gas engine is operated on producer gas. The producer is supplied with coal of the following composition: C=80.15 per cent; H₂=3.35 per cent; N₂=1.32 per cent; O₂=4.28 per cent; S=.9 per cent; ash=10 per cent. What is the efficiency for the system if the engine delivers one horse-power for each pound of coal fed to the producer?

Prob. 5. In a boiler plant it has been found that of the entire heat in the coal 60 per cent gets to the steam, 25 per cent escapes to the stack, 13 per cent to the ashpit, and the remainder is radiated from the boiler setting. For a coal of the following composition, what would be the loss in number of B.T.U. per pound of coal in each, and what would be the number actually used? C=70; O₂=2.5; N₂=2; H₂=12; S=.5; ash=13.

83. Mineral Oil and Natural Gas Fuel. Chemical and Physical Properties. Calorific Power Direct and as Calculated for Oils from Ultimate Analysis or from Density, and for Gas from Sum of Constituent Gases. Mineral oils are very much alike in ultimate composition, but entirely different in properties such as boiling-point. When clean, all are absolutely ash free, all yield vapors on heating and have calorific powers very close together, compared to the variations noted for coals. Analysis shows them to be hydrocarbon compounds or rather mixtures and solutions of a large number of hydrocarbons of the paraffine, ethylene, and naphthalene series, the former predominating in American and the latter in Russian oils, these two countries furnishing the

bulk of the world's supply. Among the paraffines, given in the Tables, there are noted four isomers, that is, compounds having the same ultimate analysis and fractional weight but different physical properties; while with the ethylenes, all of which have the same ultimate analysis but different molecular weights, are noted two naphthalenes which are isomeric modifications of the ethylenes, having the molecular arrangement indicated by the formula $C_nH_{2n-6} + H_6 = C_nH_{2n}$, which have the same weight proportions but whose properties are more closely those of the paraffines than the ethylenes. As pointed out in Section 76, the calorific power of all hydrocarbons varies with the value of n or their position in the series and is somewhat different for the same value of n in different series. It is also true that both liquid and vapor densities vary with the value of n , so it might be expected that the calorific power would vary with the density of the oils if they did not contain excessively variable proportions of different series. This was foreshadowed by the Slaby formula, Eq. (593), for the hydrocarbon gases and vapors which expresses calorific power as a linear function of density of those gases, and is confirmed by the work of Sherman and Kropff at Columbia University, for the oils. Table LIX in the Handbook of Tables gives the results of their calorimeter determinations on 64 oils, ranging from specific gravity of .7100 to .9644 or 67.7° Bé. to 15.2° Bé., together with the calculated value as a linear function of density by Eq. (624).

$$\text{B.T.U. per lb. oil} = 18,650 + 40(\text{Bé.} - 10). \quad (624)$$

In only one-ninth of the cases is the error more than one per cent, in only one-thirtieth over two per cent, and never exceeds three per cent, which is fairly satisfactory. This makes it possible to estimate the calorific power of an oil by simply taking its density.

Inspection of tables of properties of mineral oils, shows that the carbon-hydrogen ratio, which for the ethylenes and naphthalenes is constant and equal to six and for the paraffines always less than this, does sometimes exceed this value. In such cases it must be assumed that solid carbon is present, having been deposited by the decomposition of some of the oil under heat treatment before or after leaving the earth, and such oils are invariably heavy and black in color. No natural or crude oil is heavy or dense because it consists only of heavy hydrocarbons, nor is any one light because its hydrocarbons are exclusively light ones, but as all crudes contain both heavy and light constituents the density is chiefly an indication of which class of hydrocarbons, the light or the heavy, predominates. These relations, which are of fundamental importance in fixing the industrial value of an oil and in prescribing the treatment it shall receive in applying it to oil engines, boiler furnaces or the making of oil gas, will be brought out more clearly in a succeeding section under heat treatment for fractional distillation. This fractionation of oils is the only known way of reducing the range of hydrocarbon constituents. For properties of mineral oils, see Table LXII in the Handbook of Tables.

The calorific power of the oils cannot be as accurately predicted from their ultimate analysis by computation of the heat of formation of the products of combustion, as it can from the density by Sherman and Kropff's formula, though there seem to be in some cases large differences between the experimental bomb values and those from the formula. In most cases if not all, these can be traced to inaccurate experimental work, which, with oils, especially those that have light and easily volatile constituents, is a very difficult procedure. To still more clearly bring out this point, there are plotted in Fig. 122 all the tabular values of calorific powers, exclusive of the heats of formation of the products, and through them the S. and K. line is drawn according to its equation. It will be noted how much better their experimental values lie with respect to this line than do others of more doubtful accuracy.

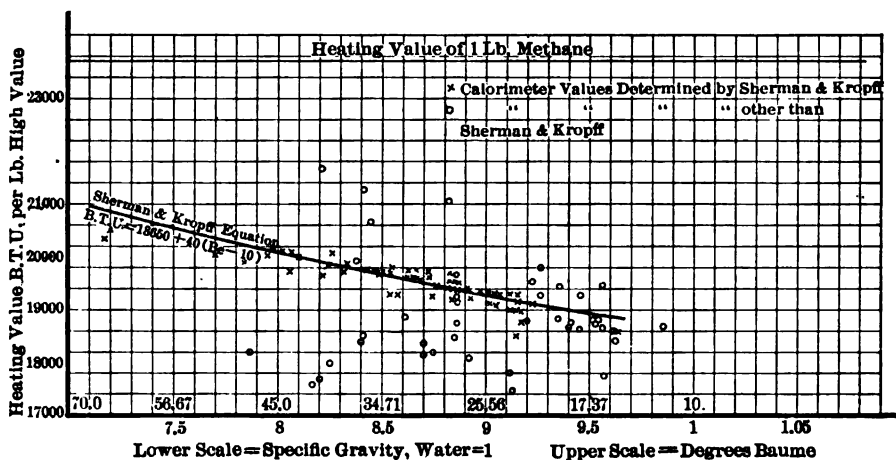


FIG. 122.—Calorific Power of Mineral Oils.

Natural gas like mineral oil is also chiefly a mixture of hydrocarbons though, of course, no such complexity of mixture can exist in the gases as in the liquids because so few of the hydrocarbons are gaseous at ordinary temperatures; most of them being liquids, and a few, solids. Accordingly, natural gas is a very definite thing and analysis shows it to be largely and in some cases almost entirely methane CH_4 . One sample, No. 1, from the series given in the Tables, is over 99 per cent methane, but another sample, No. 38, is only about half methane, and contains nearly 36 per cent by volume of free hydrogen. Practically all natural gases contain some CO , which is a product of oxygen reacting on carbon, probably in a coal, and as might be expected, there is correspondingly some nitrogen present, as would be the case if the oxygen that combined had come from the air, leaving the nitrogen free after combustion. Similarly in many cases some CO_2 is present, indicating a more complete oxidation of the carbon and accounting for more nitrogen on the assumption that the oxygen had been derived from air.

One important class of chemical constituents of some natural gases

is the higher hydrocarbons such as ethylene, but more peculiarly those that at ordinary temperatures are liquid, and are present in the gas as vapors of those liquids, together with a group that liquefy under pressure. This has led to a process of compression of natural gas with subsequent cooling for the recovery of liquid forms of hydrocarbons most of which are easily volatile and all of which are high in illuminating value.

The calorific power of natural gas, like any other gas, is very much easier to predict with reasonable precision than that for coals and oils, when its constituent gases are known. Each of these has a definite calorific power per cubic foot at standard conditions of 32° and 29.92" Hg, so that the calorific power is to be found by multiplying the volumetric proportion of each constituent by the proper calorific power. The sum will be the calorific power of the gas in B.T.U. per cubic foot (standard); and, of course, there will be both a high and a low value for every gas containing any hydrogen, free or combined, which includes all of the natural gases. For convenience, in calculations on temperature rise, due to combustion, the B.T.U. per pound is usually added. When some of the constituents are the heavier hydrocarbons, which are not separated by the absorption methods of gas analysis, they are generally reported together as heavy hydrocarbons, as illuminants, or in some equivalent terms. In all such cases, heat calculations require that a chemical constitution be assigned to these, usually ethylene and benzole in some proportion, as judgment based on experience may indicate. This is the only element of uncertainty as to calorific power that is worth noting.

To illustrate the method of determination of density and calorific power per cubic foot and per pound one case is set down in convenient tabular form.

TABLE VII
DENSITY AND CALORIFIC POWER OF NATURAL GAS FROM CONSTITUENTS
(32° F. and 29.92" Hg)

Kansas Natural Gas.	One Cubic Foot.				Summary.
	Contains		Yields B.T.U.		
	Cu.Ft.	Lbs.	High.	Low.	
Methane, CH ₄9820	.043935	1040.5	944.7	B.T.U./cu.ft. gas.high = 1046.3 B.T.U./cu.ft. gas low = 950.1
Heavy hydrocarbon, C ₆ H ₆0010	.000240	5.0	4.6	Cu.ft./lb. gas = 22.42
Carbon monoxide, CO....	.0025	.000195	.8	.8	Lbs./cu.ft. gas = .0446
Oxygen, O ₂0025	.000223	B.T.U./lb. gas high = 23458.
Total for gas.....	1.0000	.044593	1046.3	950.1	B.T.U./lb. gas low = 21301.

Prob. 1. The following oil was used in an engine and one horse-power hour (2545 B.T.U.) was obtained for each pound of the oil supplied. What was the thermal

efficiency based on the heating value, (a) as derived by calculation from the ultimate analyses, and (b) by the Sherman and Kropff formula?

Sp.gr. at 60° F. = .926. $C = 83.36$; $H_2 = 12.41$; $S = .5$; $(N_2 + O_2) = 3.83$.

Prob. 2. At one time a melting device was used to clear away snow from city streets. The fuel used was oil and the heat generated was used to melt the snow placed in a tank above the fire, the water running off to the sewer as soon as formed. Assuming 90 per cent of the heat of the fuel to be available, that the snow is put in at a temperature of 20° F. and that the water runs away at a temperature of 40° F., how many lbs. of snow will be disposed of per lb. of the following oil? $C = 85.5$; $H_2 = 14.2$; $(O_2 + N_2) = 3$.

Prob. 3. To prevent frost damage in orchards, crude oil is burned in smudge pots. If the specific heat of air is .243, how many cubic feet can be heated from 30° to 40° F. by a gallon of California crude oil, the density of which is 16.85° Bé. and the ultimate analysis by weight is $H_2 = 11.3$; $C = 85.75$; $S = .67$? Compute the results by S. and K. formula and by heat of formation of products.

Prob. 4. The following analyses are for a West Virginia and an Ohio natural gas respectively by volume. On the basis of the heat of formation of the products of each, what would be their relative heating values?

	CH ₄	H ₂	CO	C ₂ H ₄	N ₂	CO ₂	O ₂
West Virginia..	99.55
Ohio.....	93.35	1.64	.41	.35	3.41	.25	.39

Prob. 5. The oil, the analysis of which is given below by weight, may be had for 5 cents per gallon. At what price will the gas, the analysis of which is also given, but by volumes, be an equally economical fuel, the economy being based solely on the heat value?

Oil: $C = 82$; $H_2 = 14.8$; $O_2 + N_2 = 3.2$.

Gas: $CH_4 = 75.99$; $H_2 = 6.1$; $C_2H_4 = 18.12$; $CO_2 = .34$.

Prob. 6. A ton of coal occupies roughly 40 cu.ft. Compare the space occupied by a million B.T.U. in the form of fuel oil and a bituminous coal.

Prob. 7. Assuming a boiler efficiency of 70 per cent for both fuels, what will be the consumption of a crude oil and an anthracite coal per boiler horse-power?

84. Charcoal, Coke, Coke Oven and Retort Coal Gas as Products of Heating Wood and Coal. Chemical, Physical, and Calorific Properties per Pound. Calorific Power of Gases per Cubic Foot in Terms of Constituent Gases. Yield of Gas and Coke per Pound of Coal. When *wood* is heated, a distillation process begins at about 400° F. before which the discharge is mainly water vapor, but after this temperature is reached complex gases and vapors are liberated, some of which may condense. A few of these distillates were present as such in the wood, and are liberated unchanged, but most of them are compounds formed by the heat action in the fiber, sap, and ash constituents by mutual chemical reactions. The products of heating wood as given by Jüptner show the surprising complexity of the process and products which are divisible into five groups:

- (1) Hygroscopic water; (2) Gas; (3) Liquid tar; (4) Pyroligneous acid;
- (5) Wood charcoal.

When the heating of wood takes place in closed chambers, the wood melts as coking coals do, but at temperatures in the neighborhood of 600° F., yielding a sort of hard coke quite different from charcoal. The whole process is summed up concisely by M. Violette, who indicates the changes in the result brought about by gradually rising temperatures of distillation, which are briefly, an increased gas yield continuously, and a liquid yield first increasing and then decreasing, showing that at high temperatures some of the liquid is decomposed into gaseous constituents. In all cases the yield of charcoal decreases and its quality changes. The charcoal at higher temperatures contains more carbon, about the same hydrogen and regularly lessening amounts of oxygen and nitrogen, the sum of which is, however, always large and indicates why peats and lignites near the woods have volatiles so weak in combustible matter. Quick coking produces only half the charcoal that results from a slow process.

Peat on heating also gives off an equally large variety of volatile products, the nature of which is intimately related to the fuel value and its treatment in boiler fires and producers.

The most striking difference between the products of peat and wood distillation from the fuel standpoint is the appearance of heavy hydrocarbon, in the former in the gas form and an increase of it in the liquid forms, or a general rise of hydrocarbon combustible matter in the volatile, because the liquids appear first as vapors mixed with the permanent gases.

Lignite yields on heating much the same sort of products as peat does but there are two groups, one in which the hydrocarbon or bituminous substances are larger than in the other, probably because of different origin. The coke is not of very much value and being little used there are few data available. The coke from such of the lignites as do not disintegrate by reason of loss of water of crystallization will be a little less than half the coal by weight, the volatile, from one-seventh to one-third, and tar-water from one-eighth to one-fifth. Thus, in the lignites that will coke, there is an increase in volatile combustible, while those that disintegrate have combustible close to peat. The former might properly be called sub-bituminous coal by reason of the difference, and the latter solidified peat or lignite proper.

Bituminous coals are the great raw materials for roasting treatment for the manufacture of coke, (a) in beehive ovens where the volatile is burned and wasted, (b) in by-product coke ovens which yield not only valuable coke but also illuminating and fuel gases besides chemical by-products; and also for the primary manufacture of illuminating gas in retorts where the coke is a by-product. The study of the distillation or roasting of the bituminous coals, is therefore not only important as throwing light on their fuel properties, but also because of its relation to the illuminating gas and coke industries. In what follows the fuel properties will be dwelt upon rather than the chemical by-products recovery, or the production of illumination, or the metallurgical properties of coke.

While there is a very great difference between the products from the dif-

ferent bituminous coals, the following by Wagner may be taken as a basis of comparison:

Coal: 78 per cent C, 4 per cent disposable H_2 , $1\frac{1}{2}$ per cent N_2 , .8 per cent S, 5.7 per cent combined H_2O , 5 per cent hygroscopic H_2O , 5 per cent ash.

1. Tar water or ammonia water containing,
 - (a) Water, carbonate of ammonia and sulphide of ammonia.
 - (b) Chloride, cyanide, sulphocyanide of ammonia.
2. Gas containing,
 - (a) Illuminants

{	gases—	acetylene, ethylene, propylene, butylene;
	vapors {	benzole, styrole, naphthalene, propyl, butyl, acetylnaphthalene;
 - (b) Combustible non-illuminants—hydrogen, methane, carbon monoxide;
 - (c) Impurities—carbon dioxide, ammonia, cyanogin, rhoden, sulphureted hydrogen, carbon disulphide, nitrogen.
3. Liquid tar containing,
 - (a) Liquid hydrocarbons—benzole, toluol, propyl, butyl, etc.;
 - (b) Solid hydrocarbons—naphthalene, etc.;
 - (c) Substances containing oxygen—phenol, creosote, aniline, etc.;
 - (d) Asphaltic substances and resins.
4. Coke, 70 to 75 per cent of the coal containing 90 to 95 per cent of combustible matter, and 10–5 per cent ash.

These results, when compared with analyses of the lower fuels, show most clearly a rise of hydrocarbons in both gaseous and liquid form, which together constitute the volatile of the coal, so that the volatile has combustible properties very close to those of vapor oils. The diluent materials in the volatile are small in amount but may increase on excessive heating. The manner of heating, as to time and temperature, largely control the nature of the products, gases, coke or by-products. Moderate temperatures are necessary to secure high yields of ammonia and for highly illuminating gas, *high* temperatures increase the coke at the expense of the carbon of the hydrocarbons, decomposing them to methane, hydrogen and carbon. The products differ, moreover, with the period of coking, as the temperature does also as a rule, fresh cold coal being charged which is slowly heated. During the first periods most of the richer hydrocarbons forming the tar and illuminants come off, constituting the high calorific power part of the volatile. Toward the end of the coking period and that of highest temperature the gases are free of tar and low in calorific and illuminating power. It has been suggested that the first period corresponds to the formation of coke and ends when the coke has *set* or taken its permanent size and shape, while the second is a period of coke decomposition or rather distillation of the freshly set coke; this is a useful distinction to keep in mind for boiler and gas producer application of fuels.

The variation of gas with time of coking is shown in Fig. 123 which has been plotted from results reported by Blauvelt for both a high and a low volatile American coal in the Solvay by-product coke oven.

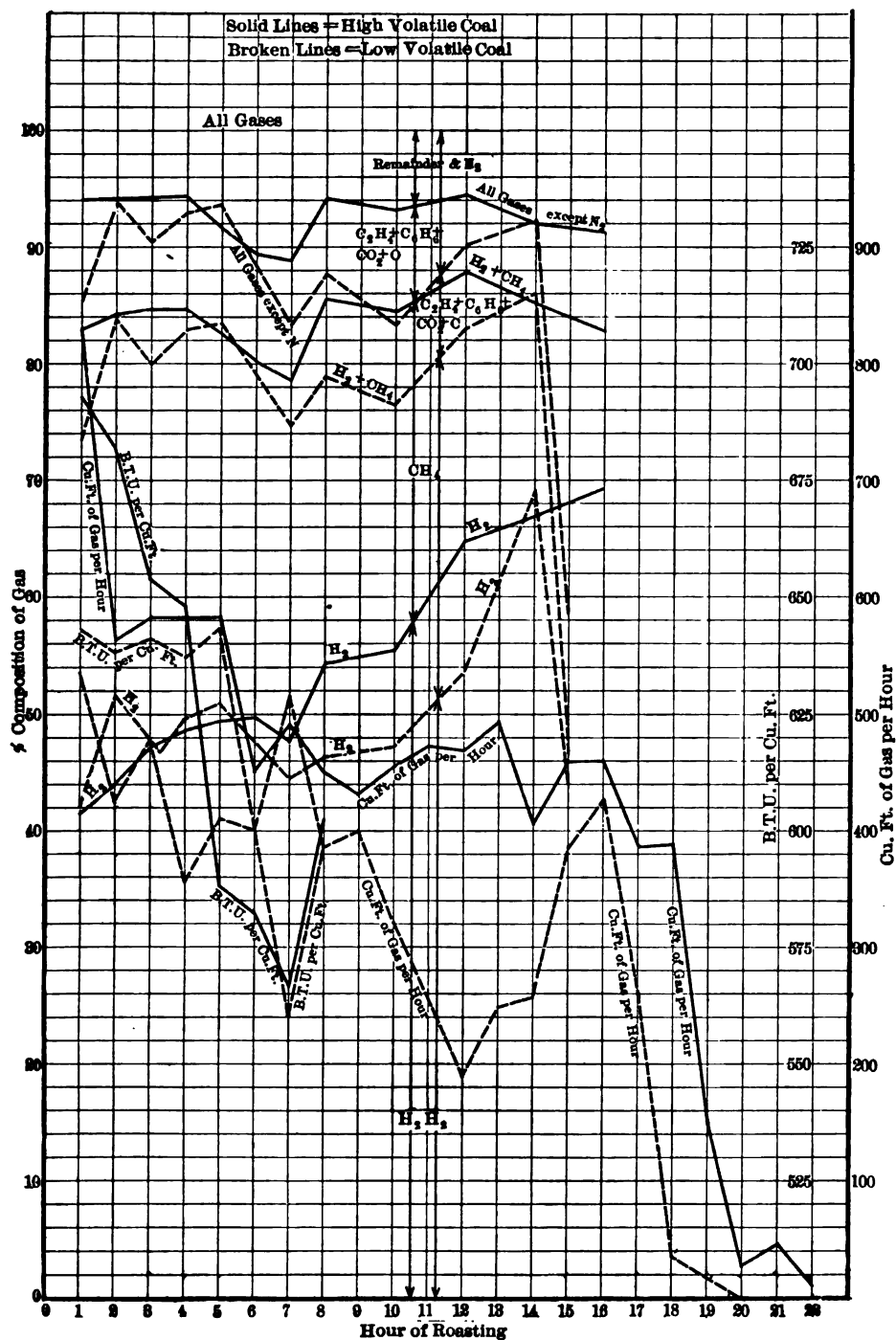


FIG 123.—Variation in the Composition of By-product Coke Oven Gas with Time of Roasting.

Coke ovens, so far as the fuel processes are concerned, are exactly the same as gas-house retorts except for size, shape and temperature. The former yield more coke than the latter where the temperatures are higher, time of gas contact with coal shorter, and with walls longer. The hydrogen is higher and hydrocarbons and tar yield lower in the retorts, probably because of the temperature and contact conditions, but just how or why these variations occur is not apparent.

A number of analyses of coal gas and by-product oven gases are given in the Handbook of Tables taken from a variety of sources to show possible limits. The following, Table VIII, gives a comparison of two analyses considered by Jüptner as typical of the two classes, with Blauvelt's typical coke oven gas and Güldner's typical retort coal gas.

TABLE VIII
COMPARISON OF COKE OVEN AND RETORT COAL GASES

Constituents.	Coke Oven Gas Per Cent by Volume.			Retort Coal Gas Per Cent by Volume.	
	Blauvelt.	Güldner.	Jüptner.	Güldner.	Jüptner.
Benzole, C_6H_6	1.2	.81	.61	1.54
Ethylene, C_2H_4	4.2	1.52	1.63	4.5	1.19
Methane, CH_4	35.5	32.40	36.11	35.0	36.00
Hydrogen, H_2	48.0	55.60	35.32	48.5	55.00
Hydrogen disulphide, H_2S43
Carbonic acid, CO_2	1.3	1.21	1.41	2.0	.87
Carbon monoxide, CO	5.1	7.08	6.49	7.0	5.40
Oxygen, O_2525
Nitrogen, N_2	4.2	1.38	2.75

The coke itself is a by-product in the gas retort and no attempt is made to obtain a good quality, the retort management and the selection of coal being both guided by the desire for low coke and high gas yield with high illuminating value. Beehive coke ovens are operated with regard to nothing but the largest production of good, hard coke of large pieces and good porosity, for use in foundry cupolas and in blast furnaces, while by-product coke ovens are designed with several ends in view: first good coke, second large yield, third good gas for heating, power, and illumination, and fourth chemical by-product recovery. It has already been shown how the gases of the retort and coke oven compare; they contain not the same quantities but nearly so and the same kinds of constituents, the differences being mainly in the hydrocarbons which for the retorts and high heats are simpler in character with more hydrogen and less carbon content. The beehive coke is generally regarded as the best for iron making but the difference compared to oven coke is not great. Retort coke is, however, very different because (a) it is made from a non-coking or semi-coking coal and (b) has much soot deposited from decomposed hydrocarbons and is very dense, close grained, hard, black and always in small pieces. In the beehive the yield from a given coal must always be less than from the

oven because some is burnt with the gases and so is less than the fixed carbon, whereas in ovens or retorts the yield is always greater than the fixed carbon. Some coke analyses are given in the tables from the U. S. Geological Survey reports of tests on American coals, mainly to show their characteristics as fuels; reference must be made to metallurgical works for analyses and properties of the coke adapted for treatment of the metals or ores.

Deulle states that the best gas coals should contain from 7.5 to 9 per cent of oxygen; less than this means a high coke yield but gas poor; more will yield bad coke, *indicating that the illuminating hydrocarbons are originally oxygenated substances like fatty acids or alcohols decomposing to form ethylene, methane, hydrogen and oxides of carbon, which are not primarily present as such.*

The calorific power and density of coke oven and retort coal gases may be calculated from the constituent gases and their respective constants as previously shown for natural gas, the operations being most conveniently set down in tabular form, as follows, Table IX.

TABLE IX
DENSITY AND CALORIFIC POWER OF COKE OVEN AND RETORT COAL GAS
FROM CONSTITUENTS (32° F. AND 29.92" Hg)

Constituents of Average Coke Oven Gas.	One Cubic Foot				Summary
	Contains		Yields B.T.U.		
	Cu.ft.	Pounds.	High.	Low.	
Hydrogen, H ₂4200	.0023607	143.220	122.640	B.T.U. per cu.ft.
Methane, CH ₄3430	.0153470	365.638	328.937	gas high..... 642.16
Carbon monoxide, CO..	.0600	.0046840	20.460	20.460	B.T.U. per cu.ft.
Heavy { C ₂ H ₄ ..	.0200	.0015880	34.000	31.900	gas low..... 579.84
Hydrocarbons { C ₆ H ₆ ..	.0200	.0043800	78.840	75.900	Pounds per cu.ft.
Carbon dioxide, CO ₂0250	.0030620	0.0	0.0	gas..... .04031
Oxygen, O ₂0110	.0009812	0.0	0.0	Cu.ft. per lb. gas 24.807
Nitrogen, N ₂1010	.0079070	0.0	0.0	B.T.U. per lb. gas
Total for gas.....	1.0000	.0403099	642.158	579.837	high..... 15930
Constituents of Average Retort Coal Gas.					B.T.U. per lb. gas
Hydrogen, H ₂5250	.0029509	179.025	153.3000	low..... 14384
Methane, CH ₄3135	.0140310	334.191	300.6400	B.T.U. per cu.ft.
Carbon monoxide, CO..	.0860	.0067140	29.326	29.3260	gas high..... 604.6
Heavy { C ₂ H ₄ ..	.0110	.0008740	18.700	17.5450	B.T.U. per cu.ft.
Hydrocarbons { C ₆ H ₆ ..	.0110	.0024100	43.362	41.7450	gas low..... 542.56
Carbon dioxide, CO ₂0150	.0018400	0.0	0.0	Lbs. per cu.ft. gas .031872
Oxygen, O ₂0035	.0003121	0.0	0.0	Cu.ft. per lb. gas... 31.375
Nitrogen, N ₂0350	.0027400	0.0	0.0	B.T.U. per lb. gas
Total for gas.....	1.0000	.0318720	604.604	542.5560	high..... 18969.5
					B.T.U. per lb. gas
					low..... 17022.7

NOTE. In the following problems the numbers refer to Table LXIII, in the Handbook of Tables.

Prob. 1. A natural gas which was practically 100 per cent CH₄ was procurable for

7 cents per 1000 cu.ft. Compare the cost of coal gases Nos. 1 and 17 at \$1.00 per 1000 cu.ft., with this natural gas on a heat-unit basis.

Prob. 2. Five cubic feet of coal gas No. 10, were made per pound of coal, and the gas was sold at \$1.10 per million B.T.U. What was the cost per ton (2000 lbs.) of coal if 50 per cent of the value of gas was charged to coal?

Prob. 3. What will be the cubic feet of gas used per horse-power per hour by an engine running on coal gas No. 71 with a thermal efficiency of 20 per cent?

Prob. 4. A hall is lighted by 50 lights each consuming 5 cu.ft. of gas per hour. If the hall be 100 ft. \times 50 ft. \times 20 ft. and the gas used be No. 43, how many times per hour must the air be changed to prevent a temperature rise of over 5° F.?

Prob. 5. A thousand cubic feet of coal gas No. 12 are being forced through a pipe per minute. How many of the following units are being transmitted per minute? (a) Foot-pounds; (b) B.T.U.; (c) Horse-power hours.

85. Distillate Oils, Kerosene, Gasolene, Residue Oils; and Oil Gas as Products of Heating Mineral Oils. Chemical, Physical and Calorific Properties. Calorific Power of Fractionated Oils in Terms of (a) Carbon and Hydrogen; (b) Density per Pound, and Estimated Value per Cubic Foot of Vapor. Calorific Power of Oil Gas per Pound and per Cubic Foot in Terms of Constituent Gases. Yield of Distillates and Oil Gas. The discovery that crude mineral oil could be made to yield by simple heat treatment such valuable products as kerosene and gasolene, was one of great industrial importance, and it is surprising to find so little scientific information available as to the precise chemical and molecular nature of such important raw materials and products. It is known that all samples contain many hydrocarbons, and these usually of more than one series, it also appears that each sample consists of a mixture of several, more or less soluble in each other. Each hydrocarbon having a different boiling-point from the others, it should be possible to separate them by distillation but not very well or completely because of mutual influences. In the first place, if one is dissolved in the other the boiling-point of the solvent will be raised by the substance dissolved and the effect will be different as the proportions vary, and these will vary with continued boiling. Again, vapors of constituents having high boiling-points will escape with other vapors coming off at their own lower boiling-points in some proportion, because each exerts its own vapor tension in a mixture. Therefore, while distillation will accomplish some sort of separation no distillate from a complex mixture of solutions can itself be simple but will be of the same general nature, except that the proportions of the constituents will be different. This explains why distillates obtained at low temperatures will leave residues when they are themselves heated even to a very much higher temperature than that at which they were obtained.

The separation and naming of the various oil products is a purely local procedure and though based on difference in boiling-points mainly, is not practiced in the different refineries in the same manner. Therefore, a product of a given name, like gasolene, may be quite different, one sample from another. Table LXVI is given by Robinson in the Tables as the average practice

in separating and naming the products of American and Russian crude oil, but must be accepted with caution as the differences mentioned above may be quite marked, though for lamp-oil kerosene, considerations of safety have led to legal restrictions as to boiling and flash points that keep it reasonably constant.

A somewhat more common distribution of products with the American names and densities as now understood in the oil business is given in the following Table X:

TABLE X
AMERICAN MINERAL OIL PRODUCTS AT 60° FAHRENHEIT

Kind of Oil.	Degrees Gravity, Baumé.	Weight of One Gallon. 231 Cu.in. in Ounces.
Gasolene.....	93.5	83.5
Gasolene.....	91.	85.
Gasolene.....	90.	85.5
Gasolene.....	88.	86.
Gasolene.....	86.	86.5
Gasolene.....	78.5	90.
Gasolene.....	74.	91.5
Sumatra naphtha.....	74.	
Gasolene.....	72.	92.
Gasolene.....	62.0	97.5
Kerosene, 120°, water white.....	49.	104.5
Kerosene, 150°, water white.....	47.5	105.5
Kerosene.....	46.5	106.5
Celcius.....	42.	
Fuel oil, Pratt's.....		108.
Fuel oil, Lima.....		109.
Limpid residue, W. P.....	34.5	114.
Gas oil, E. W.....	34.5 to 35.0	
Paraffine, .865° sp.gr.....	32.5	115.5
Water, at 62° F.....		133.4

The percentages and gravities given in these tables are supposed to be average practice, but there is a question whether there is any average practice; certainly there is no such thing as an average crude oil. For example, California oils yield practically no burnable lamp oil, that is, clean, smokeless burning, but do yield much asphalt residue, and no paraffine; practically all the lighter constituents are collected in one lot and called distillate, which has some of the properties of gasolene and some of kerosene. Also the quantity of any one product usually sold by its specific gravity, or Baumé, as a means of definition or specification, can be varied greatly by mixing lighter and heavier constituents, and this is a regular practice as can be shown by fractional distillation of the product. Densities are always given at 60° F. and are corrected for kerosene at other temperatures by adding or subtracting .0004 sp.gr. per degree F.

To illustrate, first, the variations in the products having a given trade name; and second, the complexity of product, numerous fractionation tests

have been tabulated and are given in the Tables for crude petroleums, kerosenes and gasolenes.

The distillate between any two temperatures is by no means a simple substance nor is one sample the same as another, so that the gasoline collected in original manufacture between the same temperatures from different oils will not be the same. This is a fundamental characteristic of the boiling of solutions of different substances in each other, some of the vapor of every one will come off at any temperature, and while at low temperatures the amount of heavy vapors is small, they are always present in varying amounts for different oils, especially for different ways and rates of boiling that cannot be fully explained here.

For the United States the following, Table XI, gives an estimate of the oil characteristics from the different fields, divided into two groups. The first represents in output about 15 per cent of the total for the whole country and yields from 8 to 12 per cent of gasoline, while the second represents about 85 per cent of the total production, and yields from 1 to 4 per cent of gasoline, the calorific power of all varying not over 12 per cent.

TABLE XI
U. S. GASOLENE- AND KEROSENE-BEARING CRUDE OILS

Group I (8%-12% gasoline).		Group II (1%-4% gasoline).	
Pennsylvania. . .	40-50 B _é .	Illinois.	30-34 B _é .
Kentucky.	40-44 "	Kansas.	22-32 "
Ohio.	37-40 "	Oklahoma.	22-32 "
Indiana.	37-40 "	Louisiana.	22-32 "
		Texas.	16-26 "
		California.	12-28 "

The calorific power of distillates can be estimated from their carbon, hydrogen and sulphur contents by formulas of the Dulong type, but very much closer results will be obtained from the density formula of the type derived by Sherman and Kropff. Samples of gasoline and kerosene, tested by the U. S. Geological Survey, yielded the following formulas, Eq. (625) and Eq. (626), under the manipulation of Stanton and Strong for the gasolenes and Allen and Strong for the kerosenes, both suggested by Eq. (624).

$$\text{B.T.U. per lb. gasoline} = 18,320 + 40 (\text{Bé.} - 10), \quad . . . \quad (625)$$

$$\text{B.T.U. per lb. kerosene} = 18,440 + 40 (\text{Bé.} - 10). \quad . . . \quad (626)$$

The calorific powers of these samples, as determined by calorimeter and as calculated from the various formulas, show that the error by using the Strong constants is in every case less than three-tenths of one per cent; and less than two per cent with the general Sherman-Kropff formula, Eq. (624).

Residue oils always contain carbon in the free state as suspended solid matter and are heavy and viscous in nature. However, the carbon may be removed by filtration, or the density reduced and fluidity increased by adding

some light constituent, in which case they are very different from distillates or crudes of the same specific gravity, a fact that renders their use very difficult, as oils are generally bought on a specific gravity or Baumé specification. When the free carbon is not removed, the oils will burn very much more smoky than otherwise, and such oils cannot be used in some oil engines at all, because of carbon deposits, when the same engine can successfully use another oil of the same density and perhaps same calorific power. Specifications based on temperature fractions would avoid these difficulties.

Mineral oils when heated, not in a liquid mass but by slow admission to a hot chamber the temperature of which is above the boiling-point of some or all of their constituents, undergo a decomposition which may be called *destructive distillation* which is in many ways similar to the roasting of coal. It will yield a large quantity of permanent gases if the temperature be high enough and, mixed with them, a series of hydrocarbon vapors, mainly benzole if the temperature is sufficient; there will be left some heavy liquid residue and coke. Such gas after washing and cooling is called oil gas, and as made by the Pintsch process is the main illuminating medium of over 90 per cent of all

American steam railroad cars. Usually the oil used for this purpose is the grade between the lamp oil or kerosene and the lubricating oil, but there is considerable variation in practice and price, the latter largely controlling practice in the choice of raw material. Too low a retort temperature gives poor gasification, and tar and condensible vapors in excess; too high a temperature gives fixed carbon, reduced illuminants and increased hydrogen. Each oil must be treated a little differently from the others. The gas consists mainly of hydrocarbons and hydrogen, the hydrocarbons being much the same as those from coal gas but in different proportions. A recent investigation of Pintsch oil gas at Columbia University showed it to consist of 63.1 per cent CH_4 , 5.6 per cent H_2 , 27.4 per cent illuminants, 0.8 per cent O_2 , 0.4 per cent CO , and 27 per cent N_2 . Blau gas is the name applied to the condensible constituents of the distillation rejected in the Pintsch process, and as obtained by very high compression they include only those that freely revaporize on release of pressure.

From calorimeter results the nature of the illuminating hydrocarbons in

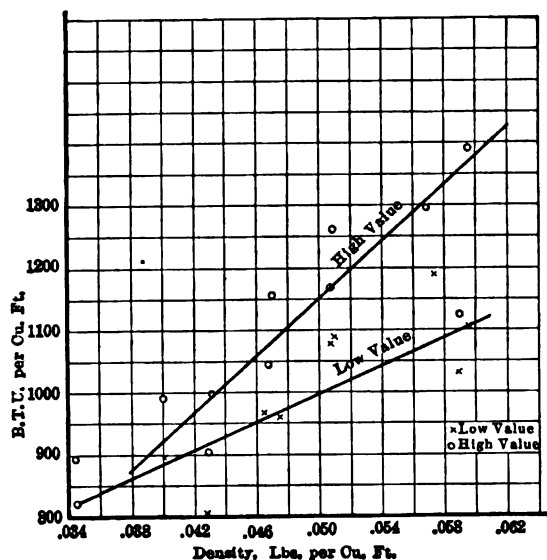


FIG. 124.—Calorific Power of Oil Gas.

the above-mentioned Pintsch gas was found to be represented by carbon 89 per cent and hydrogen 11 per cent by weight, which does not correspond to any one, but might be made up by mixtures of several hydrocarbons. In a somewhat similar way, the general conclusion has been reached that gasolene is most nearly represented by the simple hydrocarbon hexane, and kerosene by decane, but this is hardly better than an estimate to be used only when the volume of vapor per pound is to be determined by the molecular weight method, the only available means. The properties of oil gas will be found in Table LX, Handbook of Tables.

The calorific power of these oil gases is plotted to a density base in Fig. 124, which shows a straight line relation with density somewhat similar to that found for liquid oils. There are very little data available on the production or yield of oil gas except that of a confidential character, because its manufacture is not very general, but the following figures (Table XII) quoted by Guldner from the Gas Journal are useful as comparative if not as absolute data:

TABLE XII
YIELD OF RETORT OIL GAS

Oil Used.	B-naphtha Sp.gr. .730.		Kerosene. Sp.gr. .807.		Heavy Oil, Sp.gr. .847.			Heavy Oil, Sp.gr. .884.		
	1112	1562	1112	1562	1112	1472	2012	932	1112	1562
Retort temp. F.										
Cu. inches gas per cubic inch oil. . . .	451	625	469	582	401	513	594	213	368	657
Cu.ft. gas per lb. . . .	10.4	13.8	9.3	11.0	7.6	9.7	11.3	3.7	6.7	12.3
Residue % weight.	11.4	5.1	21.4	7.5	28.5	12.2	18.0	62.3	41.5	9.4

The calculation of the calorific power of oil gas with the assumption of nature of heavy hydrocarbons as half ethylene and half benzene may be laid out conveniently in tabular form as for natural and coal gas, and for this a typical analysis is given in Table XIII.

TABLE XIII
DENSITY AND CALORIFIC POWER OF OIL GAS
FROM CONSTITUENT (32° F. AND 29.92" Hg)

Constituents of Average Pintsch Oil Gas.	One Cubic Foot.				Summary.
	Contains		Yields B.T.U.		
	Cu.ft.	Pounds.	High.	Low.	
Methane, CH ₄525	.02347	559.6	504.5	B.T.U. per cu.ft. gas high..... 1290
Hydrogen, H ₂185	.00104	63.0	54.2	B.T.U. per cu.ft. gas low..... 1196.5
Heavy { C ₂ H ₄ ..	.117	.00930	198.9	186.6	Cu.ft. per lb. of gas 16.25
Hydrocarbons { C ₆ H ₆ ..	.118	.02588	465.1	447.8	Lbs. per cu.ft. of gas .06153
Carbon monoxide, CO..	.010	.00078	3.4	3.4	
Carbon dioxide, CO ₂005	.00061	B.T.U. per lb. high 20,962
Oxygen, O ₂005	.00045	B.T.U. per lb. low 19,443
Total for gas.....	.963	.06153	1290.0	1196.5	

The cooling of the distilling gases from either coal or oil in retorts produces a liquid generally termed tar, the relative character of which for the two cases is shown by the following ultimate analyses:

Tar from	Per Cent by Weight.						B.T.U. per lb.
	C	H ₂	N ₂	O ₂	Ash	S	
Coal.....	89.21	4.95	1.05	4.20	.06	.53	15,400
Oil.....	92.70	6.13	.11	.11	.05	.33	17,300

CHAPTER XI

GASIFICATION AND COMBUSTION OF FUEL.

86. Gasification of Fixed Carbon and Coke by Air Blast Reactions, Producing Air Gas, and Blast Furnace Gas. Comparative Yield per Pound of Coke and Air. Sensible Heat and Heat of Combustion of Gas. Relation of Constituents in Gas. Efficiency of Gasification. Carbon when hot enough may react with oxygen to form carbon monoxide or carbon dioxide and this is the basic principle for the gasification of carbon with the oxygen of air when the process is so controlled as to make a maximum of carbon monoxide, the product being called air gas. The principal source of air gas is the blast furnace in which the fuel is usually coke, but may be a hard coal, though the process as there carried out is not controlled with a view to getting high carbon monoxide content primarily, but rather to best permit the reduction of ore to metal. However, the conditions are generally such as to yield an air gas almost as good as if its production were the end desired.

This gasification process is characterized by a greater precision of relation between conditions and results than any dealing with hydrocarbons whether derived from coal volatile or from oils, and therefore it yields the better to prediction of results, though not so well as might be expected at first glance. While from the fundamental chemical reactions the relative weights of carbon, oxygen and gas can be set down exactly for a complete reaction, there are certain gaps to be filled by estimation, in attempting to predict a gas made from a coal or a coke even when its composition is known. In the first place *the three substances, CO, CO₂ and C are known to be in equilibrium in all proportions at some temperature*, the ratio of CO to CO₂ in the presence of an excess of C depending on the temperature. Therefore, in any fire where carbon is a fuel there will always be some CO₂ and some CO, but no one can say how much of each, even though the equilibrium proportions are known for all temperatures, because the temperature is a *resultant* and not an *imposed* condition, and to attain equilibrium requires sufficient *time* of contact, which is always unknown. No real fuel is all fixed carbon and seldom is fixed carbon in the same state, so that dense carbons like retort coke require longer time or higher temperatures for reaction than charcoals, while coals containing other constituents than carbon, such as nitrogen, oxygen, or hydrogen, both free and combined, further complicates the problem. Cokes even though they yield no gases on high heating will do so when by partial combustion the cell walls are broken down; in such cases hydrocarbons will be added to the air gas even in the late stages of coke combustion.

Accordingly, air gas while consisting mainly of carbon monoxide will also contain some carbon dioxide, some hydrogen and possibly some methane, besides the nitrogen derived from the air. Presence of oxygen in air gas is an indication of air addition after gas formation is complete, at a point where the gas has cooled below the ignition temperature of the combustible constituents. Air mixture with the gas in hot regions always results in the burning of some of the gas produced previously and is one cause of higher carbon dioxide content than the conditions of $\text{CO-CO}_2\text{-C}$ equilibrium warrant at the temperature.

Air gas as usually made is merely the product of blowing air into a *thick* bed of coal, coke or charcoal, the depth of carbon being maintained at from 2 to 6 ft. to insure *sufficient time* of contact between gases and carbon to complete the reaction. The necessary depth depends partly on the blast pressure which determines the flow velocity in a bed of given porosity and partly on the sort or condition of the fixed carbon fuel, being least for charcoal. Air gas is seldom made for itself, but as a part of other processes, for example, the blast furnace yields air gas as a by-product of iron making, called in this case blast-furnace gas; also water-gas making, to be described presently, requires a heated bed usually so prepared by a preliminary making of air gas, especially in the Lowe system of carbureted water-gas manufacture, where the air gas serves a double purpose, as it is burned to heat the carburetor after, by its formation, it has heated the carbon bed to a temperature suitable for water-gas making. Probably the nearest approach to straight air-gas making with a view to securing it for a gas engine fuel is that of the Tait producer, though in all gas producers some air gas is made to be mixed with water gas separately or simultaneously as made.

Various analyses given in the tables show that the composition varies considerably for reasons just given, the carbon monoxide ranging from about 18 to 35 per cent, carbon dioxide from 4 to 16 per cent, hydrogen from less than 1 to over 12 per cent, methane, the only hydrocarbon, from zero to 5 per cent, being least for cokes and greatest for coals.

It is of value to examine some of these results in the light of such fundamental relations as are available, to indicate what a perfect air gas should be, the corresponding yield per pound of fuel and the *efficiency of the transformation*, this latter being *defined as the ratio of the heat of combustion of the gas formed, to that of the fuel from which it came, each burning to the same final products of CO_2 and H_2O .*

The first step is to establish such relations as may be possible between the CO and CO_2 existing together, and two investigations are available for this purpose. First, that of Boudouard, who established the equilibrium ratio as a function of temperature and later determined the time factors in attaining equilibrium with carbon in three different states, which latter relations were also studied by the second authority, Clement.

By leaving amorphous carbon in contact with CO_2 for very long times Boudouard found on analyzing the gases resulting, that, as might be expected,

the relation of CO to CO₂ became finally constant for any one temperature and as temperature rose the per cent of CO increased. His numbers are tabulated in Table XIV, as calculated from the derived formula by Dowson and Larter, together with the ratio $\frac{\text{CO}}{\text{CO}_2}$ which may be termed the *carbon oxide ratio* for want of a better name.

TABLE XIV

BOUDOUARD'S EQUILIBRIUM RELATIONS CO AND CO₂ WITH TEMPERATURE

Temp. F.	Per Cent by Vol.		$\frac{\text{CO}}{\text{CO}_2}$	$\frac{\text{CO}}{\text{CO} + \text{CO}_2}$	Temp. F.	Per Cent by Vol.		$\frac{\text{CO}}{\text{CO}_2}$	$\frac{\text{CO}}{\text{CO} + \text{CO}_2}$
	CO ₂	CO				CO ₂	CO		
835	98	2	.0204	.02	1407	20.0	80.0	4.00	.80
923	95	5	.052	.05	1447	15	85	5.68	.85
1000	90	10	.11	.10	1497	10	90	9.00	.90
1090	80	20	.25	.20	1533	7.5	92.5	12.34	.925
1153	70	30	.42	.30	1582	5	95	19.00	.95
1204	60	40	.67	.40	1610	4	96	24.00	.96
1252	50	50	1.00	.50	1646	3	97	32.33	.97
1297	40	60	1.50	.60	1697	2	98	49.00	.98
1348	30	70	2.33	.70	1790	1	99	99.00	.99
1376	25	75	3.00	.75	1886	0.5	99.5	199.00	.995

To attain these per cents of CO at such temperatures, which must equal 1800° F. to exceed 99 per cent CO, requires a long time,—longer for some forms of carbon than others and longer than is available in air-gas producers or boiler fires to a lesser degree. Moreover, the process in these cases is somewhat different; if instead of making all CO₂ to be reduced at a fixed temperature to CO, air is blasted into a bed, *the temperatures are whatever result from the reaction*, and the reaction such that both CO and CO₂ are produced at once; the CO₂ which is formed later reduces, and the CO₂ arising at first partly from direct burning of C with O, and partly by burning of CO formed at one spot by O that escaped, being used up at another. However, the time factor is a most important one, and operates in such a way as to prevent the attainment of as much CO as is indicated in the preceding table at the given temperature; or if a large amount is to be obtained in a lesser time the temperature must be higher. Therefore, if a curve be plotted for CO/CO₂ with temperature for less time of contact than is necessary for equilibrium, it will be above the so-called equilibrium curve, and such data have been given by both Boudouard and Clement, reproduced in the Tables. It must be observed that the time element depends on the mode of experiment, a fact that makes the problem more complex especially when comparing results obtained from gases at rest in a closed tube with another *through which a current passes*. This latter is the more favorable as the gases sweep the carbon, and fresh CO₂ can reach it by mechanical pushing away of the CO next to it, while in the former case displacement can only take place by diffusion. Boudouard used the closed tube and Clement the open tube with current passing through; the

former knew accurately the time of contact while the latter could only estimate it from the volume of flow and temperature, a very uncertain method. See Table LXX, Handbook of Tables.

In no case do the conditions correctly represent the *gas producer* as well as in another series by Boudouard, who passed *air* through a tube packed with coke in 5 mm. pieces, held at a temperature of 1472° F. while the rate of flow was varied with the following results, Table XV:

TABLE XV
CHANGE OF O₂ IN AIR TO CO AND CO₂ AT 1472° F. (BOUDOUARD)

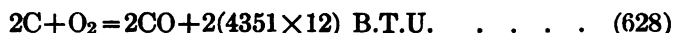
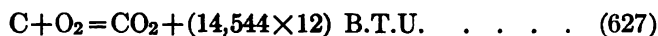
Vel. air, in Inches per Minute.	Per Cent by Volume.			CO CO ₂	CO CO + CO ₂	Per Cent of Carbon Gasified in	
	CO ₂	CO	CO ₂ + CO			CO ₂	CO
6.10	18.20	5.20	23.40	.28	.222	77.8	22.2
16.47	18.43	3.80	22.23	.205	.171	82.9	17.1
79.31	18.92	1.88	20.80	.0995	.090	91.0	9.0
89.36	19.90	1.83	21.73	.0915	.084	91.6	8.4
195.20	19.40	.93	20.33	.0478	.046	95.4	4.6

These results show that decreasing the time of contact decreases the total carbon gasified and increases the CO₂ content of the gases, indicating that a reduction to CO of some CO₂ previously made is the order of reaction and that this is more nearly complete the greater the time available, but it is quite likely that higher temperatures would materially change the result.

In using data on equilibrium it must be assumed that sufficient time has elapsed to allow equilibrium to be established, which will always be the case in producers except when blasted too vigorously, that is, overloaded.

That there should be any difference in the temperatures of excessively thick beds of fuel blasted with air is due to cooling influences, external and internal, as follows: (a) dry air is seldom used or available and frequently steam is added which on decomposition absorbs heat; (b) air and steam blasts are supplied at different temperatures, and as the reactions fix only the rise of temperature, the temperature in the bed rises directly with initial mixture temperature, other things being equal; (c) walls absorb, transmit and discharge heat to water jackets or radiate it to the air; (d) in blast furnaces chemical reactions are absorbing heat at various points of the path, and giving off gases from ore and flux changes, otherwise disturbing the purely combustion reaction. Of these influences the strongest is the hydrogen decomposition and reaction with carbon, which produces what is termed a water gas, the characteristics of which will be examined after establishing the quantitative fundamental relations for air gas.

The two fundamental relations fixing the weight and volume proportions and heats for air-gas making are given by Eqs. (627) and (628).



Accordingly for oxygen reacting with carbon, and measuring volumes at 32° F. and 29.92 ins. Hg, the proportions being the same at any other pressure,

$$1 \text{ lb. C} + \left\{ \begin{array}{l} \frac{32}{12} = 2.66 \text{ lbs.} \\ \text{or} \\ \frac{358}{12} = 29.8 \text{ cu.ft.} \end{array} \right\} \text{O}_2 = \left\{ \begin{array}{l} \frac{44}{12} = 3.66 \text{ lbs.} \\ \text{or} \\ \frac{358}{12} = 29.8 \text{ cu.ft.} \end{array} \right\} \text{CO}_2 + 14,544 \text{ B.T.U.} \quad (629)$$

$$1 \text{ lb. C} + \left\{ \begin{array}{l} \frac{16}{12} = 1.33 \text{ lbs.} \\ \text{or} \\ \frac{358}{24} = 14.9 \text{ cu.ft.} \end{array} \right\} \text{O} = \left\{ \begin{array}{l} \frac{28}{12} = 2.33 \text{ lbs.} \\ \text{or} \\ \frac{358}{24} = 14.9 \text{ cu.ft.} \end{array} \right\} \text{CO} + 4351 \text{ B.T.U.} \quad (630)$$

For air reactions taking by $\left\{ \begin{array}{l} \text{Wt., O}_2 = 23.1 \text{ per cent; N}_2 = 76.9 \text{ per cent} \\ \text{Vol., O}_2 = 20.9 \text{ per cent; N}_2 = 79.1 \text{ per cent} \end{array} \right\}$

the nitrogen to be added is $\left\{ \begin{array}{l} 3.33 \times \text{wt. of oxygen} \\ 3.78 \times \text{vol. of oxygen} \end{array} \right\}$

and the air involved is $\left\{ \begin{array}{l} 4.33 \times \text{wt. of oxygen} \\ 4.78 \times \text{vol. of oxygen} \end{array} \right\}$

These values substituted in Eqs. (629) and (630) will give the weights, volumes and heats of reaction with carbon, Eqs. (631) and (632) applying when the product is CO₂, Eqs. (633) and (634) when it is CO.

$$1 \text{ lb. C} + \left\{ \begin{array}{l} \left[\begin{array}{l} 2.66 \text{ lbs.} \\ \text{or} \\ 29.8 \text{ cu.ft.} \end{array} \right] \text{O}_2 \\ + \left[\begin{array}{l} 8.857 \text{ lbs.} \\ \text{or} \\ 112.64 \text{ cu.ft.} \end{array} \right] \text{N}_2 \end{array} \right\} = \left\{ \begin{array}{l} \left[\begin{array}{l} 3.66 \text{ lbs.} \\ \text{or} \\ 29.8 \text{ cu.ft.} \end{array} \right] \text{CO}_2 \\ + \left[\begin{array}{l} 8.857 \text{ lbs.} \\ \text{or} \\ 112.64 \text{ cu.ft.} \end{array} \right] \text{N}_2 \end{array} \right\} + 14,544 \text{ B.T.U.} \quad (631)$$

$$1 \text{ lb. C} + \left\{ \begin{array}{l} \left[\begin{array}{l} 11.517 \text{ lbs.} \\ \text{or} \\ 142.44 \text{ cu.ft.} \end{array} \right] \text{Air} \end{array} \right\} = \left\{ \begin{array}{l} \left[\begin{array}{l} 3.66 \text{ lbs.} \\ \text{or} \\ 29.8 \text{ cu.ft.} \end{array} \right] \text{CO}_2 \\ + \left[\begin{array}{l} 8.857 \text{ lbs.} \\ \text{or} \\ 112.64 \text{ cu.ft.} \end{array} \right] \text{N}_2 \end{array} \right\} + 14,544 \text{ B.T.U.} \quad (632)$$

$$1 \text{ lb. C} + \left\{ \begin{array}{l} \left[\begin{array}{l} 1.33 \text{ lbs.} \\ \text{or} \\ 14.9 \text{ cu.ft.} \end{array} \right] \text{O} \\ + \left[\begin{array}{l} 4.429 \text{ lbs.} \\ \text{or} \\ 56.32 \text{ cu.ft.} \end{array} \right] \text{N}_2 \end{array} \right\} = \left\{ \begin{array}{l} \left[\begin{array}{l} 2.331 \text{ lbs.} \\ \text{or} \\ 29.8 \text{ cu.ft.} \end{array} \right] \text{CO} \\ + \left[\begin{array}{l} 4.429 \text{ lbs.} \\ \text{or} \\ 56.32 \text{ cu.ft.} \end{array} \right] \text{N}_2 \end{array} \right\} + 4,351 \text{ B.T.U.} \quad (633)$$

$$1 \text{ lb. C} + \left\{ \begin{array}{c} \left[\begin{array}{c} 5.759 \text{ lbs.} \\ \text{or} \\ 71.22 \text{ cu.ft.} \end{array} \right] \text{Air} \end{array} \right\} = \left\{ \begin{array}{c} \left[\begin{array}{c} 2.33 \text{ lbs.} \\ \text{or} \\ 29.8 \text{ cu.ft.} \end{array} \right] \text{CO} \\ + \left[\begin{array}{c} 4.429 \text{ lbs.} \\ \text{or} \\ 56.32 \text{ cu.ft.} \end{array} \right] \text{N}_2 \end{array} \right\} + 4,351 \text{ B.T.U.} \quad (634)$$

Assuming that 1 lb. of carbon reacting with air produces both CO_2 and CO at the same time,

Let x = fraction of 1 lb. of carbon burning to CO_2 ,

$(1-x)$ = fraction of 1 lb. of carbon burning to CO.

Then if the products of reaction of 1 lb. of carbon mix, the reaction is defined by an equation which reduces to the two following forms: Eq. (635) giving the weight relations, and Eq. (636) those for volumes, each associated with the heats of reaction.

$$1 \text{ lb. C} + 5.759(1+x) \text{ lbs. air} = \left\{ \begin{array}{l} 3.66x \text{ lbs. CO}_2 \\ + 2.33(1-x) \text{ lbs. CO} \\ + 4.428(1+x) \text{ lbs. N}_2 \\ + (10,193x + 4351) \text{ B.T.U.} \end{array} \right\} \quad \dots (635)$$

$$1 \text{ lb. C} + 71.22(1+x) \text{ cu.ft. air} = \left\{ \begin{array}{l} 29.8x \text{ cu.ft. CO}_2 \\ + 29.8(1-x) \text{ cu.ft. CO} \\ + 56.32(1+x) \text{ cu.ft. N}_2 \\ + (10,193x + 4351) \text{ B.T.U.} \end{array} \right\} \quad \dots (636)$$

The final gas will have the following composition and carbon monoxide-dioxide ratio by volumes (Tables XVI and XVII). It should be noted with respect to the latter that the *ratio of carbon monoxide to dioxide by volumes in the gas is the same as the ratio of fractional weights of carbon burning to each as products.*

TABLE XVI
COMPOSITION OF HYPOTHETICAL AIR GAS. GENERAL

Constituent.	By Weight.	By Volume.	$\frac{\text{CO}}{\text{CO}_2}$ by Vol.	$\frac{\text{CO}}{\text{CO} + \text{CO}_2}$ by Vol.
CO_2	$\frac{3.66x}{5.759x + 6.759}$	$\frac{29.8x}{56.32x + 86.12}$	$\frac{1-x}{x}$	$1-x$
CO	$\frac{2.33(1-x)}{5.759x + 6.759}$	$\frac{29.8(1-x)}{56.32x + 86.12}$		
N_2	$\frac{4.428(1+x)}{5.759x + 6.759}$	$\frac{56.32(1+x)}{56.32x + 86.12}$		

If the carbon burns all to CO then $x=0$; if all to CO_2 then $x=1$ and the composition of the gas for the two cases will be given by Table XVII.

TABLE XVII
COMPOSITION OF HYPOTHETICAL AIR GAS. NO CO₂, AND NO CO

Constituent.	By Weight Per Cent.		By Volume Per Cent.		$\frac{\text{CO}}{\text{CO}_2}$		$\frac{\text{CO}}{\text{CO} + \text{CO}_2}$	
	All CO	All CO ₂	All CO	All CO ₂	All CO	All CO ₂	All CO	All CO ₂
CO ₂	0	$\frac{3.66}{12.516} = 29.2$	0	$\frac{29.8}{142.44} = 20.8$				
CO	$\frac{2.33}{6.759} = 34.47$	0	$\frac{29.8}{86.12} = 34.6$	0	∞	0	1	0
N ₂	$\frac{4.429}{6.759} = 65.53$	$\frac{8.856}{12.516} = 70.8$	$\frac{56.32}{86.12} = 65.4$	$\frac{112.64}{142.44} = 79.2$				

The general relations between the carbon and air as raw materials, and the gas produced from them are given as follows, Eq. (637):

$$\begin{array}{ll}
 1 \text{ lb. carbon makes } (5.759x + 6.759) \text{ lbs. gas} & (a) \\
 1 \text{ lb. air makes } \left(\frac{5.759x + 6.759}{5.759(1+x)} \right) \text{ lbs. gas} & (b) \\
 1 \text{ lb. carbon makes } (56.32x + 86.12) \text{ cu.ft. gas} & (c) \\
 1 \text{ lb. air makes } \left(\frac{56.32x + 86.12}{5.759(1+x)} \right) \text{ cu.ft. gas} & (d)
 \end{array} \quad (637)$$

The heats of reaction can be expressed in terms of each of the volumes or weights entering—those for the raw materials or for the gas formed—and are set down in tabular equation form below, Eq. (638), first for any proportion of CO and CO₂, next for no CO and finally no CO₂.

HEATS OF REACTION FOR AIR GAS, B.T.U.

Heat of Reaction.. General.	When CO = 0 or $x = 1$.	When CO ₂ = 0 or $x = 0$.	Unit.	
$10193x + 4351$	14544	4351	Per lb. carbon	(a)
$\frac{10193x + 4351}{5.759(1+x)}$	$\frac{14544}{11.518} = 1263$	$\frac{4351}{5.759} = 755$	" lb. air	(b)
$\frac{10193x + 4351}{5.759x + 6.759}$	$\frac{14544}{12.518} = 1162$	$\frac{4351}{6.759} = 644$	" lb. gas	(c)
$\frac{10193x + 4351}{56.32x + 86.12}$	$\frac{14544}{142.44} = 1021$	$\frac{4351}{86.12} = 51$	" cu.ft. gas	(d)
$\frac{10193x + 4351}{3.66x}$	$\frac{14544}{3.66} = 3974$	" lb. CO ₂	(e)
$\frac{10193x + 4351}{2.33(1-x)}$	$\frac{4351}{2.33} = 1867$	" lb. CO	(f)
$\frac{10193x + 4351}{4.428(1+x)}$	$\frac{14544}{8.856} = 1642$	$\frac{4351}{4.428} = 983$	" lb. N ₂	(g)
$\frac{10193x + 4351}{29.8x}$	$\frac{14544}{29.8} = 488$	" cu.ft. CO ₂	(h)
$\frac{10193x + 4351}{29.8(1-x)}$	$\frac{4351}{29.8} = 146$	" cu.ft. CO	(i)
$\frac{10193x + 4351}{56.32(1+x)}$	$\frac{14544}{112.64} = 129$	$\frac{4351}{56.32} = 77$	" cu.ft. N ₂	(j)

(638)

These heats of reaction are all positive and act to raise the temperature of the bed and the products of the reaction; the temperature rise could be calculated if the specific heat of the products were known, or on any assumption for its value. The heat of combustion of the gases formed, which is due to the CO, is the useful effect in gas producers and a loss in boiler fires, to which the preceding relations also apply. The amount of this heat of combustion is, of course, the difference between the heat of complete combustion per pound C and its heat of reaction when combustible gases are formed by it. Therefore the heat of combustion of the gas will be $14,544 - (10,193x + 4351)$ per pound of carbon, from which the value per cubic foot and pound of gas can be found as given below in tabular equation form, Eq. (639).

HEAT OF COMBUSTION OF HYPOTHETICAL AIR GAS, B.T.U.

General.	When CO = 0 or $x = 1$.	When CO ₂ = 0 or $x = 0$.	Unit.	
$10193(1-x)$	0	10193	Per lb. C	(a)
$\frac{10193(1-x)}{5.759x + 6.759}$	0	$\frac{10193}{6.759} = 1508$	" lb. gas	(b)
$\frac{10193(1-x)}{56.32x + 86.12}$	0	$\frac{10193}{86.12} = 118$	" cu.ft. gas	(c)

. . (639)

Dividing the heat of combustion of the gas formed per pound carbon, by the heat of combustion of the carbon, will give the efficiency of the producer as a fuel transformer from solid carbon to combustible gases.

$$\text{Efficiency of gasification of carbon} = \left. \begin{array}{l} \text{Efficiency of gasifi-} \\ \text{cation of carbon} = \end{array} \right\} E = \frac{10,193(1-x)}{14,544} (a); = 70\% \text{ if no CO}_2 \text{ is formed (b). (640)}$$

Therefore the efficiency of this process cannot exceed 70 per cent, the other 30 per cent of the carbon heat appearing as sensible heat of the gas, some of which can be used for the formation and dissociation of steam, adding hydrogen thereby to the gas, and raising the efficiency more and more above 70 per cent as the hydrogen increases. Such reactions, however, cool the bed and tend to make more CO₂ with its corresponding lessening air-gas efficiency effect. When the CO₂ content is not zero, the heat of combustion of the gas and efficiency of the process becomes less and the curves of Fig. 125 are plotted to show graphically the relation. On the same sheet are given the gas composition, calorific power of the gas and heats of reaction per pound of carbon, and per pound of gas. The heat of reaction per pound of carbon is a measure of efficiency, and per pound of gas a measure of the temperature of the gas when specific heats are known.

Air-gas making involves such high temperatures as to make it an impracticable process for most fuels because of the fusibility of the ash and formation of clinkers large enough to stop the flow of blast. The blast furnace, fluxing and melting everything it contains to slag and iron, does not suffer

in this way and it is not impossible that fluxing of ash and removal as liquid slag may be developed for gas making in producers. At the present time steam is almost universally introduced with the blast of air, and by its decom-

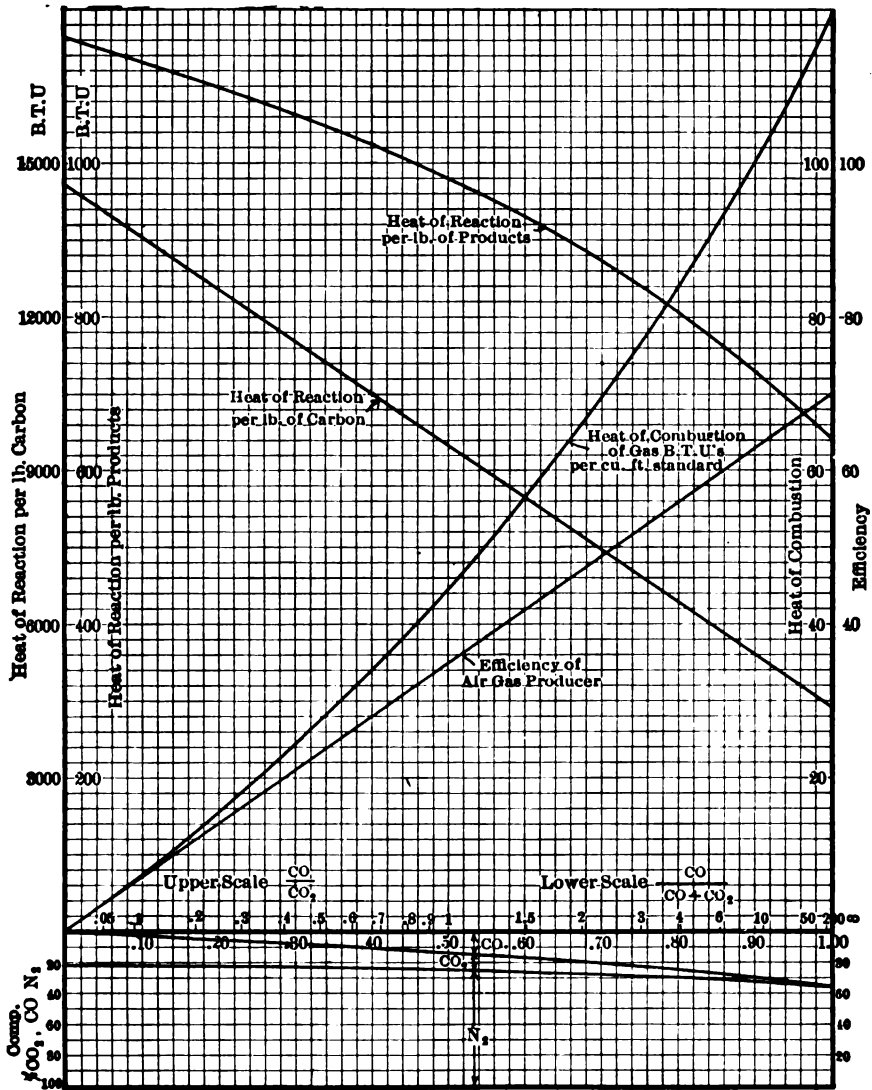


FIG. 125.—Characteristics of Air Gas.

position absorbs enough heat to prevent serious clinkering of most coals, though there are still some that give trouble.

The calorific power and density of an air gas can be determined from the properties of its constituents and such a calculation is given in tabular form below, Table XVIII:

TABLE XVIII
DENSITY AND CALORIFIC POWER OF BLAST FURNACE GAS
FROM CONSTITUENTS (32° F. AND 29.92" Hg)

Average Composition of Blast Furnace Gas.	One Cubic Foot.				Summary.
	Contains.		Yields B.T.U.		
		Pounds.	High.	Low.	
Carbon monoxide, CO.	.2861	.022336	97.560	97.560	B.T.U. cu.ft. gas high 109.035
Hydrogen, H ₂0274	.000143	9.343	8.001	B.T.U. cu.ft. gas low 107.479
Methane, CH ₄0020	.000089	2.132	1.918	Cu.ft. per lb. gas . . . 12.3132
Carbon dioxide, CO ₂1139	.013974	Lbs. per cu.ft. gas081214
Nitrogen, N ₂5706	.044672	B.T.U. lb. gas high. 1342.57
					B.T.U. lb. gas low.. 1323.41
Total for gas.	1.0000	.081214	109.035	107.479	

Example 1. A sample of air gas showed a ratio of CO to CO₂ of 5.66. What was its composition by weight and volume, pounds of carbon and air supplied per cubic foot of gas made, and the heat of combustion per cubic foot of the gas?

$$\text{Since } \frac{\text{CO}}{\text{CO}_2} = \frac{1-x}{x} = 5.66, x = .15.$$

$$\text{CO}_2 \text{ by wt.} = \frac{3.66 \times .15}{5.759 \times .15 + 6.759} = 7.2\%; \quad \text{CO by wt.} = \frac{2.33 \times .85}{5.759 \times .15 + 6.759} = 26\%;$$

$$\text{N}_2 \text{ by wt.} = \frac{4.428 \times 1.15}{5.759 \times .15 + 6.759} = 66.8\%; \quad \text{CO}_2 \text{ by vol.} = \frac{29.8 \times .15}{56.32 \times .15 + 86.12} = 4.7\%;$$

$$\text{CO by vol.} = \frac{29.8 \times .85}{56.32 \times .15 + 86.12} = 26.8\%; \quad \text{N}_2 \text{ by vol.} = \frac{56.32 \times 1.15}{56.32 \times .15 + 86.12} = 68.5\%.$$

$$1 \text{ cu.ft. gas requires } \frac{1}{56.32 \times .15 + 86.12} = .0105 \text{ lb. C.}$$

$$1 \text{ cu.ft. gas requires } \frac{5.759(1.15)}{56.32 \times .15 + 86.12} = .0705 \text{ lb. air.}$$

$$\text{The heat of combustion per cu.ft. of gas is } \frac{10,193 \times .85}{56.32 \times .15 + 86.12} = 92 \text{ B.T.U.}$$

Prob. 1. In an air gas consisting only of CO and CO₂, what would be the largest amount of CO per cubic foot attainable with a bed temperature of 1200° F.?

Prob. 2. In a producer blasted with air alone 20 per cent of the carbon burns to CO₂ and 80 per cent to CO. How many cubic feet of CO, CO₂, and N₂ will be formed per pound of carbon and how much air will be required? What heat will be developed by the reaction per pound C, per pound gases, and by the gas per pound and per cubic foot in burning?

Prob. 3. Should the carbon all burn to CO what would be the composition of the gas by weight and volume and how much air would be needed?

Prob. 4. How many cubic feet of air would be needed to make (a) 10 lbs. and (b) 10 cu.ft of air gas containing 25 per cent CO and 5 per cent CO₂?

Prob. 5. A blast furnace yielded gas containing practically nothing but N₂, CO, and CO₂, the percentages being 30 per cent for CO and 10 for CO₂. Assuming the

blast to have been preheated to a temperature of 300° F. and that the specific heat of the products was .25, what was the final temperature?

Prob. 6. What would be the heat of reaction in the above case per cubic foot and pound of gas made and per pound of carbon. What would be the heat of combustion of the gas calculated from constituents and from the general equation for the proper value of x ?

87. Gasification of Fixed Carbon, Coke and Coal, Previously Heated, by Steam Blast Reactions, Producing Water Gas. Composition and Relation of Constituents of Water Gas, Yield per Pound of Steam and Coal. Heat of Combustion of Gas and Limitation of Yield by Negative Heat of Reaction. *Water gas* is the term applied to the product obtained by blasting steam into a carbon, charcoal, coke or coal bed previously heated, and *producer gas* generally, or Dowson gas in England, and generator gas in Germany, where the heating of the bed by air-gas making proceeds at the same time as the steam reaction which, of course, is endothermic. The fundamental relations of the temperature of the bed to the reaction characteristics were studied by Bunte with the results given in Table XIX.

TABLE XIX
WATER GAS CHARACTERISTICS WITH BED TEMPERATURE (BUNTE)

Temp. F.	%SteamH ₂ O Decomposed.	Composition by Vol. of Water Gas.			$\frac{\text{CO}}{\text{CO}_2}$	$\frac{\text{CO}}{\text{CO} + \text{CO}_2}$
		H ₂	CO	CO ₂		
1245 (Dull red)	8.8	65.2	4.9	29.8	.16	.141
1396	25.3	65.2	7.8	27.0	.29	.224
1540	41.0	61.9	15.1	22.9	.65	.397
1749	70.2	53.3	39.3	6.8	5.80	.853
1850	94.0	48.8	49.7	1.5	33.1	.972
1940	98.0	50.7	48.0	1.3	36.8	.975
2057 (White)	99.4	50.9	48.5	.6	80.8	.988

These results were plotted to a base of carbon oxide ratio, together with the Boudouard equilibrium curve, from which it was shown that in no case was the equilibrium even approximately attained, as for example, at 1750° F. Bunte's ratio was less than 6, while Boudouard's equilibrium value is about 75 in round numbers. Assuming both experimenter's results to be accurate, this would be a measure of the importance of the time element, but there is considerable doubt as to the accuracy, especially of the temperatures of the bed reported by Bunte. There is, however, another explanation based on the reactions of CO with steam to form CO₂ and H₂ according to $\text{CO} + \text{H}_2\text{O} = \text{CO}_2 + \text{H}_2$, which would tend to oxidize the CO when formed in the presence of steam, and it is reported by Bunte that at the temperature named, 1750, there was 30 per cent of steam free and still undecomposed. That this is the most probable explanation is indicated by the lesser discrepancy at low temperatures at which the above steam reaction takes place feebly or not at all. For example, at Bunte's lowest temperature, 1245° F., his ratio is .16, while for equilibrium it

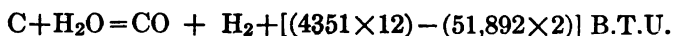
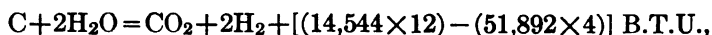
would be about five times as much as against twelve times at 1750° F. In this work of Bunte the bed, previously heated to the temperature desired, must have begun to cool at once under the steam blast influence, which is precisely what happens in straight water-gas manufacture, an essentially intermittent process. Moreover, the cooling cannot be uniform because the resistance to flow is not uniform through a fuel bed, so that where the blast can pass most easily, there most of it will pass, and a non-homogeneous condition of temperature and extent of reactions result.

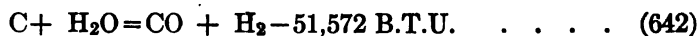
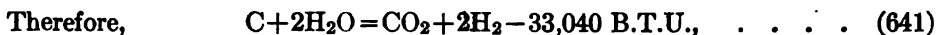
Another interesting fact shown by these data is the rapidity with which hydrogen forms even at low temperature and with only a small fraction of steam decomposed; for a very considerable temperature range the per cent hydrogen remains substantially constant between 65 per cent for 1250° F. where there is practically no CO, and 51 per cent at 2060° F. where there is nearly 49 per cent of CO. There is a retardation of CO formation until the temperatures have reached about 1600° F.

Straight water-gas making, which these experiments illustrate, proceeds with decreasing temperature, but producer-gas making is a continuous process with steady temperatures, enough air oxygen combining with carbon exothermally to balance the endothermic heat of the water-gas making, to permit the maintenance of a steady state.

In the Tables are given some analyses of water gases which show carbon oxide ratios as high as 17.5, indicating the attainment of very high bed temperatures before steam blasting, a moderately slow flow through the bed to give the CO₂ a chance to reduce, and not an excessive amount of free H₂O to react with the CO formed. In some cases CH₄ is reported, indicating the use of a coal or an incomplete coke, some oxygen and nitrogen are also reported, indicating influx of air, but it is difficult to see how oxygen could exist unless its contact with combustible H₂, CO or CH₄ were delayed till cooling occurred. The hydrogen content of these water gases ranges a little above and below 50 per cent, which is about the same as found by Bunte for all temperatures above that at which the formation of CO was appreciable, 1800° F., in which case, the volumetric amount is substantially equal for the hydrogen and carbon monoxide in Bunte's tests and in some of the analyses reported. When the analyses depart from this equality the CO is always less than H₂ and as this was found by Bunte to be so for too low a temperature, it may be assumed that when such analyses are reported, the process had been carried on for too long a time, or started with too cold a bed. Usually five minutes is a fair average time for water-gas making in a previously heated coke or anthracite fuel bed. See Table LXXII in Thermodynamic Tables.

The volumes, weights and heats involved in water-gas making are given by two fundamental equations of chemical reactions, Eqs. (641) and (642), using the low calorific value for hydrogen, and derived from,





These may be interpreted as follows:

$$1 \text{ lb. C} + \left\{ \begin{array}{c} \left[\begin{array}{c} 3 \text{ lbs.} \\ \text{or} \\ 59.6 \text{ cu.ft.} \end{array} \right] H_2O \end{array} \right\} = \left\{ \begin{array}{c} \left[\begin{array}{c} 3.66 \text{ lbs.} \\ \text{or} \\ 29.8 \text{ cu.ft.} \end{array} \right] CO_2 \\ + \left[\begin{array}{c} .333 \text{ lb.} \\ \text{or} \\ 59.6 \text{ cu.ft.} \end{array} \right] H_2 \end{array} \right\} - 2753 \text{ B.T.U.} . (643)$$

$$1 \text{ lb. C} + \left\{ \begin{array}{c} \left[\begin{array}{c} 1.5 \text{ lbs.} \\ \text{or} \\ 29.8 \text{ cu.ft.} \end{array} \right] H_2O \end{array} \right\} = \left\{ \begin{array}{c} \left[\begin{array}{c} 2.33 \text{ lbs.} \\ \text{or} \\ 29.8 \text{ cu.ft.} \end{array} \right] CO \\ + \left[\begin{array}{c} .167 \text{ lb.} \\ \text{or} \\ 29.8 \text{ cu.ft.} \end{array} \right] H_2 \end{array} \right\} - 4298 \text{ B.T.U.} . (644)$$

Assuming that 1 lb. of carbon yields both reactions at the same time and that x represents the fractional part burning to CO_2 , or $(1-x)$ the corresponding part that burns to CO , then the double reaction will be represented by Eqs. (645) and (646), the first for weights and the second for volumes, all volumes being for standard gas conditions.

$$1 \text{ lb. C} + 1.5(1+x) \text{ lbs. } H_2O = \left[\begin{array}{c} 3.66x \text{ lbs. } CO_2 \\ + 2.33(1-x) \text{ lbs. } CO \\ + .167(1+x) \text{ lbs. } H_2 \end{array} \right] + (1545x - 4298) \text{ B.T.U.} \quad (645)$$

$$= \left[\begin{array}{c} 29.8x \text{ cu.ft. } CO_2 \\ + 29.8(1-x) \text{ cu.ft. } CO \\ + 29.8(1+x) \text{ cu.ft. } H_2 \end{array} \right] + (1545x - 4298) \text{ B.T.U.} \quad (646)$$

From these equations the weight, volume, heat and temperature changes can be set down in each of the various units.

The composition of the gas will be for any proportion of CO to CO_2 , as given below in Table XX.

TABLE XX
COMPOSITION OF HYPOTHETICAL WATER GAS. GENERAL

Constituents.	By Weight.	By Volume.	$\frac{CO}{CO_2}$ (Vols.)	$\frac{CO}{CO+CO_2}$
CO_2	$\frac{3.66x}{1.5x+2.5}$	$\frac{29.8x}{29.8x+59.6}$	$\frac{1-x}{x}$	$1-x$
CO	$\frac{2.33(1-x)}{1.5x+2.5}$	$\frac{29.8(1-x)}{29.8x+59.6}$		
H_2	$\frac{.167(1+x)}{1.5x+2.5}$	$\frac{29.8(1+x)}{29.8x+59.6}$		

It appears from this that, like the case of air gas, the carbon monoxide-dioxide ratio by volume is the same as the fractional weight of carbon burning to these two constituents respectively.

When the carbon burns all to CO, $x=0$, or all to CO₂, $x=1$, the composition of the gas will be that given in Table XXI.

TABLE XXI
COMPOSITION OF HYPOTHETICAL WATER GAS. NO CO₂ AND NO CO

Constituents.	By Weight, Per Cent.		By Volume, Per Cent.		Ratio $\frac{\text{CO}}{\text{CO}_2}$		Ratio $\frac{\text{CO}}{\text{CO} + \text{CO}_2}$	
	All CO	All CO ₂	All CO	All CO ₂	All CO	All CO ₂	All CO	All CO ₂
CO ₂	0	$\frac{3.66}{4} = 91.7$	0	$\frac{29.8}{89.4} = 33$				
CO	$\frac{2.33}{2.50} = 93.3$	0	$\frac{29.8}{59.6} = 50$	0	∞	0	1	0
H ₂	$\frac{.167}{2.50} = 6.7$	$\frac{.333}{4} = 8.3$	$\frac{29.8}{59.6} = 50$	$\frac{59.6}{89.4} = 67$				

The equality of volume of CO and H₂ when no CO₂ is formed is fairly well borne out by the test results quoted, at least as perfectly as the equilibrium ratio for the temperature permits, when the temperature is high and free steam is not in contact with CO in the hot zone, while the 67 per cent by volume of hydrogen for no CO formed, is closely approached by Bunte's 65.2 per cent when there was 4.9 per cent CO at the lowest temperature of 1245° F.

The general relations between the carbon and steam as raw materials and the gas produced are given by Eq. (647).

$$\left. \begin{array}{ll} 1 \text{ lb. carbon makes } (2.5 + 1.5x) \text{ lbs. of gas} & (a) \\ 1 \text{ lb. steam makes } \left(\frac{2.5 + 1.5x}{1.5(1+x)} \right) \text{ lbs. of gas} & (b) \\ 1 \text{ lb. carbon makes } (29.8x + 59.6) \text{ cu.ft. gas} & (c) \\ 1 \text{ lb. steam makes } \left(\frac{29.8x + 59.6}{1.5(1+x)} \right) \text{ cu.ft. gas} & (d) \end{array} \right\} \quad (647)$$

The heats of reaction per unit of each quantity entering are obtainable by dividing the total heat of reaction by the volume or weight of each substance entering, and these are set down in the tabular equation on page 432, Eq. (648).

These heats all go to the lowering of temperature, being negative, and the temperature drop can be determined only by assuming an average specific heat for the products, which are 93 per cent CO and 7 per cent H₂ approximately by weight in one, and 92 per cent CO₂ and 8 per cent H₂ approximately by weight in the other limiting case, and at the same time assuming the sensible heat capacity and the weight of the large fuel bed, and the weight of gas flow through it per minute. This would give an approximation to the rate of

temperature lowering, which would be of doubtful value. In practical operation the process is controlled by observation and usually the bed drops in temperature enough in five minutes, from the highest starting temperature practicable to avoid serious clinkering, to that at which the process is no longer feasible; this last temperature may be put roughly at 2000° F. and 1700° F. if excessive CO₂ is to be avoided in the gas.

HEATS OF REACTION FOR WATER GAS, B.T.U. LOW

General Heat of Reaction.	When CO = 0, $x = 1$.	When CO ₂ = 0, $x = 0$.	Unit.	
$1545x - 4298$	-2753	-4298	Per lb. carbon	(a)
$\frac{1545 - 4298x}{1.5(1+x)}$	$-\frac{2753}{3} = -918$	$-\frac{4298}{1.5} = -2865$	" lb steam	(b)
$\frac{1545x - 4298}{1.5x + 2.5}$	$-\frac{2753}{4} = -688$	$-\frac{4298}{2.5} = -1719$	" lb. gas made	(c)
$\frac{1545x - 4298}{29.8x + 59.6}$	$-\frac{2753}{89.4} = -31$	$-\frac{4298}{59.6} = -72$	" cu.ft. gas made	(d)
$\frac{1545x - 4298}{3.66x}$	$-\frac{2753}{3.66} = -752$	" lb. CO ₂	(e)
$\frac{1545x - 4298}{2.33(1-x)}$	$-\frac{4298}{2.33} = -1842$	" lb. CO	(f)
$\frac{1545x - 4298}{.167(1+x)}$	$-\frac{2753}{.33} = -8259$	$-\frac{4298}{.167} = -25788$	" lb. H ₂	(g)
$\frac{1545x - 4298}{29.8x}$	$-\frac{2753}{29.8} = -92$	" cu.ft. CO ₂	(h)
$\frac{1545x - 4298}{29.8(1-x)}$	$-\frac{4298}{29.8} = -144$	" cu.ft. CO	(i)
$\frac{1545x - 4298}{29.8(1+x)}$	$-\frac{2753}{59.6} = -46$	$-\frac{4298}{29.8} = -144$	" cu.ft H ₂	(j)

... (648)

The pound of carbon for which the total heat of reaction is $(1545x - 4298)$ B.T.U. could give on combustion 14,544 B.T.U. so that the gas which results must yield the difference $(14,544 + 4298 - 1545x)$ B.T.U. (low) per pound of carbon with corresponding values in other units, as in tabular Eq. (649).

HEAT OF COMBUSTION OF HYPOTHETICAL WATER GAS, B.T.U. LOW

General	When CO = 0, $x = 1$	When CO ₂ = 0, $x = 0$	Unit	
$18842 - 1545x$	17297	18842	Per lb. C	(a)
$\frac{18842 - 1545x}{2.5 + 1.5x}$	$\frac{17297}{4} = 4342$	$\frac{18842}{2.5} = 7537$	Per lb. gas	(b)
$\frac{18842 - 1545x}{29.8x + 59.6}$	$\frac{17297}{89.4} = 174$	$\frac{18842}{59.6} = 316$	Per cu.ft. gas	(c)

... (649)

Dividing the heat of combustion of the gas per pound of carbon by the heat of its complete combustion per pound will give the *apparent* efficiency of the process by Eq. (650).

$$E = \left(\frac{18,842 - 1545x}{14,544} \right) (a); \quad = \left(\frac{18,842}{14,544} \right) = 130\% \text{ if no CO}_2 \text{ is formed (b).} \quad (650)$$

This is, of course, impossible continuously, and results from ignoring the heat that must be put into the bed before admitting steam to make the steam reaction, which is so strongly endothermic, a possibility. Neglecting the heat necessary for the making of steam from water, as it is possible sometimes to secure it from waste sources, the previous making of air gas can be computed in terms of the quantity that must be made, or in terms of the amount of carbon used in doing it, to permit a given amount of water-gas production subsequently.

Thus, for air-gas making, the heat of reaction per pound C is $(10,193x + 4351)$, while for water-gas making it is $(1545x - 4298)$. For these to equalize

(Heat of reaction per pound C) \times (lbs. C) used for air gas = (heat of reaction per pound C) \times (lbs. C) used for water gas. Therefore

$$\frac{\text{Heat of air gas react. per lb. C}}{\text{Heat of water gas react. per lb. C}} = \frac{\text{Lbs. C gasified to make water gas}}{\text{Lbs. C gasified to make air gas}} \\ = \left(\frac{10,193x + 4351}{1545x - 4298} \right) (a); \quad = \frac{4351}{4298} = 1.01 \text{ if CO}_2 = 0 (b). \quad (651)$$

Similarly, (Heat of reaction per lb. of air gas) \times (lbs. air gas) = (heat of reaction per lb. water gas) \times (lbs. water gas).

$$\text{Hence, } \frac{\text{Heat of reaction per lb. air gas}}{\text{Heat of reaction per lb. water gas}} = \frac{\text{Lbs. water gas made}}{\text{Lbs. air gas made}} = \frac{644}{1719} \\ = .37 \text{ if CO}_2 = 0. \quad (652)$$

Or in terms of volumes

$$\frac{\text{Heat of reaction per cu.ft. air gas}}{\text{Heat of reaction per cu.ft. water gas}} = \frac{\text{Cu.ft. water gas made}}{\text{Cu.ft. air gas made}} = \frac{51}{72} \\ = .70 \text{ if CO}_2 = 0. \quad (653)$$

Therefore, assuming no CO₂ to be formed, which is the limiting case, 2 cu.ft. of water gas can be made for each 3 cu.ft. of air gas previously made, in round numbers, or about a third of a pound of water gas for each pound of air gas, or about equal weights of carbon must be used for each. If, as in many cases, the process is intermittent and the air gas is thrown away, the efficiency would be about $\frac{130}{2} = 65$ per cent for water-gas making alone, and proportionately

higher if some of the air gas or its heat were put to useful duty.

The density and calorific power of a water gas can, of course, be calculated from its constituents by the general method, and the results for a typical water gas are tabulated below, Table XXII.

TABLE XXII
DENSITY AND CALORIFIC POWER OF WATER GAS
FROM CONSTITUENTS (32° F. and 29.92" Hg)

Constituents of Average Water Gas.	One Cubic Foot				Summary
	Contains		Yields B.T.U.		
	Cu. Ft.	Lbs.	High.	Low.	
Hydrogen, H ₂4557	.002561	155.39	133.06	B.T.U. per cu.ft. gas high.....357.04
Carbon monoxide, CO..	.4485	.035014	152.94	152.94	B.T.U. per cu.ft. gas low.....329.86
Methane, CH ₄0441	.001971	47.01	42.29	Cu.ft. per lb. gas....21.917 Lbs. per cu.ft. gas....045626
Heavy hydrocarbons, C ₂ H ₄0010	.000080	1.7	1.57	
Carbon dioxide, CO ₂ ..	.0445	.005460	B.T.U. per lb. high.....7825 B.T.U. per lb. low.....7228
Oxygen, O ₂0050	.000446	
Nitrogen, N ₂0012	.000094	
Totals.....	1.0000	.045626	357.04	329.86	

Example 1. A sample of water gas showed a ratio of CO to (CO + CO₂) of .9. What was its composition by weight and volume; pounds of carbon and of steam supplied per cubic foot of gas made, and the heat of combustion per cubic foot of the gas? Since $\text{CO} + (\text{CO} + \text{CO}_2) = 1 - x$, $(1 - x) = .9$, and $x = .1$.

$$\begin{aligned}\text{CO}_2 \text{ by wt. } & \frac{3.66 \times .1}{1.5 \times .1 + 2.5} = 13.8\%; & \text{CO by wt. } & \frac{2.33 \times .9}{1.5 \times .1 + 2.5} = 79.3\%; \\ \text{H}_2 \text{ by wt. } & \frac{.167 \times 1.1}{1.5 \times .1 + 2.5} = 6.9\%; & \text{CO}_2 \text{ by vol. } & \frac{29.8 \times .1}{29.8 \times .1 + 59.6} = 4.7\%; \\ \text{CO by vol. } & \frac{29.8 \times .9}{29.8 \times .1 + 59.6} = 42.8\%; & \text{H}_2 \text{ by vol. } & \frac{29.8 \times 1.1}{29.8 \times .1 + 59.6} = 52.5\%.\end{aligned}$$

$$1 \text{ cu.ft. gas requires } \frac{1}{29.8 \times .1 + 59.6} = .016 \text{ lb. C.}$$

$$1 \text{ cu.ft. gas requires } \frac{1.5(1 + .1)}{29.8 \times .1 + 59.6} = .026 \text{ lb. steam.}$$

The heat of combustion per cubic foot of gas is

$$\frac{18,842 - 1545 \times .1}{29.8 \times .1 + 59.6} = 298 \text{ B.T.U. (low value).}$$

Prob. 1. In a water-gas producer the bed temperature was found to be 1800° F. According to the results of Bunte's experiments, what would be the composition of the gas and the per cent of steam decomposed? How would the CO to CO₂ ratio compare with that for the air-gas producer?

Prob. 2. Lewes gives as an average analysis of water gas the following: H₂ = 51.9; CO = 40.08; CH₄ = .1; CO₂ = 4.8; N₂ = 3.13. How does the percentage of hydrogen as given compare with that found by the general equation in terms of x ?

Prob. 3. How much carbon and how much steam will be required to make 1000 cu.ft. of gas which is 50 per cent H₂, 45 per cent CO, and 5 per cent CO₂ by volume, and what will be the heat of reaction per pound of carbon and per pound of gas?

Prob. 4. A producer gives a gas in which the ratio of CO to (CO+CO₂) is .8. How many cubic feet of gas will be made per pound of C and per pound of steam?

88. Gasification of Coals by Steam and Air Blasts, Resulting in Producer Gas. Composition and Relation of Constituents of Producer Gas, Yield per Pound of Fixed Carbon, Air and Steam. Modification of Composition by Addition of Volatile of Coal. Heat of Combustion of Gas, Sensible Heat and Efficiency of Gasification. Horse-power of Gas Producers. The most important of these gasifying processes is the most complex, and as carried out in the power-gas producer is as important in gas-power systems using coal fuel as is steam generation in the boilers of steam-power systems. The gas producer is a continuously operated apparatus, having a brick-lined casing to hold its thick fuel bed, provided with means for blasting the bed with a mixture of steam and air in more or less closely regulated proportions. This gives rise to a condition of steady state, as to temperature and reactions, which are maintained as long as working conditions permit; the most important of these is the physical condition of the bed as to porosity, ash content, homogeneity and clinker. Neglecting these physical conditions, it may be assumed that the blast enters all points of the supply part of the bed uniformly, and passes through all parts at an equal rate, all parts of the blast remaining in contact with the carbon for the same length of time, and finally that all points in any cross-section of the bed at right angles to the path are at equal temperature. These things are not really as assumed, but it is difficult to fix any relations even with these assumptions and impossible without them except in a qualitative manner.

In such producers there is made a combination of water gas and air gas, the ratio depending on the amounts of carbon reacting respectively with the oxygen of air and with the oxygen of steam. In all cases the exothermic heat of the air-gas process supports and more or less balances the endothermic heat of the water-gas process. The gas will thus contain as combustibles *from the fixed carbon and steam*, carbon monoxide, hydrogen, nitrogen, carbon dioxide, and some uncombined oxygen, usually very small in amount. Mixed with this gas will be the *products of the coal volatile*, which are most complex in character and amount, as might be inferred from the discussion of retort-coal gas and coke-oven gas. The nature and amounts of these additional products of the volatile of the coal depends on the coal itself and also to a very considerable degree on the manner of treatment in producers; it is in this respect only that different gas-producer processes differ one from the other, however different they may seem, or vary in structure.

Producers may be divided into the following classes with respect to the treatment of the volatile of the coal:

1. *Up draft.* Coal is fed at top and blast at the bottom. Volatile and moisture roast off at the top layer in the presence of gases from the coke-bed reactions and in a temperature equal to that of the gas leaving.

2. *Down draft.* Coal and blast are supplied to the top. Volatile and moisture distilled at the point of supply are mixed with the air and steam of the blast

and pass down through the entire bed. Volatile is partly burned and partly decomposed in the hot zone.

3. *Combined up and down draft.* Blast enters both top and bottom, coal is fed at the top and the gas leaves at the bottom. Volatile passes through hot zones, not so hot as (2) but hotter than (1) and in the presence of some air but less than (2). Volatile is partly burned, partly decomposed and partly unchanged.

4. *Combined up and down draft, blast entering only at bottom.* Volatile and moisture are roasted off at the top, and rising, are brought around the bed to its bottom, entering there with the blast gases, the finished gas leaving at a mid point. Usually, this is imperfectly executed, some volatile passing straight down to the gas outlet. Volatile is all burned as it enters the bed.

In the first class are to be found all standard forms of anthracite and coke producers, while the others are intended for bituminous coals, yielding much tar when supplied to the first class, which cannot be completely separated from the gas, clogging mains and interfering with the operation of the engine. In no case can a caking or coking coal be properly handled in any standard producer, because of draft interference due to melting of the coal, without prohibitively costly attendance to keep the cake broken up, and in some cases not even then. Even non-caking bituminous coals give some trouble, not only because of the tar they yield, but also on account of the variability of the gas which is a mixture of water gas, air gas and more or less decomposed rich hydrocarbon volatile. Even when the ratio of air and water gas is kept constant, the volatile of the coal distilling off at a variable rate will seriously impair the constancy of the gas quality, because the volatile of bituminous coals has about six times the heating power of the gasified carbon. With lignites the high moisture is mixed with the volatile, absorbing heat in its liberation and in the steam reactions with the other constituents like CO, and so introduces a further cause of interference in a manner depending on its path.

Tables LXXI and LXXIV in the Handbook of Tables give some producer gas analyses, the first from a wide range of sources, to illustrate the ranges of composition, while the second are from Fernald, reporting for the U. S. Geological Survey the results of trials of bituminous coals and lignites, ranging from high to low grade in a common updraft anthracite or coke type of producer to which was added as an auxiliary a tar extractor of the centrifugal fan order. In these latter tests the volatile is all reported as methane and the calorific power calculated on that basis. For each case the carbon oxide ratio has been added, to indicate more clearly the relations of the CO, CO₂ and H₂, derived mainly from the coke bed reaction, the hydrocarbons being purely roasting products. In down draft and mixed draft producers some of the volatile has also reacted, part of it in a manner similar to the bed.

Inspection of the general table shows a fairly steady value of the carbon-oxide ratio between 4 and 6, except for certain special conditions, indicating that the temperatures and rates of blast feed are about the same in all;

or that where high rates of combustion are used and time of contact small, the temperatures are allowed to rise to compensate. When the ratio departs from this, a reason is always available, for example, increase of steam in the blast, produced a regular decrease in the $\frac{\text{CO}}{\text{CO}_2}$ ratio according to the Bone and Wheeler

tests on air blasts saturated with water vapor at 60° F. to 80° F., in which the ratio changed from 5.2 to 1.2, and the hydrogen, as might be expected, rose from 16.60 to 22.65 per cent by volume. The increase of hydrogen with increase

of steam and the corresponding reduction of temperature and $\frac{\text{CO}}{\text{CO}_2}$ ratio, is well

confirmed by all results on Mond gas plants, operated primarily for ammonia recovery, which requires low temperatures. In these the ratio falls between 6 and 7, indicating, according to Boudouard, temperatures above 1200° F. On the other hand reduction of steam permits the bed to attain high temperatures and the gas will contain little hydrogen; this is indicated by the Thwaite test on charcoal where the hydrogen is .2 per cent by volume, which could easily have been derived from the moisture in even fairly dry air alone, and the $\frac{\text{CO}}{\text{CO}_2}$ ratio=42.5, corresponding to a temperature somewhere above 1500° F.

These results, therefore, show effectively how the steam supply controls the bed temperature and the gas quality as to CO and H₂ content and, why, for the constant gas quality needed in power gas, accurate control of bed and blast conditions is so necessary.

In the second table, reporting the Geological Survey tests, the first remarkable feature noticeable is the regularity with which the carbon oxide ratio $\frac{\text{CO}}{\text{CO}_2}$ increases with decrease of CO₂, and the small value of the ratio

compared to that obtained in other operating tests. This is an indication of very low temperature working, for the ratio is as small as .87 and never rises above 2.4, possibly due to high rates of blasting. It is too much to expect equilibrium to be established in any producer, yet departures from it give rise to interpretations as to what was happening in one case that did not take place in the other. The general average of the ratio of CO to CO₂ being so low in the U. S. G. S., tests leads to the conclusion that (a) the producer was driven to higher capacity than the average of others, leaving less time for establishment of equilibrium; (b) more steam was used, resulting in lower temperatures and leaving a residue of undecomposed steam to react on the CO that was formed, oxydizing it to CO₂. There is no relation whatever between the efficiencies reported for these tests and any of the fundamental factors that control efficiency, owing probably to the shortness of the runs and the irregularity of working reported. Producer efficiencies cannot be obtained except by absolutely steady rate of output, which is necessary to a steady thermal state in the producers for long periods of time because of the slowness of gasification compared to the body of coal in the producer. In these tests the producer could hold about 5000

lbs. of coal in all stages from green coal to ash. The rate of combustion per hour varied from a little over 5 per cent to 12 per cent of the weight in the full producer, making possible a very large error in judging equality of conditions of the bed before and after the run, for such short periods of time. For with the 5 per cent rate a particle of coal would be roughly 20 hours in passing through, as it would take this time to consume what was under it when it was fired.

It is desirable to establish such standards of relation between the quantities involved as were found by analysis for air gas and water gas, to be applied to the producer-gas case for the purpose of comparison of results, with possibilities, and this may be done by similar methods. From what has been said it is evident that when air gas and water gas are made simultaneously from the fixed carbon or coke, and mixed with the volatile more or less decomposed, that *the final gas will depend on (a) the proportion of the air gas to water gas; (b) the volume of combined air and water gas to the volatile of the coal; (c) the degree to which the volatile is decomposed or suffers reactions.* The first step, therefore, in a fundamental analysis is the establishment of formulas for the mixed gasified carbon gases in terms of the quantities of constituents, carbon, air and steam.

From Eq. (635) of Section 86 for air gas in terms of weights the following Eq. (654) is derived by dividing by 5.759(1+x) and rearranging.

$$1 \text{ lb. air} + \left[\frac{1}{5.759(1+x)} \right] \text{ lbs. C} = \left\{ \begin{array}{l} \left[\frac{3.66x}{5.759(1+x)} \right] \text{ lbs. CO}_2 \\ + \left[\frac{2.33(1-x)}{5.759(1+x)} \right] \text{ lbs. CO} \\ + \left[\frac{4.428}{5.759} \right] \text{ lbs. N}_2 \\ + \left[\frac{10,193x+4351}{5.759(1+x)} \right] \text{ B.T.U.} \end{array} \right\}. \quad (654)$$

The corresponding volume relation follows from Eq. (636) and is given by Eq. (655).

$$1 \text{ lb. air} + \left[\frac{1}{5.759(1+x)} \right] \text{ lbs. C} = \left\{ \begin{array}{l} \left[\frac{29.8x}{5.759(1+x)} \right] \text{ cu.ft. CO}_2 \\ + \left[\frac{29.8(1-x)}{5.759(1+x)} \right] \text{ cu.ft. CO} \\ + \left[\frac{56.32}{5.759} \right] \text{ cu.ft. N}_2 \\ + \left[\frac{10,193x+4351}{5.759(1+x)} \right] \text{ B.T.U.} \end{array} \right\}. \quad (655)$$

Two similar equations are given below, Eqs. (656) and (657), derived from the water-gas relations Eqs. (645) and (646) of Section 87, for *S* lbs. of steam reacting with carbon.

$$S \text{ lbs. H}_2\text{O} + \left[\frac{S}{1.5(1+x)} \right] \text{ lbs. C} = \left\{ \begin{array}{l} \left[\frac{3.66xS}{1.5(1+x)} \right] \text{ lbs. CO}_2 \\ + \left[\frac{2.33(1-x)S}{1.5(1+x)} \right] \text{ lbs. CO} \\ + \left[\frac{.167S}{1.5} \right] \text{ lbs. H}_2 \\ + S \left[\frac{1545x-4298}{1.5(1+x)} \right] \text{ B.T.U.} \end{array} \right\} \quad (656)$$

$$S \text{ lbs. H}_2\text{O} + \left[\frac{S}{1.5(1+x)} \right] \text{ lbs. C} = \left\{ \begin{array}{l} \left[\frac{29.8xS}{1.5(1+x)} \right] \text{ cu.ft. CO}_2 \\ + \left[\frac{29.8(1-x)S}{1.5(1+x)} \right] \text{ cu.ft. CO} \\ + \left[\frac{29.8S}{1.5} \right] \text{ cu.ft. H}_2 \\ + S \left[\frac{1545x-4298}{1.5(1+x)} \right] \text{ B.T.U.} \end{array} \right\} \quad (657)$$

If 1 lb. of air and S lbs. of steam together react on carbon, and the oxygen of the air and that of the steam act the same, or in symbols x is the same for both reactions, then together they will give a result expressed by Eqs. (658) and (659), one in terms of gas volumes and the other in weights, both giving the heats of reaction and both obtained by addition.

$$\left[\begin{array}{l} 1 \text{ lb. air} \\ + S \text{ lbs. H}_2\text{O} \end{array} \right] + \left[\left(\frac{1}{5.759} + \frac{S}{1.5} \right) \frac{1}{1+x} \right] \text{ lbs. C} = \left\{ \begin{array}{l} \left[\frac{3.66x}{(1+x)} \left(\frac{1}{5.759} + \frac{S}{1.5} \right) \right] \text{ lbs. CO}_2 \\ + \left[\frac{2.33(1-x)}{(1+x)} \left(\frac{1}{5.759} + \frac{S}{1.5} \right) \right] \text{ lbs. CO} \\ + \left[\frac{4.428}{5.759} \right] \text{ lbs. N}_2 \\ + \left[\frac{.167S}{1.5} \right] \text{ lbs. H}_2 \\ + \left[\frac{10,193x+4351}{5.759(1+x)} + \frac{S(1545x-4298)}{1.5(1+x)} \right] \text{ B.T.U.} \end{array} \right\} \quad (658)$$

$$\left[\begin{array}{l} 1 \text{ lb. air} \\ + S \text{ lbs. H}_2\text{O} \end{array} \right] + \left[\left(\frac{1}{5.759} + \frac{S}{1.5} \right) \frac{1}{1+x} \right] \text{ lbs. C} = \left\{ \begin{array}{l} \left[\frac{29.8x}{(1+x)} \left(\frac{1}{5.759} + \frac{S}{1.5} \right) \right] \text{ cu.ft. CO}_2 \\ + \left[\frac{29.8(1-x)}{(1+x)} \left(\frac{1}{5.759} + \frac{S}{1.5} \right) \right] \text{ cu.ft. CO} \\ + \left[\frac{56.32}{5.759} \right] \text{ cu.ft. N}_2 \\ + \left[\frac{29.8S}{1.5} \right] \text{ cu.ft. H}_2 \\ + \left[\frac{10,193x+4351}{5.759(1+x)} + \frac{S(1545x-4298)}{1.5(1+x)} \right] \text{ B.T.U.} \end{array} \right\} \quad (659)$$

These two equations are fundamental to the gasification of the fixed carbon part of a coal in a gas producer fed by steam and air together.

The composition of the producer gas is shown in Table XXIII for the general case and for the two limiting cases when $\text{CO} = 0$ and when $\text{CO}_2 = 0$.

The general relations between the various quantities of raw materials and products are given by Eq. (660) below.

$$\begin{aligned}
 & \text{1 lb. carbon makes} \\
 & \left[\frac{1+S+\left(\frac{1}{1+x}\right)\left(\frac{1}{5.759+1.5}+\frac{S}{1.5}\right)}{\left(\frac{1}{5.759+1.5}\right)\left(\frac{1}{1+x}\right)} \right] \text{ lbs.} = \left[\frac{\left(\frac{29.8}{1+x}\right)\left(\frac{1}{5.759+1.5}+\frac{S}{1.5}\right) + \frac{56.32}{5.759} + \frac{29.8S}{1.5}}{\left(\frac{1}{5.759+1.5}\right)\left(\frac{1}{1+x}\right)} \right] \text{ cu.ft. gas (a)} \\
 & \text{1 lb. steam makes} \\
 & \left[\frac{1+S+\left(\frac{1}{1+x}\right)\left(\frac{1}{5.759+1.5}+\frac{S}{1.5}\right)}{S} \right] \text{ lbs.} = \left[\frac{\left(\frac{29.8}{1+x}\right)\left(\frac{1}{5.759+1.5}+\frac{S}{1.5}\right) + \frac{56.32}{5.759} + \frac{29.8S}{1.5}}{S} \right] \text{ cu.ft. gas (b)} \\
 & \text{1 lb. air makes} \\
 & \left[\frac{1+S+\left(\frac{1}{1+x}\right)\left(\frac{1}{5.759+1.5}+\frac{S}{1.5}\right)}{\left(\frac{1}{1+x}\right)\left(\frac{1}{5.759+1.5}+\frac{S}{1.5}\right)} \right] \text{ lbs.} = \left[\frac{\left(\frac{29.8}{1+x}\right)\left(\frac{1}{5.759+1.5}+\frac{S}{1.5}\right) + \frac{56.32}{5.759} + \frac{29.8S}{1.5}}{\left(\frac{1}{1+x}\right)\left(\frac{1}{5.759+1.5}+\frac{S}{1.5}\right)} \right] \text{ cu.ft. gas (c)} \\
 & \text{1 cu.ft. gas requires} \\
 & \left[\frac{\left(\frac{1}{5.759+1.5}\right)\left(\frac{1}{1+x}\right)}{\left(\frac{29.8}{1+x}\right)\left(\frac{1}{5.759+1.5}+\frac{S}{1.5}\right) + \frac{56.32}{5.759} + \frac{29.8S}{1.5}} \right] \text{ lbs. carbon} \quad (d) \\
 & \text{1 cu.ft. gas requires} \\
 & \left[\frac{S}{\left(\frac{29.8}{1+x}\right)\left(\frac{1}{5.759+1.5}+\frac{S}{1.5}\right) + \frac{56.32}{5.759} + \frac{29.8S}{1.5}} \right] \text{ lbs. steam} \quad (e) \\
 & \text{1 cu.ft. gas requires} \\
 & \left[\frac{1}{\left(\frac{29.8}{1+x}\right)\left(\frac{1}{5.759+1.5}+\frac{S}{1.5}\right) + \frac{56.32}{5.759} + \frac{29.8S}{1.5}} \right] \text{ lbs. air} \quad (f)
 \end{aligned}
 \tag{660}$$

In order to show the composition of the gas by volume for various proportions of steam to air in the blast, assuming all the steam to react, four curves have been plotted for different ratios which are given in the book of tables, Chart 32. The left-hand curve is for a ratio of $\frac{\text{CO}}{\text{CO}_2} = 2$, the other three for ratios of 6, 15, and infinity. The first three cover the ranges obtained in practice, extending from a value that is lower, to one that is higher than usually obtained, while the last is the limiting case. The composition by volume is plotted vertically and the value of S , which is the weight of steam per pound of air reacting, horizontally. These are separately plotted because it is not convenient to show on one curve the variations in each of the four gas constituents as a function of the two prime variables S and $\frac{\text{CO}}{\text{CO}_2}$.

Heats of reaction are plotted as a function of S alone, laid off horizontally in Chart 33, in the Handbook of Tables, and a separate curve drawn for each value of the $\frac{\text{CO}}{\text{CO}_2}$ ratio, 2, 6, 15 and infinity. The vertical distances are heats of reaction, first, per pound of gases produced, and second, per pound of

TABLE XXIII
COMPOSITION OF HYPOTHETICAL PRODUCER GAS FROM THE FIXED CARBON

Constituents	By Weight.	By Volume.	When $z=0$.		When $z=1$.	
			By Wt.	By Vol.	By Wt.	By Vol.
CO_2	$\left(\frac{3.66x}{1+x}\right) \left[\frac{1}{1+x} \left[\frac{1}{5.759} + \frac{S}{1.5} \right] \right]$	$\left[\frac{29.8x}{1+x} \right] \left[\frac{1}{5.759} + \frac{S}{1.5} \right]$	0	0	$.318 + 1.222S$ $1.087 + 1.333S$	$2.59 + 9.93S$ $12.37 + 29.8S$
CO	$\left[\frac{2.33(1-x)}{1+x} \right] \left[\frac{1}{5.759} + \frac{S}{1.5} \right]$	$\frac{29.8(1-x)}{1+x} \left[\frac{1}{5.759} + \frac{S}{1.5} \right]$	$.405 + 1.556S$ $1.174 + 1.667S$	$5.17 + 19.87S$ $14.95 + 39.73S$	0	0
H_2	$\frac{.167S}{1.5}$	$\frac{29.8S}{1.5}$.111S	$19.87S$.111S	$19.87S$
N_2	$\frac{4.428}{5.759}$	$\frac{56.32}{5.759}$	$1.174 + 1.667S$	$14.95 + 39.73S$	$1.087 + 1.333S$	$12.37 + 29.8S$
Ratio $\frac{\text{H}_2}{\text{N}_2}$	$\frac{.167 \times 5.759 S}{1.5 \times 4.428} = .144S$	$\frac{29.8 \times 5.759 S}{1.5 \times 56.32} = 2.031S$.144S	2.031S	.144S	2.031S
Ratio $\frac{\text{CO}}{\text{CO}_2}$	$\frac{2.33(1-x)}{3.66x} = .636 \left[\frac{1-x}{x} \right]$	$\frac{1-x}{x}$	∞	∞	0	0
Ratio $\frac{\text{CO}}{\text{CO} + \text{CO}_2}$	$\frac{1-x}{1 + .572x}$	$1-x$	1	1	0	0

carbon, the former being a measure of temperature rise, and the latter of efficiency of reaction. These two heats are given in the two Eqs. (661) and (662), as derived from Eq. (658).

$$\left\{ \begin{array}{l} \text{Heat of reaction B. T. U.} \\ \text{per pound of} \\ \text{gas produced.} \end{array} \right\} = \left\{ \frac{\left[\left(\frac{10,193x+4351}{5.759} \right) + S \left(\frac{1545x-4298}{1.5} \right) \right] \left(\frac{1}{1+x} \right)}{\left[1+S+\left(\frac{1}{1+x} \right) \left(\frac{1}{5.759+1.5} \right) \right]} \right\}. \quad (661)$$

$$\left\{ \begin{array}{l} \text{Heat of reaction B. T. U.} \\ \text{per pound} \\ \text{of carbon} \\ \text{gasified.} \end{array} \right\} = \left\{ \frac{\left[\left(\frac{10,193x+4351}{5.759} \right) + S \left(\frac{1545x-4298}{1.5} \right) \right]}{\left(\frac{1}{5.759+1.5} \right)} \right\}. \quad (662)$$

Air saturated with water vapor carries a definite weight of vapor per pound

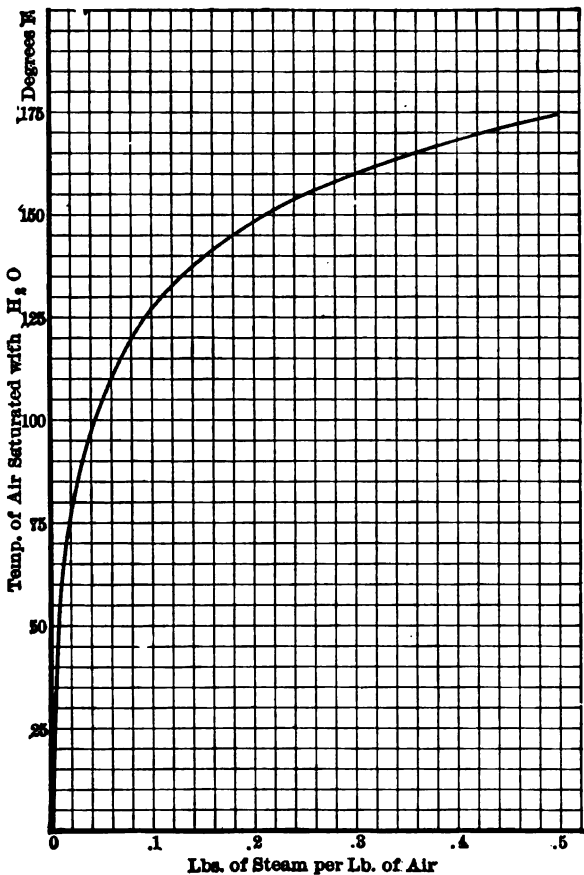


FIG. 126.—Temperature of Air Blast at which Saturated Air Carries a Given Weight of Steam per Pound of Air.

of air and as the air-blast is to carry water vapor in some proportion it is convenient to define the amount in terms of the temperature of moisture-saturated air that carries the definite quantity in question. This is most conveniently done in curve form, Fig. 126, where the temperature of water-saturated air is plotted vertically to horizontals of pounds of steam per pound of air.

As previously pointed out, the greater the quantity of steam in the blast the more endothermic heat will be taken up from the exothermic heat of the air-gas reaction, and when these two are equal there will be *no temperature rise* of the products of gasification over that of the blast. This is a limiting case, for which no reaction could take place. In order that reaction may occur some temperature rise must be permitted or, in other terms, the net heat of reaction must be exothermic and sufficient in

quantity to allow the final gas to be as hot as necessary. This final gas should not be less than 1100° F., corresponding roughly to 250 B.T.U. exothermic heat, per pound of gases, which fixes a limiting value of S (on Chart No. 33), depending on the $\frac{\text{CO}}{\text{CO}_2}$ ratio to be allowed in the gas. For $\frac{\text{CO}}{\text{CO}_2}=6$, the corresponding value of S is .215, which would be carried by air saturated at 150° F.; for $\frac{\text{CO}}{\text{CO}_2}=15$, the value of S that gives 250 B.T.U. per pound of gases exothermic heat, is .164, which would be carried by air saturated at 142° F.; while for no CO_2 in the gas or $\frac{\text{CO}}{\text{CO}_2}=\infty$, $S=.15$.

The heat of combustion of the gas produced from 1 lb. of carbon is equal to 14,544 less the heat of reaction per pound of carbon or, Eq. (663).

$$\left\{ \begin{array}{l} \text{Heat of combustion of gas from one pound of carbon B.T.U., low.} \end{array} \right\} = 14,544 - \left[\frac{\left(\frac{10,193x+4351}{5.759} \right) + S \left(\frac{1545x-4298}{1.5} \right)}{\left(\frac{1}{5.759} + \frac{S}{1.5} \right)} \right]. \quad (663)$$

Therefore the efficiency of the reaction is,

$$E = 1 - \left[\frac{\left(\frac{10,193x+4351}{5.759} \right) + S \left(\frac{1545x-4298}{1.5} \right)}{14,544 \left(\frac{1}{5.759} + \frac{S}{1.5} \right)} \right]. \quad (664)$$

From this equation it appears that the efficiency of the reaction is primarily a question of the quantity of steam used for any given value of $\frac{\text{CO}}{\text{CO}_2}$. The efficiency as given by this equation is plotted vertically to horizontal values of S , one curve each, for values of $\frac{\text{CO}}{\text{CO}_2}$ equal to 2, 6, 15 and ∞ , in Fig. 127.

The heat of combustion of the gas itself can be found by dividing the heat of combustion of the gas produced from 1 lb. of carbon by its volume, and is given by Eq. (665),

B.T.U. cu.ft. gas (standard, low)

$$= \frac{1}{1+x} \left[\frac{14,544 \left(\frac{1}{5.759} + \frac{S}{1.5} \right) - \left(\frac{10,193x+4351}{5.759} \right) - \left(\frac{1545x-4298}{1.5} \right) S}{\left(\frac{29.8}{1+x} \right) \left(\frac{1}{5.759} + \frac{S}{1.5} \right) + \frac{56.32}{5.759} + \frac{29.8S}{1.5}} \right]. \quad (665)$$

If CO_2 is absent $x=0$ and this becomes

$$\text{B.T.U. cu.ft. gas (standard, low)} = \frac{1770+6831S}{14.95+39.73S}. \quad (666)$$

If no steam is supplied $S=0$ and B.T.U. per cubic foot gas=118, which checks the value found for air gas with 70 per cent efficiency. With .10 lb. steam per pound air, B.T.U. per cubic foot gas=130, for which condition the reac-

tion efficiency is 86.6 per cent. These figures show that steam in the blast increases both calorific power of gas and the thermal efficiency of its production. The steam-efficiency relations have been shown in Fig. 127, so, to correspond, the calorific power of gas is plotted in Fig. 128 to the same horizontal, pounds of steam per pound of air.

To complete the series of graphic results, the gas production in cubic feet standard, per pound of fixed carbon, is plotted in Fig. 129, to horizontal of S , one curve each for $\frac{\text{CO}}{\text{CO}_2}$ values of ∞ , 2, 6, and 15, Eq. (660).

So far the results all relate to the pure process of gasifying carbon with steam and air, but in real producers there are additional factors to be accounted

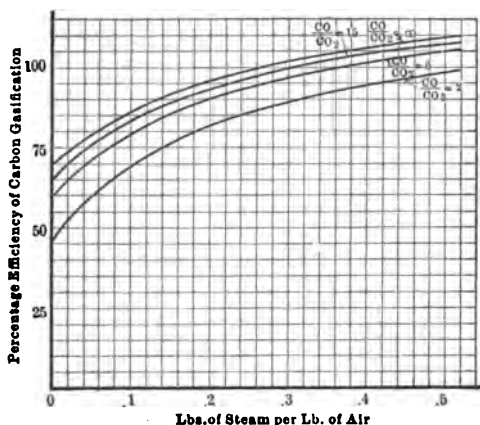


FIG. 127.—Efficiency of Hypothetical Gas Producer Gasifying Fixed Carbon.

for, but this cannot be done quantitatively. There must be considered the heat losses of radiation, steam making, distillation of volatile, escape of unchanged steam and heats of reaction of volatile parts that suffer chemical change after being liberated. The production of the volatile always consumes less heat per B.T.U. in the gas formed than the gasification of carbon, so that the coals which yield much volatile in the producer should give a higher efficiency than those that do not yield as much, unless some of the volatile is burned or its

heat used up in other endothermic reactions. In fact, this gain by the volatile generally serves to balance the other heat losses just mentioned, so that the real producer can have efficiencies fairly close to those calculated for the gasification of carbon if the volatile is 10 per cent and over. The addition of the volatile or its products to the gasified carbon will not change the relation of the CO, CO₂, H₂ and N₂ constituents very much, if at all, but will reduce the per cent of each. The following general relation must hold:

$$\left\{ \begin{array}{l} \text{Cubic feet gas} \\ \text{per pound coal} \end{array} \right\} = \left\{ \begin{array}{l} \left[\frac{\text{cu.ft. volatile}}{\text{per lb. volatile}} \right] \times \left[\frac{\text{Weight volatile}}{\text{per lb. coal}} \right] \\ + \left[\frac{\text{cu.ft. gasified}}{\text{C per lb. C}} \right] \times \left[\frac{\text{Wt. fixed carbon}}{\text{per lb. coal}} \right] \end{array} \right\} \quad \dots (667)$$

This Eq. (667) must be more or less exactly evaluated before the effects of volatile addition on each constituent can be estimated, and to do this it is necessary to have some data on the cubic feet of gas per pound of volatile and the proportions of constituents making it up. These can be found by considering it to behave as coal gas from retorts or as coke oven gas. English cannel coals yield the largest amount of volatile; they give on the average about 4.5 cu.ft.

gas per pound coal, and are about 60 per cent by weight of volatile, so that the cu. ft. of gas per pound of volatile = 7, approximately. Gas coals in retorts or coke ovens will yield about 8 or 9 cu.ft. gas per pound volatile, and about the same for quite a range of volatile per cent. The gasified fixed carbon yield is somewhere near 90 cu.ft. per pound. Therefore, a coal with 30 per cent volatile and 60 per cent fixed carbon would give in round numbers 56 cu.ft. of mixed volatile and gasified carbon, of which the volatile makes up 4 per cent by volume, which has small influence on the proportion. The real influence may, however, be less, as in an up-draft producer some of the volatile will condense to tar; several of the U. S. G. S. tests show about .2 lb. tar per pound volatile of the proximate analysis, so that one-fifth of the volatile was condensed. With down-draft producers the volatile may completely change and the gas show no trace except a small per cent of CH_4 .

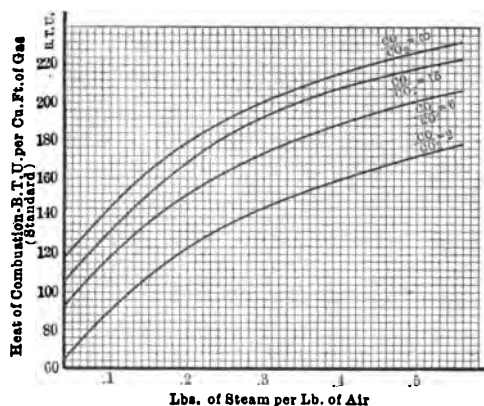


FIG. 128.—Calorific Power of Hypothetical Producer Gas from Fixed Carbon. Low Value.

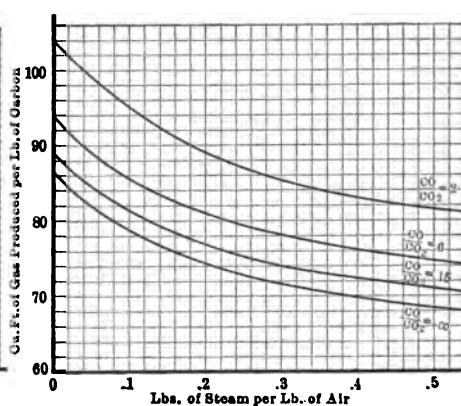


FIG. 129.—Quantity of Hypothetical Producer Gas (Standard) per Pound of Fixed Carbon.

Two most important factors in the gasifying process which have only recently come within the scope of experimental measurement are the physical condition of the bed as to porosity, homogeneity, uniformity of gas flow, and the temperature of the bed with its homogeneity of distribution. Clement and Grine of the U. S. G. S. measured the temperature in the bed of a producer 7 ft. internal diameter, up-draft, steam jet blower, delivering the blast under a hood in the center of the producer bottom at a point normally covered with ash. The results are plotted in Fig. 130 in the form of constant temperature lines. The highest temperature is found in the bottom the same as would be found in a boiler fire when supplied with steam in the air. The temperature falls slowly and then more rapidly toward the top of the fuel bed, kept at 5½ ft. above the blast entrance. The curvature of these lines shows that the blast rises toward the sides more than in the center. The explanations and discussion of gas analyses, which were made at the same time from the bed interior, show first, that a non-homogeneous condition exists, partly because the blast is impelled outward by the hood

and also because the resistance at the walls is less than anywhere else; second, that the temperatures actually attained are very much higher than correspond to equilibrium of the $\frac{\text{CO}}{\text{CO}_2}$ ratio reported. This ratio for the gases leaving, which were a mixture of different gases from different parts of the bed, was $\frac{\text{CO}}{\text{CO}_2}=3$, for which the equilibrium temperature is 1376° F., while in this bed the temperature was 2200° F. at which value the equilibrium value of the ratio $\frac{\text{CO}}{\text{CO}_2}$ is over 200.

It has been found possible to generate a usable power gas from a jet of coal dust or oil blasted into a combustion chamber with air insufficient for complete combustion, if the walls and blast be so arranged as to keep the hot

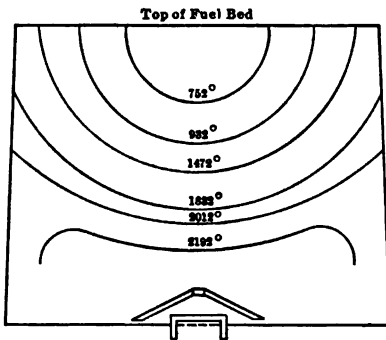


FIG. 130.—Portion of Producer Section showing Temperature Variation.

products in contact with glowing free carbon liberated from the fuel by its decomposition. Analyses of such gas are given in the Tables for heavy Western oils, (Table LXIII) which show a relatively low CO content, though not so high a per cent CO₂ as might be expected from the bed type of solid carbon producer. As the temperatures in these chambers are quite high, it must be concluded that equilibrium between CO and CO₂ has not been established and the reason may be either too high a velocity of the gases, which is the same as

insufficient time, or what is more likely, *not enough free carbon present at the right time and place for CO₂ reduction*. These gases if passed through a coke bed would be very much improved, in fact would become almost perfect as to combustibles. Another characteristic of these analyses is the large CH₄ content and with it also a large amount of heavy hydrocarbons. These two things are characteristic of the oil fuel as raw material, the methane having come from decomposition of higher hydrocarbons, yielding free carbon, some of which went into CO and some was left behind as coke in the producer.

An almost identical operation with pulverized coal that has recently been tried gave the gas analyses as in Table LXXV in the book of tables. The powder and air are blown into a large combustion chamber without steam, all hydrogen coming from coal volatile, or from moisture in air and coal. As the coal was bituminous, ranging from 60 to 77 per cent fixed carbon and 18 to 33 per cent volatile, the small quantity of illuminants or heavy hydrocarbons shows that these were attacked by simple decomposition or by oxygen, more likely the latter, as the methane is also low, while in coal gas it would be much higher and even higher also in up-draft producer gas with the same coal. The carbon monoxide-dioxide ratios are very much the same as for the oil-gas

producer and probably for the same reason, insufficient amounts of free carbon for the more complete reduction of CO_2 to CO , because the temperatures developed are quite high enough.

Horse-power, as a term applied to a gas producer, is just as much out of place but just as necessary as when applied to a steam boiler. It is out of place because no mechanical work is done by either piece of apparatus, both being concerned with and used for supplying the means for doing work in steam and gas engines. This gives the reason for the necessity of a name as engines are rated at, and develop a given horse-power, so it is natural to associate the horse-power name and number with the producer or boiler that makes it possible, however indirect or scientifically unsound it may be. There is therefore necessary some standard of producer as of boiler horse-power, and it should be based on the capacity of either to supply respectively the requisite amount of gas or steam per hour that an engine may need to develop and maintain one horse-power of output. No such standard has, however, been generally accepted, but one can be easily created.

TABLE XXIV
DENSITY AND CALORIFIC POWER OF PRODUCER GAS

Low Volatile Anthracite	Constituent.	One Cubic Foot.				Summary.
		Contains		B.T.U.		
		Cu.ft.	Lbs.	High.	Low.	
	Carbon monoxide, CO..	.261	.020376	89.0	89.0	B.T.U. per cu.ft. gas, high 142.3
	Hydrogen, H ₂1500	.000843	51.2	43.8	B.T.U. per cu.ft. gas, low 134.7
	Methane, CH ₄0020	.000089	2.1	1.9	
	Carbon dioxide, CO ₂053	.000052	Cu.ft. per lb. gas..... 14.36
	Oxygen, O ₂002	.000178	Lbs. per cu.ft. gas..... .0696
	Nitrogen, N ₂532	.041650	
	Total	1.000	.069638	143.3	134.7	B.T.U. per lb. gas, high. 2043 B.T.U. per lb. gas, low... 1934
Peat	Carbon monoxide, CO..	.272	.021235	92.8	92.8	B.T.U. per cu.ft. gas, high 128.9
	Hydrogen, H ₂009	.000051	3.1	2.6	B.T.U. cu.ft. gas, low.... 125.1
	Methane, CH ₄031	.001385	33.0	29.7	
	Carbon dioxide, CO ₂121	.014845	Cu.ft. per lb. gas..... 12.21
	Nitrogen, N ₂567	.044390	Lbs. per cu.ft. gas..... .0819
	Total	1.000	.081906	128.9	125.1	B.T.U. lb. gas, high.... 1574 B.T.U. lb. gas, low.... 1627
Bituminous, Updraft	Carbon monoxide, CO..	.253	.019751	86.3	86.3	B.T.U. per cu.ft. gas, high 164.3
	Hydrogen, H ₂092	.000517	31.4	26.9	B.T.U. per cu.ft. gas, low 155.5
	Methane, CH ₄031	.001385	33.0	29.7	
	Heavy hydro., C ₂ H ₄008	.000636	13.6	12.6	Cu.ft. per lb. gas..... 13.88
	Carbon dioxide, CO ₂034	.004171	Lbs. per cu.ft. gas..... .0720
	Nitrogen, N ₂582	.045565	
	Total072025	164.3	155.5	B.T.U. per lb. gas, high.. 2280 B.T.U. per lb. gas, low.. 2158.

It will be safe to say that the average good gas engine of almost any size above 100 H.P. can develop a horse-power on 10,000 B.T.U. per hour

in the gas, low value, and many can do better—as low as 8500 having been proved possible, though a few two-cycle engines may use as high as 12,500. The figure 10,000 B.T.U. per hour being an easily remembered round number and well within the range of practical performance with a fair margin of capacity for very good engines, may well be adopted as the equivalent of a gas-producer horse-power. Therefore,

$$\left. \begin{aligned} \text{H.P. of gas producer} &= \frac{\text{B.T.U. per hr. (low) in gas produced}}{10,000} & (a) \\ &= \frac{(\text{Cu.ft. gas per hr.}) \times (\text{B.T.U. cu.ft. gas (low)})}{10,000} & (b) \end{aligned} \right\} (668)$$

In Table XXIV are given some average density and calorific properties for the producer gases, illustrating their calculation from the volumetric constituents in the usual way.

Example 1. A producer is blasted with air and steam in the ratio of 6 to 1 by weight. The ratio of CO to (CO₂+CO) is .8. What will be the composition of the gas by volume and weight, the pounds of carbon, air, and steam required per cubic foot of gas and the heat of combustion per cubic foot of gas?

For an air-steam ratio of 6, S will be $\frac{1}{6} = .1667$. Since $\text{CO} + (\text{CO} + \text{CO}_2) = 1 - x = .8$ or $x = .2$.

$$\text{The CO}_2 \text{ by weight is } \left[\frac{\frac{3.66 \times .2 \left(\frac{1}{5.759} + \frac{.1667}{1.5} \right)}{1.2}}{1 + .1667 + \frac{1}{1.2} \left(\frac{1}{5.759} + \frac{.1667}{1.5} \right)} \right] = 12.4 \text{ per cent.}$$

$$\text{The CO by weight is } \left[\frac{\frac{2.33 \times .8 \left(\frac{1}{5.759} + \frac{.1667}{1.5} \right)}{1.2}}{1 + .1667 + \frac{1}{1.2} \left(\frac{1}{5.759} + \frac{.1667}{1.5} \right)} \right] = 31.6 \text{ per cent.}$$

$$\text{The H}_2 \text{ by weight is } \left[\frac{\frac{.167 \times .1667}{1.5}}{1 + .1667 + \frac{1}{1.2} \left(\frac{1}{5.759} + \frac{.1667}{1.5} \right)} \right] = 1.3 \text{ per cent.}$$

$$\text{The N}_2 \text{ by weight is } \left[\frac{\frac{4.428}{5.759}}{1 + .1667 + \frac{1}{1.2} \left(\frac{1}{5.759} + \frac{.1667}{1.5} \right)} \right] = 54.8 \text{ per cent.}$$

$$\text{The CO}_2 \text{ by volume is } \left[\frac{\frac{29.8 \times .2 \left(\frac{1}{5.759} + \frac{.1667}{1.5} \right)}{1.2}}{29.8 \left(\frac{1}{5.759} + \frac{.1667}{1.5} \right) + \frac{56.32}{5.759} + \frac{29.8 \times .1667}{1.5}} \right] = 7 \text{ per cent.}$$

$$\text{The CO by volume is } \left[\frac{\frac{29.8 \times .8 \left(\frac{1}{5.759} + \frac{.1667}{1.5} \right)}{1.2}}{29.8 \left(\frac{1}{5.759} + \frac{.1667}{1.5} \right) + \frac{56.32}{5.759} + \frac{29.8 \times .1667}{1.5}} \right] = 28.1 \text{ per cent.}$$

$$\text{The H}_2 \text{ by volume is } \left[\frac{\frac{29.8 \times .1667}{1.5}}{\frac{29.8 \left(\frac{1}{5.759} + \frac{.1667}{1.5} \right) + \frac{56.32}{5.759} + \frac{29.8 \times .1667}{1.5}} \right] = 16.4 \text{ per cent.}$$

$$\text{The N}_2 \text{ by volume is } \left[\frac{\frac{56.32}{5.759}}{\frac{29.8 \left(\frac{1}{5.759} + \frac{.1667}{1.5} \right) + \frac{56.32}{5.759} + \frac{29.8 \times .1667}{1.5}} \right] = 48.5 \text{ per cent.}$$

$$1 \text{ cu.ft. gas requires } \left[\frac{\left(\frac{1}{5.759} + \frac{.1667}{1.5} \right) \frac{1}{1.2}}{\frac{29.8 \left(\frac{1}{5.759} + \frac{.1667}{1.5} \right) + \frac{56.32}{5.759} + \frac{29.8 \times .1667}{1.5}} \right] = .0116 \text{ lb. C.}$$

$$1 \text{ cu.ft. gas requires } \left[\frac{1}{\frac{1}{.1667} \left[\frac{29.8 \left(\frac{1}{5.759} + \frac{.1667}{1.5} \right) + \frac{56.32}{5.759} + \frac{29.8 \times .1667}{1.5} \right]} \right] = .0083 \text{ lb. steam.}$$

$$1 \text{ cu.ft. gas requires } \left[\frac{1}{\frac{29.8 \left(\frac{1}{5.759} + \frac{.1667}{1.5} \right) + \frac{56.32}{5.759} + \frac{29.8 \times .1667}{1.5}} \right] = .0495 \text{ lb. air.}$$

The heat of combustion of the gas per pound of carbon is, low value,

$$14,544 - \left[\frac{\frac{10,193 \times .2 + 4351}{5.759} + .1667 \left(\frac{1545 \times .2 - 4298}{1.5} \right)}{\left(\frac{1}{5.759} + \frac{.1667}{1.5} \right)} \right] = 13,471 \text{ B.T.U.}$$

From the above, 1 cu.ft. of gas requires .0116 lb. of carbon; hence the heat of combustion of the gas is $13,471 \times .0116$ or 156 B.T.U. per cubic foot.

Prob. 1. The following analysis of coke producer gas was taken when the producer was receiving little steam. $\text{CO} = 28$; $\text{H} = 4$; $\text{CO}_2 = \text{CH}_4 = 4$; $\text{N}_2 = 60$. What was the ratio of steam to air?

Prob. 2. If the air-steam ratio had been raised to 4, what would have been the composition of the gas for the same CO to CO_2 ratio?

Prob. 3. For an air-steam ratio of 6 and a CO to $(\text{CO}_2 + \text{CO})$ ratio of .9, what will be the carbon, steam, and air required for 1000 cu.ft. of gas and what will be the heating value of the gas?

Prob. 4. A coal used in a producer consists of 85 per cent fixed carbon, 5 per cent volatile, and 10 per cent ash. The CO to CO_2 ratio and the air-steam ratio are both five. If the volatile has a heating value of 20,000 B.T.U. per pound and it is assumed that it comes off uniformly, what will be the heat value of the gas per pound and per cu.ft.?

Prob. 5. A coke producer blast is saturated at 140°F . What is the maximum CO to CO_2 ratio which could be had for a continuous run? What is the efficiency?

Prob. 6. For a ratio of CO to CO_2 of 10, what would be the minimum steam ratio for a continuous process and the corresponding efficiency of reaction for a fixed carbon bed?

89. Combustion Effects. Final Temperature, Volume and Pressure for Explosive and Non-Explosive Combustion. Estimation of Air Weights and

Heat Suppression Due to CO in Products from Volumetric Analysis. When used for industrial purposes fuels are burned in one of two radically different characteristic ways, designated as, (a) explosive, and, (b) non-explosive combustion; while in every case there is a rise of temperature it may be accompanied by a volume or by a pressure change or both, rise of pressure being especially characteristic of explosive combustion. In any case the fundamental effect of combustion is a temperature rise which can be *only estimated* as to amount because of the great uncertainty of specific heats, coupled with the dissociation phenomenon, which retards further oxidation of fuel elements after the temperature has reached a certain value. Accordingly, all calculations of combustion effects must be regarded as more or less approximate, and to the results factors must be applied to bring them into accord with direct observation of effects, which factors measure by their magnitude the errors of the method and the uncertainty of fundamental physical constants. The application of such factors, which is necessary even with the most precise methods, opens the way for quick estimation of results by shorter methods that are almost universally used in engineering work concerned with explosions, especially those in the cylinders of internal-combustion engines.

Usually *non-explosive combustion* takes place at constant pressure, and explosive at constant volume; but these are not essential relations as either kind may proceed under any conditions of pressure or volume relation. The essential distinction between *explosive* and non-explosive combustion is to be found in the *self-propagation* of the former, or the progress of the flame or reaction causing it. For combustion to be explosive there must be a *mixture* of fuel and oxygen which may be diluted with neutral gases like nitrogen, carbon dioxide or water vapor within certain limits. Moreover, the mixture must be of the most intimate and homogeneous sort though the fuel may be in the solid, liquid, or gaseous state. Each particle of oxygen must be in contact with the fuel particle the combustion of which it is to support, so that if the fuel is solid it must be in powdered form suspended in the oxygen; if liquid, it must be atomized, in as fine a mist form as possible. The ideal condition of the fuel for explosive combustion is, however, the gaseous or vapor form, mixed homogeneously with the supporting oxygen. Mixtures that comply with these specifications will explode, that is, combustion once started by an ignition of any part of the mass at a single point, will propagate itself throughout the entire mass if it is at rest. The flame will at any instant be on a surface ideally of no thickness, but actually of measurable thickness if there be some inactive gases present, which surface separates the burnt from the unburnt. This surface advances through the mixture at a speed entirely independent of any motion of the mixture mass, and the combustion proceeds at a rate determined by this speed of propagation peculiar to the mixture itself, and not by any surrounding conditions. If the mixture be in a closed chamber the whole mass will burn in a short time; if it be supplied to a combustion chamber continuously, as sometimes happens in gas and oil furnaces and those consuming powdered coal, the combustion will proceed at the same rate against the current of supply, the velocity of which must be greater

than the rate of propagation to prevent *back flash*, and be subsequently reduced in the furnace below the rate of propagation to prevent *blow-off*.

If the mixture contain not enough oxygen to support combustion of all the fuel present, then the combustion may still be explosive within certain limits, to be discussed later, but the combustion cannot be complete when the explosion takes place in a closed chamber; it can only be complete when supplied in a stream, as in Bunsen burners or from furnace nozzles or burners, provided a supporting atmosphere of extra air be supplied externally to the jet or flame. In such cases the rate of combustion will be less because some of the fuel must wait until it can find oxygen; the flame will accordingly be longer and fill greater volumes of the combustion chamber, the rate will be least and the flame volume greatest for no previous mixture, all air for combustion in this case being derived from the external supporting atmosphere. This latter condition is found in common illuminating gas flames, and in the fires of coal with much volatile which rises from the bed practically without air mixture, all air necessary for combustion being admitted above the fire, meeting the gas when it can, the last particles burning in an atmosphere much diluted with the products of previously burnt fuel. Such conditions promote escape of unburnt gases, hydrocarbons, carbon monoxide or hydrogen, even when some oxygen is still unused and appears as free oxygen in the products of combustion.

Similarly, for solid fuels the combustion proceeds as fast as the air can reach the fuel, the rate of combustion being fixed by the rate of air supply, but not directly proportional thereto, because of the stream form of the air between the particles of fuel and the diluting, separating influence of previously burnt products tending to keep air and fuel apart. Thus, solid fuel combustion always must take place with excess air or be incomplete, for if just the right amount were supplied some oxygen would slip between the coal particles without touching and the quantity of combustible gases formed above would always be greater than this slippage air could burn. More air supplied with a view to burning this gas is ineffective because of lack of means of pre-mixing, so that while the fuel might all be burnt, partly in the solid and partly in gaseous form, it can be done only with excess air, absence of free oxygen in the flue gas being accompanied by much unburnt fuel.

While it is possible to calculate the weight of air necessary to complete the combustion of a pound of fuel from the reaction equation, or similarly, the volume of air per cubic foot of fuel, such calculations have little value either for explosive or non-explosive, continuous or intermittent fires. In all cases, however, the temperature rise should be given by the ratio of heat liberated by combustion to the product of the specific heat of the products into the weight of products. The weight of the products will always be the weight of air per pound of fuel chemically necessary for combustion, plus the excess air per pound of fuel, plus one, and the heat liberated is the calorific power per pound of fuel if it is all burnt. When, however, the air is insufficient to burn the fuel, and sometimes in the case of solid fuel, when it is also sufficient, the full calorific power cannot be developed and heat will be lost.

Furthermore, the full heat of combustion will not be available for temperature rise if in the course of that rise the dissociation temperature for the products is reached, for after that time further union, heat liberation and temperature rise will cease. Abstraction of some heat will permit combustion to proceed but without any more rise of temperature. These considerations show that the calculation of temperature rise on combustion or the consequent increase of volume at constant pressure, or increase of pressure at constant volume, when based on purely physical constants, is hardly more than a rough estimate even when the constants are positively known; decidedly so at the present time, when the dissociation constants and specific heats at high temperature are practically unknown.

Accordingly, to calculate *probable* effects of combustion experimentally determined factors must be relied upon.

Let Q = B.T.U. per pound fuel;

t_2 and t_1 = final and initial temperatures F.;

T_2 and T_1 = final and initial temperatures absolute;

V_2 and V_1 = final and initial volumes cu.ft.

P_2 and P_1 = final and initial pressures lbs. sq.ft.;

p_2 and p_1 = final and initial pressures lbs. sq.in.;

w_P = lbs. products per lb. fuel = wt. air chemically necessary +
wt. of excess air per lb. fuel + 1;

β = fraction of B.T.U. per lb. of fuel actually causing rise of
temperature;

C_p , C'_p , C_v , C'_v = specific heats at and mean specific heats, at constant
pressure and volume for the products of combustion.

$$\text{Then } \left. \begin{aligned} T_2 - T_1 = t_2 - t_1 &= \frac{\beta Q}{w_P C'_p}, \text{ when the pressure is constant (a)} \\ &= \frac{\beta Q}{w_P C'_v}, \text{ when the volume is constant (b)} \end{aligned} \right\}. \quad (669)$$

$$\text{For explosions at constant volume, } \frac{P_2}{P_1} = \frac{p_2}{p_1} = \frac{T_2}{T_1} = 1 + \left(\frac{\beta Q}{w_P C'_v T_1} \right), \quad (670)$$

$$P_2 - P_1 = \frac{P_1}{T_1} \left(\frac{\beta Q}{w_P C'_v} \right). \quad (671)$$

$$\text{Also for constant pressure combustion, } \frac{V_2}{V_1} = \frac{T_2}{T_1} = 1 + \left(\frac{\beta Q}{w_P C'_p T_1} \right). \quad (672)$$

$$V_2 - V_1 = \frac{V_1}{T_1} \left(\frac{\beta Q}{w_P C'_p} \right). \quad (673)$$

These equations as set down are perfectly correct but of no practical use, without evaluating β , C'_p and C'_v . To illustrate the point: consider the combustion of carbon monoxide in oxygen, the temperature of which, as measured by Mallard and Le Chatelier, was found to be 5800° F. The heat of combustion of CO burning to CO₂ is 4369 B.T.U. per pound CO and the weight of products is $\frac{44}{28} = 1.57$ lbs. Taking the specific heat of carbon dioxide

at constant volume at .2025 the value at 32° and constant, and assuming $\beta = 1$, that is all heat developed,

$$T_2 - T_1 = t_2 - t_1 = \frac{4369}{1.57 \times .2025} = 13,740^\circ \text{ F.}$$

This is quite different from the measured value of 5800, which latter is really only 40 per cent of that derived by the calculation based on constant specific heat. Introducing the variable specific heat the result is a little better but still fails to agree with the measurement by a very wide margin, as will appear from what follows.

The most commonly accepted, but still uncertain form for the specific heat of gases at high temperatures in terms of the value at 32°, is given by a first or second degree equation, from which the mean value between 32° and any temperature can be derived, and this multiplied by the temperature range will give the corresponding quantity of heat. To solve this equation for the final temperature in terms of the quantity of heat causing it is troublesome by algebraic methods but quite simple by a diagram such as Chart No. 34, given in the book of tables. Here one set of the Mallard and Le Chatelier values for the mean specific heat of various gases given in Eq. (674) has been used to calculate the temperature rise above 32° for various quantities of heat, and plotted. For any heat increment per pound of gases there is a corresponding temperature increment that can be read off directly. Thus for CO₂, consider 1 lb. to receive 1000 B.T.U., starting at 32° F., the temperature rise would be 3290° F. - 32° F. = 3258°; whereas from 1000° F. as a starting point this same 1000 B.T.U. would yield a temperature of 3690° F. or a rise of 2690°.

$$\left\{ \begin{array}{l} \text{Mean specific heat} \\ \text{Mallard \& Le Chatelier} \\ \text{Pressure constant.} \end{array} \right\} = \left\{ \begin{array}{ll} .149 + .000048t & \text{for CO}_2 \dots (a) \\ .433 + .000060t & \text{" H}_2 \dots (b) \\ .242 + .000024t & \text{" N}_2 \dots (c) \\ .212 + .000021t & \text{" O}_2 \dots (d) \\ 3.3 + .000015t & \text{" H}_2 \dots (e) \end{array} \right\}. \quad (674)$$

Applying this to the combustion of CO to CO₂, yielding $\frac{4369}{1.57} = 2783$ B.T.U. per pound of CO₂, the temperature rise above 32° is 6190 and the final temperature 6222° F., a little better than for constant specific heats but not good enough.

In place of the mean specific heats of the products taken by the variability law for the several constituents, the specific heat of air may be substituted at its value for low temperatures: $C_s = .170$ and $C_p = .243$ approximately. Then introducing a new constant C' by the relation ($C' \div$ constant specific heat of air) = ($\beta \div$ specific heat of products), the temperature rise will be given

$$\left. \begin{aligned} \text{by } T_2 - T_1 = t_2 - t_1 &= C' \frac{Q}{.170w_p}, \text{ for constant volume} & (a) \\ &= C' \frac{Q}{.243w_p}, \text{ for constant pressure} & (b) \end{aligned} \right\} \dots (675)$$

Estimation of temperature rise by this second method, Eq. (675), is possible whenever a direct observation on a similar case permits the determination of the value of C' for such cases. This is always possible in furnaces into which pyrometers may be inserted, but it is not possible for explosions in closed chambers because of the shortness of the time, but in such cases the pressures of explosion can be quite accurately determined. From such pressure observations the coefficient can be found from relations of the following kind:

Let H = B.T.U. per cu.ft. gas at 32° and 14.7 lbs. sq.in., low value;

a = cu.ft. air per cu.ft. gas;

n = cu.ft. neutral added to air-gas mixture containing one cu.ft. gas.

Then from Eq. (671) if the specific heat is assumed constant

$$P_2 - P_1 = \frac{P_1}{T_1} \left(\frac{\beta Q}{w_p C} \right) = \frac{R}{V_1 C_s} \frac{\beta Q}{C_s} = \frac{778(C_p - C_s)}{C_s} \frac{\beta Q}{V_1} \quad (676)$$

But $\frac{Q}{V_1}$ = B.T.U. per lb. fuel, divided by cubic feet of gases containing or formed from one pound of fuel, which is equal to B.T.U. per cubic foot mixture at pressure P_1 and temperature T_1 . Hence

$$\frac{Q}{V_1} = \left(\frac{H}{1+a+n} \right) \times \frac{p_1}{14.7} \times \frac{492}{T_1} \quad (677)$$

$$p_2 - p_1 = \frac{778}{144} (\gamma - 1) \beta \left[\frac{H}{1+a+n} \times \frac{p_1}{14.7} \times \frac{492}{T_1} \right] \quad (678)$$

Introduce the constant C and the value of γ for air according to the equation of condition,

$$[\gamma(\text{for mixture}) - 1] \beta = [\gamma(\text{for air}) - 1] C = .4C \quad (679)$$

$$\begin{aligned} p_2 - p_1 &= \frac{778 \times .4 \times 492}{144 \times 14.7} C \frac{p_1}{T_1} \left(\frac{H}{1+a+n} \right) = 72.33 C \frac{p_1}{T_1} \left(\frac{H}{1+a+n} \right) \\ &= 72.33 C \left(\frac{\text{initial press. lbs. per sq.in. abs.}}{\text{initial temp. F. abs.}} \right) \times \left\{ \frac{\text{B.T.U. per cu.ft. mixture at } 32^\circ \text{ F. and}}{14.7 \text{ lbs. per sq.in. abs.}} \right\} \quad (680) \end{aligned}$$

For explosions the pressure rise given by Eqs. (678) and (680) have been determined experimentally for all the fuels used in gas-engine cylinders, which include practically every one of the gases previously examined, as well as some vapors like kerosene, gasolene and alcohol, separately and mixed in various proportions; for all ranges of fuel and initial pressure an extraordinary constancy of the value of C has been observed, which may be summarized by Eq. (681).

$$\left\{ \begin{array}{l} \text{Heat suppression factor} \\ \text{for pressure rise} \end{array} \right\} = C = \left\{ \begin{array}{l} .45 \text{ minimum} \\ .55 \text{ maximum} \end{array} \right\} \quad (681)$$

To illustrate the procedure in determining the heat suppression factor the following calculation and experimental determinations for an oil gas will serve the purpose. In all cases the lower calorific power is taken:

Gas analysis per cent by volume—	CO ₂ =0	Of this CO ₂ =0.00 N ₂ =2.95 Total, neutral,=2.95
	C ₂ H ₄ =20.43	
	C ₆ H ₆ = 7.42	
	H ₂ = 6.	
	CH ₄ =62.15	
	O ₂ = 0.75	
	CO= 0.3	
	N ₂ = 2.95	
	Total, 100.00	

Air required=11.68 volumes per vol. gas, therefore, cu.ft. best mixture per cu.ft. gas= 12.68, of which

Neutral in gas	= .029
Neutral in nitrogen in air	= 9.24
Neutral in mixture	= 9.269 vols. in 12.68 vols. mixture = 73.2%

by volume. Therefore the detonating or neutral-free gas mixture in the best mixture = 26.8% by volume. The low heating value of the gas was 1060 B.T.U. per cu.ft., whence that of the best mixture is $\frac{1060}{12.68} = 89$ B.T.U. per cu.ft.

With these data the following mixture characteristics for various proportions of air and gas will follow for the explosion ranges, as given in Table XXV. These limiting proportions for explosive combustion, constituting the first and last columns of the table, were determined by direct experiment.

TABLE XXV

CHARACTERISTICS OF EXPLOSIVE MIXTURES OF PINTSCH OIL-GAS AND AIR

	1	1	1	1	1	1	1	1	1
Volume of gas.....	8	9	10	11	12	13	14	15	16
Volume of air.....	8	9	10	11	12	13	14	15	16
Percentage of gas in mixture.....	11.1	10	9.08	8.34	7.7	7.14	6.67	6.25	5.88
Excess or neutral air.....
Active air.....	8	9	11	11	11.68	11.68	11.68	11.68	11.68
Excess or inactive gas.....	.315	.229	.144	.057	1	1	1	1	1
Active gas.....	.685	.771	.856	.943	1	1	1	1	1
Neutral in active air.....	6.33	7.12	7.91	8.69	9.24	9.24	9.24	9.24	9.24
Neutral in active gas.....	.0202	.0227	.0252	.0278	.0295	.0295	.0295	.0295	.0295
Total inactive.....	6.665	7.372	8.079	8.777	9.59	10.59	11.59	12.59	13.59
% inactive or excess.....	74.1	73.72	73.5	73.0	73.8	75.6	77.3	78.7	79.9
% detonating gas.....	25.9	26.28	26.5	27.00	26.2	24.4	22.7	21.3	20.1
B.T.U. per cu.ft. of mixture (avail.).....	80.6	81.7	82.5	83	81.5	75.7	71.6	66.3	62.5
Calculated pres. rise.....	163	165	166.6	167.6	164.6	153	144.6	134	126
Observed pres. rise.....	73.5	80	81	79.5	77.5	70.0	64.5	59	50.5
Ratio observed pres. calculated pres. = C	.45	.485	.486	.476	.472	.46	.446	.44	.40

It is quite remarkable how small is the variation in the quantity of active or detonating mixture for all the ranges of explosive proportions, and this explains why the pressures vary to so small a degree.

To facilitate calculations with characteristic analyses of common gases the data of Table LXXVII, in the Tables, have been prepared. These also illus-

trate a remarkable fact regarding explosive mixtures of such gases with air, viz., the small variations in the B.T.U. per cu.ft. of standard air-gas mixture notwithstanding a tenfold, or greater variation in the B.T.U. per cu.ft. of gas, which explains why explosions of so called rich gases give rise to pressures but little greater than those for the so-called lean, poor or weak gases. The lowest value is about 60, that for blast-furnace gas, the highest value is 107, that for kerosene, not twice as much, though the calorific of the fuel vapor per cu.ft. is for kerosene nearly ten times that for blast-furnace gas.

In this table gasolene is assumed to be heptane, C_7H_{16} ; kerosene to be dodecane, $C_{12}H_{26}$; alcohol to be pure ethyl, C_2H_5OH , and the hydrocarbon illuminants of other gases to be distributed as indicated in the table. *It is very uncertain just what these hydrocarbons are, but it seems to make very little difference so far as the mixture heats are concerned, as shown by the following:*

For the paraffine series, C_nH_{2n+2} :

$$\text{B.T.U. per cu.ft. air-vapor mixture} = \frac{777.4n + 290}{7.15n + 3.38} \quad \dots \quad (682)$$

$$= 101 \text{ app. for } n=1; \quad = 105 \text{ app. for } n=20; \quad = 108 \text{ app. for } n=100.$$

This indicates that from the lightest, methane, to the heaviest of the series, the heat per cubic foot of combining, or best mixture with air, will be between 102 and 110 approximately and the same will be true for any group of hydrocarbons such as make a real oil.

For the ethylene series C_nH_{2n} :

$$\text{B.T.U. cu.ft. of mixture} = \frac{758n}{7.15n + 1} \quad \dots \quad (683)$$

$$= 99 \text{ for } n=2; \quad = 105 \text{ for } n=20; \quad = 106 \text{ for } n=100.$$

This shows the same is true with even narrower limits for this series, *hence for any hydrocarbon oil, B.T.U. per cubic foot best mixture with air measured at standard conditions, will be between 100 and 108, no matter what it is.*

The weight characteristics of the mixture are added to the volumetric in Table LXXVII for convenience in the use of the relations indicated by the preceding equations and summarized as:

Temperature rise is proportional to B.T.U. per lb. of products or of mixture. (684)

Pr. rise for explosions is proportional to B.T.U. per cu.ft. of init. mixture. (685)

In furnace work the temperature rise determination, as well as the final temperature, is made in a similar manner to that used for the pressure rise due to explosions in closed chambers. High temperature attainment is always facilitated by initial heating of the blast or mixture supplied, as the combustion determines the temperature rise, and final temperature is the sum of *initial temperature* and temperature rise. The final temperature does not rise the same amount, however, as the initial temperature, except for low temperatures, indicating the operation of the dissociation limitation, but it does

rise appreciably, showing that all heat suppression is not due to dissociation and may be charged to higher specific heats than are believed to hold.

In furnaces, especially hand-fired boiler furnaces, the temperature is constantly fluctuating with the air supply, draft, condition of bed and other minor influences. This is admirably illustrated in the diagram, Fig. 131, taken from one of the boiler tests made by the U. S. Geol. Survey. According to this diagram the furnace temperature rose to a maximum of about 3000° F. and fell to about 2000° F., as limits, due to operating conditions, the rise and fall being cyclic with firing. Opening the door chills the fire unless previously the gases had insufficient air. Depositing coal results in rapid distillation, as the coal was bituminous (in one case 36 per cent volatile and in the other 21 per cent), and the gases in burning raise the temperature in the combustion chamber beyond the bridge wall, while that of the bed drops

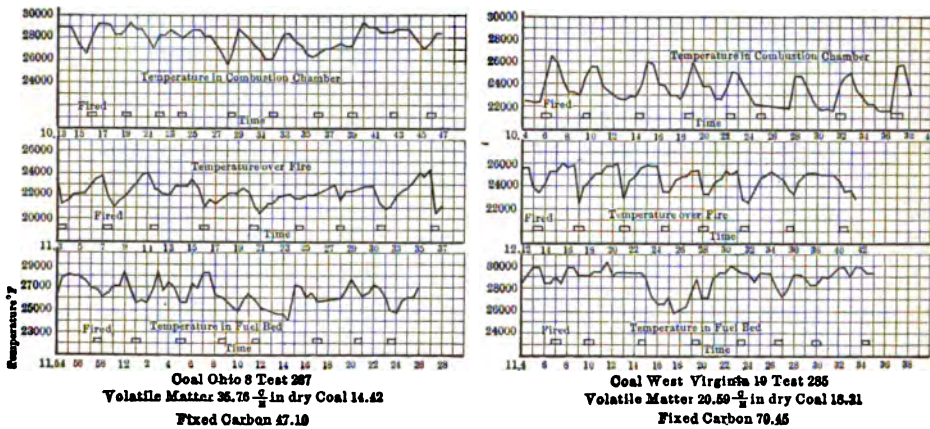


FIG. 131.—Temperatures in a Boiler Furnace.

due to less air passage and larger per cent of CO formed in it, as well as to the heat absorption of volatile distillation. All temperatures in the furnace would be lower with anthracite coal, especially in its smaller sizes.

As carbon yields 14,544 B.T.U. per pound when completely burned to CO₂, and in so burning requires 11.52 lbs. air per pound, yielding 12.52 lbs. of products, the value of β in Eq. (669), assuming the temperature rise to be at the high value of 2400° F., will be

$$\beta = \left[\frac{t_2 - t_1}{\frac{Q}{.243w_F}} \right] = \frac{2400 \times .243 \times 12.52}{14,544} = .5. \quad . \quad . \quad . \quad (686)$$

It thus appears that for ordinarily high coal furnace temperatures the heat suppression coefficient is of the same order of magnitude as the corresponding factor for explosion pressures, but it must be noted that for lower temperatures its value will be higher, as for example, when a little carbon, oil or gas is burned

in a large excess of air, limiting the temperature rise. Thus, if there were 100 per cent excess air in the furnace, the temperature rise would be only 1200° for the same heat suppression factor, whereas experience shows that the temperature will be closer to 2000° ; the most rational explanation of this is that the temperature attainable cannot rise much after dissociation sets in.

It might be expected in furnaces that if at the time dissociation is limiting combustion, the gases be *suddenly* cooled below their ignition temperature, so suddenly as to leave insufficient time for union which would take place by gradual heat abstraction, they would show unburned fuel elements. This is always found to be the case in boiler fires and is one of the explanations of smoke and of the presence of CO and some hydrogen and hydrocarbons in the gases when smoke is much in evidence, even though there is free oxygen present at the same time, the sudden cooling being done by contact of flame with the cold boiler tubes and plates.

In no boiler furnace is combustion ever complete without excess oxygen indicated by free O_2 in the analysis of flue gas, and seldom so, even with appreciable amounts of it, so that analysis of flue gases of boiler furnaces or even other furnace fires is a most important practical method of indicating the attainment of good or bad fire conditions, leading to correction of faults in adjustment of air supply above and below the grates, and at the same time giving a measure of the amount of unburnt fuel escaping, the amount of excess air being used and the weight of products per pound of coal. Flue-gas analysis is, therefore, the indicator of furnace efficiency, and it, together with the temperature of escaping gases, measures the heat discharged to the chimney and unavailable for evaporating water.

The weight of combining air, excess air and total products, can be computed from the volumetric relations of the flue-gas constituents approximately for the fixed carbon of the fuel but not so easily for its volatile. Considering the fixed carbon alone, the fundamental relation for no excess air is given by Eqs. (635) and (636), Section (86) which is,

1 lb. C + 5.759(1+x) lbs. of air give a gas consisting by volume of

$$\left(\frac{1}{56.32x + 86.12} \right) \left[\begin{array}{l} 29.8x \text{ of } CO_2 \\ 29.8(1-x) \text{ of } CO \\ 56.32(1+x) \text{ of } N_2 \end{array} \right] \text{ or } \left(\frac{1}{56.32x + 86.12} \right) \left(\frac{29.8(CO + CO_2)}{56.32(1+x)N_2} \right).$$

The weight of air per pound of carbon is 5.759(1+x) lbs. which can be evaluated numerically in terms of flue-gas volumetric relations when (1+x) can be found in terms of their CO and CO_2 contents. From these volumetric relations,

$$\frac{\% \text{ by volume } N_2}{\% \text{ by volume } CO + \% \text{ by volume } CO_2} = \frac{56.32(1+x)}{29.8} = 1.89(1+x). \quad (687)$$

$$\text{Hence,} \quad (1+x) = \frac{1}{1.89} \times \frac{\% N_2}{\% CO + \% CO_2}. \quad (688)$$

Denoting by N_2 , CO, and CO_2 the volumetric per cent of each,

$$\text{Pounds air per pound C} = \frac{5.759}{1.89} \left(\frac{N_2}{CO + CO_2} \right) = 3.1 \left(\frac{N_2}{CO + CO_2} \right). \quad (689)$$

If there is excess air there should be no free CO in the flue gases, but as the gases flow in streams the oxygen may not have come in contact with the unburned CO, its presence indicating unused air though not necessarily in excess chemically. It is, however, customary to consider all unused air as excess air whether chemically in excess or not, although uncombined air might better describe it.

If A = weight of the excess or uncombined air in pounds per pound of carbon, then if N'_2 , CO' , and CO'_2 are volumetric per cents exclusive of the excess air of the flue gas,

$$\text{Pounds air per pound C} = 3.1 \left(\frac{N'_2}{CO' + CO'_2} \right) + A \quad . \quad . \quad . \quad (690)$$

But the presence of the O_2 and N_2 of the extra air will change all the ratios. Each pound excess air per pound C will add 12.387 cu.ft. (std.) to the flue gas of which 21 per cent or 2.701 cu.ft. is oxygen and 79 per cent or 9.686 cu.ft. is nitrogen. The total nitrogen will then be $[56.32(1+x) + 9.686A]$ for A pounds excess air, and 2.701 A cu.ft. oxygen will be with it. The total weight of air per pound C is $= 5.759(1+x) + A$, the evaluation of which requires an expression for A and $(1+x)$ in terms of volumetric relations, which is not possible exactly. By trial the following is found to express the relation fairly well:

$$\frac{N_2 + .055O_2}{CO + CO_2} = \frac{56.32(1+x) + 9.686A + .055 \times 2.7A}{29.8}$$

$$= 1.89(1+x) + \left(\frac{9.834}{29.8} \right) A = 1.89(1+x) + .33A = .33[5.7(1+x) + A],$$

or

$$\frac{3N + .165O_2}{CO + CO_2} = 5.7(1+x) + A \quad . \quad . \quad . \quad . \quad . \quad (691)$$

Similar relations can be found for fuels consisting partly of hydrogen, but they are too complex to be of much help, so that the estimation of weight of air and excess air from the volumetric flue-gas analysis is possible *by a simple formula, only for fuels that are all or mainly fixed carbon*. The hydrogen relations can be worked out by expressions similar to those for producer gas, and as flue-gas analysis is always made with cold gases none of the H_2O formed will show in the analysis. There will be added 337 cu.ft. of nitrogen (std.) for each pound of hydrogen and it will be derived from 34.64 lbs. of air required. This will have the effect of increasing the nitrogen per cent and decreasing the CO_2 , CO and O_2 per cents in the flue gases from high volatile coal, oil or hydrocarbon gases, for the same disposition and perfection of carbon combustion.

The heat loss due to unburned CO in the flue gas can be expressed in terms of the volumetric relations of CO to CO_2 . If the carbon burns partly to CO_2 and partly to CO, and x represents the fraction burning to CO_2 , then $(1-x)$ represents the fraction burning to CO and 10,193 $(1-x)$ is the heat lost due to CO per pound of carbon. But in the gases there will be

29.8 cu.ft. of CO_2 per lb. C, and 29.8 $(1-x)$ cu.ft. of CO per lb. C.

Hence designating by CO and CO₂ the respective per cents of each by volume

$$\frac{\text{CO}}{\text{CO} + \text{CO}_2} = \frac{29.8(1-x)}{29.8x + 29.8(1-x)} = \frac{29.8(1-x)}{29.8} = 1-x. \quad \text{Therefore}$$

$$\text{B.T.U. per pound C, heat loss, due to unburned CO} = 10,193 \left(\frac{\text{CO}}{\text{CO} + \text{CO}_2} \right) \quad (692)$$

In the Tables are some careful determinations as compiled by Uehling, that may be regarded as representative of the resultant of all the complex conditions and reactions in a boiler fire. His conclusions, which are most reliable are quoted: "There is a definite relation between the per cent of CO₂ and O₂ contained in the flue gas. The percentage of O₂ falls almost exactly in the same ratio as CO₂ goes up and vice versa. The analysis also shows conclusively that there is no relation between either O₂ or CO₂ and CO. High O₂ or low CO₂ is no evidence that CO will not be present or that the combustion is more complete with high CO₂ or that low O₂ means incomplete combustion. There is a tendency toward larger CO contents with larger CO₂ contents."

The total per cent of CO, CO₂ and O₂ together in the U. S. Geological Survey tests varied from 16.7 to 20 per cent, and of 1130 analyses only one total fell below 17 per cent, four below 18 per cent, 20 between 18 and 20 per cent, 287 between 19 and 20 per cent, 277 between 20 and 20.8 per cent. The best per cent of CO₂ varies with the carbon-hydrogen ratio of the fuel, being smaller for large carbon-hydrogen ratios. As a matter of interest in comparison with gas producer work the ratios of CO to CO₂ and of CO to (CO+CO₂) is added to the tables. See Table LXXVI in the Handbook of Tables.

The utter impossibility of completely burning without excess air, which carries off heat otherwise usefully available, is clearly shown by these analyses, which represent unusually good adjustment of fires, though expert adjustment with no end in view but perfect combustion, would improve results to some extent. In actual working, boiler capacity is just as important as efficient combustion and actual conditions represent attainable compromises.

Prob. 1. The flue-gas analysis taken on a boiler trial gave O₂ = 10 per cent; CO₂ = 10 per cent; H₂ = 80 per cent. If the coal used consisted of 80 per cent C and 20 per cent ash and the stack temperature was 400° F. above the room, what was the stack loss per pound of coal?

Prob. 2. 1000 heat-units are added to a pound of gas at 32° F. at constant pressure. The gas consists of 30 per cent CO, 40 per cent N₂, 5 per cent CO₂, and 25 per cent H₂. Calculate the final temperature on basis of constant specific heat and give what you consider a likely value for the actual temperature.

Prob. 3. A pound of air is compressed from 1 to 5 atmospheres above atmosphere and then 1000 heat-units are added at constant volume. What will be the final temperature?

Prob. 4. A gas requires 5 cu.ft. of air to completely burn it. Its heating value is 900 B.T.U. per cubic foot under standard conditions. A mixture of one part of this gas with 5, 6 and 7 cu.ft. of air at a temperature of 100° F. and at a pressure of 25 lbs. per

square inch gage is ignited in a closed vessel. What is the probable pressure after combustion?

Prob. 5. A pound of coal gave the following analysis: C=70 per cent; H₂=5 per cent; O₂=5 per cent; ash=20 per cent. If twice the air chemically needed were supplied, what would be the maximum temperature as calculated on basis of constant specific heats? What would be the flue-gas analysis?

Prob. 6. In a boiler test it was found that 40 per cent of the heat in the coal went up the flue as sensible heat and in unburned carbon. The coal contained 13,500 B.T.U. per pound. The flue gas analysis showed CO₂=12 per cent, CO=6 per cent, O₂=2 per cent, N₂=80 per cent. If the coal is assumed to be 80 per cent C and the boiler-room temperature is 80° F., what was the flue temperature?

Prob. 7. The pressure in a gas-engine cylinder before ignition was 70 lbs. gage and after ignition 350 lbs. gage. The temperature before ignition was 300° F. For a mixture containing 110 B.T.U. per cubic foot under standard conditions, what was the ratio of observed to calculated pressure rise?

Prob. 8. Take an analysis of coal gas from Table LXIII and on the basis of there being .5 of the calculated pressure rise, find the pressure exerted on the side of a room 50'×10' if there should be an explosion of the gas in it with atmospheric pressure and temperature before the explosion and with best air mixture.

90. Temperature of Ignition and Its Variation with Conditions. Limits of Proportion Air-Gas-Neutral, or Detonating Gas and Neutral, for Explosive Combustion of Mixtures. Limits of Adiabatic Compression for Self-Ignition of Mixtures. It is only with gaseous fuel previously mixed with measured air that best combustion results can be obtained—perfectly complete combustion without excess air, and this is easily managed in engine cylinders. Gas and oil and powdered coal-fired boilers may approximate this, but with the burners in common use explosive mixtures cannot be used without endangering the settings should the flow be momentarily interrupted and then continue, and just such serious accidents have happened. With specially designed burners and furnaces, explosive combustion is perfectly feasible in boiler and other fires but as yet these are not widely used. Mixtures of fuel and air are explosive at the chemical proportions and for a considerable range beyond, usually more on the excess fuel side than that of excess air. Two of the most authoritative tables of limits, those of Eitner and Bunte, are relatively but not absolutely correct. This is true because the limit of explosive proportions is said to be reached when a mixture will not explode, and this is as much a question of how the ignition is attempted as it is one of proportions. Flames will ignite mixtures that will not explode with electric sparks, some sparks will ignite mixtures that others will not, and mixtures will ignite when a flame is plunged into the mass when they cannot be ignited at a touch hole or at one corner of their chamber.

When, however, experimental observations are considered, the complexity of the ignition becomes more apparent. With carbon it is known that the state determines the temperature at which combustion proper sets in, it being lower for porous varieties of charcoal, higher for denser varieties and very high for graphitic forms, but it is also known that all forms will react

with oxygen, yielding carbon dioxide at some temperature. This suggests that there is no such thing as the *ignition temperature* of carbon and probably also for

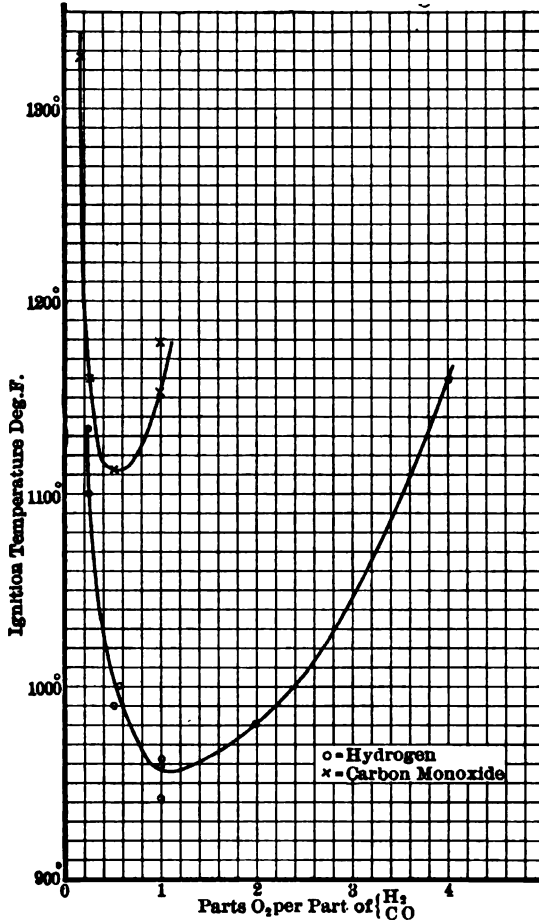


FIG. 132.—Temperature of Ignition for H_2 and CO mixed with O_2 .

other fuel elements and compounds. Who can say just what takes fire when ethylene is ignited, is it ethylene as a whole or is it methane or acetylene or hydrogen separated out by the attempt to ignite? A little light can be thrown on the question by some recent experiments prompted by previous work of Davy followed by Mallard and Le Chatelier, who observed that for gaseous mixtures the *ignition temperature true or apparent, was different for different proportions of air and fuel and likewise still different when neutrals were present*. Results, however, were quite variable with the method of procedure and quite inconsistent. No real progress could be said to have been made until Falk, by rapidly compressing a previously determined mixture, heated it by compression uniformly throughout to ignition,—the temperature of which was calculated by the adiabatic law. His results are shown graphically in Fig. 132 for carbon-

monoxide-oxygen and hydrogen-oxygen mixtures, which are the principal fuel elements in power producer gas.

From the curve it is clear that there is a regular variation of ignition temperature with proportion and a minimum point for each fuel which corresponds to the $H_2 + O_2$ proportions for hydrogen, indicating that it reacts in two stages first to H_2O_2 , and later to $2H_2O$, whereas, for carbon monoxide the minimum point corresponds to the detonating mixture.

When the combustible and its oxygen must be ignited in the presence of neutral matter, like excess oxygen or excess combustible which when not combining are in effect neutral, or real neutrals like nitrogen, the ignition temperature may be calculated from the two following formulas, Eqs. (693) and

(694), working out each independently of the other and accepting whichever is the lower value.

Ignition temperature

$$\left\{ \begin{array}{l} \text{of H}_2 \text{ with O}_2 \\ \text{and N}_2 \end{array} \right\} = \left\{ \begin{array}{l} [\text{Ignition temperature for H}_2 \text{ with O}_2 \text{ alone}] \\ + \left[\frac{54 \times \text{volume of inert gases}}{\text{Vol. of H}_2 \text{ or O}_2 \text{ (whichever is smaller)}} \right] \end{array} \right\} . \quad (693)$$

Ignition temperature

$$\left\{ \begin{array}{l} \text{of CO with} \\ \text{O}_2 \text{ and N}_2 \end{array} \right\} = \left\{ \begin{array}{l} [\text{Ignition temperature for CO with O}_2 \text{ alone}] \\ + \left[\frac{144 \times \text{volume of inert gases}}{\text{Volume of CO}} \right] \end{array} \right\} . \quad (694)$$

By the use of these equations the ignition temperature for five samples of hypothetical producer gas have been calculated as an example, the various steps being set down in the following Table XXVI:

TABLE XXVI

CALCULATED IGNITION TEMPERATURES FOR PRODUCER GAS

No.	Producer Gases, Composition by Volume.				Volumes O ₂ to Burn 100 Volumes Gas.	Volumes Air to Burn 100 Volumes Gas.	Total Volume Mixture.	
	CO ₂	CO	H ₂	N ₂				
1	.0	38.9	12.6	48.5	25.75	123.08	223.08	
2	3.7	34.2	16.1	46.0	25.15	120.22	220.22	
3	7.2	29.9	19.4	43.5	24.65	117.82	217.82	
4	10.7	25.6	22.7	41.0	24.15	115.44	215.44	
5	13.6	21.5	25.8	39.1	23.65	113.05	213.05	

No.	Volume of Inert Gas when		n'	n	Temperature for Zero Inert Gas from Fig. 132.		Calculated Ignition Tem- perature for the Mixture.	
	CO Burns.	H ₂ Burns.			For CO.	For H ₂ .	Based on CO.	Based on H ₂ .
1	164.73	204.18	4.13	16.2	1112	982	1708	1857
2	168.42	196.07	4.93	12.18	1118	963	1824	1620
3	172.97	188.72	5.93	9.73	1128	979	1984	1503
4	177.04	181.39	6.92	7.99	1148	954	2146	1385
5	180.80	174.35	8.4	6.76	1175	957	2385	1322

The results show that in all but the first case with H₂ = 12.6 per cent, which increases to 26 per cent, it is the hydrogen that controls the ignition temperature, and that the difference for wide range of proportions is really not great.

The same method and apparatus was used for determining the ignition temperature of gasolene and ethyl alcohol, and gave the following values which were fairly constant for explosive mixtures through a 100 per cent range of proportions.

$$\left. \begin{array}{l} (\text{Ignition temperature of gasolene mixtures with air}) = 986^\circ \text{ F. (a)} \\ (\text{Ignition temperature of alcohol mixtures with air}) = 1292^\circ \text{ F. (b)} \end{array} \right\} . \quad (695)$$

These temperatures of ignition would never be reached by compression of cold charges adiabatically to the pressures used in engine cylinders, yet for even small compressions the charges ignite themselves or preignite, proving that the compression temperatures are very high in such cylinders and must become so *by reason of heat added to the charge before or during compression*. As the temperature rise before compression is multiplied by compression, it may be assumed that this is the controlling factor, and the following calculation shows to what extent this may be expected to take place.

Let T_1 , t_1 , and T_2 , t_2 be absolute and scale temperatures before and after compression to ignition temperature from one atmosphere, to P_2 atmospheres absolute, adiabatically.

$$\text{Then assuming the exponent, } s = 1.4 \quad \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{.29} \quad \text{and} \quad T_1 = \frac{T_2}{\left(\frac{P_2}{P_1}\right)^{.29}}$$

From this the initial temperature for mixtures that really preignite at a known compression in engine cylinders can be calculated as shown in tabular form for the three cases cited.

Fuel.	Compression Ratio in Cylinder when Preigniting, $\frac{P_2}{P_1}$	Temperature of Ignition, Falk.	Corresponding Initial Temperature.	
		T_1 Absolute F.	T_1 Absolute F.	t_1 Scale F.
Producer gas $\left\{ \begin{array}{l} \text{CO } 29.9 \\ \text{H}_2 \text{ } 19.4 \\ \text{CO}_2 \text{ } 7.2 \end{array} \right\}$	9	$1080 + 460 = 1540$	816	356
Alcohol.	13	$1292 + 460 = 1752$	833	373
Gasolene.	7	$986 + 460 = 1446$	824	364

This shows that in using these fuels properly mixed with air in cylinders the amount of compression allowable without self-ignition *varies with the fuel mixture, ignition temperature and with the initial temperature*, which cannot be allowed for the three fuels above to rise over 350° F. , and does reach fairly close to this value, probably 300° , as some heat will be derived from the walls during compression. This temperature, when charges are taken in at about 60° F. on the average, is a measure of the heating effect of the hot walls during suction, coupled with the heat added by the residue hot gases from a previous explosion, and left in the clearance.

In the following Table XXVII are given some of the usually practiced limits of compression in gas engines; these are subject to considerable variations with type of engine and its management, which are controlled as much by cylinder heating as by the natural temperatures of ignition of the mixtures themselves.

The setting on fire of a combustible, or more scientifically, its ignition, is, therefore, not the simple operation that it might seem, especially when the fuel is in the form of explosive gaseous mixture. It has been the custom to say

that the fuel, when raised to its ignition temperature in the presence of oxygen, will at once begin to burn, and ignition temperature tables like those given in Table XXXIII in the Handbook of Tables, though common, are probably wrong—surely so for gases.

TABLE XXVII
COMPRESSIONS COMMONLY USED IN GAS ENGINES

Fuel.	Type of Engine.	Compression, Pounds per Square Inch Absolute.	Average Compression, Pounds per Square Inch Abs.
Gasolene.....	Automobile.....	60-95	75
Gasolene.....	Stationary.....	70-100	90
Kerosene.....	Hot bulb, 250-500 r.p.m.....	30-75	60
Kerosene.....	Vaporized before entering cylinder...	45-85	65
Natural gas.....	Medium and large engines.....	75-130	115
City gas.....	Small engines.....	60-110	85
Producer gas.....	Medium engines.....	100-160	135
Blast gas.....	Large engines.....	120-190	155

Not only do the Falk data prove the controlling influence of proportions on ignition temperatures for gases, but Violette shows that for solid charcoal the ignition temperature is dependent on its origin, which may be assumed to control its state. In this case the prime variable is the temperature at which the charcoal was formed by wood distillation, which it has been pointed out controlled the density and the character of the residue, and probably similar conditions obtain for other fuels.

Temperatures of ignition for explosive gaseous mixtures are intimately related to the limiting proportions for explosive combustion, because inflammation will propagate or pass of itself, from particle to particle in the mass, *only when the heat liberated by combustion at one point is able to raise the next one to its temperature of ignition*, but it is impossible yet to state algebraically just what this relation is.

A series of values for the limiting proportions of air-gas mixtures is given in the tables which show wide differences, for different fuels with no apparent relation between them, nor are any controlling influences indicated, though some are known to exist.

Pressure and temperature variations act irregularly, but in general tend to widen the limits; rise of temperature and pressure, more particularly the latter, will render explosive, proportions otherwise not so, while neutral dilution always narrows the limits. A series of experiments made by Dr. Lucke indicated a tendency toward a limit of explosibility when the active or detonating mixture was equal to, or greater than, about 17 per cent of the total, or the neutral inactive gases not in excess of 83 per cent. This active mixture consists of the combustible constituents of the gas with just the right amount of oxygen to burn them, and is generally called the *detonating gas* or *detonating mixture*. All excess oxygen or excess fuel is classed as neutral and the following table shows the relations in various mixtures of (air-gas-neutral) of the detonating mixture to the whole mixture.

This limitation of explosibility by limiting quantities of detonating mixture, or total inactive constituents noted above for all ranges of one gas, to air-gas mixtures alone, was observed by Bunte for all kinds of gases with a comparatively small range of variation, as in Table XXVIII.

TABLE XXVIII

PER CENT DETONATING MIXTURE IN TOTAL AIR-GAS MIXTURE AT
EXPLOSIVE LIMITS OF PROPORTION (BUNTE)

Gas.	When Air is in Excess.	When Gas is in Excess.
Carbon monoxide.....	24.75	15.78
Hydrogen.....	14.17	21.16
Water gas.....	18.80	20.94
Acetylene.....	11.72	14.03
Coal gas.....	17.40	31.15
Ethylene.....	16.40	23.91
Alcohol.....	15.00	24.17
Marsh gas.....	18.30	27.47
Ether.....	19.25	22.61
Benzene.....	19.87	22.25
Pentane.....	21.60	22.47

These figures taken in conjunction with those for large neutral additions suggest the possibility of a *universal law for all explosive gaseous mixtures whatever the fuel or the nature of constituents associated with it: that explosion is no longer possible when there are sufficient inactive particles of gas present to prevent a heat propagation that will allow one group of burning fuel and oxygen particles to set fire to the next separated from it by an insulating layer of dead gas or even allow the oxygen to reach the fuel, and that this per cent of inactive is nearly constant for all.*

A remarkable condition brought out by the table illustrating this general law is the narrowness of the range for rich hydrocarbons, especially on the excess air side. Practically no excess air is permissible, a fact that indicates the necessity in gasoline and kerosene engines for most accurate adjustment of proportions, without which there is almost a certainty that excess fuel will be used by an engine operator, all excess being direct waste.

Prob. 1. Acetylene gas is escaping into a room which is 15'×20'×10'. How many cubic feet of gas must escape into the room and mix with the air before there is a possibility of an explosion? Should there be nothing to ignite the mixture, how many cubic feet of the gas must there be in the room before the upper limit of explosibility is reached? Should the gas have been coal gas, what would have been the quantities?

Prob. 2. A sample of gas gave the following analysis by volume: CO=35 per cent; CO₂=6 per cent; H₂=14 per cent; N₂=35 per cent. If this gas be mixed with the combining proportion of air to what temperature must it be heated to cause it to ignite?

Prob. 3. A gas engine is running on a natural gas which may be considered as all methane. The combining proportions of this gas with air are in the ratio of 9.5 to 1. Taking the limits of proportion for explosive mixture from the general law what would be

the cubic feet of air per cubic foot of gas for the leanest and richest mixture which would still ignite?

Prob. 4. Gasolene is stated to have an ignition temperature of 986°F . If the temperature of the mixture in the cylinder of an engine using this fuel is 400°F . at the beginning of compression, what will be the maximum compression which can be carried without preignition? The value of s may be taken as 1.4.

Prob. 5. If instead of gasolene there be used in the engine of Prob. 4, the following gas, how much could the compression be raised without preignition occurring? $\text{CO}=37.6$ per cent; $\text{H}_2=11.3$ per cent; $\text{CO}_2=3.7$ per cent; $\text{N}_2=47.4$ per cent.

91. Rate of Combustion of Solid Fuels with Draft. Propagation Rates, Normal and Detonating for Explosive Gaseous Mixtures. The last combustion characteristic to be examined is the rate of combustion or speed with which the combination, once started, proceeds. In boiler fires the coal rests on grates and the rate with which it may burn determines the amount of steam that the boiler can make per hour, assuming a constancy of heat losses to flues and setting. When gases or oils are burned under boilers or in other furnaces the rate of combustion determines the size of furnace necessary for the combustion of the requisite fuel for the duty expected of the heat; while in gas engine cylinders the time of combustion is extremely small if good efficiency is to be secured. For the latter case the explosion should always be as nearly complete as possible before expansion begins, and should not start too soon toward the end of compression, or the work of compression will be too great. To get an idea of the time available, assume an automobile engine running at 1200 R.P.M. or 20 revolutions per second, completing one revolution in .05 second. Then if ignition and combustion must take place during 18° of crank rotation near the dead center, or $\frac{18}{360} = .05$ of the revolution, the time for the whole process to

be completed is $.05 \times .05 = .0025$ or about $\frac{1}{400}$ second, for a path not over 6 ins., corresponding to a flame speed of .005 second per foot or 200 ft. per second. The very largest engine with a cylinder of 4 ft. diameter, turning at 60 R.P.M. or 1 revolution per second, for the same crank angle requires .05 second, whence for 1 ft. of mixture the speed must be $\frac{.05}{4} = .012$ second approximately and the flame speed about 85 ft. per second.

By elaborate apparatus Mallard and Le Chatelier photographed the progress of a flame passing through an explosive mixture of known characteristics in a glass tube. The photographic plate was moved across the tube during the flame passage giving a diagram of speed of propagation. It appears, that at the beginning of combustion in the open tube the speed is first uniform then undulating or wavelike and sometimes at the end very great, as indicated by a nearly vertical record line. *This rapid mode of propagation is termed the explosive, or better, the detonating wave*, and is always reached in good mixtures if there is distance enough to be traversed, or if there is agitation of the mixture or pressure waves developed equivalent to agita-

tion, which is quite common in engine cylinders. It was at first believed to follow the speed law of sound or other impact waves in gases, but Dixon has shown that this is not true, the speed being much greater than that of sound, but it is constant.

Therefore, in explosive mixtures the rate of combustion is at first uniform and slow, and rapidly accelerates to a high constant value.

Various attempts have been made to derive a fundamental formula for the speed of the wave in terms of physical constants of the mixture, and while some are reasonably satisfactory no one of them seems to fit all cases or give results agreeing with measurements. If in cylinders, as often happens, the combustion seems to be too slow, then it is certain that the wave has not been set up; as a matter of fact it is undesirable to have it develop because of the accompanying shocks.

For the slow, uniform propagation the best data on the conditions and rates are given by Mallard and Le Chatelier, whose results show that the walls have an influence, the nature of which is a cooling one, abstracting heat at various rates compared to that of generation. *When cooling is strong enough to take heat away as fast as it is generated, propagation of explosion is impossible*, and this always takes place in narrow slits between plates, in very small tubes and in cold wire gauze screens. Rise of temperature always increases the rate, thus for H_2 and air with 30 per cent H_2 , $r = 10.75$ ft.sec. at $60^\circ F.$; $r = 14.26$ ft.sec. at $212^\circ F.$ The greatest influence in fixing the rate of propagation is the nature and proportion of the mixture.

In all cases neutral dilution has a strong influence on reduction of rate of propagation, reducing it for all air-gas mixtures to zero when the limit of proportion has been reached at somewhere about 83 per cent of total neutral. In general the uniform or slow normal rate of propagation r never exceeds 14.1 ft. per second and occurs with 40 per cent of gas or 30 per cent excess H_2 ; for C_2H_4 and air the maximum is 2 ft. per second and occurs in the mixture containing 12.2 per cent gas or an excess of 42 per cent; for CO and air the maximum is 6.56 ft.per second, and for illuminating gas and air the maximum is 4.1 ft. per second, occurring when the mixture contains 15 per cent excess gas, all at atmospheric pressure.

These figures show that in engine cylinders, pressure and temperature rise by compression are necessary to make the mixtures burn in the time available, as the rate of combustion at atmospheric pressure is not high enough. If the detonating wave is set up, there will be more than enough time.

Rate of combustion of coal on grates is measured in pounds per square foot per hour; if the coal burned as fast as the air flowed through the bed, the rate should increase directly with the square root of the draft, but would be different for each condition of porosity and thickness of bed; or in general, resistance to flow. As a matter of fact coals with much volatile are burned as much above the grate, in the gasified form of hydrocarbon and carbon monoxide as on it. The gas thus made may easily equal in weight that of the fixed carbon left on the grate. In this case the rate of combustion will

not vary with the square root of the draft, especially if the coal cakes some in which case the rate will be constantly varying from minute to minute

On the assumption that the coal will burn as fast as the air passes through the bed, and that air will pass with a velocity proportional to the square root of the draft measured between ash pit and furnace, then

$$(\text{Lbs. coal sq.ft. grate per hr.}) = C\sqrt{(\text{draft in inches of water})}. \quad (696)$$

The coefficient C may be determined from experiment approximately, as it is not a true constant for a number of reasons. Consider a given coal on a given grate, then C will vary with thickness of fire, age of the fire, amount and condition of ash and clinker and amount of cake on the surface if the coal is of the caking variety. A change of size of coal with no other change will affect the air flow and the speed of carbon union with the air that does pass, and a change in proportion of surface and bottom air will also modify C , as will a change in grate. A grate that breaks up a bed like the step stoker will pass much more air than one that does not like a chain grate, and a fireman that hand slices often, lets more air pass than one that does not. A coal that has much volatile may burn as much above as on the grate, and its rate of combustion be very different at the same draft from another with less volatile, everything else being the same.

However the rate follows fairly well the relation of Eq. (696), though

Values of C in Rate of Combustion, Eq. (696).	Coal name.
17.3	Anthracite rice
24.5	No. 1, anthracite buckwheat
31.5	Pea anthracite
81.6	Run-of-mine bituminous
50.2	Run-of-mine semi-bituminous
59.3	Bituminous slack

the coefficient must be selected with care. One general statement of the relations for a variety of coals is given by the Stirling Boiler Company, compiled from tests on their boiler and reproduced with some additions as Chart No. 35, in the Handbook of Tables. Assuming the middle rate reported for each coal to be correct, the square root curve is

drawn through this point for higher and lower drafts and from it the accompanying coefficients, C , are found for hand-fired furnaces.

To show how far actual cases depart from these averages, the results of some special determinations are separately plotted in Fig. 133 for anthracites, and in Fig. 134 for bituminous coals.

Prob. 1. A boiler rated at 180 H.P. has 60 sq.ft. of grate area. If 70 per cent of the heat of the coal is absorbed by the water, what must be the rate of combustion for No. 1 buckwheat containing 11,000 B.T.U. per pound? What draught will be required for the rate? When the capacity was raised to 270 H.P. the efficiency fell to 60 per cent. What draught would then be required to burn the necessary coal?

Prob. 2. A gasoline engine has a cylinder 5×5 ins. and runs at 1000 R.P.M. Ignition occurs 12 degrees ahead of dead center. What must be the velocity of flame travel to give complete inflammation at dead center with cylinder clearance = 20% of the stroke in length?

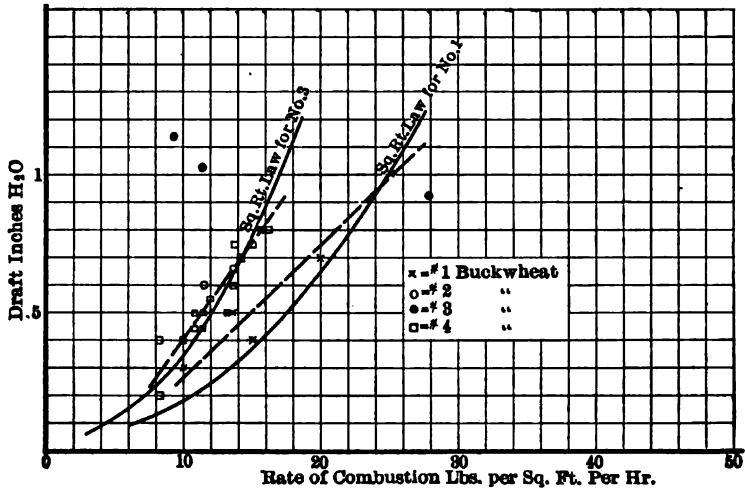


FIG. 133.—Rate of Combustion of Anthracite Coal.

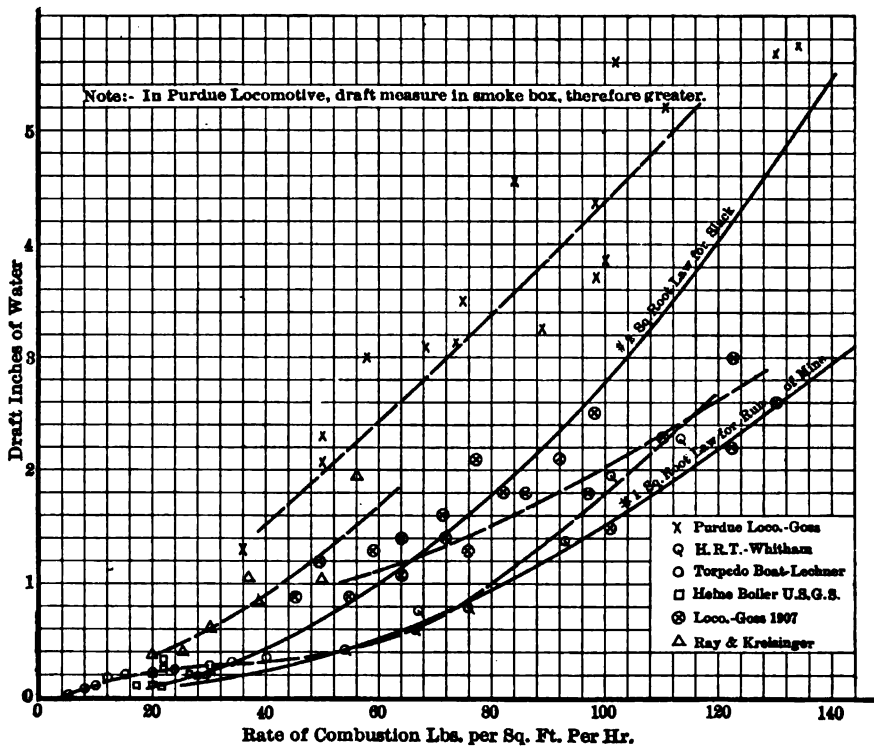


FIG. 134.—Rate of Combustion of Bituminous Coal.

Prob. 3. A stationary boiler had a grate area of 60 sq.ft. and was rated at 180 H.P. A locomotive boiler having a grate area of the same amount was rated at 500 H.P. Assuming that both boilers were fired with run-of-mine bituminous and that the respective efficiencies were 65 and 50 per cent, what draughts would be needed in each?

Prob. 4. 100,000 ft. of natural gas are burned per hour under a boiler. Assume a natural gas from the Table LXI, in the Handbook of Tables, and an anthracite coal from Table LV, and find what rate of combustion must be maintained with the coal on a grate area of 100 sq.ft. to develop the same heat?

Prob. 5. Ten barrels of fuel oil are burned per hour under a boiler. At what rate must bituminous coal be burned under another boiler having 60 sq.ft. of grate surface to produce the same heat? Take any fuel oil from Table LIX and bituminous coal from Table LV.

Prob. 6. No. 2 buckwheat anthracite, having a heating value of 11,600 B.T.U., is being burned with a draught of .7 in. on a grate with 80 sq.ft. of surface. On another grate run-of-mine bituminous is being burned with .5 in. draught. If the calorific power of the latter is 14,000 and the rate of combustion follows the square root law, what is the necessary area of the second grate for equal heat generation?

CHAPTER XII

STEAM BOILER EVAPORATION. HORSE-POWER AND EFFICIENCY

92. Steam Boiler Evaporative Capacity and Horse-power. Horse-power Units, Equivalent Rates of Evaporation and of Heat Absorption. Factors of Evaporation. Relation between Absorption Rates and Rates of Heat Generation. Influence of Heating and Grate Surface, Calorific Power of Fuels, Rates of Combustion and Furnace Losses. The difficulties in dealing with boiler processes are all concerned with the establishment of some fundamentally sound relations for the capacity of heating surface to absorb heat in terms of the amount of heat developed by the burning of coal, oil or gas in the furnace when the dimensions are known. After many years of experience with steam boilers it is not possible to-day to calculate from any fundamental relations either the amount of steam that will be developed per hour with a given fuel, or the weight of steam that will be produced per pound of fuel. Of course, from empiric relations this can be done by a comparison of the boiler in question and its fuel, with exactly or nearly similar ones for which tests have been made, yet even here it is possible to make blunders by forgetting differences in operating conditions such as available draft or methods of firing. It must be said, therefore, that there is no generally accepted fundamental theory of either steam boiler capacity or efficiency, but there are some relations between certain important factors, that are established, so a discussion of steam boilers must be almost entirely confined to these relations which cannot yet be grouped to constitute a general theory of the subject. To illustrate, it is possible to explain why a given boiler gives more steam with one coal than with another, or more steam at .4 in. draft than with .3 in. draft with the same coal; why, when twice the coal is burned, less than twice the steam is made, and even to account for and measure the losses that are responsible for the difference; also in some cases, why with the same coal and draft one boiler makes more steam than another, either in pounds per hour or in pounds of steam per pound of coal: but it is not possible to compare two different kinds of boilers as to type and size with different methods of firing different coals with different drafts, in such a way as will explain or permit of prediction of difference in results due to all influences acting together. To state it a little differently there is no *absolute* measure of boiler performance as to capacity or efficiency as a basis of comparison to measure the value of a boiler as a boiler; comparisons must therefore be between one and another boiler, or one and another service condition; one boiler may

be said to be better than another, or one condition more favorable and another worse, for the results desired, but hardly more than this is possible.

In this section will be discussed the various factors that seem to be determining influences in fixing the capacity, or as it is generally measured in terms of horse-power, the horse-power of boilers. Boiler horse-power, like gas-producer horse-power, is scientifically a bad term. It appears in the literature of the subject first during the period when steam pressures were low, about 70 lbs. gage and when *the average steam consumption of engines* was about 30 lbs. per hour per horse-power. Accordingly a boiler was said to have 100 horse-power capacity when it could make steadily 3000 lbs. of steam per hour. But variations in both engine and boiler conditions made it absurd to talk of average water rates of steam engines, and introduced correspondingly large differences in the amount of heat necessary for the production of a pound of steam, so that an adjustment of *boiler horse-power* definition became necessary. This was done in America by the American Society of Mechanical Engineers adopting a double definition, which was

- (a) *The evaporation of 34.5 lbs. of water per hour from and at 212° F.;*
- (b) *The absorption by the water between feed condition and that of the steam leaving the boiler, of 33,305 B.T.U. per hour per pound.*

This last heat definition was most fortunate, as it is an absolute unit, and whether it has any relation to engine requirements or not is a matter of no importance whatever. It was believed to be the equivalent of the weight definition and would be, if the latent heat of evaporation at 212° were $\frac{33,305}{34.5} = 965.36$, and

also equivalent to the evaporation of 30 lbs. from 100° F. feed water to steam at 70 lbs. gage. The language of the committee of the Society adopting the standard in 1885 is worth quoting: "Your committee, after due consideration, has determined to accept the Centennial Standard, and to recommend that in all standard trials (boiler tests) the commercial horse-power be taken as an evaporation of 30 lbs. of water per hour from a feed-water temperature of 100° F. into steam (dry saturated) at 70 lbs. gage pressure (above standard atmosphere), which shall be considered to be equal to $34\frac{1}{2}$ units of evaporation; that is, to $34\frac{1}{2}$ lbs. of water evaporated from a feed-water temperature of 212° F. into steam (dry saturated) at the same temperature. This standard is equal to 33,305 thermal units per hour." A later committee, 1899, retained the $34\frac{1}{2}$ lbs. from and at, but changed the heat equivalent to 33,317 B.T.U. per hour, based on a latent heat of 965.7, so that $965.7 \times 34.5 = 33,317$ B.T.U.

Now that the latent heat at 212° F. is by recent research known to be higher than 965.7, and the use of superheat is quite general, it is necessary to once more reconcile the double definition. Considering the history of the term it is believed that the retention of the $34\frac{1}{2}$ lbs. evaporation from and at 212° is desirable, and according to the generally accepted Marks and Davis Steam Tables, the latent heat corresponding is 970.4, therefore, *the heat equivalent of a boiler horse-power is $970.4 \times 34.5 = 33,478.8$ B.T.U. per hour.* This unit

is adopted in this work, and is applicable as well to superheated as to wet or dry saturated steam. Therefore,

$$\text{one boiler horse-power} = \left\{ \begin{array}{l} \text{Absorption by water and steam in the boiler} \\ \text{of 33,478.8 B.T.U. per hour} \end{array} \right\} \cdot (697)$$

With the heat basis as a standard, the weight of water evaporated per hour per boiler horse-power will, of course, vary regularly with the initial water temperature and final steam condition, and to facilitate practical work a *factor of evaporation* is calculated for reduction of weights according to the relation of Eq. (699) derived by the following relations illustrating its use:

$$\begin{aligned} \left(\begin{array}{l} \text{Lbs. water evaporated} \\ \text{per hour per B.H.P.} \end{array} \right) & \times \left(\begin{array}{l} \text{B.T.U. per lb. steam above feed} \\ \text{water temperature} \end{array} \right) = 34.5 \times 970.4. \\ \left(\begin{array}{l} \text{Lbs. water evaporated} \\ \text{per hour per B.H.P.} \end{array} \right) & = \frac{34.5 \times 970.4}{\text{B.T.U. per lb. of steam above feed temp.}} \quad (a) \\ & = \frac{34.5}{\text{B.T.U. per lb. of steam above feed temp.}} \quad (b) \\ & = \frac{34.5}{\text{Factor of evaporation}} \quad (c) \end{aligned} \quad (698)$$

$$\begin{aligned} (\text{Factor of evaporation}) & = \frac{\text{B.T.U. per lb. of steam above feed temp.}}{970.4} \quad (a) \\ & = \frac{\left\{ \begin{array}{l} \text{Total heat per lb. of} \\ \text{steam above 32° F.} \end{array} \right\} - \left\{ \begin{array}{l} \text{Heat of liquid at feed} \\ \text{temp. above 32° F.} \end{array} \right\}}{970.4} \quad (b) \end{aligned} \quad (699)$$

The values of the factor of evaporation and equivalent pounds of water per hour per boiler horse-power are to be found from the steam tables or directly from the curves, Chart No. 36, in the Handbook of Tables, which also give the heat per pound for dry saturated, wet or superheated steam above any feed-water temperature by the following simple operations. Each of the upper curves gives directly the total heat per pound of steam above 32°, and the distance between them and the lower curve intercept, that for any feed-water temperature by a vertical distance. If, therefore, *AB* be the total heat for the steam above 32° at 100 lbs. per sq.in. absolute and 20° superheat, and *DE* the heat of liquid at 200° F. feed temperature above 32°, then *AC* the vertical distance between these two points is the heat per pound of steam above the feed temperature 200° F. for 100 lbs. steam with 20° superheat. This can be marked on a slip of paper and read off on the extra scale to the right in terms of heat in B.T.U., or factor of evaporation, or actual weight of water that must be evaporated per hour to give a boiler horse-power.

A sort of commercial rating of boilers has grown up as a convenience to purchase and sale, based solely on the surface of tubes and plates exposed to the heat, or so-called *heating surface*, and the figure used is either 10 or 12 sq.ft.

for common forms, but varies from 8 to 16 for others. This is the result also of old average evaporations per square foot of surface for stationary boilers, and suggests the dependence of capacity on the heat-absorbing power of the surface *which in such ratings is assumed to be constant*. To show what is the evaporation in pounds per hour per square foot of surface when 10 or 12 sq.ft. are allowed per horse-power, with the corresponding rate of heat absorption in B.T.U. per hour per square foot, some extra scales are added to the right of the chart showing all relations at a glance.

The figures on evaporation per square foot of heating surface per hour, or better still, the heat absorbed per square foot of heating surface per hour, is a proper basis for comparing two sets of data for different boilers at equivalent capacities. Thus different boilers may be said to be operating at the same capacity when their heating surface rates of heat absorption are the same, though one may be developing 50 and the other 1000 boiler horse-power. The rate of heat absorption by boiler heating surface is a quantity that must be dependent on two factors or terms, one an absolute figure and the other expressing the rate of change with generation. It is clear that as heat is generated faster it must certainly be absorbed faster unless the capacity of the water side of the heating surface for heat is lessened at the same time, and this is not the case with ordinary boilers, but not beyond the bounds of possibility.

The experimental data on the transfer of heat show conclusively that the capacity of water in tubes or tanks to absorb heat very much exceed the capacity of gases to give it to the surface under a wide range of conditions, so it is not unreasonable to suppose that as gases bring more and more heat to water surfaces, it will be taken up by them at a rate which is some function of the rate of supply. If, therefore, by more rapid fuel combustion there is a regularly increasing quantity of heat available for the water in the form of hot gases, then more will be absorbed per hour, more steam made per hour, and more boiler horse-power developed per square foot of heating surface, for the whole boiler, and these things must be functions of the rates of combustion and heat generation. In two different boilers the rate of increase of absorption may or may not bear the same relation to the rate of increase in generation, but it is quite possible for them to be similar in this respect, but at a given rate of generation in each it is more likely that the actual rates of absorption should differ by reason of the differences in structure. This latter should require a different surface per horse-power for the same amount of heat generated, even though a doubling of the rate of heat development produced the same *fractional increase* in horse-power of both.

Heat generation is, of course, essential to absorption, but it is necessary to distinguish between apparent and real generation in seeking relations between absorption and generation in accordance with the following definitions:

$$\left(\begin{array}{l} \text{Apparent heat generation in} \\ \text{boiler fires, B.T.U. per hr.} \end{array} \right) = \left(\begin{array}{l} \text{Lbs. fuel supplied} \\ \text{per hr.} \end{array} \right) \times \left(\begin{array}{l} \text{B.T.U. per lb. of} \\ \text{fuel as fired} \end{array} \right) (700)$$

$$\left(\frac{\text{Real heat generation in boiler fires B.T.U. per hr.}}{\text{B.T.U. per hr.}} \right) = \left(\frac{\text{Lbs. fuel supplied}}{\text{per hr.}} \right) \times \left(\frac{\text{B.T.U. per lb. of fuel as fired}}{\text{B.T.U. per lb. of fuel as fired}} \right) \times \left[1 - \left(\frac{\text{Fraction lost}}{\text{in furnace}} \right) \right]. \quad (701)$$

The fraction of heat lost in furnaces is that part of the calorific power represented by unburnt fuel in ashes, in cinders, in soot discharged through the flues, by unburned gases, carbon monoxide, hydrogen and hydrocarbons, by evaporation of moisture in the fuel and that conducted away from the *hot gases* by the furnace walls and settings between the point of firing and the place where absorption begins. This fraction is always appreciable and often large though very difficult to exactly measure, 20 per cent being not uncommon, in which case only 80 per cent of the calorific possibilities of the coal are actually available for absorption. These losses are mentioned here because of their importance, but will be more fully discussed in the next section with others which together make the whole boiler efficiency less than 100 per cent.

With some fraction of the fuel heat available for absorption in the form of *hot gases*, it may be absorbed in one of two characteristic ways, first as radiant heat from the glowing coal, brickwork, or incandescent floating carbon particles in hydrocarbon flames, and second by actual contact of gases with surface. It is convenient and usual to characterize these as fire-box and tube absorption, respectively, although some tubes may get radiant heat as well as conducted heat, and all internal fire-boxes get some gas contact heat. Now, the rate at which fire-boxes, actual or equivalent, absorb heat, being governed by the laws of radiation, is proportional to the fourth power of the temperature difference, according to the Stefan and Boltzmann law, so it would seem to be quite independent of the rate of generation or rate of fuel combustion, except as this may affect the temperature and extent of the radiant matter. This independence of radiant heat absorption with respect to rate of combustion or the constancy of radiant heat for all good fires seems to be fully established. On the other hand, the absorption by tubes or plates of heat from *hot gases* passing through or around them is governed by the laws of heat transfer from *hot gases* to water, whatever they may be. It is certain that higher mean temperatures of the *gases* should increase the rate of absorption per hour, and it is also certain that any influences that make the dead gas film thinner, or promote contact between *fresh* *hot gases* and the tube itself, will increase the rate of absorption. One such influence on reduction of dead gas film is the velocity of the *gases*, which is roughly proportional to the amount of air supply to the furnace, or the rate of combustion. *Whatever these laws are, they should be the same in kind for all boilers, but comparing any two, certain factors affecting the rate of absorption are fixed by the construction and may be very different in the two cases.* For example, the disposition of the heating surface may be such as to offer a long narrow passage-way for the *gases* between the surfaces, or a short wide one, or a narrow short one, or a wide long one. If the *gases* passed the water surface in infinitely thin streams, it seems likely that they would give

up their heat almost instantly, so the length of such a passage would not make much difference. On the other hand, if the passage were wide, the gases at the center of the stream might have to wait a long time to come into surface contact and might never do so, in which case they must give up their heat through surrounding gas layers, which would take a long time and require long tubes.

It appears, therefore, since the disposition of the heating surface, the proportions of the gas passages between the absorbing surface, the relations of fire-box to flue surface and such matters as are fixed by construction, may vary through almost infinite ranges, that, however well known might be the fundamental laws of radiant heat flow and of transfer by conduction and convection, the rate of heat absorption in boilers would resist any definite generalization though offering a most attractive field for theorizing. This makes it seem all the more remarkable that experimental results should show such a consistency as demonstrated below, even though the constants in the relation differ because of the *structural* influences noted.

The locomotive boiler is typical of internal fire-box construction and of operation with greatest "forcing," that is, highest rates of combustion and highest rates of evaporation per square foot of heating surface, while the ordinary stationary water-tube boiler is typical of much slower rates of operating conditions and of practically all tube construction. The first data selected for study are on 18 tests of an American locomotive boiler at the Purdue testing plant, with experienced and skilful experimenters, as reported by Goss, Bul. 402 U. S. G. S. These tests used two different, but both good quality coals, under four different boiler pressures and with different rates of combustion for each, all high. The boiler had a heating surface of 1023 sq.ft. and 193 sq.ft. of superheating surface. The grate surface was 17.25 sq.ft.

The heat-absorbing capacity of the heating surface with respect to the heat developed in the furnace was plotted with the result that the experimental points were shown to lie on straight lines whether the water surface be considered alone, the superheater surface alone, or the entire heating surface as one, within the limit of experimental error. By taking the line which represented the whole surface as characteristic of the whole heat-absorbing process or the boiler itself as a heat absorber, the following equations were obtained:

$$\left\{ \begin{array}{l} \text{B.T.U. absorbed} \\ \text{per hour per} \\ \text{sq.ft. H.S.} \end{array} \right\} = 1450 + .66 \left\{ \begin{array}{l} \text{B.T.U. developed in fire and} \\ \text{available for absorption} \\ \text{per hour per sq.ft. H.S.} \end{array} \right\} (a)$$

$$\left\{ \begin{array}{l} \text{Boiler horse-power} \\ \text{per sq.ft. H.S.} \end{array} \right\} = .0433 + .0000197 \left\{ \begin{array}{l} \text{B.T.U. developed in} \\ \text{fire and available for} \\ \text{absorption per hour} \\ \text{per sq.ft. H.S.} \end{array} \right\} (b)$$

$$\left. \begin{array}{l} (a) \\ (b) \end{array} \right\} \cdot (702)$$

Similar results are found for a very different type of boiler, the Heine water tube, by the tests made by Breckenridge and reported to the U. S. Geological Survey, though it was operated at very much lower rates, and on a great variety of coals ranging from low grade lignite to good bituminous.

Over two hundred tests are given which when plotted as before are represented by an almost perfect straight line within the limits of experimental procedure; it is worth noting that the different coals were well distributed over the curve, showing that the heating surface performance is independent of the fuel used. The boiler had 2031 sq.ft. of heating and 40.55 sq.ft. of grate surface.

This line is represented by Eq. (703).

$$\left\{ \begin{array}{l} \text{B.T.U. absorbed} \\ \text{per hour per} \\ \text{sq.ft. H.S.} \end{array} \right\} = 615 + .511 \left\{ \begin{array}{l} \text{B.T.U. developed in fire and} \\ \text{available for absorption per} \\ \text{hour per sq.ft. H.S.} \end{array} \right\} (a)$$

$$\left\{ \begin{array}{l} \text{Boiler horse-power} \\ \text{per sq.ft. H.S.} \end{array} \right\} = .0185 + .0000152 \left\{ \begin{array}{l} \text{B.T.U. developed in} \\ \text{fire and available for} \\ \text{absorption per hour} \\ \text{per sq.ft. H.S.} \end{array} \right\} (b) \quad (703)$$

The data show that from two totally different boilers, driven at very different rates, 4000 B.T.U. per sq.ft. heating surface maximum, were absorbed in the Heine water-tube and nearly 15,000 maximum in the locomotive fire-tube boiler, or about 4 to 1; with all sorts of coal in the former, some about the same as in the latter, the *heat absorbed varies by a straight-line law with the amount developed*, for both. Furthermore, though the constant for the water tube is about half that of the locomotive fire-tube boiler the *rate* of increase of absorption with increase of development or evolution of heat is nearly the same for both.

Extending the investigation a little further, more confirmatory results are obtained, and for this three boilers are selected, the first representing the largest and the second the smallest for which authentic tests are available, while the third gives data for oil fuel to further check the influence of kind of fuel.

The largest boiler is that of the Detroit Edison Co., tests for which are reported by Jacobus, having a heating surface of 23,650 sq.ft. and of the curved tube Stirling type, fired with coal mechanically, on both Roney and Taylor stokers. To the same coordinates as were used for the other cases the test data were also plotted with a similar result of an almost perfect straight-line. This line is represented by the linear relation Eq. (704):

$$\left\{ \begin{array}{l} \text{B.T.U. absorbed} \\ \text{per hour persq.ft.} \\ \text{H.S.} \end{array} \right\} = 235.3 + .8235 \left\{ \begin{array}{l} \text{B.T.U. developed in fire} \\ \text{and available for absorp-} \\ \text{tion per hr. per sq.ft. H.S.} \end{array} \right\} (a)$$

$$\left\{ \begin{array}{l} \text{Boiler h.p.} \\ \text{per sq.ft. H.S.} \end{array} \right\} = .0703 + .0000245 \left\{ \begin{array}{l} \text{B.T.U. developed in fire and} \\ \text{available for absorption} \\ \text{per hr. per sq.ft. H.S.} \end{array} \right\} (b) \quad (704)$$

The smallest boiler is represented by one used on the White steam automobile, for which tests are reported by Carpenter. This is of the "flash" type, gasoline fuel, carrying 45.8 sq.ft. of heating surface, the steam being of high pressure 200–500 lbs. gage, and high superheat—400° F. These results were plotted, and the resulting straight line is represented by Eq. (705):

$$\left. \begin{aligned} \left\{ \begin{array}{l} \text{B.T.U. absorbed} \\ \text{per hour per sq.ft.} \\ \text{H.S.} \end{array} \right\} &= .78 \left\{ \begin{array}{l} \text{B.T.U. developed in fire and avail-} \\ \text{able for absorption per hour per} \\ \text{sq.ft. H.S.} \end{array} \right\} & (a) \\ \left\{ \begin{array}{l} \text{Boiler horse-power} \\ \text{per sq.ft. H.S.} \end{array} \right\} &= .000026 \left\{ \begin{array}{l} \text{B.T.U. developed in fire and} \\ \text{available for absorption per} \\ \text{hour per sq.ft. H.S.} \end{array} \right\} & (b) \end{aligned} \right\} (705)$$

These relations are absolutely independent of the fuel. This is also shown by the tests of a Hohenstein water-tube boiler fired with liquid fuel. This boiler had 2130 sq.ft. of heating surface and was worked under quite a wide range of conditions, nevertheless a straight-line relation maintains as given by Eq. (706):

$$\left. \begin{aligned} \left\{ \begin{array}{l} \text{B.T.U. absorbed} \\ \text{per hour per sq.ft.} \\ \text{H.S.} \end{array} \right\} &= 1367.8 + .613 \left\{ \begin{array}{l} \text{B.T.U. developed in} \\ \text{fire and available for} \\ \text{absorption per hour} \\ \text{per sq.ft. H.S.} \end{array} \right\} & (a) \\ \left\{ \begin{array}{l} \text{Boiler horse-power} \\ \text{per sq.ft. H.S.} \end{array} \right\} &= .0408 + .0000183 \left\{ \begin{array}{l} \text{B.T.U. developed in} \\ \text{fire and available for} \\ \text{absorption per hour} \\ \text{per sq.ft. H.S.} \end{array} \right\} & (b) \end{aligned} \right\} (706)$$

There appears to be no doubt of the existence of a linear relation between heat absorbed and heat available for absorption in boilers, regardless of boiler structure, of kind of fuel or rate of forcing. The only difference is to be found in the two constants for which there is adequate explanation. When rate of heat development or evolution becomes zero there appears to be some absorption in all but the White automobile boiler, because the line does not pass through zero, but cuts the coordinate of absorption rate at a finite value. This value then must be a measure of the constant rate of absorption due to *radiant heat*, whether any hot gases carry heat to the absorbing surface or not. It is natural that this should differ for different boilers, as it is a structural feature depending on the exposure of heating surface to heat rays from both incandescent fuel and *brickwork*. The rate at which the rate absorption increases with the rate of evolution of heat is a result of the proportions of the gas passages through the heating surface tubes, or around them; again a structural factor for which existing data are insufficient to permit a numerical valuation in terms of dimensions alone.

It is often impossible to secure knowledge of furnace losses giving the amount of heat not available for absorption but still supplied in the fuel. The heat in the fuel is the product of the weight fired per hour, and the calorific power of the fuel as fired. The furnace loss, so called, is the difference between this product and that available for absorption in the form of hot gases approaching the heating surface. Algebraically these relations are given by Eqs. (700) and (701).

Accordingly, if the furnace loss as a fraction of the heat supplied be represented by L , then, Fraction of combustion heat really generated $= 1 - L$, . . . (707)

Whence

$$\left\{ \begin{array}{l} \text{B.T.U. developed} \\ \text{in fire and avail-} \\ \text{able for absorption} \\ \text{per hr. per sq.ft.} \\ \text{H.S.} \end{array} \right\} = (1-L) \times \left\{ \begin{array}{l} \text{Lbs. of fuel sup-} \\ \text{plied per hour} \\ \text{per sq.ft. H.S.} \end{array} \right\} \times \left\{ \begin{array}{l} \text{B.T.U. per} \\ \text{lb. of fuel} \\ \text{as fired} \end{array} \right\} \quad (708)$$

Substituting this in the general linear equation between rate of absorption and rate of true generation, Eq. (709), gives the following relation between rate of absorption and rate of apparent generation, Eq. (710).

$$\left\{ \begin{array}{l} \text{B.T.U. absorbed per} \\ \text{hour per sq.ft. H.S.} \end{array} \right\} = a + b \left\{ \begin{array}{l} \text{B.T.U. developed in fire and avail-} \\ \text{able for absorption per hour per} \\ \text{sq.ft. H.S.} \end{array} \right\} \quad (709)$$

$$= a + b(1-L) \left\{ \begin{array}{l} \text{Lbs. of fuel sup-} \\ \text{plied per hour} \\ \text{per sq.ft. H.S.} \end{array} \right\} \times \left\{ \begin{array}{l} \text{B.T.U.} \\ \text{per lb.} \\ \text{of fuel} \\ \text{as fired} \end{array} \right\} \quad (710)$$

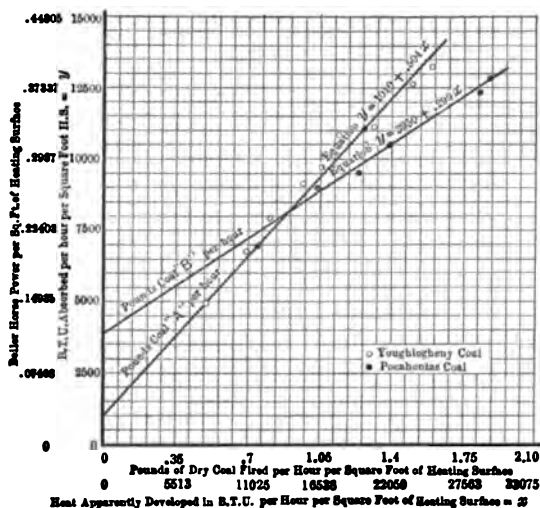


FIG. 135.—Relation between Rate of Heat Absorption or B.H.P. and Rate of Combustion or Apparent Heat Generation for a Locomotive Boiler.

of coal per hour per square foot of H.S. is directly proportional to the rate of combustion. For a fuel of constant calorific power the last term of Eq. (710) will be constant, and a linear relation is established between rate of absorption or boiler horse-power and rate of combustion, a most valuable relation and true if,

- (a) The furnace losses are a constant fraction of the heat in the fuel as supplied;
- (b) The calorific power of the fuel is constant.

If now L is a constant, then b being also a constant, $b(1-L)$ is likewise a constant, so that this relation would also be linear, and experimental determinations should be on a straight line. Should it happen in any case that a curve results, then *this is in itself a proof of a variable fractional furnace heat loss*. As a general rule a straight line does result, so that constant fractional furnace heat losses are rather the rule than the exception. It should be noted that as the ratio of heating surface to grate surface is fixed in any given boiler, the weight

Referring to Fig. 135, representing the data of the locomotive boiler tests, vertical distances give the rate of heat absorption and boiler horse-power to a double scale; horizontals, rate of combustion and equivalent rate of heat apparently developed. The difference in the slopes for the lines representing the two different fuels shows clearly the effect of different furnace losses in one case compared with the other, but in each the fractional loss is constant, as proved by the linear relation.

A similar conclusion was indicated by plotting a series of tests on four U. S. Navy water-tube boilers intended respectively for U. S. S. *Denver*, *Cincinnati*, *Nebraska* and *Virginia*.

These straight-line relations give the following values for the constants of Eq. (710):

Boiler.	Value of Constants, in Eq. (710).	
	<i>a.</i>	<i>b</i> (1 - <i>L</i>).
B. & W., U.S.S., <i>Cincinnati</i>	1350	.606
B. & W., U.S.S. <i>Nebraska</i>	50	.722
Niclausse, U.S.S. <i>Virginia</i>	1760	.566
Hohenstein, U.S.S. <i>Denver</i>	1090	.56
Locomotive, coal A.	1010	.504
Locomotive, coal B.	3950	.299

When the furnace losses are not a constant fraction of the heat supplied in the form of fuel, then the graph between rate of absorption, or boiler horse-power, and rate of combustion, or of apparent heat generation, is really a curved not a straight line.

The general fundamental Eq. (711 *a*, *b*, *c*.) may be changed as to form into others containing different but related variables, and in some one of these different forms they apply to existing test data.

$$\left. \begin{aligned}
 &\left\{ \begin{array}{l} \text{B.T.U. absorbed per hour} \\ \text{per square foot of heat-} \\ \text{ing surface} \end{array} \right\} = a + b \left\{ \begin{array}{l} \text{B.T.U. developed in fur-} \\ \text{nace and available for ab-} \\ \text{sorption per hour per sq.ft.} \\ \text{of heating surface} \end{array} \right\} \quad (a) \\
 &\left\{ \begin{array}{l} \text{Lbs. evaporation from} \\ \text{and at 212° F. per} \\ \text{square foot of H.S. per} \\ \text{hour} \end{array} \right\} = \frac{a}{970.4} + \frac{b}{970.4} \left\{ \begin{array}{l} \text{B.T.U. developed} \\ \text{in furnace and} \\ \text{available for ab-} \\ \text{sorption per hr.} \\ \text{per sq.ft. heating} \\ \text{surface} \end{array} \right\} \quad (b) \\
 &\left\{ \begin{array}{l} \text{Boiler horse-power per} \\ \text{square foot of heating} \\ \text{surface} \end{array} \right\} = \frac{a}{33479} + \frac{b}{33479} \left\{ \begin{array}{l} \text{B.T.U. developed} \\ \text{in furnace and} \\ \text{available for ab-} \\ \text{sorption per hr.} \\ \text{persq.ft. of heat-} \\ \text{ing surface} \end{array} \right\} \quad (c)
 \end{aligned} \right\} \quad (711)$$

Multiplying throughout by the heating surface in square feet gives three more formulas, Eq. (712):

$$\left. \begin{aligned} \left\{ \begin{array}{l} \text{B.T.U. absorbed} \\ \text{per hour} \end{array} \right\} &= a \left\{ \begin{array}{l} \text{Sq.ft. of heating} \\ \text{surface} \end{array} \right\} + b \left\{ \begin{array}{l} \text{B.T.U. developed in fur-} \\ \text{nace and available for} \\ \text{absorption per hr.} \end{array} \right\} \quad (a) \\ \left\{ \begin{array}{l} \text{Lbs. evapo-} \\ \text{rated per hr.} \\ \text{from and at} \\ \text{212° F.} \end{array} \right\} &= \frac{a}{970.4} \left\{ \begin{array}{l} \text{Sq.ft. of heating} \\ \text{surface} \end{array} \right\} + \frac{b}{970.4} \left\{ \begin{array}{l} \text{B.T.U. developed} \\ \text{in furnace and} \\ \text{available for ab-} \\ \text{ sorption per hr.} \end{array} \right\} \quad (b) \\ \left\{ \begin{array}{l} \text{Boiler horse-} \\ \text{power} \end{array} \right\} &= \frac{a}{33479} \left\{ \begin{array}{l} \text{Sq.ft. of heating} \\ \text{surface} \end{array} \right\} + \frac{b}{33479} \left\{ \begin{array}{l} \text{B.T.U. developed} \\ \text{in furnace and} \\ \text{available for ab-} \\ \text{ sorption per hr.} \end{array} \right\} \quad (c) \end{aligned} \right\} \quad (712)$$

Various other equations involving fuel consumption, grate surface and ratio of grate to heating surface may be obtained in a similar manner.

One of the best analytical investigations of the laws of boiler-surface heat absorption is that of Professor John Perry, who arrives at conclusions very similar to the above. According to his mathematical analysis, based on the kinetic theory of gases and using the fundamental idea suggested by Osborne Reynolds, *the flue part of a boiler, that is, all surface obtaining its heat from hot gas contact, will absorb always the same fraction of the heat that gets to it.* The fraction depends on the relation of surface to cross-section of gas passage, that is, its mean hydraulic depth and on nothing else, except a constant of proportionality, the nature of which is somewhat uncertain.

$$\frac{\text{Heat absorbed by flue}}{\text{Heat supplied to flue}} = \left(\frac{\text{Constant depending on dimensions of gas}}{\text{passage between heating surface}} \right) = C. \quad (713)$$

From an analysis of the data of an old French locomotive test made on a boiler built up in sections, the evaporation in each of which could be measured, Perry concludes that the absorption or the evaporation in the fire-box alone is given by

$$\left\{ \begin{array}{l} \text{B. T. U. absorbed} \\ \text{per hour by fire-box} \end{array} \right\} = A \times (\text{grate surface}) + B \times (\text{pounds coal per hour}). \quad (714)$$

Assuming that the flues can absorb a constant fraction of what is left after the fire-box has taken out some of the heat of combustion from the hot gases, then, neglecting furnaces losses,

$$\left\{ \begin{array}{l} \text{B.T.U. ab-} \\ \text{sorbed per} \\ \text{hour by} \\ \text{flues} \end{array} \right\} = C \times \left\{ \begin{array}{l} \left(\begin{array}{l} \text{B.T.U. developed} \\ \text{in furnace and} \\ \text{available for ab-} \\ \text{sorption per hr.} \end{array} \right) - \left(\begin{array}{l} \text{B.T.U. absorbed} \\ \text{by fire-box per} \\ \text{hour} \end{array} \right) \end{array} \right\} \quad (a) \\ = C \times \left\{ \begin{array}{l} \left(\begin{array}{l} \text{B.T.U. developed in} \\ \text{furnace and avail-} \\ \text{able for absorption} \\ \text{per hour} \end{array} \right) - A \left(\begin{array}{l} \text{Grate} \\ \text{surface} \end{array} \right) - B \left(\begin{array}{l} \text{Lbs. coal} \\ \text{per hour} \end{array} \right) \end{array} \right\} \quad (b) \end{aligned} \right\} \quad (715)$$

$$\begin{aligned}
 \left\{ \begin{array}{l} \text{Total B.T.U. absorbed} \\ \text{per hour by boiler} \end{array} \right\} &= \left\{ \begin{array}{l} \text{B.T.U. absorbed per hour} \\ \text{by (fire-box+flues).} \end{array} \right\} \\
 &= \left\{ \begin{array}{l} A \text{ (Grate surface)} + B \text{ (Lbs. coal per hour)} \\ + C \left\{ \begin{array}{l} \text{B.T.U. developed in furnace} \\ \text{and available for absorption} \\ \text{per hour} \end{array} \right\} - \left(\begin{array}{l} A \text{ (Grate surface)} \\ + B \text{ (Lbs. coal)} \\ \text{per hour} \end{array} \right) \end{array} \right\} (a) \\
 &= C \left(\begin{array}{l} \text{B.T.U. developed in fur-} \\ \text{nace and available for} \\ \text{absorption per hour} \end{array} \right) + \left\{ \begin{array}{l} A(1-C) \text{ (Grate surface)} \\ + B(1-C) \left(\begin{array}{l} \text{Lbs. coal} \\ \text{per hour} \end{array} \right) \end{array} \right\} (b)
 \end{aligned} \quad (716)$$

This Eq. (716), reached by Perry partly by mathematical analyses starting with the kinetic theory of gases and partly from fire-box evaporation data for one boiler, is substantially the same as Eq. (712a), which is purely empiric, based on many tests of different boilers, conducted with far more than usual accuracy. To show this similarity it is only necessary to assume that

$$M \times \left\{ \begin{array}{l} \text{B.T.U. developed in furnace} \\ \text{and available for absorp-} \\ \text{tion per hour} \end{array} \right\} = \text{Lbs. coal per hour.}$$

$$\text{Also that } \text{Grate surface} = \frac{\text{Heating surface}}{\text{Ratio of heating surface to grate surface}},$$

which on substitution in Eq. (716b) gives

$$\begin{aligned}
 (\text{B.T.U. absorbed per hour}) &= C \left(\begin{array}{l} \text{B.T.U. developed in furnace and} \\ \text{available for absorption per hour} \end{array} \right) \\
 &+ \frac{A(1-C)}{R} \left(\begin{array}{l} \text{Sq.ft. of} \\ \text{heating} \\ \text{surface} \end{array} \right) + B(1-C)M \left(\begin{array}{l} \text{B.T.U. developed in fur-} \\ \text{nace and available for} \\ \text{absorption per hour} \end{array} \right) \\
 &= \frac{A(1-C)}{R} \left(\begin{array}{l} \text{Sq.ft. of} \\ \text{heating} \\ \text{surface} \end{array} \right) + \{B(1-C)M + C\} \left(\begin{array}{l} \text{B.T.U. developed in fur-} \\ \text{nace and available for} \\ \text{absorption per hour} \end{array} \right). \quad (717)
 \end{aligned}$$

As the ratio of heating surface to grate surface R is a constant for one boiler, the two coefficients above become the constants (a) and (b) of Eq. (712a). To state it otherwise, his conclusion is identical with the new facts in the case if the ratio R is constant and if the furnace losses are a constant fraction of the apparent generation. If the approximate identity above noted is real then his most interesting conclusion, the most promising of all boiler transfer theories, is supported and *flues always will take out a constant fraction of the heat carried to them by the hot gases*, the fraction being larger, as they are longer and the cross-section of gas passage smaller compared to the heating surface at its perimeter or edge.

Prob. 1. A boiler receives 10,500 lbs. of water per hour at a temperature of 50° F. and turns it into steam at 150 lbs. per sq.in. gage, with 100° of superheat. What boiler horse-power is being developed and what is the factor of evaporation for this case?

Prob. 2. A boiler has a heating surface of 1050 sq.ft. On the basis of 10 sq.ft. per horse-power, how many pounds of steam will it make per hour from feed-water at 150° F. and steam 95 per cent dry at 125 lbs. per sq. in. gage?

Prob. 3. A boiler with 1000 sq.ft. of heating surface makes 4000 lbs. of steam per hour from feed-water at 200° F. and steam at 200 lbs. gage with 3 per cent moisture. What is the number of square feet of heating surface per horse-power?

Prob. 4. What would be the factors of evaporation for the following cases:

Feed Water Temperature.	Steam Pressure Gage.	Moisture in Per Cent or Superheat in ° F.
50	110	5%
150	50	100°
212	0	0
250	200	150°
300	400	3%
125	100	50°
212	0	10%
100	150	5%
70	125	200°
212	0	500°

Prob. 5. 3000 lbs. of coal having a heating value of 12,500 B.T.U. per lb. are fired per hour. If the cinder loss is 5%, CO loss 3%, moisture loss $\frac{1}{2}$ %, and radiation and other furnace losses are 5%, what is the rate of real generation per hour in B.T.U.?

Prob. 6. If 15 per cent of the heat actually generated is lost to the stack and the efficiency of the heating surface is 80 per cent, what will be the horse-power of boiler, efficiency of boiler as a whole, and pounds of steam made per hour from feed water at 100° F., and steam at 100 lbs. gage, dry and saturated?

Prob. 7. For a boiler, the constants of Eq. (712a) were, $a=1000$ and, $b=.55$. The ratio of heating surface to grate surface was 60 and grate was 60 sq. ft. in area. The coal fired per hour was 1500 lbs. and 90% of the heat in the coal was actually generated. If the B.T.U. per pound of coal were 13,450, what was the horse-power of boiler?

Prob. 8. A boiler has 3500 sq.ft. of heating surface and a grate surface of 50 ft. The constants in the straight-line equation are $a=1000$ and $b=.5$. Show how the horse-power will vary with the rate of combustion for coal containing 12,000 B.T.U. per pound and a constant furnace loss of 15 per cent.

Prob. 9. The rate of combustion in a boiler varies from 20 to 80 lbs. per hour per square foot of grate surface. The grate area is 150 sq.ft. and ratio of heating surface to grate surface is 60. For a constant furnace loss of 15 per cent and a coal having a heating value of 14,000 heat-units, what will be the variation in horse-power, the relation being the same as for the tests in Fig. 135 with coal A?

93. Steam-boiler Efficiency, Furnace and Heating-surface Efficiency. Heat Balances and Variation in Heat Distribution. Evaporation and Losses per Pound of Fuel. Defining the efficiency of a boiler, inclusive of grate, furnace, setting and heating surface, as the ratio of the heat that is actually absorbed and retained by the steam per pound of coal, to the calorific power of the coal as fired, it may be said that there is no fundamental theory or

absolute standard, as was the case for evaporative capacity; but there are known and established relations between the factors that together make it less than 100 per cent. Study of boiler efficiency must be based therefore, not on what might be expected of the heat absorption surface but rather on the nature and extent of the losses, or the several parts of the original heats that do not get into the water or steam, whether operating to reduce the amount available for absorption before the gases reach the surface or, on the other hand, to leave a residue of sensible heat in the gases after they have swept the surface.

Boiler heat losses can be divided, grouped and classified in many different ways; the following is a very useful classification:

1. *Furnace and Setting Loss.* This includes all those amounts of heat that tend to reduce the sensible heat of the gases available for heating surface absorption except radiant heat taken up directly by the heating surface, such as:

- (a) That necessary to evaporate the moisture in the coal;
- (b) That due to burning hydrogen to vapor instead of to water, the difference between high and low calorific power of hydrogen burnt per pound of coal;
- (c) That radiated (1) from furnace and setting before absorption by water or steam;
- (d) That due to unburned gases CO, H₂ and hydrocarbons in the flue gases;
- (e) That due to unburned fixed carbon, in ash dropping through grate, or in soot and cinders in the flues, or discharged from the stack.

2. *Flue Loss.* This includes the amount of sensible heat still carried by the gases that have swept over the heating surface; it is the product of the total weight of gases per pound of coal, including water vapor and excess air, into the mean specific heat and excess of temperature over that of the air supply. Obviously, this is more or less governed by the steam pressure and temperature since the leaving gases can never be made cooler than the last boiler surface they touch, and if this is superheater surface the temperature may be high, but if it is feed-water supply surface it may be low. Otherwise this loss is governed by the control of air supply, and, more fundamentally, by the absorbing laws of heating surface

On the assumption that the rest of the heat goes into the steam and water, all the heat is thus accounted for. *That which gets into the water and steam may be regarded as the useful effect*, but this may not be warranted if some steam and water leakages exist or radiation occurs from the steam or water surfaces themselves. In these cases there may be a difference of opinion as to whether the losses should be credited or debited because from the standpoint of boiler effectiveness as a heat absorber all heat carried off by leakages credit heat, for it has been taken from gases; but on the other hand from the steam user's standpoint this heat is of no value as it cannot run an engine. The steam and water surface radiation is also a steam user's loss, but the

boiler absorbed it from the gases and, considering the boiler as an absorber rather than a conserver of heat, it is a credit. No confusion is likely to result in practical work if these items be clearly understood, circumstances in each case will indicate where they belong.

The preceding classification indicates that there may be more than one standard of *efficiency of boilers* or rather that boiler efficiency is divisible into parts, which is a valuable way of keeping the processes clearly divided and of establishing a basis of analysis. The following definitions will apply:

1. *Furnace Efficiency* E_f , as a term is properly applied to the ratio of the heat of the gases available for heating surface absorption per pound of fuel to the calorific power of the fuel, or

$$\left. \begin{aligned} E_f &= \frac{(\text{B.T.U. per lb. fuel}) - (\text{Furnace and setting loss per lb. fuel})}{\text{B.T.U. per lb. fuel}} & (a) \\ &= 1 - \frac{\text{Furnace and setting losses per lb. fuel}}{\text{B.T.U. per lb. fuel}} & (b) \end{aligned} \right\} \quad (718)$$

2. *Heating-surface Efficiency* E_s , as a term is properly applied to the ratio of the heat absorbed by the boiler water and steam to the sensible heat brought to the heating surface by the hot gases, radiated to it per pound of fuel. As the heat usefully absorbed is the difference between the sensible heat of gases, as developed in the furnace, and the flue loss, this may be set down in two ways:

$$\left. \begin{aligned} E_s &= \frac{(\text{B.T.U. absorbed by water or steam per lb. fuel})}{(\text{B.T.U. per lb. fuel}) - (\text{furnace and setting losses per lb. fuel})} \dots \dots (a) \\ &= \frac{(\text{B.T.U. per lb. fuel}) - (\text{furnace and setting losses per lb. fuel}) - (\text{flue losses per lb. fuel})}{(\text{B.T.U. per lb. fuel}) - (\text{furnace and setting losses per lb. fuel})} & (b) \\ &= 1 - \frac{\text{Flue loss per lb. fuel}}{(\text{B.T.U. per lb. fuel}) - (\text{furnace and setting losses per lb. fuel})} \dots \dots (c) \end{aligned} \right\} \quad (719)$$

3. *Boiler Efficiency* E_b is most commonly applied to the ratio of heat absorbed by water and steam per pound of fuel to its calorific power per pound, or it is the product of heating surface and furnace efficiencies.

$$\left. \begin{aligned} E_b &= \frac{\text{B.T.U. absorbed by water per lb. fuel}}{\text{B.T.U. per lb. fuel}} \dots (a) \\ &= 1 - \left[\frac{\text{furnace and setting losses per lb. fuel}}{\text{B.T.U. per lb. fuel}} \right] \dots \dots (720) \\ &\quad - \left[\frac{\text{Flue loss per lb. fuel}}{\text{B.T.U. per lb. fuel}} \right] (b); \quad = E_f \times E_s. (c) \end{aligned} \right\}$$

The experimental determination of all these losses and efficiencies is continually going on, and from the data boiler designers have continuously sought to improve results. As a consequence one might expect to find modern boilers turned out by the best engineers much more efficient than others, but this is not the case, nor can it be said that one type is any more efficient than another, if small portable and other special forms be excluded, nor are recent

designs more efficient than old ones. This is well illustrated by the summaries made by Donkin after studying some four hundred tests, of all kinds of boilers, which are quoted from his book in Table XXIX.

TABLE XXIX
BOILER EFFICIENCY SUMMARIES (DONKIN)

Type of Boiler.	No. of Tests.	Mean of Best Two Efficiencies Per Cent.	Lowest of One Test, Efficiency Per Cent.	Mean of All Tests, Efficiency Per Cent.
Water tube, 1½-inch tubes.....	6	81.4	66.6	77.4
Locomotive.....	37	83.3	53.7	72.5
Lancashire.....	10	74.4	65.6	72.0
Two-story.....	9	76.1	57.6	70.3
Two-story.....	29	79.8	55.9	69.2
Dry back.....	24	75.7	64.7	69.2
Return-smoke tube.....	11	81.2	56.6	68.7
Cornish.....	25	81.7	53.0	68.0
Cornish.....	9	81.0	55.0	67.0
Wet back.....	6	69.6	62.0	66.0
Elephant.....	7	70.8	58.9	65.3
Water tube, 4-inch tubes.....	49	77.5	50.0	64.9
Lancashire.....	40	73.0	51.9	64.2
Cornish.....	3	65.9	60.0	62.7
Lancashire.....	107	79.5	42.1	62.4
Dry back.....	6	73.4	54.8	61.0
Lancashire 3-flue.....	6	66.7	52.0	59.4
Elephant.....	8	65.5	54.9	58.5
Lancashire.....	8	74.3	45.9	57.3
Vertical.....	5	76.5	44.2	56.2

These data certainly demonstrate that boiler efficiency, however many things may determine it, cannot be associated with boiler type, nor with size. Admitting that in some cases the efficiencies may be high, that is, somewhere near 80 per cent, and that in other cases they may be low, 50 per cent or even less, the causes of such a possible range from good to bad when the average good performance of all types of boilers may be and is about the same, are worth investigating even if prediction of efficiency for specific conditions is impossible. The charge of bad management will not account for all the facts even though a poor fireman may make the performance of the best boiler as poor as the worst; there can be absolutely no question as to very considerable differences in efficiency with the most skillful management, that can be traced partly to form and proportions of the boiler structure and partly to operating conditions more or less beyond the control of the operator. In a great many cases gains or losses in efficiency are found chargeable to almost absurdly irrelevant things; the literature of the subject, especially that circulated by makers of boiler attachments is full of such false conclusions, due no doubt to unskillful or over skillful testing that failed to discover which one of a possible dozen variables that may affect boiler efficiency was responsible for an improvement obtained.

A complete statement of results of a boiler performance accounting for supply and distribution of heat, termed the heat balance of the boiler, is almost impossible, yet may be closely approximated by careful experimental work. The degree to which the approximation fails to represent the true distribution is indicated by an item containing *unaccounted for losses* often grouped with leakage of steam and water, and radiation. To illustrate, three different heat balances are reported in Table XXX for, first—the U. S. Geological Survey Heine water tube boiler of 210 H.P. and 2031 sq.ft. of heating surface, as reported by Breckenridge; second—the fire tube locomotive boiler of 400 H.P. and 1216 sq.ft. of heating surface as reported by Goss, and, third—a Stirling water tube boiler.

TABLE XXX
THREE EXAMPLES OF HEAT BALANCE FOR BOILERS

Distribution of Heat of Coal, Per Cent.	210 H.P. Heine.	400 H.P. Locomotive.	510 H.P. Stirling.
1. Absorbed by water and steam	60.30	57.00	74.87
2. Loss due to vaporization of coal moisture26	5.00	.24
3. Loss due to sensible heat of stack gases	15.19	14.00	10.73
4. Radiation leakage and unaccounted for	3.97	7.00	8.24
Total per cent of fuel heat developed	79.72	83.00	94.08
5. Loss due to unburned CO in stack gases	2.17	1.00	.42
6. Loss due to other forms of incomplete combustion, H_2 , C_mH_n in gases and fuel in ash cinder, in flues, setting and discharged from stack	18.11	16.00	5.50
Total per cent of fuel heat not developed	20.28	17.00	5.92
	100.00	100.00	100.00

About some of these items there may be a difference of possible interpretation, especially as to whether a given loss represents heat developed or undeveloped and more particularly as to whether heat developed, as indicated, is really available for boiler heating-surface absorption. For example, when the fuel contains hydrogen, the steam from which does not get a chance to cool below 212° F., the low value only is developed, whereas the high value is reported by the calorimeter as the calorific power of the fuel. Again, the heat absorbed by vaporizing moisture in coal is developed heat, but it is unavailable for heating surface absorption because it is taken up by coal-water.

To avoid conflicts in commercial transactions in which the efficiencies and losses in boilers are sometimes the subject of legal guarantees, standards must be available as subjects of agreement whether correct scientifically or not, and such standards are established by the American Society of Mechanical Engineers, which should be consulted by everyone interested. Assuming that the loss due to vaporization of coal moisture is unavailable for absorption, and that the items of radiation, leakage and "unaccounted for," (in which nearly all errors are concentrated), represent available heat, the furnace and

heating-surface efficiencies can be estimated, and it must be understood that it is never possible to do more than estimate these. Charging all losses representing heat not available for absorption against furnace efficiency, these efficiencies decrease.

	Stirling	Heine	Locomotive
Efficiency of furnace, per cent.	94	79	78
“ “ heating surface, per cent..	81	76	73
“ “ boiler complete, per cent..	75	60	57

These figures are given not as typical of these kinds of boilers or of any particular service conditions, though they are as fair as any, but rather to show how division of performance may be made. In actual service and even during the conduct of tests, conditions are continually changing, so that a whole series of tests under conditions that seem identical will produce different results and show that the conditions were not identical—the causes of variation being due almost entirely to the impossibility of fire control or the maintenance of steady conditions therein. The ash is constantly accumulating and more in one spot than in another because air passes most freely where bed resistance is least, and burns the coal faster at that point; the thickness of fire is varying, coal cannot be supplied with absolute uniformity and the air supply is not only varying all the time but the ratio of top to bottom air and side leakage also varies: all these things change furnace and fire conditions and thus disturb the heat balance.

Even if the furnace efficiency were kept constant through a more perfect fire and air control than is possible except in tests, the conditions for heat absorption by the heating surface may vary considerably. Probably the greatest of the disturbing influences that tend to baffle attempts at analysis is the *delayed and long-continued combustion of the gases beyond the fire-bed or the relation of flame to completely burned hot gases.*

All theories of heating-surface absorption that have ever been proposed are based on the assumption that the gases passing over the heating surface are immediately and progressively cooled according to some assumed law. Pyrometric investigations in flues show that so long as there is a clear, visible flame there is practically no cooling, and even where the flame becomes of the flickering, irregular sort, the cooling of the gases is very much slower than beyond the limit of flame, for this reason the generation of heat continuing during part of, sometimes all of the absorption period, makes it practically impossible to formulate correct absorption laws. It must not be understood, however, that this theorizing is of no value; attention has already been called to Perry's idea of absorption as most useful, and reasonably in agreement with observation on rate of heat absorption related to rate of heat generation.

The efficiency of heating surface being the ratio of the amount absorbed to that generated and available for absorption, can be derived from the relations between rates of generation and absorption, whether these relations are derived from a theory or represent average experimental data. According to the

experimental data analyzed in the last section the relation between absorption and generation rates could be expressed in either of the following ways:

$$\left\{ \begin{array}{l} \text{B.T.U. absorbed per hr. per} \\ \text{sq.ft. of heating surface} \end{array} \right\} = a + b \left\{ \begin{array}{l} \text{B.T.U. developed in fire and} \\ \text{available for absorption} \\ \text{per hr. per sq.ft. of heat-} \\ \text{ing surface} \end{array} \right\}$$

$$\left\{ \begin{array}{l} \text{B.T.U. absorbed per hr. per} \\ \text{sq.ft. of grate surface} \end{array} \right\} = aR + b \left\{ \begin{array}{l} \text{B.T.U. developed in fire} \\ \text{and available for absorp-} \\ \text{tion per hr. per sq.ft. of} \\ \text{grate surface} \end{array} \right\}$$

from which two possible expressions for efficiency of the heating surface follow both of the same general form, Eq. (712), since efficiency is

$$E_s = \frac{\text{B.T.U. absorbed}}{\text{B.T.U. developed in fire and available for absorption}}$$

$$= b + \frac{a}{(\text{B.T.U. developed in fire and available for absorption per hr. per sq.ft. H.S.})} \quad (a)$$

$$= b + \frac{aR}{(\text{B.T.U. developed in fire and available for absorption per hr. per sq.ft. G.S.})} \quad (b) \quad (721)$$

These are equations of an equilateral hyperbola asymptotic to 100*b* per cent efficiency as indicated by *AB* in Fig. 136. They seem to be at fault because they show that efficiency may exceed 100 per cent with small amounts of generation, which means, however, only that the relation is not true for very small generations. For the working ranges of generation rates the heating surface efficiency line is substantially straight and falls with increase of generation. According to Perry's division of absorption between fire-box and flue, or their equivalents for the particular structure in question, the efficiency of the flue part of the heating surface is independent of the rate of heat supply to it, so that all variations in the efficiency of the whole heating surface must be chargeable to the part receiving radiant heat. Just how true this is, or just how the whole efficiency of heating surface should, and really does, vary with rate of generation cannot be settled with any available data, though there are enough to prove almost anything with a little skillful stretching.

The curve *AB* of Fig. 136 does, however, within the limits of the experimental points from which it was plotted—those for the Hohenstein oil-fuel boiler—represent the facts of these experiments, and comparison with Eq. (721) for heating surface efficiency in terms of losses indicates a most interesting meaning for the vertical distance from the curve *AB* to the 100 per cent line. This distance must stand for the flue loss per pound of fuel divided by the generation per pound fuel, or the calorific power less the furnace and setting loss, and shows that for these experiments the ratio is increasing with increase of total generation or rate of combustion.

If on the same diagram, the line *CD* represents the overall boiler efficiency then the vertical distance from it to the 100 per cent line must represent the sum

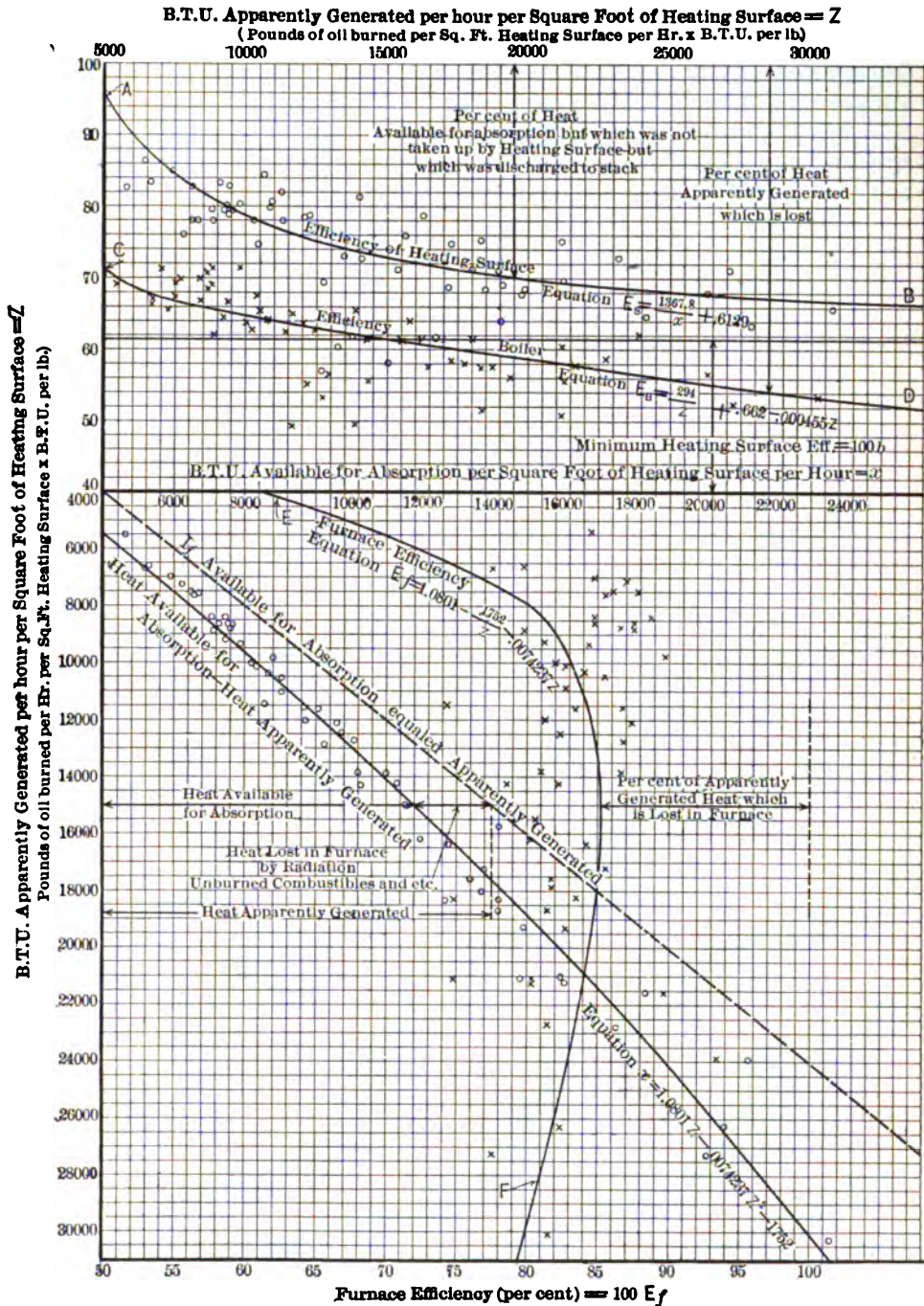


FIG. 136.—Curves showing the Relations which Exist between Quantities Entering into Boiler Efficiency.

of the losses in the flue as sensible heat, together with the furnace and setting losses. Similarly, the vertical distance from CD to AB must represent the furnace and setting losses. If the two curves are parallel these latter losses are constant in amount; if they converge toward the right they must decrease with increase in heat generation, and if they diverge the losses are increasing with increase of production. In this particular case the losses first increase and then decrease, and are separately plotted below, line EF .

In all cases the flue losses increase in quantity with increase in heat generation according to these data, and it is worth while to investigate what meaning may be attached to this.

Setting down the expression for flue loss which is represented by the distance from the heating-surface efficiency line AB to the 100 per cent line, in symbols

$$\frac{\text{Flue loss per lb. fuel in sensible heat}}{\text{B.T.U. per lb. fuel - furnace and setting losses per lb. fuel}} = 1 - b - \frac{aR}{\text{B.T.U. generated per square foot grate}} \quad (722)$$

For the above to be true either the numerator must increase or denominator decrease, that is, with increase of combustion rate the sensible heat per pound of fuel must increase or the furnace and setting losses per pound of fuel must increase, or both. This shows clearly how variations in the furnace conditions affect the heating-surface efficiency in spite of any supposed constancy of the efficiency of the flue part. As the experiments from which the conclusion is derived include all possible ranges in rate of combustion and kind of coal and several very different types of boiler, it must be accepted notwithstanding the apparent failure of the law of relation for low rates of generation where the curve passes the 100 per cent line.

The flue loss in sensible heat is the product of specific heat of gases, their weight per pound of fuel and the excess of temperature over the air supply, and if the flue part of the heating surface takes out always the same part C , then

$$\text{Heat absorbed by flues} = C \times (\text{heat supplied to flue}). \quad (723)$$

$$\text{Flue loss} = (1 - C) \times \text{heat supplied to flue}, \quad (724)$$

$$\text{or} \quad C_p w (t_{\text{stack}} - t_{\text{air}}) = (1 - C) C_p w (t_{\text{furnace}} - t_{\text{air}}). \quad (725)$$

$$[(\text{stack temp.}) - (\text{air temp.})] = \text{constant} \times [(\text{furnace temp.}) - (\text{air temp.})], \quad (726)$$

provided the weight of gases per pound of coal, w , is constant. Experimental observation proves that the stack temperature always rises with increase in rate of combustion unless excess air be supplied at the same time, and as a rule the excess air and weight of gases per pound of fuel decreases with increased rate of combustion or increased draft if the fire is thick enough and free from holes, otherwise; this is most common with moving-bar stokers and with anthracite small sizes, whether hand or mechanically fired, the weight of gases per pound of coal quickly decreases and the stack temperature increases less quickly, or not at all.

In the U. S. Geological Survey series on the Heine boiler the combustion chamber temperature increased somewhat irregularly with rate of combustion and rate of heat evolution, for a considerable variety of coals and varying ratios of flue temperature to combustion chamber temperature. Individual readings show that the ratio of excess temperature of combustion chamber over the boiler temperature, to the excess of flue temperature, was found to range from 4 to 14, the best average being 7.3. This could hardly be called a constant ratio, and the reason may be due to the fact that the flame extended between the tubes more in some cases than in others, or it may be that the expectation of constancy is based on wrong hypotheses, and yet these are the best data on the subject available.

Examples of specific relations like the above could be cited for other variables, and at the end, the problem would be little if any nearer solution.

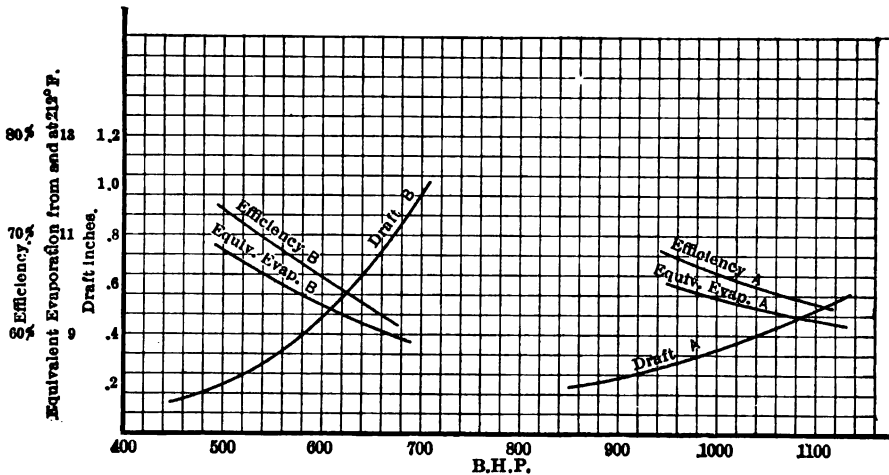


FIG. 137.—Relation between Horse-power and Draft on Efficiency.

However difficult division or analyses of losses may be, there is little doubt as to the sort of effect an increase or decrease of any one factor may have, and it is, furthermore, usually possible to explain an overall efficiency curve when obtained.

The overall efficiency curves are not always given in terms of per cent efficiency, nor is capacity in terms of B.T.U. absorbed per hour, but for efficiency the evaporation actual or equivalent per pound of fuel is more common, and for capacity, boiler horse-power or per cent boiler rating or rate of combustion. One such curve of considerable significance is that of Fig. 137, representing the results from the Interborough Railway boilers equipped with two Roney stoker furnaces, one 70 per cent of the grate area of the other and burning good semi-bituminous coal, first on one grate alone and then on both together. Increased capacity causes a decrease in efficiency regularly along a line almost straight with one grate, but when for a given capacity the second grate is used, the efficiency becomes higher after which it again decreases along a

nearly parallel line, that is, in accord with a similar law. The only way this could happen is through a change in furnace conditions, probably traceable to excess air variations, as the draft suddenly decreases for a given amount of generation, when passing from the single to the double grate, as indicated by the draft curves. If there are holes or air leaks, as is the case with such stokers, this would cause a decrease of excess air and an increase of efficiency exactly as occurred when the draft changed from a high to low value for the *same or even greater* total rate of heat evolution. In the Goss locomotive tests the evaporation is reported per pound of dry coal as a function of equivalent evaporation per square foot of heating surface, and shows a similar nearly straight line relation as in Fig. 135, but different for the two coals used. The reason for this appears if the whole heat balance be plotted to the same base, and is mainly due to a difference in cinder loss which in both cases increases with boiler capacity and draft that produces it. The essential difference between this fire and that last discussed is in its very superior thickness, lack of agitation by stoker bars, and lack of air holes at sides or end. See Chart No. 49, in the Tables.

Another good illustration of the controlling nature of some one condition is given by Myers, reporting results of boiler performance with tan bark as a fuel in leather-making works, the chief characteristic of the fuel being the large amount of moisture in proportion to the calorific power, which makes it unburnable in an ordinary furnace. The fuel showed about 9500 B.T.U. per pound when dry and ten samples contained $65\frac{1}{2}$ per cent moisture average, leaving only 2665 B.T.U. per pound of fuel as fired, which gave a boiler efficiency over all of 71 per cent when referred to dry tan bark and 54 per cent when referred to the bark as fired. Use of this wet fuel absolutely prevents the attainment of high efficiencies even in the best apparatus because of the moisture heat loss which is the controlling condition.

Another illustration of this fact is furnished by a test of the well-known B. and W. water-tube boiler by the Pittsburgh Testing Laboratory, using blast-furnace gas fuel which yielded a little over 54 per cent efficiency because of insufficient air and premixture with the gas, the flue gas analysis showing as much as 20 per cent escaping unburnt. With exactly the same boiler, except as to furnace and combustion chamber, an efficiency in excess of 80 per cent is not difficult with an oil fuel if care be exercised to control air supply in proportion to oil, and means be provided to mix the oil and air; failure to do this may yield just as poor results as noted for the blast-furnace gas. These incidents will suffice to show that boiler efficiency is controlled not so much by the boiler as by the furnace, not so much by boiler designer as by fireman, but the analysis shows that there is a problem of design, more concerned, however, with high capacity of surface, and therefore cost and size of boiler, than with efficiency improvements.

A number of boiler tests are given in graphical form in the Tables with some one item of importance selected to show the effect of various conditions of service and fuels in the same and different boilers, all of which are self explanatory. See Charts Nos. 38 and 39, Handbook of Tables.

Prob. 1. A furnace is fired with coal containing 12,500 B.T.U. per pound which needs 14 lbs. of air per pound of coal. In the ash there is 5 per cent of the weight of the coal in unburned carbon; the coal has 5 per cent moisture with it; there is 3 per cent of CO in the flue gas; the stack temperature is 500° above that of the room, which is 80° F., and twice the combining proportion of air is supplied. For the above data and allowing for 5 per cent radiation, of which 4 per cent is from the setting and 1 per cent from steam surface, what is the boiler efficiency?

Prob. 2. The heating surface efficiency of a boiler is 80 per cent when making 7 lbs. of steam at 100 lbs. per sq.in. gage per hour, 97 per cent dry from feed at 100° F. per lb. coal. The coal used contains 12,500 B.T.U. per pound. What is the furnace loss and boiler efficiency?

Prob. 3. If in the Hohenstein boiler test, for which curves are plotted in Fig. 136, the oil used had a heating value of 18,000 B.T.U. per pound and the heating surface was 2130, what was the efficiency of the boiler when 1.5 tons of oil were burned per hour, and what the furnace efficiency and the heating-surface efficiency?

Prob. 4. The efficiency of a certain boiler is 72 per cent. The steam pressure is 125 lbs. gage, feed-water 180° F. and there is 120° of superheat; 100 per cent excess air is supplied. Flue temperature is 100° above steam temperature and the room is 70° F. The evaporation per pound of coal is 6.5. The coal used is No. 87 of Table LV given in the Handbook of Tables. What is the heating surface and furnace efficiency?

Prob. 5. A boiler makes 6 lbs. of steam per hour at 100 lbs. gage pressure from feed-water at 60° F. per pound of coal fired. The coal is No. 66 of the general table referred to in Prob. 4; the ash loss is 8 per cent, radiation 5 per cent, and the CO loss 2 per cent. What is the heating surface efficiency, the furnace efficiency, the boiler efficiency?

PART III

TRANSFORMATION OF HEAT AND WORK BY CHANGES OF SUBSTANCE CONDITION.

CHAPTER XIII

HEAT AND WORK. FUNDAMENTAL RELATIONS. THERMAL PHASES.

94. General Heat and Work Relations. Thermal Cycles. Work and Efficiency Determination by Heat Differences and Ratios. Graphic Method of Temperature-entropy Heat Diagram. Whatever is known of the existence of energy, its different manifestations or forms, the essential identity of the different forms, the possibility of converting one into another, and the laws of this transformation expressing quantitative relations between amounts of energy and changes of conditions of substances, has been all derived from very simple observations of the form and condition of substances, by a somewhat elaborate process of comparison and judgment, assisted by certain established mathematical principles. Experiments on substances suffering thermal changes are recorded directly in tables of observational data. The numerical values of any two series of readings are compared for the purpose of establishing, if possible, a general relation between them, a proceeding materially assisted by plotting one against the other on cross-section paper yielding a curve. The various points will seldom lie all on the curve because of experimental error whether personal or instrumental. The smooth curve passing through or near the greatest number of points is considered as representing the true conditions of relation, especially when repetition tests of outlying points bring them closer to the curve. Such a curve is a graphic representation of a general relation between the two quantities involved, and its equation is the mathematical expression for the *law* of relation. Two different laws so derived may, by similar methods, be combined with each other if any essential relation between the respective quantities really exists, and such a process will, if carried far enough, lead to more and more inclusive statements of the relation between various quantities. Each generalization is a law, whether summarizing one set of observations, or combining two or more sets. These laws may be stated in three ways: (a) by equations; (b) by curves, graphically; (c) by words; and of these, the statement in words is least valuable because of the inadequacy of language to express mathematical truths or to interpret physical mathematical facts. For solving numerical problems with actual substances, those laws which are least general and most concrete are most useful, because as a rule they involve a lesser number of corrections and hypotheses than the broader generalizations which are reached only after the introduction of more and more assumptions, conditions and hypotheses.

The relations of heat energy to work and the conditions for the transformation of one into the other, have been found by experience to yield quite

readily to the process of generalization, into more and more broad statements, crystallizing finally into two, under the masterly manipulation of great physicists: at the same time a tendency has developed to call these two statements *the laws of thermodynamics*. Thermodynamics is defined by Rankine, who helped to establish it, as "the reduction of the laws according to which such (heat and work) phenomena take place, to a physical theory or corrected system of principles."

Thermodynamics, as thus defined, is concerned with no numerical quantities, nor with any particular substance, nor for that matter with any actual substance whatever, but it is a physical theory of energy in relation to matter; expressible by two all-inclusive laws known as the first and second laws of thermodynamics, and as such, is a branch of natural philosophy. These two laws are inclusive of individual laws of particular thermal relations, only in the sense that the concrete cases *do not contradict them*, or have served to create them, but not in the sense that any particular concrete fact can be derived from the two laws of thermodynamics; in most cases it cannot.

Engineering is concerned with real substances and with numerical quantities, so that alone the two laws of thermodynamic philosophy will not yield a solution of any practical numerical problem of design, or of analysis of test performance of actual heat machines or thermal apparatus. To guide such numerical thermal computations dealing with actual substances and apparatus, which is the province of Engineering Thermodynamics, two differences must be noted: first, numerical values for heat effects must be available for the various units involved, especially for that class known as physical constants; and second, their values must be known for actual substances or the materials of engineering. These physical constants include, for example, such quantities as the coefficients of expansion, the specific heats, latent heats of fusion and vaporization, the ratio of the pressure-volume product to absolute temperature, and the exponent " s " in adiabatic expansion of gases and vapors. These so-called thermal constants must be accepted as given in authoritative physical tables until such time as they are themselves expressible by laws with numerical coefficients, when the special law replaces the table either wholly or in part.

All the thermal constants are derivable by direct experiment, but in some cases, for example the specific heat of gases or superheated vapors at constant pressure and the latent heat of dry saturated vapors, the experimental difficulties are so great as to render indirect methods of determination preferable. From the general laws of heat and work a few important relations between these constants can be found, permitting the determination of one of them, more difficult to observe, from the values for others more easily measured; for example C_p can be found from C_v and R , or from C_v and γ , and the specific volume of dry saturated steam may be found by Clapeyron's equation, Eq. (548). Not only are relations between the constants desirable for the determination of one from others, but also for purposes of checking observations made on different quantities, perhaps at different times and places and by different experimenters. Even though some of these coefficients of heat effect may not prove to be really constant, nearly all of them being variables in fact, they are

still called constants for want of an equally short and descriptive name. These general heat and work relations are extremely simple in conception and easy to use, but when mathematically stated by a series of differential equations, may lead to a most complex series of derived equations to express which completely would require a vast amount of work, but for engineering purposes this is not at all necessary nor even desirable.

Experiment has shown that the state of a body is related to its heat content, and that a given body with an unknown heat content at any particular state or time may receive heat, may expand, may be compressed, or may lose heat in all sorts of ways, but if at the end of all these processes it finally is brought back to its original state, it must then have the same heat content as it had in the beginning, even though the numerical value of this heat content cannot be measured. This is a most significant fact because it provides a means of discussing heat taken in by the body, heat lost by it, and work done, each in its relation to the other. As energy is non-destructible, any work that has been done must be the exact equivalent of some heat that is not accounted for as heat, so that, assuming a body to pass through a series of thermal changes and to return to its original state, and therefore have the same heat content as before, it follows that the relations are as given in Eq. (727):

$$(\text{Heat added}) = (\text{Heat abstracted}) + (\text{Work done}). \quad (727)$$

To calculate the work done by a *complete* series of thermal changes it is only necessary to be able to calculate the heat added to the body or taken from it, during such a complete series of thermal processes, or, algebraically,

$$(\text{Work done}) = (\text{Heat added}) - (\text{Heat abstracted}). \quad (728)$$

Defining *any single thermal process as a thermal phase*, such as evaporation at constant pressure, or expansion at constant temperature, and defining a *complete series of thermal processes bringing the substance back to its original physical state as a thermal cycle*, then a thermal cycle consists of a closed series of thermal phases, or on a diagram of changes the cycle is represented by a closed figure consisting of a number of separate connecting lines, each representing a phase. The work done by a thermal cycle is to be found directly by subtracting from the algebraic sum of the heats added in each heat-adding phase, the corresponding algebraic sum of the heats abstracted in each heat-abstracting phase, so that the problem of finding the work done is resolved into:

- (a) The establishment of a thermal cycle, representative of the processes;
- (b) The heat evaluation for each phase of that cycle.

This work is, of course, done at the expense of heat added to the substance and the ratio of the amount transformed into work to the amount supplied, is the *thermal efficiency*, which is algebraically defined by Eq. (729).

$$\left. \begin{aligned} \left(\begin{array}{l} \text{Thermal efficiency of transforma-} \\ \text{tion of heat into work} \end{array} \right) &= \left(\frac{\text{Work done}}{\text{Heat added}} \right) \quad (a) \\ &= \frac{(\text{Heat added}) - (\text{Heat abstracted})}{(\text{Heat added})} \quad (b); \quad = 1 - \left(\frac{\text{Heat abstracted}}{\text{Heat added}} \right) \quad (c) \end{aligned} \right\} \quad (729)$$

To evaluate the various phase heats, positive and negative, graphic methods involving diagrams are of great assistance, but algebraic methods may also be used; for any particular problem that method is used which saves the most time. As algebraic methods are dependent directly on the relations discussed in Part II, it is only necessary to explain the graphic methods before proceeding to the detailed work of evaluating each phase by both methods.

It was found convenient in the study of work as a function of pressure-volume changes, to use diagrams the coordinates of which were the two factors, pressure and volume, whose product, represented by an area, indicated to scale the quantity of work—the diagram showing how the work varied with either volume change or pressure change. A similar diagram can be developed, equally, if not more, useful for studying quantities of heat as functions of temperature changes, by taking as one coordinate absolute temperature, and for the other the heat per degree of absolute temperature, so that the area will indicate heat as the product of absolute temperature and heat per degree of temperature. As there is ordinarily little interest in the total heat content of a substance, counting from the state of no heat, and as the question under examination is usually concerned only with the heat received or lost between two thermal conditions or between two temperatures, it is common to take for the coordinates of the heat diagram absolute temperature for vertical distances, and the heat gained or lost per degree of temperature starting anywhere, for horizontal distances. This horizontal distance—heat gained or lost per degree of absolute temperature—was named by Rankine the *thermodynamic function*, and by Clausius the *entropy*; this latter and simpler name is now always used. Therefore the diagram on which heat is represented by areas is known as the temperature-entropy, or $T\Phi$ diagram, just as the other diagram on which work was represented by areas is known as the pressure-volume or PV diagram, the coordinate names, vertical and horizontal, being used for the name of the diagram. As the pressure-volume diagram was also called the work diagram, so may the temperature-entropy diagram be similarly called the heat diagram.

Horizontal distances on the temperature-entropy, or heat diagram are, from the above specifications of the diagram, *derived* quantities and not directly measurable physical quantities like volumes or temperatures. When a substance in condition A , receives or gives up heat to reach condition B , then the horizontal distance between the two corresponding diagram points is given by Eq. (730).

$$(\text{Entropy change from } A \text{ to } B) = \frac{\text{B.T.U. } (A \text{ to } B)}{\left(\begin{array}{c} \text{Absolute temperature } F \text{ at which the} \\ \text{heat exchange took place} \end{array} \right)} \quad (730)$$

These horizontal distances or entropies can be laid off from an absolute zero of entropy or from any given value as a starting point, but as practical problems deal only with cyclic heat changes and not with total heat content, it is customary to plot only entropy differences in passing from one state to another, creating thus an artificial or *arbitrary zero* of entropy at the point where the heat content

of the body is *least*. Entropies measured from this point are really differences in entropy between any point and that point where the body has least heat content. To avoid confusion the abscissa of any point will be set down as a difference. Introducing symbols,

Let ϕ_a , ϕ_b , etc. = the absolute and indeterminate entropy of the substance in the condition represented by points A , B , etc.;

$(\phi_b - \phi_a)$ = change in entropy, in changing condition from A to B , equal to the horizontal distance between the diagram points;

T = absolute temperature F at which the heat is exchanged;

Q_{ab} = the quantity of heat added to, or abstracted from the body in passing from state A to state B .

Then

$$(\phi_b - \phi_a) = \frac{Q_{ab}}{T}. \quad (731)$$

It is evident that as substances may change temperature during a change of heat content, T will vary with Q , and as T is the temperature *at* which the heat

is received, it is unknown, so that solution of Eq. (731) is possible, first, by the use of an average temperature for the process, second, by what is preferable and equivalent, the substitution of a differential form, Eq. (732).

$$(\phi_b - \phi_a) = \int_a^b \frac{dQ}{T}. \quad (732)$$

This formula, Eq. (732), applies to all thermal processes in which the temperature changes with the heat content, such as heating solids, liquids, gases, and vapors, without change of state. When, however, the temperature does not change with heat reception or its loss, as for the boiling of liquids, condensing of vapor, freezing of liquids, melting of solids, and for isothermal expansion of gases or superheated vapors, then Eq. (731), the simpler form, applies directly.

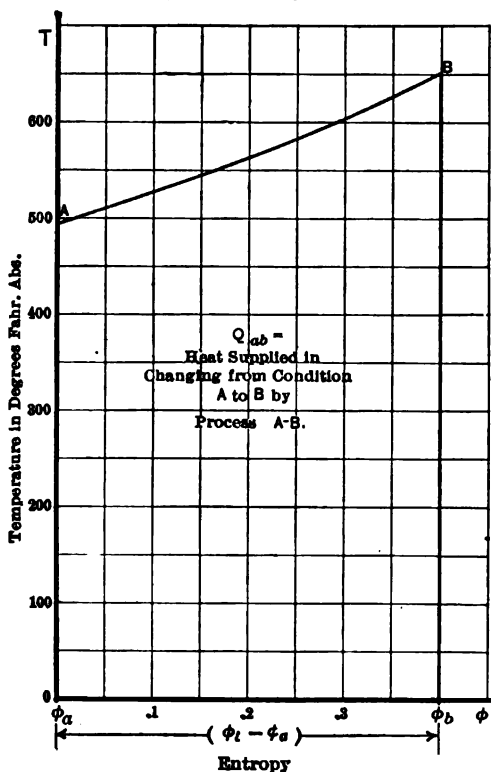


FIG. 138.—Heat Diagram to Temperature-entropy Coordinates Representing Quantity of Heat by the Area under the Thermal Process Curve.

Referring to the diagram, Fig. 138, the point A represents the original state, B the final state after receiving the quantity of heat Q_{ab} , represented to scale by the area under the change of state curve, AB , down to the

horizontal axis. It will be observed that heat added is laid off to the right and this is the usual way, accordingly heat abstracted is to be laid off to the left of the point at which it begins. This arrangement makes it possible to easily deal with cycles or complete thermal series of processes, as illustrated in Fig. 139, which represents the following four-phase cycle:

- (1) Heat addition with rising temperature from A to B ;
- (2) Heat addition at constant temperature from B to C ;
- (3) Heat abstraction with falling temperature from C to D ;
- (4) Heat abstraction at constant temperature from D to A .

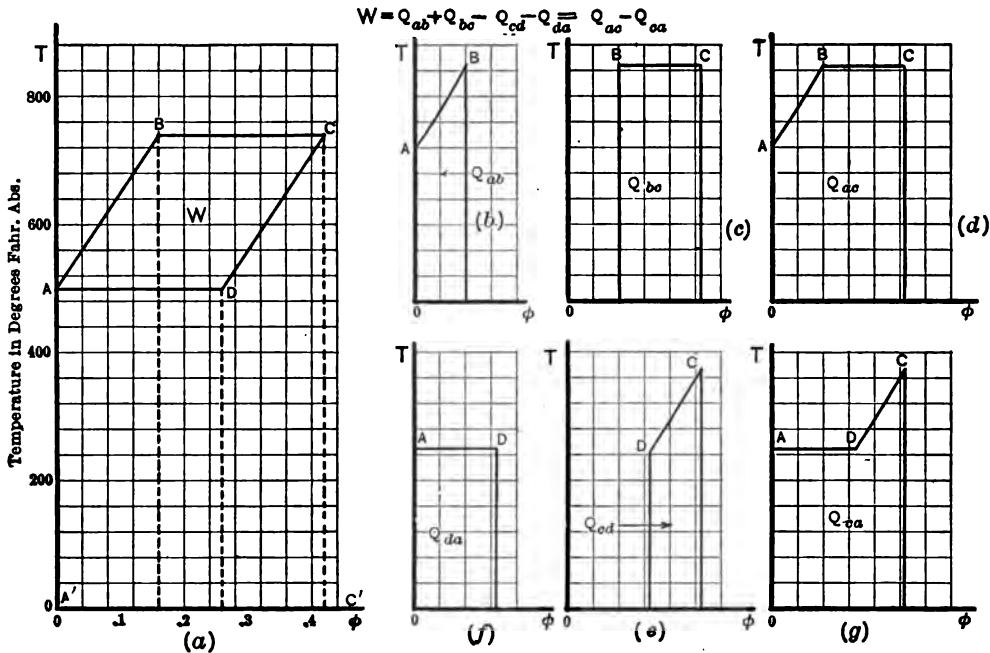


FIG. 139.—Heat Diagram of Complete Thermal Cycle to Temperature-entropy Coordinates, showing Net Work done as the Algebraic Sum of Heat Areas under the Thermal Process Curves.

Each of these heats is represented separately in diagrams (b), (c), (e), (f); the algebraic sum of all heats added, in diagram (d); of all heats abstracted in diagram (g); so that the heats added less the heats abstracted, is the difference between the areas under the curves ABC of (d), and CDA of (g), and this is the area enclosed by the cycle $ABCD$ on the main diagram (a).

It therefore appears that the evaluation of work done, or the thermal efficiency of transformation, becomes easy with the assistance of the temperature-entropy diagram; the work is the area enclosed by the cycle diagram $ABCD$ and the efficiency the ratio of enclosed area $ABCD$ to all that area under the top boundary line $ABCC'A'A$.

Practical application of this method of study of efficiency of heat transformation into work requires a preliminary establishment of the character-

istics of the separate phases, which in turn involves the physical constants discussed in Part II, where certain relations between them were assumed, that must be proved on rational grounds.

Example. Evaluate the heat added to the substance in the change of condition represented by the line *AB* in Fig. 138, stating the results in heat units (B.T.U.) and in foot-pounds.

The area of the figure below the line *AB* down to absolute zero temperature measured by the planimeter or by adding narrow vertical strips, was found to be 5.67 square inches as originally plotted. On the diagram, 1 inch represents 200° F. absolute in height, and .2 entropy units horizontally, so that 1 square inch represents, $200 \times .2 = 40$ B.T.U. The entire area represents, therefore,

$$40 \times 5.67 = 226.8 \text{ B.T.U.} = 226.8 \times 778 = 176,500 \text{ ft.-lbs.}$$

Prob. 1. In Fig. 139 evaluate by the heat added from *A* to *B*, and from *B* to *C*, the heat removed from *C* to *D*, and from *D* to *A*, and find the net work done by the cycle, first in B.T.U. and then reduced to foot-pounds. What is the thermal efficiency of the cycle?

Prob. 2. A certain quantity of gas passes through a cycle in which it receives 1000 B.T.U. of heat. The mechanical work of the cycle is 90,000 ft.-lbs. How much heat is removed during the cycle, and what is the thermal efficiency?

Prob. 3. A steam engine has a thermal efficiency of 9.7 per cent. Each pound of steam entering the engine has 1050 B.T.U. more heat than the condensate leaving the condenser. How much heat must the condenser remove from each pound of steam?

Prob. 4. If an engine were of 100 per cent thermal efficiency, how many B.T.U. must be supplied per hour per H.P.? How many, if the efficiency were 20 per cent? If the efficiency were 5 per cent?

Prob. 5. A hypothetical engine receives 1000 B.T.U. at a constant temperature of 900° F. and rejects heat at a constant temperature of 90° F. No heat is added nor removed while the temperature is changing. What is the thermal efficiency?

Prob. 6. A power plant using coal of 13,000 B.T.U. per lb., requires 2 lbs. of coal per hour per K.W. measured at the switchboard. What is the efficiency of the plant for converting heat of fuel into electrical energy?

Prob. 7. If a pound of water be evaporated at a constant temperature of 212° F., the heat to completely vaporize it will be 970.4 B.T.U. What is the increase in its entropy?

Prob. 8. To evaporate a pound of water at a temperature of 400° F. (247 lbs. per square inch absolute) requires 827.2 B.T.U. Find the increase of entropy due to evaporation.

Prob. 9. Anhydrous ammonia evaporating at a temperature of 0° F. absorbs 555 B.T.U. per pound. What is its increase in entropy due to evaporation?

Prob. 10. Plot to scale the $T\phi$ diagrams for each of these numerical problems.

Prob. 11. It has been proposed to apply the heat of condensation of steam after exhausting from a steam engine to the generation of ether vapor to work in an ether engine, the exhaust of which is to be condensed by water. Describe and sketch the steam and ether cycles showing what phases they have in common, what similar in kind, and what different, using $T\phi$ and PV coordinates.

95. General Energy Equation between Heat Change, Intrinsic Energy Change, and Work Done. Derived Relations between Physical Constants for Gases and for Changes of State, Solid to Liquid, and Liquid to Vapor. As previously stated, work done by a substance is always done at the expense of its heat, and while doing work the substance may be receiving heat or it may not. If no heat is being added at the time the work is being done, that work is derived directly from the heat content of the substance and as a consequence the heat content is lessened, or the energy reduced, by an amount exactly equivalent to the work done. Should heat be added while work is being done, the change in the energy content of the substance is the difference between the heat added and work simultaneously done. It has been the custom to call the whole energy content of a body its intrinsic energy, so that the above expression becomes for a case of heat addition,

$$(\text{Change in intrinsic energy}) = (\text{Heat added}) - (\text{Work done}). \quad (733)$$

For complete thermal cycles, the body returns to the original state, has the same intrinsic energy at the end as at the beginning, and the change in intrinsic is zero. During the cycle, heat may have been received at one time and lost at another, so that for the "heat added" term in Eq. (733) there must be substituted a more general one representing the algebraic sum of all heats added, giving abstracted heats a negative sign, and similarly for the work term, there must be substituted one representing the algebraic sum of the works, work of compression being negative. Hence for a complete cycle,

$$\left. \begin{aligned} (\text{Algebraic sum of heats added}) - (\text{Algebraic sum of works done}) &= 0, (a) \\ \left(\begin{array}{l} \text{Algebraic sum of} \\ \text{heats added} \end{array} \right) &= (\text{Heat added}) - (\text{Heat abstracted}) = \left(\begin{array}{l} \text{Net work} \\ \text{done.} \end{array} \right) (b) \end{aligned} \right\} \quad (734)$$

It was on this form of the expression that the work and efficiency methods described in the last section, were based; it is, however, a special case of a more general expression for any kind of thermal process whether a complete cycle or not and consisting of any number of phases whether overlapping or separate, which is derivable as follows:

Let \bar{U} = intrinsic energy of a body or its total energy of condition defined as the heat it possesses above a condition of absolute zero of heat, in B.T.U.

Q = the heat gained by a body in passing from any state to any other, which is the sum of all heat directly added less all abstracted as such, in B.T.U.

W = work done in passing from the first to the second state, which is the sum of all expansion less all compression work, or equal to the *net* work done, in ft.-lbs.;

J = Joule's equivalent of one B.T.U. in ft.-lbs. = 777.52 or approximately 778.

Then the more general expression for the relation between heat, work, and intrinsic energy takes the following form, Eq. (735), which is one statement

of the so-called first law of thermodynamics, namely, that heat and mechanical energy are mutually convertible:

$$d\bar{U} = dQ - \frac{1}{J} dW. \quad \dots \dots \dots (735)$$

This equation will be called for convenience the general energy equation for substances and is true for all kinds of thermal changes except those including as a part an irreversible process. In such cases the failure of the equilibrium of the state of the substance destroys the dependence of intrinsic energy on state alone, which is otherwise true and assumed in the above equation.

Applying this equation directly to *perfect gases* gives some useful relations between the physical constants. For such hypothetical substances, which all actual gases closely approximate at some time, $PV = RT$, for 1 lb., in foot-pound units in which R is the gas constant. See Eq. (522). The differential form of this is given by Eq. (736).

$$d(PV) = PdV + VdP = RdT. \quad \dots \dots \dots (736)$$

When gases receive a little heat dQ , it is conceivable that it may be divided into two parts, one part raising temperature without doing work, and therefore given by $C_v dT$ B.T.U. for 1 lb., and a part transformed directly into work and equal to PdV foot-pounds, hence $dQ = C_v dT + \frac{1}{J} PdV$. $\dots \dots \dots (737)$

$$\text{Combining Eqs. (735) and (737), } d\bar{U} = C_v dT + \frac{1}{J} PdV - \frac{1}{J} dW. \quad \dots \dots (738)$$

$$\text{But as } dW = PdV, \quad d\bar{U} = C_v dT. \quad \dots \dots \dots (739)$$

which shows that *for perfect gases the intrinsic energy depends on the absolute temperature only*. Furthermore, the intrinsic energy of a perfect gas is equal to the product of its specific heat at constant volume into its absolute temperature plus a constant of integration, if this specific heat is not a function of temperature, which it probably is. Combining Eqs. (736) and (737) by elimination of dV gives $dQ = C_v dT + \frac{R}{J} dT - \frac{1}{J} VdP = \left(C_v + \frac{R}{J}\right) dT - \frac{1}{J} VdP$. (740)

$$\text{But the specific heat at constant pressure is given by } C_p = \left(\frac{dQ}{dT}\right)_{P=C}. \quad \dots (741)$$

$$\text{Whence by Eq. (740), } C_p = C_v + \frac{R}{J}, \text{ and } R = J(C_p - C_v). \quad \dots \dots (742)$$

This Eq. (742) proves the assumption made previously that the gas constant R is equal to the difference between the two specific heats expressed in foot-pounds.

Substitution of this value for R in Eq. (740) gives another form to the heat expression, in terms of dT and dP , variables, $dQ = C_p dT - \frac{1}{J} VdP$. $\dots (743)$

$$\text{Giving to } dT \text{ in Eq. (735) its value in (736), } dQ = \frac{C_v}{R} d(PV) + \frac{1}{J} PdV. \quad (744)$$

This can be put into an integral form, Eq. (745), directly, and the factor simplified by using the relation, $\frac{C_p}{R} = \frac{C_p}{J(C_p - C_v)} = \frac{1}{J\left(\frac{C_p}{C_v} - 1\right)}$,

$$\left. \begin{aligned} \left(\begin{array}{l} \text{Heat added from state} \\ A \text{ to state } B \text{ in foot-} \\ \text{pounds} \end{array} \right) &= \left(\frac{1}{\frac{C_p}{C_v} - 1} \right) (P_b V_b - P_a V_a) + \int_a^b P dV \quad (a) \\ &= \left(\frac{1}{\frac{C_p}{C_v} - 1} \right) (P_b V_b - P_a V_a) + (\text{Work done}) \quad (b) \end{aligned} \right\} \quad (745)$$

When gases expand they follow the general law $PV^s = K$, and in so expanding the work done is given by Eq. (746).

$$W = \int_a^b P dV = K \int_a^b \frac{dV}{V^s} = \frac{P_a V_a^s}{1-s} (V_b^{1-s} - V_a^{1-s}). \quad (746)$$

Substitution of this in Eq. (745) gives

$$\begin{aligned} JQ_{ba} &= \left(\frac{1}{\frac{C_p}{C_v} - 1} \right) (P_b V_b - P_a V_a) + \left(\frac{P_a V_a^s}{1-s} \right) (V_b^{1-s} - V_a^{1-s}) \\ &= \left(\frac{P_a V_a^s}{\frac{C_p}{C_v} - 1} \right) (V_b^{1-s} - V_a^{1-s}) + \left(\frac{P_a V_a^s}{1-s} \right) (V_b^{1-s} - V_a^{1-s}) \\ &= \left[\frac{\left(\frac{C_p}{C_v} - s \right)}{\left(\frac{C_p}{C_v} - 1 \right)} \right] \left(\frac{P_a V_a^s}{1-s} \right) (V_b^{1-s} - V_a^{1-s}) \quad (747) \end{aligned}$$

$$\text{Hence} \quad Q = \frac{1}{J} \left[\frac{\frac{C_p}{C_v} - s}{\frac{C_p}{C_v} - 1} \right] \times (\text{Work of expansion}). \quad (748)$$

To proceed further with the investigation of relations between physical constants requires a still more general energy equation involving more of them. One of the basic definitions of heat makes a quantity of heat proportional to the amount of its effect on pressure, on volume and on temperature, so that taking these quantities in pairs as independent variables, two fixing the third by the general property relations of matter, any heat increment may be equated to effect, by the three sets of algebraic relations of Eq. (749).

$$dQ = a dT + b dV, \quad (a); \quad = c dT + d dP, \quad (b); \quad = e dP + f dV, \quad (c). \quad (749)$$

The change in intrinsic energy is given by the following set, Eq. (750), corresponding to the above, in which the factors (a), (b), (c), etc., are unit thermal effects or physical constants. Substituting Eq. (749) in Eq. (735).

$$\left. \begin{aligned} d\bar{U} &= a dT + b dV - \frac{1}{J} P dV = a dT + \left(b - \frac{P}{J} \right) dV \quad (a) \\ &= c dT + d dP - \frac{1}{J} P dV \quad (b); \quad = e dP + f dV - \frac{1}{J} P dV = e dP + \left(f - \frac{P}{J} \right) dV \quad (c) \end{aligned} \right\} \quad (750)$$

These are, of course, partial differential equations and each must be a complete differential, that is, it must be possible to solve or integrate it. Therefore the differential coefficients of each coefficient with respect to the other variable, when its own variable is constant, must be equal to each other. This is expressed symbolically in Eq. (751); there are two others similar, but not set down because they are not to be used.

$$\frac{d}{dV}(a)_{T-C} = \left(\frac{da}{dV}\right)_{T-C} = \frac{d}{dT}\left(b - \frac{P}{J}\right)_{V-C} \quad (751)$$

The change of heat per degree of absolute temperature has been defined as the entropy change, as in Eq. (752), for the differential increments,

$$d\phi = \frac{dQ}{T} \quad (752)$$

Combining Eq. (752) with the original general equation, Eq. (749), there results some new relations between constants, Eq. (753), that constitute a mathematical form of enunciation of the second law of thermodynamics, which has been thus defined: "If all of the heat be absorbed at one temperature and rejected at another lower temperature, the heat transformed to external work is to the total heat absorbed in the same ratio as that of the difference between the temperatures of absorption and rejection to the absolute temperature of absorption." As will be shown later this is in reality an expression for the efficiency of a perfect elementary engine. The following statement of the second law by Rankine is most specific:

If the actual total heat of a uniformly hot substance be conceived to be divided into any number of equal parts, the effects of those parts in causing work to be performed are equal.

$$d\phi = \frac{a}{T}dT + \frac{b}{T}dV \quad (a); \quad = \frac{c}{T}dT + \frac{d}{T}dP \quad (b); \quad = \frac{e}{T}dP + \frac{f}{T}dV \quad (c). \quad (753)$$

These are also complete differentials, so that the differential coefficient relations yield the following, Eq. (754), and two others similar, not given here.

$$\frac{d}{dV}\left(\frac{a}{T}\right)_{T-C} = \frac{d}{dT}\left(\frac{b}{T}\right)_{V-C} \quad (754)$$

Performing the operations indicated in Eqs. (751) and (754), they may be combined.

$$\text{From Eq. (751),} \quad \left(\frac{da}{dV}\right)_{T-C} = \left(\frac{db}{dT}\right)_{V-C} - \frac{1}{J}\left(\frac{dP}{dT}\right)_{V-C} \quad (755)$$

From Eq. (754),

$$\frac{1}{T}\left(\frac{da}{dV}\right)_{T-C} = \frac{1}{T}\left(\frac{db}{dT}\right)_{V-C} - \frac{b}{T^2}, \quad \text{or} \quad \left(\frac{da}{dV}\right)_{T-C} = \left(\frac{db}{dT}\right)_{V-C} - \frac{b}{T} \quad (756)$$

$$\text{Equating Eq. (755) to Eq. (756)} \quad \frac{b}{T} = \frac{1}{J}\left(\frac{dP}{dT}\right)_{V-C} \quad (757)$$

This is a perfectly general expression, as no defining conditions or limiting

hypotheses have yet been introduced except that of excluding non-reversible processes. Similar treatment will yield a long list of relations, omitted here because they may be found in works on the philosophy of thermodynamics and because they are not needed here. These particular ones are selected since they lead to values for specific heats which are defined by Eq. (758).

$$C_v = \left(\frac{dQ}{dT} \right)_{v=c} (a); \quad C_p = \left(\frac{dQ}{dT} \right)_{p=c} (b). \quad (758)$$

Referring to Eq. (749) it is evident that, $C_v = a$, and $C_p = c$.

One step has been taken in evaluating the constant (a), by establishing a value for the constant (b) in Eq. (757); substituting this in Eq. (758), the complete evaluation of the constant $a = C_v$, is given by Eq. (759),

$$\left(\frac{da}{dV} \right)_{T=c} = \frac{T}{J} \left(\frac{d^2P}{dT^2} \right)_{v=c}, \quad \text{whence} \quad C_v = a = \frac{T}{J} \int \frac{d^2P}{dT^2} dV + K'. \quad (759)$$

Therefore the specific heat at constant volume for *perfect* gases is a function of absolute temperature only, or it is constant, because the constant of integration K' must be constant or a function of temperature only, and it may be zero. By exactly similar reasoning, using other equations of the general group, it can be shown that a similar conclusion is applicable to the constant, $c = C_p$, by Eq. (760),

$$C_p = c = \frac{T}{J} \int \frac{d^2V}{dT^2} dP + K''. \quad (760)$$

These relations are derived here for two reasons: first, to show the general process of thermodynamic computation of relations between physical constants; and second, because this particular relation admirably illustrates how utterly misleading some of the most laboriously reached of these conclusions may be. It was because of such a demonstration as this that for so long nobody even sought a variation in the specific heat of superheated steam with pressure, which was ultimately found after engineering and not physical calculations showed that the old accepted values must be wrong. This does not indicate that conclusions based on such mathematical analyses will always be misleading because they are absolutely true for perfect gases, but they are to be adopted for real gases only with necessary restrictions based on how much departure there is by the real substance from the perfect gas.

When a substance changes state, a similar analysis starting with the general energy equation yields some useful relations between the new constants involved for these processes. From the general statement, Eq. (737) or Eq. (749a), that heat added is equal to something, (a) multiplied by the temperature change, added to the product of something else (b) and the volume change, it was found, Eq. (757), by various transformations involving the two laws of thermodynamics, that the coefficient (b) was proportional to the product of absolute temperature into the rate of change of pressure with temperature which when inserted in Eq. (749a) gives

$$dQ = adT + \frac{T}{J} \left(\frac{dP}{dT} \right) dV. \quad (761)$$

This equation may be applied to a change of state from solid to liquid and liquid to vapor at constant temperature, for which case it takes the form of Eq. (762), since $dT=0$.

$$dQ = \frac{T}{J} \left(\frac{dP}{dT} \right) dV. \quad (762)$$

If the process be one of evaporation of 1 lb. and this be complete, then the heat added is the latent heat L , the final volume that of dry saturated vapor V_v , and the initial volume that of liquid at the temperature of boiling V_L , whence integration gives an expression for the physical constant, latent heat, Eq. (763),

$$L = \frac{T}{J} \left(\frac{dP}{dT} \right) (V_v - V_L). \quad (763)$$

This is the Clapeyron equation previously used, Eq. (548), and now established, by methods used by Clausius and Clapeyron, who derived it independently.

For fusion it is believed that latent heats are constant for all pressures, in which case the change in the freezing- or melting-point with pressure is given in degrees per unit of pressure by Eq. (764), where V_L = specific volume of liquid and V_s the volume of solid.

$$\frac{dT}{dP} = \frac{T(V_L - V_s)}{JL}. \quad (764)$$

It is interesting to note that for ice and water, ice being lighter than water, the difference above is negative, so that increase of pressure lowers rather than raises the melting- or freezing-point. Rewriting the equation in the following form, for vapor-liquid changes, another interesting conclusion becomes possible:

$$\frac{JL}{V_v - V_L} = T \frac{dP}{dT}. \quad (765)$$

This indicates that latent heat in foot-pounds divided by the volume change in vaporization—which is in the nature of a pressure and may be regarded as the pressure resisting or opposing a change of state—is equal to the product of absolute temperature and the rate of change of vapor pressure with temperature. It has been remarked that this quantity, given by Eq. (765), is nearly constant for several different vapors at the same pressure, so that each term is constant, that is

$$\frac{JL}{V_v - V_L} = K, \text{ and } T \frac{dP}{dT} = K, \text{ Approx. when } P \text{ is const.} \quad (766)$$

It is noted, however, that the constant K rises with pressure so that at any pressure

$$T \frac{dP}{dT} = f(P), \text{ whence on integration, } \log_e T = F(P) + K'. \quad (767)$$

This Eq. (767) is the origin of form of some of the numerous equations for the relation between saturated vapor temperatures and pressures.

When two different substances A and B follow similar laws, according to Eq. (766), then the boiling-points at equal pressures follow the law of constant ratio of Eq. (768),

$$\left(\frac{\text{Boiling-point of one substance}}{\text{Boiling-point of another substance}} \right) = (\text{Constant, at same pressure}). \quad (768)$$

It also follows that for substances having the same " K " or constant $\left(\frac{L}{V_v - V_L}\right)$ values, that latent heats are inversely proportional to the density of vapors when V_L is small compared to V_v and, therefore, that the generation of a cubic foot of any vapor in the group requires the same amount of heat.

These applications of general laws to vapors are no more nearly correct than the hypotheses on which they are based. As Mellor remarks in his *Chemical Statics and Dynamics*, "*No process of reasoning can establish a law of Nature. The elements of sameness—the law—must be actually discovered in the facts.*"

Example. Air expands under a piston from a volume of 1 cu.ft. and a pressure of 300 lbs. per square inch absolute to a volume of 5 cu.ft. at a pressure of 40 lbs. per square inch absolute. Assuming that expansion follows the law of constant PV^s , find the heat absorbed in B.T.U. Ratio of specific heats for air = 1.4.

The work done from Eq. (746) is $W = \frac{P_a V_a^s}{1-s} (V_b^{1-s} - V_a^{1-s})$, and $s = \frac{\log \left(\frac{P_a}{P_b}\right)}{\log \left(\frac{V_b}{V_a}\right)}$.

Substituting numerical values, $s = \frac{\log \frac{300}{40}}{\log 5} = \frac{.875}{.699} = 1.25$.

Therefore, $W = \frac{300 \times 1}{1-1.25} \times 144 \left(\frac{1}{5^{1.25-1}} - 1 \right) = \frac{43,200}{-.25} \times (.667 - 1) = 57,600 \text{ ft.-lbs.}$

From Eq. (747)

$$(\text{Heat})_{ba} = \frac{1}{J} \left\{ \frac{C_p - 1.25}{C_p} \right\} W = \frac{1}{778} \times \frac{1.4 - 1.25}{1.4 - 1} \times 57,600 = 27.7 \text{ B.T.U.}$$

Prob. 1. A pound of air at 60° F. and atmospheric pressure receives 500 heat-units at constant pressure. How much of this heat will be expended in raising the temperature of the air and how much in doing work?

Prob. 2. Show that when air expands isothermally the work done is equal to the heat added.

Prob. 3. A pound of air at 70° F. and a pressure of 12 lbs. per square inch absolute is compressed adiabatically until its volume is one-fourth of the original volume. One thousand heat-units are added at constant volume and the air then expands to the original pressure, whereupon it is cooled to the original temperature at constant pressure. Show numerically that the algebraic sum of heats added is equal to the work done.

Prob. 4. A pound of gas having the specific heat at constant volume of .2, changes in volume from 100 cu.ft. to 150 cu.ft., while the pressure remains constant at 100 lbs. per square inch absolute. How much heat was added to the gas?

Prob. 5. Prove that for a perfect gas $R = J(C_p - C_v)$ and show how much departure there is for air, methane, oxygen, nitrogen, carbon dioxide and steam.

Prob. 6. One thousand heat-units are supplied to a pound of air at 60° F. and 15 lbs. per square inch absolute pressure. What will be the temperature at the end of the heat reception, and the work done, if (a) heat is received at constant volume, (b) if it is received at constant pressure, and (c) if it is received during expansion according to $PV^{1.1} = \text{Const.}$

Prob. 7. Show that for adiabatic compression the work done is $C_v(T_2 - T_1)J$ foot-pounds.

Prob. 8. It has been stated that $\frac{JL}{V_v - V_L}$ is approximately constant. Compare the results obtained for steam, ammonia and carbon dioxide at pressures of 10, 25, 50, 75 and 100 lbs. per square inch absolute. See tables for values of L , V_v and V_L .

Prob. 9. If air expands in a cylinder according to constant $PV^{1.3}$, find an expression for the heat gained or lost.

Prob. 10. Determine whether heat is being gained or lost by ammonia vapor during compression when it is found that s has the value 1.2, and derive a quantitative expression for it.

Prob. 11. Compare the change in intrinsic energy during vapor formation for one pound of steam, ammonia and carbon dioxide at 32°F .

Prob. 12. Derive from the tables a formula for vapor pressure in terms of temperature, of the form of Eq. (767) for steam, ammonia and carbon dioxide.

Prob. 13. From the equation derived in Problem 12, find $\frac{d^2P}{dT^2}$ and substituting in Eq. (759), find C_v for steam, compare it with the values given in Marks and Davis, and explain the differences found.

96. Quantitative Relations for Primary Thermal Phases, Algebraic, and Graphic, to PV and T Φ Coordinates. Constancy of P, V and T for Gases and Vapors, Wet, Dry and Superheated. Heating of solids, liquids, gases and vapors without change of state takes place according to the fundamental relation between temperature change and quantity of heat, Eq. (769).

$$(\text{Heat quantity}) = (\text{Specific heat}) \times (\text{Temperature change}) \times (\text{Weight}). \quad (769)$$

If the specific heat is constant this is a very simple expression, but it has been shown that specific heat is variable and given generally by an equation of the form, Eq. (770), in which a , b and c are constants, different for each substance.

$$\text{Specific heat} = C = a + bT + cT^2. \quad (770)$$

In this equation, if b and c are very small, it reduces to the form

$$\text{Specific heat} = C = a \text{ (substantially)}, \quad (771)$$

From these equations the entropy change can be determined for all substances suffering heat change without change of state, from Eq. (752), as follows:

$$\phi_2 - \phi_1 = \int_1^2 \frac{dQ}{T} = w \int_1^2 \frac{(\text{Specific heat})}{T} dT, \quad (772)$$

$$\text{but Specific heat} = \frac{dQ}{dT}; \text{ or } dQ = (\text{Specific heat})dT; = (a + bT + cT^2)dT.$$

$$\begin{aligned} \text{Whence } \phi_2 - \phi_1 &= w \left\{ a \int_1^2 \frac{dT}{T} + b \int_1^2 dT + c \int_1^2 T dT \right\} \\ &= w \left\{ a \log_e \frac{T_2}{T_1} + b(T_2 - T_1) + \frac{c}{2}(T_2^2 - T_1^2) \right\}. \quad (773) \end{aligned}$$

This Eq. (771) is the entropy change for all heat gains or losses by solids,

liquids, gases and vapors not changing state when the specific heat is a function of absolute temperature of the form given in Eq. (770), which reduces to the following special forms, Eq. (773, a, b, c, d), for the condition stated.

$$\left. \begin{aligned}
 & \text{(For constant specific heat } C) & (\phi_2 - \phi_1) &= wC \log_e \frac{T_2}{T_1} & (a) \\
 & \text{(For water if the specific heat be taken} & (\phi_2 - \phi_1) &= w \log_e \frac{T_2}{T_1} & (b) \\
 & \quad \text{as unity)} \\
 & \text{(For gases and superheated vapors at} & (\phi_2 - \phi_1) &= wC_v \log_e \frac{T_2}{T_1} & (c) \\
 & \quad \text{constant volume with constant} & & & \\
 & \quad \text{specific heat)} \\
 & \text{(For gases and superheated vapors at} & (\phi_2 - \phi_1) &= wC_p \log_e \frac{T_2}{T_1} & (d) \\
 & \quad \text{constant pressure with constant} & & & \\
 & \quad \text{specific heat)}
 \end{aligned} \right\} \quad (774)$$

To illustrate the relation of these changes to each other there are plotted to PV and $T\phi$ coordinates in Fig. 140 the following cases each for 1 lb. of substance:

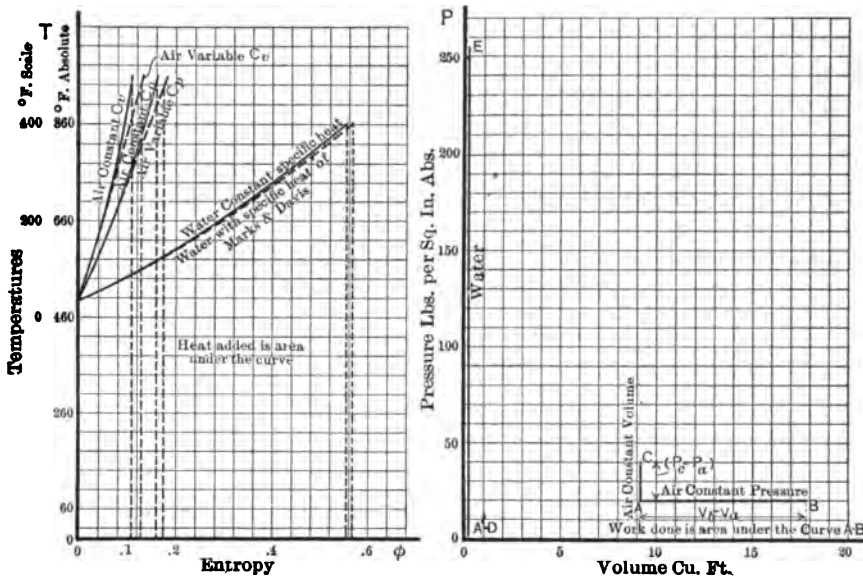


FIG. 140.—Changes in P , V , T , ϕ , Heat, and Work, for *Constant Pressure* and *Constant Volume* Phases of Liquids, Gases and Vapors, without Change of State.

- (1) Heating of water from 32° to 400° F. with specific heat = 1;
- (2) Heating of water from 32° to 400° F. with specific heat as given in the Marks and Davis Steam Tables using the columns called heat and entropy of liquid (above 32° F.);
- (3) Heating of air from 32° F. and 20 lbs. sq.in. to 500° F. at constant volume with constant specific heat = (.17);
- (4) Heating of air from 32° F. and 20 lbs. sq.in. to 500° F. at constant volume with variable specific heat, mean $C_v = .174 + .0000675(t - 32)$;

- (5) Heating of air at constant 20 lbs. per square inch pressure from 32° to 500° F. with constant specific heat, $C_p = .239$;
 (6) Heating of air at constant 20 lbs. per square inch pressure from 32° to 500° F. with variable specific heat, $C_p = .239 + .0000675(t - 32)$.

For constant-pressure heating of a gas illustrated here by air, there will be an increase of volume depending on the temperature rise and work done, which latter is measured by the volume increase, but as there are two possible final temperatures for the same amount of heat, one for the constant and one for the larger variable specific heat, there will be two possible volume increases and quantities of work done. In the example the temperature has been fixed so that the heat quantities differ by the differences between the areas on the $T\Phi$ diagram under the solid curve for constant, and under the dotted curve for variable specific heat. These differences are so small, and in the study of heat transformation into work, so inconsequential that except when otherwise specifically stated, the constant value of specific heat will be used. This practice will save great labor and also avoid the introduction of doubts, which have real foundation, as to the accuracy of the determination of the variability laws so far announced. One special case of difference in practice is that for water and superheated steam, the entropy changes for which are calculated and form part of the steam tables under the heading "entropy of the liquid (above 32° F.)" and "entropy of the steam," but in approximate calculations no serious error will be introduced by assuming constancy of specific heat for water and unity as its value, but the variability for the steam cannot be so safely ignored.

Accordingly, with the above reservations the following relations of tabular Eq. (775) are found between the quantities for the gas heated at constant pressure from A to B , Fig. 140.

Related Quantities for Constant Pressure Heating	Formula	
Heat and Temp.	$Q_{ab} = C_p(T_b - T_a)$	(a)
Temp. and Volume	$V_b - V_a = V_a \left(\frac{T_b}{T_a} - 1 \right)$	(b)
Heat and Volume	$V_b - V_a = V_a \frac{Q_{ab}}{C_p T_a} = \frac{R}{C_p} \left(\frac{Q_{ab}}{P_a} \right)$	(c)
Work and Temp.	$W = P_a(V_b - V_a) = P_a V_a \left(\frac{T_b}{T_a} - 1 \right) = R(T_b - T_a)$	(d)
Work and Heat	$W = \frac{P_a V_a}{T_a} \frac{Q_{ab}}{C_p} = \frac{R}{C_p} Q_{ab}$	(e)
Entropy and Temp.	$\phi_b - \phi_a = C_p \log_e \frac{T_b}{T_a}$	(f)

(775)

A similar set may be written down, Eq. (776), almost by inspection for the case of constant volume heating from A to C , Fig. 140, which of course causes a pressure rise, during which no work can be done, or $W = 0$.

Related Quantities for Constant Volume Heating	Formula	
Heat and Temperature	$Q_{ac} = C_v(T_c - T_a)$	(a)
Temperature and Pressure	$P_c - P_a = P_a \left(\frac{T_c}{T_a} - 1 \right)$	(b)
Heat and Pressure	$P_c - P_a = P_a \left(\frac{Q_{ac}}{C_v T_a} \right) = \frac{R}{C_v} \left(\frac{Q_{ac}}{V_a} \right)$	(c)
Entropy and Temperature	$\phi_c - \phi_a = C_v \log_e \frac{T_c}{T_a}$	(d)

$$\left. \begin{array}{l} (a) \\ (b) \\ (c) \\ (d) \end{array} \right\} \dots (776)$$

The case presented by the water is peculiar inasmuch as it makes so little difference whether the water be heated at constant pressure or constant volume or at what pressure, its expansion being so small. For the purpose of illustration it is assumed in the diagram that the water is at the same 20 lbs. pressure and that it is kept (a) at constant pressure, expanding from A' to D , and (b) at constant volume, its pressure increasing from A' to E . In ordinary practical work no account is taken of the difference at all; first, because water in use is rarely heated at constant volume, and second, because the work of expansion is vanishingly small.

Constancy of temperature characterizes all changes of state, solid to liquid and liquid to vapor, if the pressure be constant; it is also characteristic of the expansion of gases when the product of pressure and volume is constant, for in this case $PV = RT = \text{Constant}$. For melting or boiling, then, the heat added is proportional to the weight of substance changed, or the *quality*.

Let L = latent heat of fusion or vaporization per pound;

w = weight of substance present;

x = fraction of total weight liquefied from the solid, or vaporized from the liquid, and to be called the *quality*;

$1 - x$ = fraction of total weight remaining in the solid state in contact with liquid, or in the liquid state in contact with vapor.

Hence for the changes of state at constant temperature and pressure the quantity of heat is the product of weight, quality and latent heat, $Q = wxL$. . (777)

The latent heat L is to be taken from tables of physical properties of substances but will not be the same at different pressures. The representation of the case of vaporization on the PV and $T\Phi$ diagrams, especially the latter, is usually associated with the heating of liquid which precedes vaporization, and with the line of vapor superheat which succeeds vaporization if the heating is long continued. Accordingly the successive processes of heating liquid, evaporating it, and superheating its vapor, are represented by three lines joining each other, for a single pressure, and by a family of lines if vaporization at all pressures are to be included.

This is illustrated in Fig. 141 where, starting with liquid at 32° F. (A) heating at constant pressure to the boiling-point, B , will cause a slight expansion, shown

A to B on the PV diagram, doing the work shown by the area $ABB'A'A$ and requiring the heat shown by $ABB'A'A$ on the $T\Phi$ diagram, as already explained, but repeated here to assure continuity. The formation of vapor at the temperature represented by BB' on the $T\Phi$, and at the corresponding pressure represented by BB' on the PV diagram, is represented by the line BC horizontally on both, until all liquid has been just vaporized, which condition is denoted by the point D . Therefore, the area under the line of complete vaporization BD which is $BDD'B'B$ on the $T\Phi$ diagram, is a graphic representation of the latent heat of vaporization to scale. Similarly, on the PV diagram the corresponding area $BDD'B'B$ is the work of vaporization, or $P(V_v - V_L)$, since \overline{AB} is the specific volume of the liquid V_L and \overline{AD} the specific volume of the dry saturated vapor. Should the vaporization be not complete but stop at the point C , there will be present a mixture of liquid and vapor, to be called for

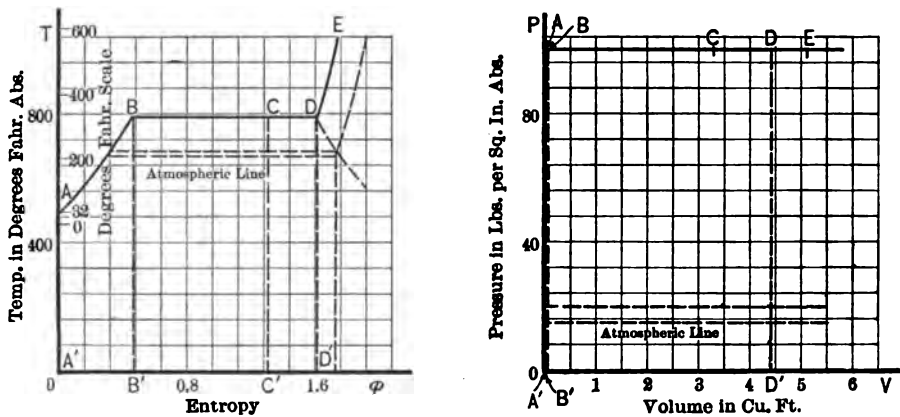


FIG. 141.—Changes in Pressure, Volume, Temperature, Entropy, Quality, Heat Added, and Work done for *Constant Temperature* Heating of Liquid-vapor with Change of State.

want of a better name *wet vapor*. For such wet vapor as represented by the point C the quality is given by $\frac{\overline{BC}}{\overline{BD}} = x$ on the $T\Phi$ diagram, and the heat used in making it at the boiling-point is represented by the area $BCC'B'B$ and is equal to xL . On the PV diagram the point C for wet vapor is located in a similar way, \overline{BD} corresponding to the quality $x=1$, so that $\frac{\overline{BC}}{\overline{BD}} = x$, and the wet vapor will occupy a volume \overline{AC} per pound of mixed vapor and liquid. Tabular values for the increase of entropy for vaporization alone or for the entropy of dry saturated steam over liquid at 32° , which includes the liquid part, are very useful in solving problems with wet vapors by the above relation.

Continued heating at the same pressure after the completion of vaporization, of course superheats the vapor along a line DE . If the whole process had taken place at some lower pressure it could be represented as in the dotted lines leading to the family of curves mentioned. It is not possible to express

algebraically the relations between heat of vaporization or work and temperature, except empirically and then only by an expression too complicated for use. Problems must therefore be solved by the use of tabular values given in the Handbook, and other Steam Tables.

Gases receiving heat at constant temperature behave quite differently, and the relation between the quantities involved can be found directly from the general equation, Eq. (748) for heat and work of gases. If in the equation, s be given the value unity, which it must have for this case, the relation between heat and work for isothermal gas processes follows as given by

$$Q_{ab} = \frac{1}{J} (\text{Work of expansion}) (a); \quad = \frac{1}{J} P_a V_a \log_e \frac{V_b}{V_a} (b). \quad (778)$$

Therefore *when gases expand isothermally the heat added is equal to the work done in the same units; or the work done in foot-pounds is equal to the heat added in foot-pounds.* This is equivalent to saying that the internal energy remains constant and heat is converted directly and completely into work as fast as added

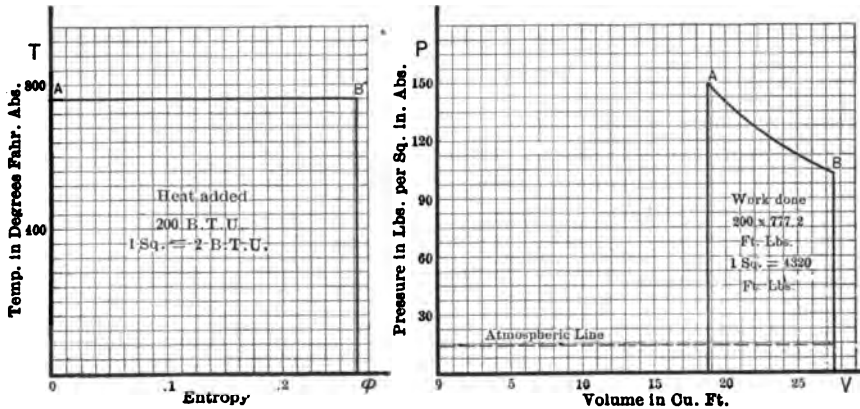


FIG. 142.—Changes in Pressure, Volume, Temperature, Entropy, Heat added, and Work done, for *Constant Temperature* Heating of Gases and Vapors without Change of State.

during isothermal expansion, or inversely for isothermal compression. To illustrate this case, the diagrams, Fig. 142, are plotted for the case of 1 lb. of air at 150 lbs. per square inch absolute and temperature 300° F. expanding isothermally under the reception of 200 B.T.U.'s from A to B. The area under the curve or isothermal AB on the TΦ diagram is equivalent to 200 B.T.U. and the area under the hyperbola AB on the PV diagram represents the mechanical equivalent of 200 B.T.U. or 155,504 ft.-lbs. For such cases as this, the set of relations given in Eq. (779), apply.

These same equations and general relations apply to superheated vapors provided they do not approach the region of wetness which they will do on isothermal compression if it is carried far enough. Condensation of vapors by isothermal compression is shown by the curve, Fig. 143, for the case of steam, on which the point A represents steam in the condition of high superheat at low pressure. Isothermal compression causes the change of state represented by the hyperbola

AB on the *PV*, and by the horizontal *AB* on the *TΦ* diagram. When the point *B* is reached the vapor has acquired the pressure and volume occupied by dry saturated vapor so that further abstraction of heat at constant temperature results in condensation from *B* to *C* without any further pressure change, and this would continue until all vapor had so disappeared, after which isothermal

Related Quantities for Constant Temperature Heating of Gases	Formula	
Heat and Volumes	$Q_{ab} = \frac{1}{J} P_a V_a \log_e \frac{V_b}{V_a} = \frac{1R}{J} T_a \log_e \frac{V_b}{V_a}$	(a)
Heat and Pressures	$Q_{ab} = \frac{1}{J} P_a V_a \log_e \frac{P_a}{P_b} = \frac{1R}{J} T_a \log_e \frac{P_a}{P_b}$	(b)
Volumes and Pressures	$P_a V_a = P_b V_b$	(c)
Work and Heat	$Q_{ab} = \frac{1}{J} W_{ab}$	(d)
Entropy and Temperature	$\phi_b - \phi_a = \frac{Q_{ab}}{T_a} = \frac{W_{ab}}{J T_a}$	(e)

(779)

abstraction of heat would be impossible, because a liquid cannot give up heat without losing temperature.

Constant-volume cooling of a vapor will cause a change of state just as iso-

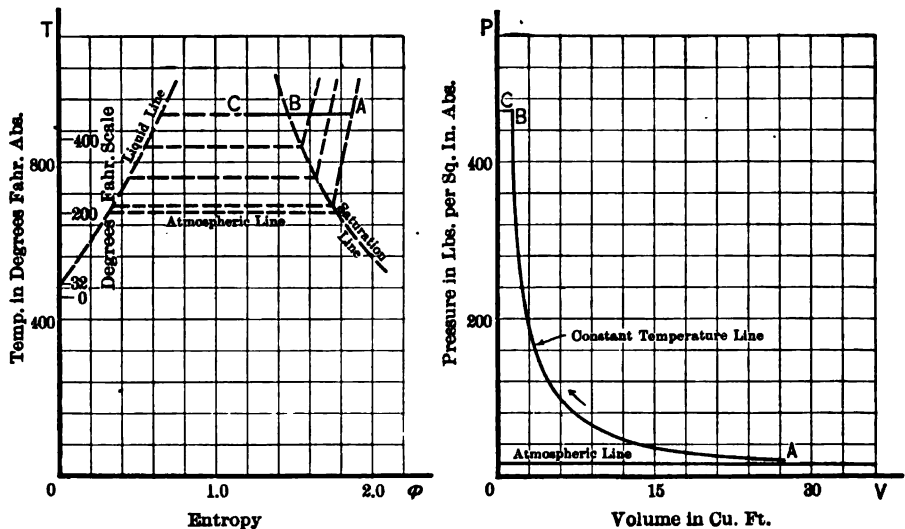


FIG. 143.—Constant Temperature or Isothermal Compression of Superheated Steam Reducing Superheat, and Subsequently Causing Partial or Complete Condensation.

thermal compression does, as will also a constant-volume heating of a mixture of vapor and liquid, but constant-volume heating or cooling of a vapor within those limits which keep it always superheated follows the laws for gases.

The departure from the gas laws for constant-volume changes of vapors is clearly shown by the $T\Phi$ diagram in its relation to the PV , as in Fig. 144. On the diagram AB represents liquid heating to both $T\Phi$ and PV coordinates, BC complete vaporization, so that the length of BC on the $T\Phi$ diagram is proportional to latent heat and on the PV diagram to the specific volume of dry saturated steam for the pressure. Similarly, \overline{GD} represents the latent heat of condensation at a lower pressure to $T\Phi$ coordinates and the specific volume of low-pressure dry saturated steam to PV coordinates. Changes of temperature, pressure and

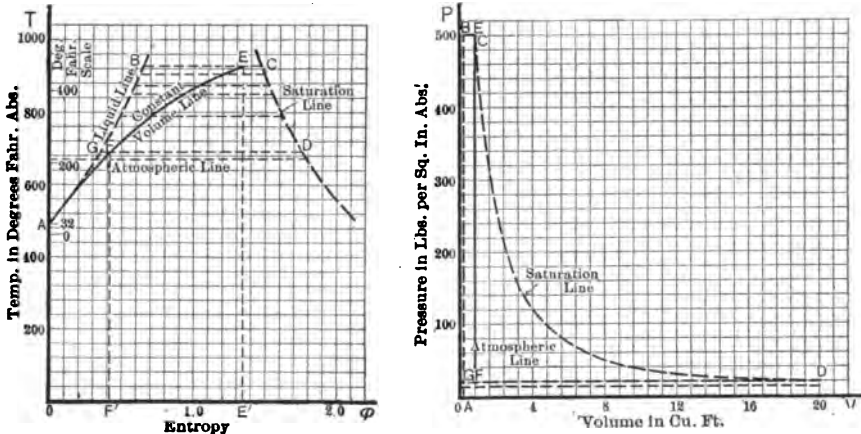


FIG. 144.—Constant Volume Changes for Steam Showing Corresponding Changes of Quality.

volume for steam continuously dry and saturated is represented by the saturation line CD .

A condition of wetness of the higher pressure steam is indicated by the position of a point E on both diagrams, and the following properties are defined by the relations of line lengths:

$$\text{Quality of steam at } E = \left(\frac{\overline{BE}}{\overline{BC}} \right)_{T\Phi} = x;$$

$$\text{Wetness of steam at } E = \left(\frac{\overline{EC}}{\overline{BC}} \right)_{T\Phi} = 1 - x;$$

$$\text{Volume of steam-water mixture} = (\overline{BE})_{PV} = \left(\frac{\overline{BE}}{\overline{BC}} \right)_{T\Phi} \times (\overline{BC})_{PV}.$$

$$\frac{\text{Heat of wet steam formation}}{\text{Heat of dry steam formation}} = \frac{xL}{L} = \left(\frac{\overline{BE}}{\overline{BC}} \right)_{T\Phi} = \left(\frac{\text{Vol. of wet steam}}{\text{Vol. of dry steam}} \right) = \left(\frac{\overline{BE}}{\overline{BC}} \right)_{PV}.$$

These relations permit the location of the line representing heat abstraction at constant volume and its heat evaluation by the $T\Phi$ area under it. This phase is represented by a vertical, EF , to PV coordinates and the position of point F on the PV diagram indicates not only the volume the steam will have after heat abstraction to the low pressure, but the relation it bears to the volume of dry saturated steam at the same pressure. If $(V_v - V_L)_1$ is the high-

pressure volume increase for dry saturated steam over its liquid, and $(V_v - V_L)_2$ the same at the low pressure, then the original volume of the wet steam is $(V_v - V_L)_1 x_1 + (1 - x_1) V_{L1}$ and the final volume of the wet steam is $(V_v - V_L)_2 x_2 + (1 - x_2) V_{L2}$; since these are to be equal,

$$(V_v - V_L)_1 x_1 + (1 - x_1) V_{L1} = (V_v - V_L)_2 x_2 + (1 - x_2) V_{L2}.$$

The volume of the liquid is, however, so small as compared to the volume of the vapor that it is negligible and so will hereafter be neglected.

Calling the specific volume of dry saturated steam V_v to be taken from the tables, there results $V_v x_1 = V_v x_2$, or the product of quality and specific volume of dry vapors is constant. Points may be located along $(EF)_{T\Phi}$ by applying the equality between the ratio of partial to total latent heats on $T\Phi$, and ratio of saturation to wet volume on PV coordinates. The heat abstracted is, of course, the area under FE or $(FEE'F'F)$ and is best evaluated graphically by cross-section paper plotting when needed, which is not often.

A series of such constant-volume lines is drawn over a $T\Phi$ diagram, Chart No. 40 in Tables, which indicates the change in the character of the line at different degrees of wetness and its reversal of curvature from concave down to concave up, as it passes into the region of superheat, where it is the same as that for a gas.

Example. *Calculation and use of the diagram, Chart No. 40, giving constant-volume lines for steam.* To illustrate the method the location of the line of constant volume of 2 cu.ft. will be traced. Let the first temperature be taken at 800° F. absolute for the first point A, corresponding to 340° F. From the steam tables, dry saturated steam at 340° F. has a specific volume of 3.787 cu.ft., so that the quality when the volume is 2 cu.ft. is $\frac{2}{3.787} = 52.8$ per cent. Therefore the entropy increase in making this steam from 32° F. and at 340° F. = entropy of the steam + entropy of water content - entropy at 32° = $\phi_a - \phi_{s1} = (.528 \times 1.0984 + .4903) - 0 = 1.0703$. Another point B is located by assuming a temperature $t_b = 440^\circ$ F. or $T_b = 900$, for which $\phi_b - \phi_{s1} = 1.5602$ by the same method.

To illustrate the use of the diagram in solving problems, suppose 1 lb. of wet atmospheric pressure steam, occupying 10 cu.ft. be enclosed in tank and heated to raise the pressure to 30 lbs. per square inch absolute, to find the final temperature, entropy and dryness. From 14.7 lbs. per square inch on the pressure scale project to point P on the constant volume line of 10 cu.ft. and follow this line to the point C for 30 lbs. per square inch absolute pressure. Projecting from C to D the absolute temperature is found to be 710° or $t = 250^\circ$ F., and projecting from C to E the entropy $\phi_c - \phi_{s1} = 1.332$.

The final quality = $\frac{CM}{OM} = 72.4$ per cent.

Again, if heat be added to raise the temperature to 842° absolute the entropy is found by following the 10 cu.ft. line to the point K opposite the temperature, and projecting down from K to Q the entropy is found $\phi_k - \phi_{s1} = 1.724$. The quality may be read off directly from Chart No. 44 in the Handbook of Tables, which carries lines of constant quality that might be superimposed on this constant-volume chart.

Prob. 1. Ten pounds of water are heated from 50° F. to 300° F. without change of state. What is the change of entropy?

Prob. 2. What will be the entropy change for 100 lbs. of the following gases per 1000° temperature rise from 0° F. at constant pressure and at constant volume, assuming a constancy of specific heats as given in the Tables? Hydrogen, air, nitrogen, and oxygen.

Prob. 3. Mallard & Le Chatelier give for the specific heat of O_2 , $C_p = (.303 + .000037T)$. Compare the entropy change for 100 lbs. of this gas from 0° to 1000°, having this value of specific heat, with the result of Prob. 2.

Prob. 4. A pound of air at 800° F. and a pressure of 150 lbs. per square inch gage expands at constant pressure until the temperature becomes 60° F. Find the volume at the end of expansion, the work done, and the change in entropy. A pound of air at 32° F. and 14.7 lbs. per square inch gage weighs .0807 lb. and the specific heat at constant pressure is .243, assumed constant.

Prob. 5. If the same amount of heat be removed at constant volume from a pound of air at 150 lbs. per square inch gage and 800° F., as removed in Problem 4 at constant pressure, what will be the pressure, entropy and temperature change? The specific heat at constant volume is .17.

Prob. 6. Water at 50° F. is changed to steam at 100 lbs. per square inch gage with 5 per cent moisture. What is the entropy change? What would it be if ammonia had been the liquid instead of water? What would be the value if the vapor were superheated 100° instead of being wet?

Prob. 7. Draw a $T\phi$ and PV diagram for the following case. Ice is heated from 0° F. to 32° F. and then melted, the water being heated to 212° and evaporated. The steam formed is then superheated 200° F. The specific heat of ice is .5 and the latent heat of fusion 142.

Prob. 8. One pound of air receives 1000 B.T.U. while expanding isothermally from 100 lbs. per square inch gage and 60° F. Draw the PV and the $T\phi$ diagrams for this case.

Prob. 9. A pound of steam at 50 lbs. gage and 200° superheat is compressed isothermally until it becomes 10 per cent wet. Show by a $T\phi$ diagram the loss in entropy and work done.

Prob. 10. A closed tank contains nothing but superheated steam at 200 lbs. per square inch absolute at a temperature of 550° F. If the volume of the tank be 10 cu.ft. and it be cooled to 32° F., plot the $T\phi$ and PV diagrams to scale and find the volumes of liquid and vapor at every 10 lbs. pressure and the corresponding heat abstracted for each 10 lbs. per square inch pressure reduction.

97. Quantitative Relations for Secondary Thermal Phases. Adiabatics for Gases and Vapors. Constant Quality, Constant Total Heat, and Logarithmic Expansion Lines for Steam. Adiabatic expansion and compression for both gases and vapors, whether superheated or wet, are characterized or defined by constancy of entropy, which corresponds to zero heat change, represented on the temperature-entropy diagram as a straight vertical line for all cases. On the pressure-volume diagram this is an exponential curve, $PV^s = \text{Constant}$. It is possible now to find the value of s for both gases and vapors, including vapor liquid mixtures, previously designated wet vapors. Making the heat zero, in the general equation, Eq. (748).

$$Q_{ab} = \frac{1}{J} \left[\frac{C_p}{C_v} - s \right] \times (\text{Work of Expansion}) = 0.$$

It follows that since neither the work of expansion nor the specific heats are themselves zero, the s must be equal to the ratio of specific heats designated by γ , that is, for adiabatic changes in gases, $s = \frac{C_p}{C_v} = \gamma$ (780)

For exponential expansion of gases in general, and for adiabatic in particular, the following relations between the temperatures and pressures, or temperatures and volumes, are derived from the fundamental pressure-volume relation,

$$P_a V_a^s = P_b V_b^s = \text{Constant} = P_a V_a V_a^{s-1} = P_b V_b V_b^{s-1}.$$

Putting $P_a V_a = RT_a$, and $P_b V_b = RT_b$, there results the relation between temperatures and volumes, $T_a V_a^{s-1} = T_b V_b^{s-1}$.

Also $\left(\frac{T_a}{T_b}\right)^{\frac{1}{s-1}} = \frac{V_b}{V_a} = \left(\frac{P_a}{P_b}\right)^{\frac{1}{s}}$. Therefore, $T_a P_b^{\frac{s-1}{s}} = T_b P_a^{\frac{s-1}{s}}$.

Collecting the series for convenience and giving to the adiabatic the special exponent, all are given by the tabular equation, Eq. (781),

Related Quantities for Exponential and Adiabatic Changes in Gases and Dry Vapors.	Formula	
	General Exponential Change.	Adiabatic Change.
P and V	$\frac{P_a}{P_b} = \left(\frac{V_b}{V_a}\right)^s; \quad \frac{V_a}{V_b} = \left(\frac{P_b}{P_a}\right)^{\frac{1}{s}}$	$\frac{P_a}{P_b} = \left(\frac{V_b}{V_a}\right)^\gamma; \quad \frac{V_a}{V_b} = \left(\frac{P_b}{P_a}\right)^{\frac{1}{\gamma}} \quad (a)$
P and T	$\frac{P_a}{P_b} = \left(\frac{T_a}{T_b}\right)^{\frac{s}{s-1}}; \quad \frac{T_a}{T_b} = \left(\frac{P_a}{P_b}\right)^{\frac{s-1}{s}}$	$\frac{P_a}{P_b} = \left(\frac{T_a}{T_b}\right)^{\frac{\gamma}{\gamma-1}}; \quad \frac{T_a}{T_b} = \left(\frac{P_a}{P_b}\right)^{\frac{\gamma-1}{\gamma}} \quad (b)$
V and T	$\frac{V_a}{V_b} = \left(\frac{T_b}{T_a}\right)^{\frac{1}{s-1}}; \quad \frac{T_a}{T_b} = \left(\frac{V_b}{V_a}\right)^{s-1}$	$\frac{V_a}{V_b} = \left(\frac{T_b}{T_a}\right)^{\frac{1}{\gamma-1}}; \quad \frac{T_a}{T_b} = \left(\frac{V_b}{V_a}\right)^{\gamma-1} \quad (c)$

To illustrate the case of adiabatic processes and to bring out clearly the striking difference between the isothermals and adiabatics for gases on $T\Phi$ diagrams in contrast to the small difference to PV coordinates, there is replotted in Fig. 145 the isothermal example of Fig. 142, and beside it is added an adiabatic for air, $PV^{1.41} = \text{Constant}$, from A to E , between the same pressures; as well as two general exponentials according to $PV^{1.2} = K$ from A to C , and $PV^{1.3} = K$ from A to D . The PV diagram shows little variation in work done compared to the variation in heats required as shown on the $T\Phi$ diagram where for

$$\begin{aligned} s = 1 & \quad (\text{Heat added}) = (\text{Work done}); \\ s = 1.2 & \quad (\text{Heat added}) = .51 \times (\text{Work done}); \\ s = 1.3 & \quad (\text{Heat added}) = .27 \times (\text{Work done}); \\ s = 1.41 & \quad (\text{Heat added}) = 0. \end{aligned}$$

This heat and work relation is given, from the general equation (748),

$$(\text{Heat added}) = \frac{1}{\gamma} \left(\frac{\gamma - s}{\gamma - 1} \right) \times (\text{Work done}),$$

in which the coefficient has the following values:

For $s=1; 1.2; 1.3; 1.41;$
 $\left(\frac{\gamma-s}{\gamma-1}\right)=1; .51; .27; .0.$

To facilitate calculations of P, V, T relations for expansion and compression having various values of s , diagrams have been plotted to a vertical scale of $\left(\frac{P_1}{P_2}\right)$, and with a double horizontal scale for the corresponding $\left(\frac{V_2}{V_1}\right)$ and $\left(\frac{T_1}{T_2}\right)$. See Charts 41 and 42 in the Tables. Each curve is drawn for a different value of s , as marked on it. These are also given on logarithmic cross-section paper

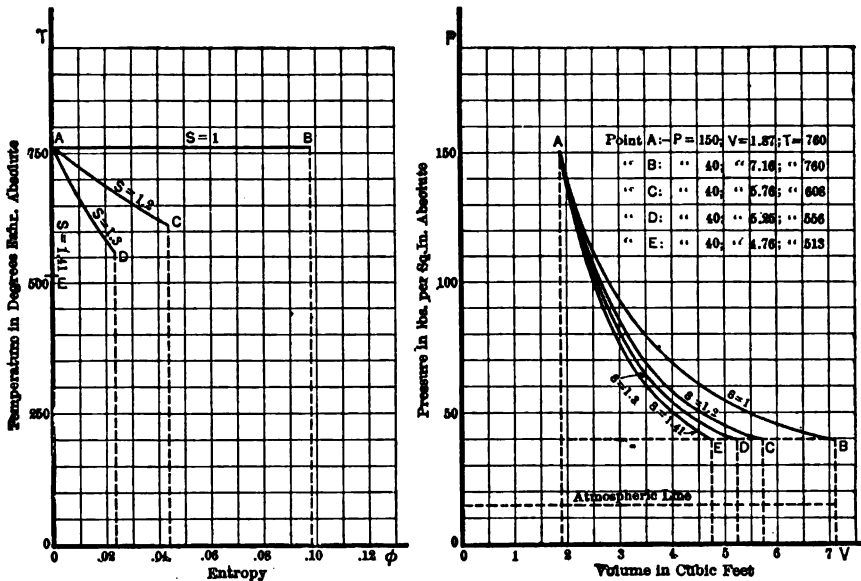


FIG. 145.—Adiabatic and Exponential Changes for Gases and Comparison with Isothermal.

in Chart No. 43, as arranged by Gunn, where all lines become straight; to which an entropy scale is added.

The simplicity of these relations for adiabatics of gases or superheated vapors not nearing the condensation condition, is in striking contrast to the complexity characterizing the adiabatics for vapors passing into the wet region. It was shown that isothermal compression of a superheated vapor would wet the vapor, and inversely, isothermal expansion dry it or increase the superheat; it now appears that adiabatic expansion of superheated vapors makes them lose superheat and become wet, or adiabatic compression dries them or increases their superheat, just the inverse of isothermal action. From the diagram, Fig. 146, on which OA is the liquid line and EJ is the saturation line to both $T\phi$ and PV coordinates it is clear that there are five typical cases of adiabatic change, either compression or expansion between any two pressures, depending on the relation of the adiabatic to the saturation and liquid lines.

First, AB passing through the state of *all liquid* at the high pressure and temperature, indicates that liquid at the boiling-point will evaporate or make vapor, by adiabatic expansion. The weight formed will be, $\left(\frac{\overline{OB}}{\overline{OJ}}\right)_{T_\Phi} \times (\text{weight of substance})$, while its volume will be likewise, $\left(\frac{\overline{OB}}{\overline{OJ}}\right)_{T_\Phi} \times (\text{volume of dry saturated vapor}) = \left(\frac{\overline{OB}}{\overline{OJ}}\right)_{T_\Phi} \times (\overline{OJ})_{PV}$, neglecting the liquid volume.

Second, CD passing through a wet region at the high pressure also passes through the wet region at the low pressure, but the quality will not be the same at all points, though it may be at two or more. The quality at the beginning is $\left(\frac{\overline{AC}}{\overline{AE}}\right)_{T_\Phi}$, and at the end $\left(\frac{\overline{OD}}{\overline{OJ}}\right)_{T_\Phi}$, while the initial volume before expansion is $\left(\frac{\overline{AC}}{\overline{AE}}\right)_{T_\Phi} \times (\overline{AE})_{PV} = (\overline{AC})_{PV}$, and the final volume is $\left(\frac{\overline{OD}}{\overline{OJ}}\right)_{T_\Phi} \times (\overline{OJ})_{PV} = (\overline{OD})_{PV}$.

Third, EF passing through the state of dry saturated vapor at the high pressure, enters the region of wet vapor immediately for all lower pressures, showing that dry saturated vapors get wet by adiabatic expansion and, in addition, providing means of evaluating the quality or moisture. Thus, for expansion, the init. quality is 1 and the vol. $(\overline{AE})_{PV}$; the final quality, $\left(\frac{\overline{OF}}{\overline{OJ}}\right)_{T_\Phi}$ and corresponding vol. $\left(\frac{\overline{OF}}{\overline{OJ}}\right)_{T_\Phi} \times (\overline{OJ})_{PV} = (\overline{OF})_{PV}$, while the final moisture is $\left(\frac{\overline{FJ}}{\overline{OJ}}\right)_{T_\Phi}$.

Fourth, GH passing into the superheat region at high, and into the wet region at low pressures, crossing the saturation line *EJ* on both diagrams at a point *M*. The high-pressure superheat is $(T_g - T_s)$ degrees, the low-pressure quality $\left(\frac{\overline{OH}}{\overline{OJ}}\right)_{T_\Phi}$ and wetness, $\left(\frac{\overline{HJ}}{\overline{OJ}}\right)_{T_\Phi}$. Assuming that superheated volumes vary as absolute temperatures, then $\frac{V_g}{V_s} = \frac{T_g}{T_s}$ or, $V_s = V_g \frac{T_s}{T_g}$, then the initial volume before expansion is $(\overline{AG})_{PV} = (\overline{AE})_{PV} \times \frac{T_g}{T_s}$, while the final volume is $\left(\frac{\overline{OH}}{\overline{OJ}}\right)_{T_\Phi} \times (\overline{OJ})_{PV} = (\overline{OH})_{PV}$.

Fifth, IJ passing through the state of dry saturated vapor at the low pressure, indicating that dry saturated vapors immediately superheat on adiabatic compression, or that highly superheated vapors may become dry saturated and then wet by adiabatic expansion.

Another case might have been added, represented by *KL* for adiabatic expansion entirely in the superheat region, but this is exactly the same as that previously considered for gases and has all its characteristics.

The graphic construction of such diagrams is quite simple when the saturation line is used as a reference, being plotted to both *PV* and *T Φ* coordinates

directly from the steam tables, because in the wet region the volume of wet vapor divided by the volume of dry saturated is equal to the quality of the steam, and in the superheat region the volume of superheated steam divided by the volume of dry saturated is equal to the ratio of actual steam temperature to saturation temperature absolute, very nearly; the exact relations due

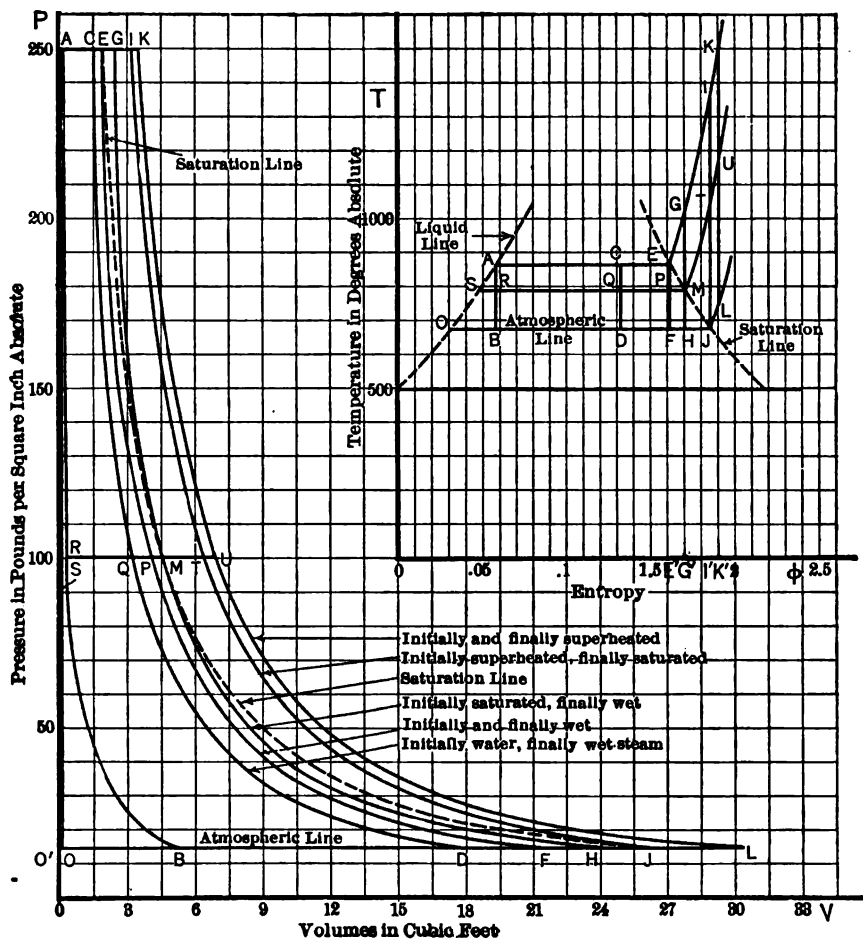


FIG. 146.—Adiabatics for Steam showing Changes of Quality and Illustrating a Graphic Method of Finding "s" for Wet Steam.

to imperfection of steam gas are given in Marks and Davis' Steam Tables. These principles applied to the diagrams give equal line ratios on both, so one can be transformed into the other, or a point fixed on one located correspondingly on the other. Thus for saturated steam, Wet steam compared to dry saturated steam at the same pressure, Fig. 146,

$$\left(\frac{SR}{SM}\right)_{T\phi} = \left(\frac{SR}{SM}\right)_{PV}; \quad \left(\frac{SQ}{SM}\right)_{T\phi} = \left(\frac{SQ}{SM}\right)_{PV}; \quad \left(\frac{SP}{SM}\right)_{T\phi} = \left(\frac{SP}{SM}\right)_{PV}.$$

Similarly, for the superheated region the ratios are between verticals on the $T\Phi$, and horizontals on the PV , since the former represent absolute temperatures and the latter specific volumes; thus for

Superheated steam compared to dry saturated steam at same pressure, Fig. 146,

$$\begin{aligned} \left(\frac{GG'}{EE'}\right)_{T\Phi} &= \left(\frac{AG}{AE}\right)_{PV}; & \left(\frac{II'}{EE'}\right)_{T\Phi} &= \left(\frac{AI}{AE}\right)_{PV}; \\ \left(\frac{TI'}{MG'}\right)_{T\Phi} &= \left(\frac{ST}{SM}\right)_{PV}; & \left(\frac{UK'}{MG'}\right)_{T\Phi} &= \left(\frac{SU}{SM}\right)_{PV}; & \left(\frac{LK'}{JI'}\right)_{T\Phi} &= \left(\frac{OL}{OJ}\right)_{PV}. \end{aligned}$$

From any of these adiabatics located on the PV diagram from the vertical line on the $T\Phi$ diagram, the exponent s can be determined by measuring the simultaneous pressures and volumes at any pair of points and substituting them in Eq. (14), Chapter I. Taking the two points E and F on Fig. 146, the exponent for adiabatic expansion of originally dry saturated steam can be found from

$$s_{EF} = \frac{\log \left(\frac{P_e}{P_f} \right)}{\log \left(\frac{V_f}{V_e} \right)} = \frac{\log P_e - \log P_f}{\log V_f - \log V_e}.$$

This gives a sort of overall value for s which is not uniform over the line EF , since other values from $s_{ep} = \frac{\log P_e - \log P_p}{\log V_p - \log V_e}$, and $s_{pf} = \frac{\log P_p - \log P_f}{\log V_f - \log V_p}$, for the same adiabatic using intermediate points, are not the same.

To illustrate this, tables have been prepared which give the values of s for different conditions, as outlined in Table XV, in the book of Tables. Case *B* shows the results for dry saturated steam expanding in 10-lb. drops from 200 lbs. to 1 lb. per square inch absolute. After the first drop the steam is wet and becomes more wet for the next. Case *A* is for the case of all water initially, the percentage of steam increasing as expansion continues. In case *C* the steam is originally superheated to such a degree that it remains superheated throughout the range of the expansion. In case *D* the original superheat is small and at some point during the expansion the steam is momentarily dry and saturated, and then becomes wet.

Hot water shows by its adiabatic expansion, values from $s=.10$ to $s=1.042$ for the first and last 10-lb. interval, while for dry saturated steam the values are from $s=1.145$ to $s=1.126$. In neither case does the variation for the whole range agree with that for the 10-lb. interval corresponding. Steam always superheated shows values slightly decreasing from $s=1.342$ to $s=1.322$ over the whole range, but when the saturation condition is passed there is a rapid change, as shown in the last table.

Owing to the incompleteness of the necessary data the results as calculated and as given in the second column of the table showed some inconsistencies. These former values were plotted and the values given in the third column of the table were read from a smooth curve drawn through the calculated results.

Inspection of these tables shows conclusively the variability of s along one

expansion line which is due to the condensation that is proceeding at a variable rate. This variability of s is also dependent on the pressure at which expansion begins and on the original amount of moisture or quality; to show this, s has been determined for steam expanding from 250, 200, 100 and 50 lbs. to 1 lb., from the initial and final volumes only and for original qualities of 100, 90, 80, 50, 20 per cent, the results being set down in Table XXXI.

TABLE XXXI
VALUES OF " s " FOR ADIABATIC EXPANSION OF STEAM
(Determined from initial and final volumes only).

Pressure Range.	Initial Quality.						Water.
	250° F. Superheat.	100%.	90%.	80%.	50%.	20%.	
From 250 lbs. to 1 lb....	1.183	1.119	1.110	1.102	1.060	.961	.662
From 200 lbs. to 1 lb....	1.185	1.124	1.117	1.107	1.066	.966	.638
From 100 lbs. to 1 lb. . .	1.192	1.125	1.118	1.109	1.070	.969	.565
From 50 lbs. to 1 lb. . . .	1.206	1.124	1.118	1.111	1.073	.963	.490

If for any possible condition of expansion fixed by pressures and quality, an average value of s were known for the whole range, then work could be calculated by the PV methods discussed in Part I, but even then the labor would be considerable and there would be necessary an almost infinite number of values of s . If there were no other better way of proceeding this would be satisfactory, but the entropy diagram and steam tables provide methods of *exact* calculation of such simplicity as to command admiration, and warrant the entire abandonment of all other methods of calculation of work of adiabatic expansion. As a matter of fact the thermal method, whether executed graphically or by means of tabular values, involves practically no calculation at all, and it is now possible to read directly from a chart the work done for a complete cycle with adiabatic expansion, so that labor is concentrated in the preparation of the chart itself. Indirectly the work of the expansion alone as a single phase may be found from the same data, as will be explained.

For a *complete cycle* of water heating, evaporation at constant pressure, adiabatic expansion, and condensation at constant pressure, the work done is equal to the heat added less the heat abstracted, which, per pound of steam, is the difference between the total heats at the two different conditions before and after adiabatic expansion. Therefore, the work of such complete cycles is to be determined from the total heats of steam. Charts of the total heat to which are added lines of constant pressure, temperature and quality for the same entropy, are graphic means for reading off the work directly.

For expansion alone as a single phase the work can be calculated from the total heats from the general relation,

$$(\text{Heat added}) = (\text{Change in intrinsic energy}) + (\text{Work done}).$$

The expansion being adiabatic the work done will be the difference between

the intrinsic energy before and after, with opposite sign, because the heat added algebraically is zero. Therefore,

Let H_1 and H_2 = Total heat, B.T.U. per lb. dry sat. steam, init. and final;

q_1, q_2 and L_1, L_2 = Heats of liquid and latent heats, tabular values;

\bar{U}_1 and \bar{U}_2 = Internal energy at initial and final conditions;

x_1 and x_2 = Quality or dryness fractions;

V_{v_1} and V_{v_2} = Specific volume of dry saturated steam at high and low temperature, tabular values;

V_{L_1} and V_{L_2} = Specific volume of liquid at high and low temperature which may generally be neglected.

Then W_{1-2} = Work done, ft.-lbs., by adiabatic expansion = $J(\bar{U}_1 - \bar{U}_2)$. (782)

But during the formation of steam from water

(Change in internal energy) = (Heat added) - (Work done),

so that the internal energy of a pound of steam above 32° F. is the amount of heat it takes to make the steam starting with water at 32° F., less the work the steam does in coming into existence, $\frac{1}{J} \times P(V_v - V_L)$. Therefore the work done in adiabatic expansion is the difference between the total heats before and after, less the difference between the work of steam formation at the two states before and after in B.T.U. When steam is wet,

$$\text{Total heat} = q_1 + x_1 L_1 \text{ B.T.U.} \quad (783)$$

$$\text{Vol. wet steam} = x_1 V_{v_1} + (1-x) V_{L_1} \text{ cu.ft.} \quad (784)$$

$$\begin{aligned} \text{Therefore, (Work of steam formation)} &= P_1[x_1 V_{v_1} + (1-x_1) V_{L_1} - V_{L_1}] \\ &= x_1 P_1(V_{v_1} - V_{L_1}) \text{ ft.-lbs.} \end{aligned} \quad (785)$$

$$\bar{U}_1 - \bar{U}_{32} = q_1 + x_1 L_1 - \frac{1}{J} x_1 P_1(V_{v_1} - V_{L_1}) \text{ B.T.U.} \quad (786)$$

$$\bar{U}_2 - \bar{U}_{32} = q_2 + x_2 L_2 - \frac{1}{J} x_2 P_2(V_{v_2} - V_{L_2}) \text{ B.T.U.} \quad (787)$$

Substituting in Eq. (782) the values of the intrinsic energy above 32° F., before and after adiabatic expansion as given by Eqs. (786) and (787), the work of expansion is given by Eq. (788),

$$W_{1-2} = J(q_1 - q_2 + x_1 L_1 - x_2 L_2) - [x_1 P_1(V_{v_1} - V_{L_1}) - x_2 P_2(V_{v_2} - V_{L_2})] \text{ ft.-lbs.} \quad (788)$$

Neglecting liquid volumes, this takes the form of Eq. (789) for the special case of expansion illustrated by the line from C to Q, Fig. 146, which is the case of constantly wet steam,

$$W_{cQ} = J(q_c - q_s + x_c L_c - x_s L_m) - (x_c P_c V_{v_c} - x_s P_s V_{v_s}). \quad (789)$$

But the quality after expansion, x_s , is a function of the quality before, x_c , which may be expressed in terms of entropies or the corresponding heats.

The entropy relations are given by Eq. (790) derived by the following steps:

$$x_c = \frac{\phi_c - \phi_a}{\phi_s - \phi_a}; \quad x_s = \frac{\phi_s - \phi_s}{\phi_m - \phi_s}; \quad (\phi_s - \phi_s) = (\phi_c - \phi_s) + (\phi_s - \phi_s);$$

hence

$$x_s(\phi_m - \phi_s) = x_c(\phi_s - \phi_a) + (\phi_a - \phi_s).$$

Adding to each side the entropy difference from 32° to the point S ,

$$x_s(\phi_m - \phi_s) + (\phi_s - \phi_{32}) = x_c(\phi_s - \phi_a) + (\phi_a - \phi_{32}). \quad (790)$$

This equation may be stated as follows: for points on the same adiabatic the sum of the entropy of the liquid from 32° to any temperature, and the product of the quality into the entropy of complete vaporization at that temperature is a constant. From Eq. (790) the value of x_s , the final quality after adiabatic expansion, can be found for use in Eq. (789) by using the tables of entropy for any initial quality of steam.

A similar relation between condition after and that before adiabatic expansion can be expressed in terms of heats and temperatures. If the specific heat of water be taken as unity then from Eq. (790) this relation will be given by

$$x_s \frac{L_m}{T_s} + \log_e \frac{T_s}{492} = x_c \frac{L_s}{T_c} + \log_e \frac{T_c}{492}; \quad x_s \frac{L_m}{T_s} = x_c \frac{L_s}{T_c} + \log_e \frac{T_c}{T_s}. \quad (791)$$

In some steam tables, notably Peabody's, are given the values of x , the quality for a given entropy at any pressure and temperature; these values are extremely useful as they eliminate the solution of Eq. (788) to Eq. (791); constancy of entropy characterizing adiabatic expansion, the tabular values solve such expansion problems directly. If corresponding qualities for equal entropies are not given directly in tables, which is the case with the Marks and Davis tables, it is possible to estimate without calculation the quality at one temperature, for a condition defined by an entropy the same as for some other quality at another temperature.

Constant quality lines on the temperature entropy diagram are directly useful in this sort of problem work and permit direct reading of the quality at any temperature, after adiabatic expansion from another temperature and quality; they are especially useful in the region of superheat or for passing from superheat to wetness, which is the case in nearly all practical steam turbine problems. Such a set of lines is laid out on Chart No. 44, in the book of tables, graphically by dividing each horizontal vaporization line proportionally and joining the one-quarter, middle and three-quarter points and others located correspondingly between, in the wet region. In the region of superheat constancy of quality implies constancy of superheat, so that on each pressure line points Z and Z' are so located that $\overline{ZY} = \overline{Z'Y'}$. Changes of quality along adiabatics represented by verticals, are given by the intersection of the vertical adiabatic with the quality curve.

Constant total heat lines may be located directly on such a chart and serve to save the labor of computation of work, which is still appreciable even when quality changes during adiabatic expansion are known.

When the end sought is work done by the whole cycle there are two forms for such charts, first, the ordinary temperature entropy chart with constant total heat lines located on it; and second, the total heat plotted as ordinates and entropy as abscissæ, known as the Mollier diagram. See page 529.

into work, represented by the area $CDEBC$. If this energy be converted back into heat it will reduce the wetness by evaporation from condition E to F , the point F being located so that area $EFGHE$ = area $CDEBC$. Another point K would be located so that area $CDIJC$ = $IKLHI$. In this way a line DK can be drawn, on which the steam has always the same total heat, or along which it suffers no change in heat content, so that it is a constant internal energy line for steam. So long as the heat equivalent of expansion work is all consumed in drying wet steam the construction is relatively simple, but if the steam is originally dry or nearly so it will become superheated, and if superheated it will acquire more superheat. In fact for superheated steam, if it behaved like a perfect gas, the internal energy would be a function of temperature only and constant energy lines would be isothermals. It will be observed from the diagram that this is not the case. To illustrate the location of a point in the superheat region, consider an original condition, represented by M , to be the beginning of an adiabatic expansion ending at N , then the point O will be located so that area $CMNJC$ = area $NROPQN$, and drying has proceeded along NR , followed by superheating RO to the constant total heat line MSO .

When the points of such a diagram are replotted so that total heats above 32° are ordinates and entropy from 32° are abscissa there will be a family of curves such as first plotted by Prof. Mollier, known as the Mollier diagram. On this diagram, given in the Handbook of Tables as Chart No. 45, the vertical distance from any pressure, temperature or quality, to any other, is the work done in heat units, by the whole cycle including an adiabatic expansion which can be marked off on a strip of paper and referred to the scale of heat to permit the work to be read directly, or the ordinate of the low can be subtracted from that of the high point. As this is so convenient for turbine work a scale of corresponding steam jet velocities is usually plotted beside that for total heats. A large scale chart of this sort is very necessary when many calculations of this nature are to be made, and such may be plotted from the steam tables.

Algebraic evaluation of the heat added during any expansion that is not adiabatic is often desirable, especially in dealing with cycles and their efficiency as heat converters into work. Unfortunately this is not always possible with precision, but approximations are possible for some cases, good enough for comparative purposes. It is necessary for this work that the total heat be expressed as a function of temperature which is possible only by algebraic expressions of the empiric sort, complicated in form, derivable from the general expression,

$$(\text{Heat added during expansion}) = \int T d\phi = \int T \frac{d\phi}{dT} dT \dots (792)$$

Integration of Eq. (792) is possible only when the differential coefficient $\frac{d\phi}{dT}$ can be expressed as a function of T alone, and by not too complex a function, if the final formula is to be of any practical value. Along the saturation curve the total heat of the steam is the sum of heat of liquid and latent heat at any given temperature above 32° F., so that the abscissa of any point on the $T\phi$ dia-

gram from 32° as an origin, will be given by Eq. (793), if C_s is the specific heat of water, which may be taken as constant and equal to unity or as a function of T .

$$\phi_a - \phi_{32} = \int_{492}^{T_a} C_s \frac{dT}{T} + \frac{L}{T} \quad (793)$$

Latent heat can be expressed as a function of T approx. by $L = \alpha + \beta T + \gamma T^2$, (794)

so that the entropy increase above 32° for saturated steam is a function of temperature given by $(\phi_a - \phi_{32}) = \int_{492}^{T_a} C_s \frac{dT}{T} + \frac{\alpha}{T} + \beta + \gamma T$. (795)

Corresponding to this integral expression is a differential form from which the differential coefficient of ϕ with respect to T is, $\frac{d\phi}{dT} = \frac{C_s}{T} - \frac{\alpha}{T^2} + \gamma$. (796)

This takes the two following forms (a) and (b), Eq. (797), according as $C_s = 1$ approximately, or is a series function of the temperature.

$$\left. \begin{aligned} \text{For specific heat of water} = 1, \quad \frac{d\phi}{dT} &= \frac{1}{T} - \frac{\alpha}{T^2} + \gamma \quad (a) \\ \text{For specific heat of water } C_s &= a + bT + cT^2, \quad \frac{d\phi}{dT} = \frac{a}{T} + b + cT - \frac{\alpha}{T^2} + \gamma \quad (b) \end{aligned} \right\} (797)$$

Substituting the simpler form Eq. (797a), in Eq. (792), the heat added to keep steam dry and saturated as it expands will be given by Eq. (798),

Heat added during expansion from B to A along saturation line

$$= \int_{T_b}^{T_a} \left[1 - \frac{\alpha}{T} + \gamma T \right] dT \text{ (approx.)} = \alpha \log_e \frac{T_b}{T_a} - (T_b - T_a) - \frac{\gamma}{2} (T_b^2 - T_a^2). \quad (798)$$

A more nearly exact, but still approximate result, may be obtained by substituting Eq. (797 b), in Eq. (792).

Numerical problems cannot be solved without evaluation of the constants $a, b, c, \alpha, \beta, \gamma$, which can be done approximately as follows: the Dieterici equation for the specific heat of water is

$$C_s = .9983 - .0000288(t-32) + .0000002133(t-32)^2;$$

$$= .9994 - .0000424t + .000000213t^2; \quad = .9738 + .0001536T - .000000213T^2;$$

whence $a = .9738$; $b = .0001536$; $c = .000000213$.

Similarly, the Davis formula for the total heat of steam above 32° F. is

$$H = 1150.3 + .3745(t-212) - .00055(t-212)^2; \quad = 1046.187 + .6077t - .00055t^2.$$

From the specific heat of water the heat of liquid, q , becomes,

$$\begin{aligned} q &= .9994(t-32) - .0000424t(t-32) - .000000213t^2(t-32); \\ &= 1.0007t - 31.981 - .0000356t^2 \text{ app.} \end{aligned}$$

From these it follows that as $L = H - q$,

$$L = 1078.168 - .393t - .00051t^2; \quad = 1151.0 + .076T - .00051T^2;$$

whence $\alpha = 1151.0$; $\beta = .076$; $\gamma = -.00051$.

Should the expansion proceed along any constant quality line in the wet region, the same method will apply, if to the latent heat, or to each term derived from it separately, the quality x be applied as a multiplying factor. Hence

$$\left\{ \begin{array}{l} \text{Heat added during wet steam} \\ \text{expansion from } B \text{ to } A \text{ for} \\ \text{constant quality } x \end{array} \right\} = ax \log_e \frac{T_b}{T_a} - (T_b - T_a) - \frac{\gamma x}{2} (T_b^2 - T_a^2). \quad (799)$$

Logarithmic expansion of steam, it has been stated, is the old assumption for cylinders, and it is of interest to see what changes in quality correspond to it. In Fig. 148 is plotted the saturation line $ABCDE$ to $T\phi$ and PV coordinates dotted. On the PV diagram are located a number of logarithmic lines for wet and superheated steam which it is found tend to cross the satura-

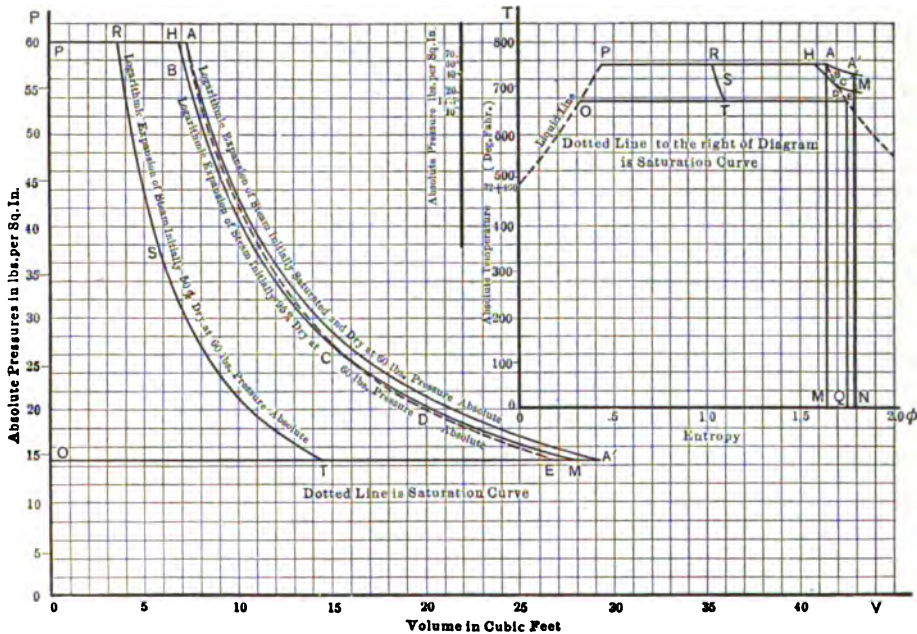


FIG. 148.—Logarithmic Expansion of Steam Showing Changes of Quality.

tion line and so prove that evaporation takes place with logarithmic expansion of wet steam, and superheating follows. In fact, for steam initially superheated the logarithmic expansion line is an isothermal, and the superheat increases as the difference between saturation temperature for any pressure and the original temperature of the superheated steam. Thus, a logarithmic line AA' through A , a point of original dryness, is an isothermal as nearly as the approach of superheated steam to a perfect gas permits. It is located by the ratio

$$\left(\frac{A'N}{EQ} \right)_{T\phi} = \left(\frac{A'O}{EO} \right)_V$$

Points in the wet region are located by proportionality of horizontal intercepts as for adiabatics. It is of no interest to compute algebraically the heat that

must be added during expansion to keep it logarithmic, but for any case where it is needed it can best be evaluated graphically by areas.

Example 1. *Calculation and use of Temperature-Entropy Diagram, Chart No. 44 (Tables), lines of constant pressure and quality.* Let it be assumed that the line of quality 80 per cent is to be located, starting with the pressure of 200 lbs. per square inch absolute, point *A*. From the steam tables $t = 381.9^\circ \text{ F.}$ or $T_s = 841.9$, the entropy of the liquid is .5437, of evaporation complete, 1.0019, so that $\phi_s - \phi_{s2} = .8 \times 1.0019 + .5437 = 1.3452$. To locate a point *B* in the superheat region at the same pressure and for 100° of superheat, the steam tables are found to give directly $\phi_s - \phi_{s2} = 1.6120$.

As an example of the use of the diagram the following problem will serve. Steam at a pressure of 160 lbs. per square inch absolute, dry and saturated, expands adiabatically to atmospheric pressure and to some unknown quality to be found. From the point *C* representing the initial condition project vertically down to the pressure line 14.7, at point *D*. By interpolation the quality is found to be 86.5 per cent, as point *D* lies between the two lines of 80 per cent and 90 per cent quality.

Another example will illustrate the passage into the superheat region. Atmospheric exhaust steam at 20 lbs. per square inch absolute, is superheated 120° by a reheater and then expands adiabatically in an exhaust steam turbine to an absolute pressure of half a pound per square inch absolute, to find the final quality. The initial condition is represented by point *E*, from which projecting downward to the low-pressure line at *H*, lying between 80 per cent and 90 per cent, the quality is found by interpolation to be 88.4 per cent and the temperature by projecting to *K*, is $T = 540^\circ$. The corresponding volumes may be read off Chart No. 40 of the Tables.

Example 2. *Method of calculating the Diagram, Fig. 148, for logarithmic expansion of steam.* Assume for initial data, quality 50 per cent, initial pressure 60 lbs. and final pressure 14.7 lbs. sq.in. absolute. From the steam tables $V_r = 3.585$ cu.ft., $T_r = 752.7^\circ$, $\phi_r = \phi_{s2} = 1.0352$. The point *T* is located as to pressure from the data, as to temperature from the steam tables, and as to volume from the hypotheses,

$$V_t = \frac{P_r V_r}{P_t} = \frac{60 \times 3.585}{14.7} = 14.63 \text{ cu.ft.}$$

The quality at point $T = \frac{14.63}{26.79} = .546$, and from the steam tables $\phi_s - \phi_{s2} = .3118$, $\phi_s - \phi_s = 1.4447$, whence $\phi_r - \phi_{s2} = 1.4447 \times .546 + .3118 = 1.1006$.

The curve *HCM* is drawn between the same pressures, but from an initial quality of 95 per cent. From the steam tables $V_h = 6.812$, $\phi_h - \phi_{s2} = 1.5824$ and $V_m = \frac{P_h V_h}{P_m} = \frac{60 \times 6.812}{14.7} = 27.8$ cu.ft. But dry saturated steam at the low pressure has a specific volume of $V_s = 26.79$, so that the expanding steam has become superheated. The amount of superheat may be estimated from the Marks and Davis superheat tables by seeking the steam temperature for this pressure having a specific volume of 27.8, or it may be estimated by the perfect gas law $\frac{T_m}{T_s} = \frac{27.8}{26.79}$. In this way it was found that $T_m = 695^\circ$ and $\phi_m - \phi_{s2} = 1.7723$.

Prob. 1. A tank containing 500 cu.ft. of compressed air at 60° F. and twenty atmospheres pressure is drawn upon to supply a compressed-air engine. After the pressure has fallen to five atmospheres without heat exchange, what is (a) the air temperature, (b) the volume of atmospheric air withdrawn, measured at 32° F. ; (c) the work that

has been done if all air withdrawn completely expanded; (d) the heat necessary to raise the remaining air to the original temperature?

Prob. 2. If the working gases having $\gamma = 1.4$ in a gas-engine cylinder expand according to the law $PV^{1.3} = \text{constant}$, the cylinder having a clearance of 15 per cent of its displacement, 20-inch bore and 30-in. stroke, initial pressure of 400 lbs. per square inch absolute and temperature of 2500° F., find (a) the final pressure and temperature, (b) the work done by expansion alone, and (c) the heat gained or lost during the expansion, (d) the net work of the cycle consisting of adiabatic compression from one atmosphere, combined with the above expansion and two constant-volume lines.

Prob. 3. Steam at 200 lbs. per square inch absolute and 150° of superheat adiabatically expands to 100 lbs., then receives 100 B.T.U. at constant pressure and again expands to atmosphere. Find, (a) the quality after the first expansion and before and after the second, (b) the work done in each expansion thermally, (c) the value of s for each expansion, (d) the work done in each expansion by PV methods compared with that found thermally.

Prob. 4. Ammonia is being compressed adiabatically from 30 lbs. to 175 lbs. per square inch absolute. What quality must it possess originally to have the final condition of, (a) all liquid, (b) 50 per cent vapor, (c) dry saturated vapor? (d) What will be its superheat if originally it were dry saturated?

Prob. 5. Find the value of s for each of the cases of Problem 4 and determine the work of compression alone and of the whole cycle consisting of low-pressure evaporation preceding compression, high-pressure condensation and complete liquid cooling to low temperature, using both PV and $T\phi$ methods.

Prob. 6. Plot to PV and $T\phi$ coordinates compression of 80 per cent quality ammonia and carbon dioxide from 0° F. to 70° F., according to the law $PV = \text{Const.}$, and find (a) how much heat must be added or abstracted per pound, by evaluation of $T\phi$ areas and (b) the work done by both PV and thermal methods.

Prob. 7. Steam in a cylinder has an initial pressure of 100 lbs. per square inch absolute and a quality of 90 per cent. It expands logarithmically to 30 lbs., is reheated to original temperature and then expands adiabatically in a turbine to 1 lb. per square inch absolute. Plot the $T\phi$ and PV diagrams and find (a) the heat added during logarithmic expansion and reheating, (b) the quality before and after adiabatic expansion, (d) the work done by expansion alone, in each stage checking calculations by diagram areas.

Prob. 8. Using the thermal method entirely compare the work of admission, compression and delivery for CO_2 and NH_3 compressors per pound of vapor originally of quality 75 per cent, between -20° F. and 75° F.

Prob. 9. On a $T\phi$ diagram plot the constant volume and constant quality lines for CO_2 and NH_3 , starting with superheated vapor at 300° F. and the pressure corresponding to saturated vapor at 75° F., ending the lines at 0° F.

Prob. 10. For the steam cycle consisting of liquid heating, evaporation at 150 lbs. per square inch absolute, superheating 100°, adiabatic expansion to 2 ins. Hg, absolute, and constant pressure condensation, (a) plot the PV and $T\phi$ diagrams, (b) find the work done by the five thermal methods and compare with the PV area, (c) modify the diagram by assuming adiabatic expansion to end at 50 lbs. per square inch absolute, closing with a constant volume line, and find the work so lost by $T\phi$ and PV methods.

Prob. 11. If the steam of the first part of Prob. 10 expanded through an orifice and impinged on a plate, what would be its final quality and velocity if (a) its final velocity were unchanged by impact, (b) reduced to half, (c) reduced to zero?

CHAPTER XIV

THERMAL CYCLES FOR STEAM.

98. Thermal Cycles Representative of Heat-engine Processes. Cyclic Efficiency. A Reference Standard for Engines and Fuel-burning Power Systems. Classification of Steam Cycles. In practice, heat conversion into work has narrowed down to a comparatively few processes out of the infinite number possible, considering all kinds of substances as possible *heat carriers* or work *executors*. The substances of importance are only two in general, first, water-steam, and second, a substantially perfect gas, which latter may be superheated steam or any other vapor, or a mixture of any number of so-called permanent gases behaving as one. The processes through which water-steam may pass, will, when studied, yield certain principles of heat treatment of the fluid, fundamental to the steam-engine performance, while similarly basic ideas with regard to the gas engine follow from the study of the processes through which gases may pass. There will result from such analysis of gas and vapor cycles, a definite idea of the limiting possibility of the amount of conversion or maximum possible efficiency, and it will appear that the efficiency depends on how the heat is added and abstracted, how the substance changes state, how compression and expansion are related to each other and to the heating and cooling, how much heat is added per pound of substance, and the total range of variation of pressure, volume and temperature. In short, the efficiency can be found for any substance passing through any cycle and will be found to be not the same for all, sometimes quite high, often very low. Therefore, assuming that the object of engineers dealing with power generation is to convert as much heat into work as possible or practicable, this cyclic analysis will show what is desirable and what thermal actions must be avoided. It will moreover establish a numerical limit to the possible conversion or efficiency for any one series of processes that may be embodied in a machine, with which limit the actual performance of a machine may be compared, thus giving a measure of the perfection of the mechanism as an executor of the cycle and showing how far improvements may be carried. For example, for given conditions of pressure and temperature, it may be found that a given steam engine is converting into work 20 per cent of the heat supplied to it in the form of steam; this may look very low and great improvements appear to be possible, whereas on examination of the processes being carried out within the pressure and temperature limits imposed, it may be found that not more than 25 per cent efficiency is possible in the most perfect mechanism, in which case the mechanism may be regarded as $\frac{20}{25} = 80$ per cent perfect and no amount

of ingenuity or expense can make it very much better than it is. This is a good illustration of the fact that the efficiency of a power generator is limited by two things, or is the product of two other efficiencies; first, the efficiency of the thermal cycle being carried out by the substance, and second, the efficiency of the mechanism as a cycle executor. The former is conveniently termed the *cyclic* or *system efficiency*, and the second the *mechanism efficiency*, but this latter must not be confused with mechanical efficiency, which measures only mechanical friction losses, whereas the other measures all energy losses in the mechanism not fundamental or necessary to the cycle itself.

The great advantage of the cyclic analysis to engineering consists in the establishment of *standards* of performance that serve as guides to improvement, telling clearly and unmistakably when one cycle or system must be abandoned in favor of another to attain a desired end, or where and how to operate in the reduction of losses to improve a given piece of mechanism. With this as a criterion, it is evident that only those cycles that may be carried out by mechanism are worth studying; it is therefore of importance that this thermodynamic study of efficiency of conversion be so extended as to show what processes will yield efficiencies higher than by the processes now considered good practice. Cyclic analysis then has two criteria by which the *worth while* cycles are to be selected: first, their present practicability, and second, the potential high efficiency, realization of which depends on discovery of suitable mechanism to carry them out. No better illustration can be given than that supplied by the development of the gas engine in the last half century. As originally built it was just a novelty, about the possibilities of which no one had any definite ideas; then thermodynamic study of its cycle showed two things: first, that its possible efficiency was very high, compared to the steam systems then in use; and second, that in all gas engines for efficient conversion, the cycle must include a compression of the working gases before heating. Both the higher possible efficiency with respect to steam, and the necessity for precompression, were known before their realization in gas engines of good construction and competitive size, and this knowledge was the real incentive to practical development. Not only is this so, but as higher efficiency began to be realized in gas engines than had been the rule with steam, so, as a direct consequence, the improvement of the steam system was stimulated, every loss carefully studied as never before, and means taken to reduce each, so that today in large-sized plants, especially with high pressure reciprocating and low-pressure turbine units, the efficiencies of the steam and gas system on the same coal fuel are substantially the same, and twice as good as those which were considered satisfactory when the gas engine began its stimulating career.

With this preliminary survey, the cycles for analytical study may be selected from among the great number of possibilities which are greater for gases than for steam, since in the latter case there is only one practical way of adding heat, that at constant pressure and temperature, if the liquid heating and vapor superheating be excepted. It would perhaps be better to say that the bulk of the heat in the steam system, or in fact any vapor system, must necessarily

be added at constant pressure and temperature. Heat abstraction likewise is essentially a constant-pressure and temperature process for vapor systems if the expansion is complete, and in the establishment of standards of possibility there is no reason for dwelling on any but complete expansion because this is always realizable in engines if desired. Variation in vapor cycles one from the other can come in for consideration mainly as due, not to differences in modes of heat addition or abstraction directly, but rather from the differences in expansion between primary heating and heat abstraction, and in compression or its equivalent between heat abstraction and primary heating. The use of the term *primary heating* indicates that there may be secondary or minor heating in vapor cycles, and this is a convenient way of distinguishing between the bulk of the heat received at constant temperature, always to be considered as primary, and that received otherwise to be considered as secondary or minor. To illustrate, the primary heat added will be the latent heat of vaporization of so much of the fluid vaporized if it is left wet. The secondary heat will be, (a) heat of liquid if liquid heating is part of the cycle; (b) heat of superheat if the vapor is superheated at the pressure of steam generation before expansion begins; (c) heat added during expansion when it is not adiabatic, as for example constant quality logarithmic, or constant temperature for superheated vapor.

With complete condensation of vapor as a result of heat abstraction at constant pressure and temperature, heating of liquid is necessarily one phase of the vapor cycles, but should the heat abstraction or condensation cease before condensation is complete and the remaining vapor be adiabatically compressed so that it just liquefies on reaching the high pressure and temperature, then there will be no liquid heating.

All vapor cycles can be divided into two groups with respect to the mode of transition from condensation to vaporization phases; the first group will include those that completely condense and so involve liquid heating, and the second those that substitute complete adiabatic compression for liquid heating. Of course, there may be cycles with part of one, and the rest of the other process, but as their characteristics will lie between these two limits they need not be separately studied in a general investigation.

Expansion may be adiabatic or may not, but if not, it must be according to some law for which there are algebraic relations, otherwise algebraic cyclic analysis is impossible, and without this, principles are difficult to devise, though concrete problems may be solved. The only case of expansion that is of practical interest and yet yields even approximately to algebraic analysis is that of constant quality: that is, for steam initially wet, with constant wetness during expansion; for steam initially dry and saturated, constantly so during expansion; and for steam initially superheated, with constant superheat during expansion. These cases of expansion for various constant qualities of steam are usually considered as nearly representative of the possible action of steam in jacketed cylinders, and the heat added during expansion is supposedly representative of the heat lost by the jacket steam and gained by the cylinder steam at the rate required

for constant quality. As a matter of fact it is very doubtful if jackets could so add heat to cylinder steam; while they might give to it the total amount equivalent to constant-quality expansion, it is probable that the cylinder steam even if it received this amount of heat would not receive it at the rate corresponding to constant quality. To just about the same degree does the constant-quality expansion represent the skin friction heating effect of nozzles; part of the work developed and appearing as kinetic energy is converted back into heat by velocity reduction. These cycles with constant-quality expansion are not of sufficient practical importance, compared to those with adiabatic expansion, to warrant equal treatment, and while they will be indicated they will not be subjected to analysis.

Adiabatic expansion is the most important, because it is typical of heat addition completed before expansion begins, and heat abstraction delayed until expansion is over. This is what would happen in cylinder engines if the walls were non-conductors of heat and did not absorb heat from the steam on steam admission, or return it when the steam temperature dropped; it would also happen in steam turbines if the nozzles were non-conductors of heat and the surfaces in contact with the steam frictionless. Comparison of cycles otherwise the same, in which expansion is adiabatic, with those in which quality is constant, will show whether it is better to add all the heat before or continue addition during expansion. This will be taken up later. Adiabatic expansion may (a) start with wet and end with wet steam; (b) start with dry saturated and end with wet steam; (c) start with superheated and end with wet steam; (d) start with superheated and end with dry saturated steam; (e) start with superheated and end with superheated steam; so there may be five cases of adiabatic expansion cycles, *each differing from the other fundamentally in the amount of heat added per pound of steam at the same pressure.*

From the preceding it appears possible to conveniently divide the steam cycles worthy of study into four, each with special cases for various degrees of initial quality or heat added per pound of steam at the high pressure, these four being the possible combinations of, (a) zero compression, (b) complete adiabatic compression; with (c) adiabatic expansion, and (d) constant quality expansion defining the typical phases, represented by the corresponding diagrams of Fig. 149 to both $T\phi$ and PV coordinates. The relations of these various lines should be evident from the explanations given in the analysis of various phases; to make them clearer, the saturation curve is extended across each diagram as a line of reference.

Referring to the PV diagram of Cycle I, it will be noted that vaporization B to C , or C' , whether complete or incomplete, and including superheating C' to C'' if there is any, is represented by the same sort of line as admission of steam to a cylinder, in which the initial volume V_b is that of the liquid from which the amount of steam admitted is formed, which is so small as to be negligible in numerical work, but is retained here to make the cycle operation clear. Similarly, the condensation line D''' , D'' , D' , D to A , preceded by loss of superheat D'' to D''' if there is any, is represented to PV coordinates, exactly as

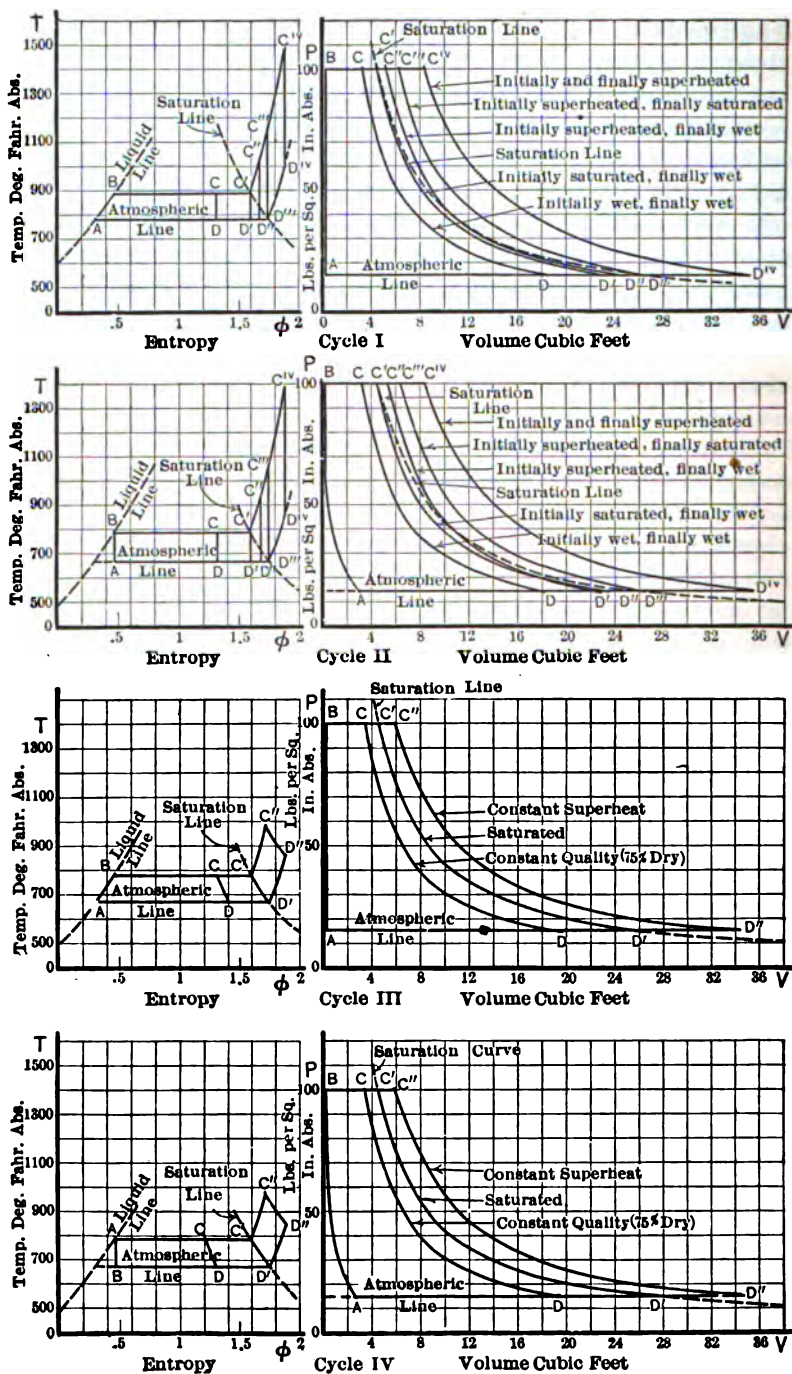


FIG. 149.—Steam Cycles.

would be a constant-pressure exhaust. Therefore, although in cylinders there occurs admission and exhaust, the efficiency of heat conversion can be studied by imagining as substitutes, constant-pressure heat addition or abstraction, which is equivalent to imagining the operations which take place in a whole plant—from boiler-feed, steam-making, cylinder action, condensation in condensers, and return of condensate to boiler—to take place completely in the cylinder. There are no unwarranted assumptions involved in this substitution of equivalent thermal phases, as it is obvious that the cylinder will do the same work in either case and there will be necessary the same amount of heat to make it possible. However, as ordinarily carried out in separate apparatus, these various processes involve some losses and some compensations, losses for example in pipe lines, and compensations from flue gas and exhaust steam heat. The compensations do not change the total heat required by the steam to put it in condition, but this amount is not all derived from fresh fuel when waste heats are partially returned.

This same diagram represents in a similar way the action in a steam turbine plant in which the whole process is imagined as concentrated in the nozzles of the turbine, generation and superheating of steam represented by B to C , C' , C'' , C^{IV} , does work represented by the area under this line to PV coordinates, and expansion does more work represented by the area under the expansion line, but in escaping from the nozzle this work is done only by pushing away some steam at the low pressure, which involves negative work under the low or back-pressure line or its thermal equivalent, the condensation line. The net work is the area enclosed by the PV diagram, as for cylinders, and is equal to the area enclosed by the $T\Phi$ diagram of the equivalent thermal processes.

Various minor modifications of these and other cycles can be studied when special cases require it; for example, incomplete expansion followed by constant-volume release could be shown by a diagram to $T\Phi$ and PV coordinates for various degrees, ranging from no expansion at all to complete expansion.

Some of these cycles adopted as representative and worthy of analytical study have received names associating them with some person who studied them either first or most effectively. Thus the name, Rankine, is applied to the Cycle I, which is characterized by adiabatic expansion and no compression; the name, Carnot, to those cases of Cycle II, that are without superheat and which are characterized by adiabatic compression and expansion, with heat added all at the high, and abstracted all at the low temperature. To other cases there is either no name applied or there is no agreement as to the name, so where confusion is likely to result from the application of a name to a cycle which elsewhere is applied to some others, the name will be omitted. There is absolute agreement as to the meaning of the Carnot cycle, and fairly general though not absolute acceptance of the name, Rankine, as signifying the cycle above defined, though some writers limit this name to the case of initially dry saturated steam, to which others apply the name Clausius. As the adiabatic-expansion and no-compression cycle is so closely representative of actual steam practice, it must be frequently referred to, and this will be called the Rankine cycle

for wet, dry or superheated steam as a better term than Cycle I. In all other cases the cycle will be defined by its phases or number when discussed.

The following analyses of these cycles will all be based on one pound of substance of which varying amounts will be in the vapor and in the liquid states; *it will also be assumed that when there is such a mixture it is of the same temperature throughout or thermally homogeneous.* In cylinders it is certain that moisture collects next the walls and is cooler than the steam; if superheated steam or gases are in cylinders that part near the walls will not have the same temperature as the rest of the mass; this is also the case in nozzles, as the heat of friction is first imparted to the skin fluid and may or may not be later communicated to the rest by conduction diffusion or mechanical mixture.

Another assumption that is necessary is that of constancy of specific heat of water at unity, and of superheated steam at a mean value for the temperature range, but not necessarily the same for different pressures or temperatures.

STEAM CYCLE I (Rankine). First phase, from *A* to *B*. Heating liquid (substantially constant volume). Second phase, from *B* to *C*. Heat addition at constant pressure, vaporizing at constant temperature and possibly followed by superheating of vapor at rising temperature. Third phase, from *C* to *D*. Adiabatic expansion. Fourth phase, from *D* to *A*. Heat abstraction at constant pressure, condensation.

STEAM CYCLE II (Carnot). First phase, from *A* to *B*. Adiabatic compression of vapor and liquid to all liquid. Second phase, from *B* to *C*. Heat addition at constant pressure, vaporizing at constant temperature and possibly followed by superheating of vapor at rising temperature. Third phase from *C* to *D*. Adiabatic expansion. Fourth phase from *D* to *A*. Heat abstraction at constant pressure, condensation.

STEAM CYCLE III. First phase, from *A* to *B*. Heating liquid (substantially constant volume). Second phase, from *B* to *C*. Heat addition at constant pressure, vaporizing at constant temperature and possibly followed by superheating of vapor at rising temperature. Third phase, from *C* to *D*. Expansion with constant quality. Fourth phase, from *D* to *A*. Heat abstraction at constant pressure, condensation.

STEAM CYCLE IV. First phase, from *A* to *B*. Adiabatic compression of vapor and liquid to all liquid. Second phase, from *B* to *C*. Heat addition at constant pressure, vaporizing at constant temperature and possibly followed by superheating of vapor at rising temperature. Third phase, from *C* to *D*. Expansion with constant quality. Fourth phase, from *D* to *A*. Heat abstraction at constant pressure, condensation.

Prob. 1. Plot to scale to PV and $T\phi$ coordinates steam Cycle I modified by expansion in two stages, the first extending for half the temperature range, and with reheating in the receiver to the initial quality.

Prob. 2. Plot to scale steam Cycle I, modified by logarithmic expansion.

Prob. 3. Plot to scale steam Cycle I, modified by logarithmic expansion in two stages as in Prob. 1, but with reheating to initial temperature.

Prob. 4. Plot to scale steam Cycle II, modified as in Prob. 1.

Prob. 5. Plot to scale steam Cycle II, modified by logarithmic expansion in the first stage for half the pressure range and cooling in the receiver to a quality of 90 per cent, followed by adiabatic expansion in the last stage.

Prob. 6. Modify Cycle II by two-stage compression adiabatic with intercooling and three-stage adiabatic expansion with reheating to original temperature. Intercooling to begin at half the temperature range and to completely condense the steam, reheating to be at $\frac{1}{3}$ and $\frac{2}{3}$ the temperature range and to just dry the steam.

Prob. 7. Modify steam Cycles II and III by two-stage expansion, dividing the temperature range equally, the first to be at constant quality as in a jacketed cylinder and the second adiabatic as in a low-pressure turbine.

Prob. 8. Modify steam Cycles I and II by stopping adiabatic expansion* at half the pressure range, closing by a constant-volume line.

Prob. 9. Plot to scale the following cycle: Water heated under pressure sufficient to prevent vaporization at 1000 lbs. per square inch absolute, the pressure being 100 lbs. per square inch higher than saturated vapor pressure. Hot water allowed to escape through a valve to a receiver at 100 lbs. per square inch absolute, there brought to rest and adiabatically expanded to atmospheric pressure. Condensation from and at 200° F. followed by return of water to high-pressure tank and there again heated.

99. The Rankine Cycle. Work, Mean Effective Pressure, Jet Velocity, Water Rate, Heat Consumption and Efficiency of Steam Cycle I. Adiabatic Expansion, Constant-pressure Heat Addition and Abstraction, No Compression.

Let

Q_1 = heat added in B.T.U. per lb. for whole cycle;
 Q_2 = heat abstracted in B.T.U. per lb. for whole cycle;
 W = work in ft.-lbs. per lb. for whole cycle;
 E = efficiency of conversion;

$\left\{ \begin{array}{l} P_c \\ V_c \\ T_c \\ x_c \\ (\phi_c - \phi_{32}) \end{array} \right\} = \left\{ \begin{array}{l} \text{pressure volume, temperature, quality, and entropy} \\ \text{above } 32^\circ \text{ F., for steam in condition represented by} \\ \text{position of point } C \text{ and with corresponding sub-} \\ \text{scripts for other points. Pressures in lbs. sq.ft., vol-} \\ \text{umes in cu.ft., temperature absolute Fahrenheit,} \\ \text{quality in ratio of weight of dry saturated steam to} \\ \text{total steam and water weight.} \end{array} \right.$

$\left\{ \begin{array}{l} q_1 \\ L_1 \\ h_1 \\ H_1 \end{array} \right\} = \left\{ \begin{array}{l} \text{heat of liquid, latent heat, heat of superheat, and total} \\ \text{heat above } 32^\circ \text{ per lb. of dry steam, tabular value,} \\ \text{at the high temperature and with subscript } (2) \text{ for} \\ \text{low temperature.} \end{array} \right.$

$S_{c''} = \text{constant mean specific heat of superheated steam from } T_c \text{ to } T_{c''}.$

Then since the change in intrinsic energy is zero for a complete cycle, the work done is equal to the heat change or the difference between heat put in and that abstracted for the whole cycle, which is algebraically expressed in Eq. (800):

$$W = J(Q_1 - Q_2). \quad (800)$$

As the efficiency of conversion of heat into work is the ratio of work done to heat supplied in the same units, it is given by $E = \frac{W}{JQ_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$. (801)

Both these expressions, Eq. (800) and (801), are perfectly general and true for any cycle, differences entering only when the heats supplied or abstracted are evaluated in accordance with the particular specifications for the cycle. For this cycle, Fig. 150, there are two forms of expression for the heats per pound, one, when superheat is initially present and the other when it is not, given as follows by Eq. (802),

$$\left. \begin{aligned} Q &= q + xL = H - (1-x)L \text{ for saturated steam of quality } x & (a) \\ Q &= q + L + h = H + h \text{ for superheated steam} & (b) \end{aligned} \right\} \dots (802)$$

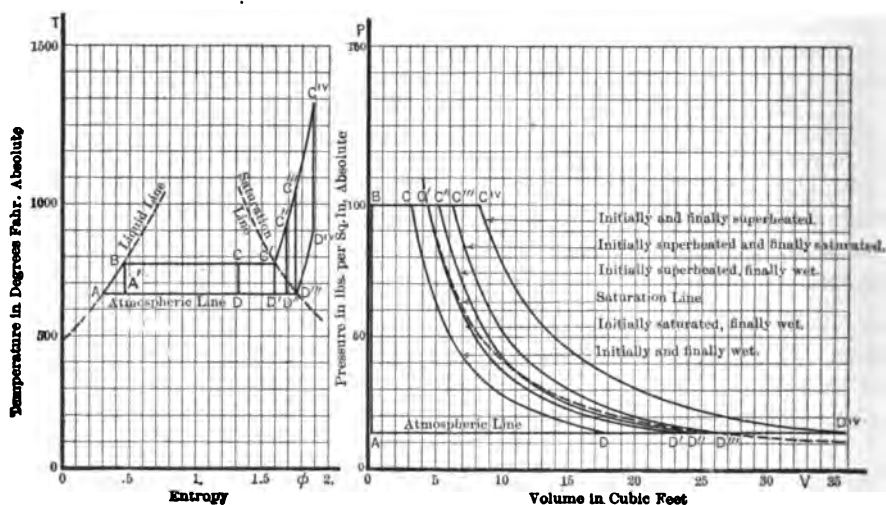


FIG. 150.—Rankine Steam Cycle.

These expressions will apply to all the special cases of this cycle which arise from different initial qualities and changes of quality due to adiabatic expansion and lettered (a), (b), (c), (d), (e) in the last Section.

Case a. Diagram ABCD, Fig. 150, steam initially wet, finally more wet.

$$Q_1 = q_1 - q_2 + x_c L_1, \quad Q_2 = x_d L_2. \quad \text{Therefore} \quad W = J(q_1 - q_2 + x_c L_1 - x_d L_2), \quad (803)$$

and

$$E = 1 - \frac{x_d L_2}{q_1 - q_2 + x_c L_1}. \quad \dots (804)$$

These two expressions Eqs. (803) and (804) for work and efficiency do not contain all independent variables because the quality x_d is a function of the original quality x_c , and of the extent of the adiabatic expansion. If tables or charts are available, all these quantities may be read off and the answer obtained at this point. It is, however, desirable that this as well as similar expressions for other cycles be reduced to a function of temperature, or other fundamental variable, so that they may be compared without proceeding to

numerical substitution. It has been shown in Section (97), Eq. (790) that for two points on the same adiabetic, the product of low-temperature quality, into the entropy of complete vaporization, is equal to the same for the high temperature added to the entropy change of the liquid from low temperature to high, or for this case, $x_s(\phi_{\phi''} - \phi_a) = x_c(\phi_c - \phi_b) + (\phi_b - \phi_a)$, (805)

$$\text{and} \quad x_d L_2 = \frac{T_a}{T_b} x_c L_1 + T_a \log_e \frac{T_b}{T_a} \quad (806)$$

Substitution of Eq. (805) in Eq. (802) gives two forms for work,

$$W = J \left[q_1 - q_2 + x_c L_1 \left(1 - \frac{T_a}{T_b} \right) - T_a \log_e \frac{T_b}{T_a} \right], \quad (807)$$

$$= J \left[(T_b - T_a) + x_c L_1 \left(1 - \frac{T_a}{T_b} \right) - T_a \log_e \frac{T_b}{T_a} \right] (\text{approx.})$$

$$= J \left[(T_b - T_a) \left(1 + \frac{x_c L_1}{T_b} \right) - T_a \log_e \frac{T_b}{T_a} \right] (\text{approx.}) \quad (808)$$

The first form, Eq. (807), gives work, in terms of heats of liquid, temperatures, initial quality, and corresponding latent heat; and the second form, Eq. (808), gives work in terms of the temperatures, the high-pressure latent heat and the initial steam quality, on the assumption of unity as the specific heat of water. By putting the latent heat as a series function of temperature as in Eq. (794), Section 97, and substituting it in the last form for work, Eq. (808), a third form results, Eq. (809) in terms of initial steam quality and temperatures,

$$\dot{W} = J \left\{ (T_b - T_a) \left[1 + x_c \left(\frac{\alpha}{T_b} + \beta + \gamma T_b \right) \right] - T_a \log_e \frac{T_b}{T_a} \right\} (\text{app.}) \quad (809)$$

The efficiency may also be put in terms of temperatures, initial latent heat and quality, or entirely in terms of temperatures and initial quality by similar substitution. The first form results from the substitution of Eq. (806) in Eq. (804), giving Eq. (810a), which is exact, and Eq. (810b), which is approximate, because the specific heat of water is assumed to be unity in the latter.

$$E = 1 - \left[\frac{\frac{T_a}{T_b} x_c L_1 + T_a \log_e \frac{T_b}{T_a}}{q_1 - q_2 + x_c L_1} \right] (a); \quad = 1 - \left[\frac{\frac{T_a}{T_b} x_c L_1 + T_a \log_e \frac{T_b}{T_a}}{(T_b - T_a) + x_c L_1} \right] (\text{app.}) (b) \quad (810)$$

The first form of Eq. (810a) gives the efficiency in terms of liquid heats, temperatures, initial quality and corresponding latent heat, which reduce in Eq. (810b) to temperatures, initial quality and corresponding latent heat, and in Eq. (811) by a final modification to temperatures and initial quality only, by introducing the temperature function of latent heat,

$$E = 1 - \left[\frac{\frac{T_a}{T_b} x_c (\alpha + \beta T_b + \gamma T_b^2) + T_a \log_e \frac{T_b}{T_a}}{(T_b - T_a) + x_c (\alpha + \beta T_b + \gamma T_b^2)} \right] (\text{app.}) \quad (811)$$

In these equations for work and efficiency, $\alpha = 1151.0$; $\beta = .076$; $\gamma = -.00051$.

Case b. Diagram ABA', Fig. 150. No steam initially present. If the initial condition is that of water at the boiling-point, $x_c=0$, and the three expressions for work and efficiency take the form,

$$\left. \begin{aligned} W &= J \left[q_1 - q_2 - T_a \log_e \frac{T_b}{T_a} \right] \text{ from Eq. (807)} & (a) \\ &= J \left[T_b - T_a - T_a \log_e \frac{T_b}{T_a} \right] \text{ from Eq. (808) or (809)} & (b) \end{aligned} \right\} \quad (812)$$

$$\left. \begin{aligned} E &= 1 - \left[\frac{T_a \log_e \frac{T_b}{T_a}}{q_1 - q_2} \right] \text{ from Eq. (810a)} & (a) \\ &= 1 - \left[\frac{T_a \log_e \frac{T_b}{T_a}}{T_b - T_a} \right] \text{ from Eq. (810b) or (811)} & (b) \end{aligned} \right\} \quad (813)$$

Case c. Diagram ABC'D'A, Fig. 150. Steam initially dry saturated. If the initial condition is that of dry saturated steam, $x_c=1$, and the work and efficiency equation take the following special forms:

$$\left. \begin{aligned} W &= J \left[q_1 - q_2 + L_1 \left(1 - \frac{T_a}{T_b} \right) - T_a \log_e \frac{T_b}{T_a} \right] \text{ from Eq. (807)} & (a) \\ &= J \left[(T_b - T_a) \left(1 + \frac{L_1}{T_b} \right) - T_a \log_e \frac{T_b}{T_a} \right] \text{ from Eq. (808)} & (b) \\ &= J \left\{ (T_b - T_a) \left[1 + \left(\frac{\alpha}{T_b} + \beta + \gamma T_b \right) \right] - T_a \log_e \frac{T_b}{T_a} \right\} \text{ from Eq. (809)} & (c) \end{aligned} \right\} \quad (814)$$

$$\left. \begin{aligned} E &= 1 - \left[\frac{\frac{T_a}{T_b} L_1 + T_a \log_e \frac{T_b}{T_a}}{q_1 - q_2 + L_1} \right] \text{ from Eq. (810a)} & (a) \\ &= 1 - \left[\frac{\frac{T_a}{T_b} L_1 + T_a \log_e \frac{T_b}{T_a}}{T_b - T_a + L_1} \right] \text{ from Eq. (810b)} & (b) \\ &= 1 - \left[\frac{\frac{T_a}{T_b} (\alpha + \beta T_b + \gamma T_b^2) + T_a \log_e \frac{T_b}{T_a}}{(T_b - T_a) + (\alpha + \beta T_b + \gamma T_b^2)} \right] \text{ from Eq. (811)} & (c) \end{aligned} \right\} \quad (815)$$

Case d. Diagram ABC'C''D''A, Fig. 150. Steam initially superheated and finally wet. Steam initially superheated to a degree indicated by condition C'', that is, such as will allow it to become wet during expansion, will have different work and efficiency equations, the general form of which is,

$$W = J(q_1 - q_2 + L_1 + h_1 - x_{c''} L_2). \quad (816) \quad E = 1 - \frac{x_{c''} L_2}{q_1 - q_2 + L_1 + h_1}. \quad (817) \quad (816)$$

As before, the final quality $x_{c''}$ is not an independent variable, but related to initial superheat by the equal entropy relations,

$$(\phi_{c''} - \phi_a) = (\phi_{c'} - \phi_a) \quad \text{and} \quad x_{c''}(\phi_{c''} - \phi_a) = (\phi_{c'} - \phi_{c'}) + (\phi_{c'} - \phi_b) + (\phi_b - \phi_a),$$

or

$$x_{e''} \frac{L_2}{T_a} = S_{e''} \log_e \frac{T_{e''}}{T_b} + \frac{L_1}{T_b} + \log_e \frac{T_b}{T_a}.$$

$$\therefore x_{e''} L_2 = S_{e''} T_a \log_e \frac{T_{e''}}{T_b} + \frac{T_a}{T_b} L_1 + T_a \log_e \frac{T_b}{T_a}. \quad (818)$$

This Eq. (818) will on substitution in Eqs. (816) and (817) give the work and the efficiency in terms of completely independent variables which may be reduced entirely to temperatures, as the only two.

Case e. Diagram ABC'C''D''A, Fig. 150. Steam initially superheated and finally dry saturated. Steam with enough initial superheat to become just dry and saturated after expansion will have characteristics similar to the last case, but in which $x_{e'}$ becomes $x_{e''}$, and $x_{e''}=1$, which finally yield:

$$W = J(q_1 - q_2 + L_1 + h_1 - L_2). \quad (819) \quad E = 1 - \frac{L_2}{q_1 - q_2 + L_1 + h_1}. \quad (820) \quad (819) \quad (820)$$

In these equations the heat of superheat, $h_1 = S_{e''}(T_{e''} - T_b)$ is not an independent variable, being contingent on dryness without superheat after expansion, therefore, the temperature $T_{e''}$ is fixed by the entropy relation that leads to Eq. (819) which, making $x_{e''}=1$, and $T_{e'} = T_{e''}$ becomes Eq. (821),

$$\frac{L_2}{T_a} = S_{e''} \log_e \frac{T_{e''}}{T_b} + \frac{L_1}{T_b} + \log_e \frac{T_b}{T_a}. \quad (821)$$

However, the two cases of small superheat just reviewed will give work per cycle and efficiency values between those for the two cases of, initially no superheat, and initially high superheat sufficient to leave the steam superheated after expansion, so that the equations of the two cases of small superheat are omitted and those for high superheat set down.

Case f. Diagram ABC'C''D''D'''A, Fig. 238. Steam constantly superheated. When superheat is initially high enough to leave some superheat after expansion, however small the amount, the work and efficiency equations become

$$W = J(q_1 - q_2 + L_1 - L_2 + h_1 - h_2). \quad (822) \quad E = 1 - \frac{L_2 + h_2}{q_1 - q_2 + L_1 + h_1}. \quad (823) \quad (822) \quad (823)$$

In these two equations the heats of superheat h_2 and h_1 are not independent, but related by their temperatures through the equal entropy relations,

$$\phi_a^{IV} - \phi_a = (\phi_c^{IV} - \phi_a),$$

or

$$S_{e''d} \log_e \frac{T_d^{IV}}{T_a} + \frac{L_2}{T_a} = S_{e''c} \log_e \frac{T_c^{IV}}{T_b} + \frac{L_1}{T_b} + \log_e \frac{T_b}{T_a}. \quad (824)$$

From this Eq. (824) the relation between T_c^{IV} and T_d^{IV} can be found, which on substitution in Eq. (825), giving the final heat of superheat, in terms of the initial for substitution in Eqs. (822) and (823), will leave only independent variables.

$$h_2 = h_1 \left[\frac{S_{e''d}^{IV} (T_d^{IV} - T_a)}{S_{e''c}^{IV} (T_c^{IV} - T_b)} \right]. \quad (825)$$

This is a very cumbersome transformation and is only approximate. It helps a little to assume a constant value for specific heat of superheated steam $S_{d''d'v} = S_{c'c'v}$, but the work is much involved and will therefore not be given here.

These methods of work and efficiency determination furnish a thermal means of deriving some other important quantities, whether W and E are calculated from equations such as have been developed here, or from the shorter, more exact, and, therefore, more practical means, provided by the tables and charts of properties as related to entropy. Thus

$$\left(\begin{array}{c} \text{Mean effective pressure in pounds} \\ \text{per square inch} \end{array} \right) = \left(\frac{W}{144 \times (\text{vol. per lb. at low press.})} \right) \quad (826)$$

If V_2 be taken as the specific volume (tabular value) of dry saturated steam at the low pressure then this will have the two forms (a) and (b) of Eq. (827), which may be substituted in Eq. (826) for mean effective pressure. The second form is only approximate, since it assumes the superheated steam to behave as a perfect gas.

$$\left. \begin{aligned} (\text{Vol. per lb. at low press.}) &= V_2 x_2 \text{ if wet finally;} & (a) \\ &= V_2 \left(\frac{\text{Temp. of superheated}}{\text{Temp. of dry saturated}} \right) \text{ if superheated.} & (b) \end{aligned} \right\} (827)$$

Again, since in round numbers the horse-power of 33,000 ft.-lbs. per minute is practically equivalent to 2545 B.T.U. per hour (more exactly 2546.56), the hourly heat consumption of the cycle per horse-power is given by,

$$(\text{B.T.U. per hr. per I.H.P.}) = \frac{2545}{E} \quad \dots \dots \dots (828)$$

Also if WR is the water rate or pounds steam per hour per horse-power

$$\left. \begin{aligned} (\text{Cyclic water rate}) = WR &= \frac{(\text{B.T.U. supplied per hr. per I.H.P.})}{(\text{B.T.U. per lb. steam})} & (a) \\ &= \frac{(\text{B.T.U. supplied per hr. per I.H.P.})}{\left(\begin{array}{c} \text{B.T.U. per lb. st.} \\ \text{at high press.} \\ \text{above } 32^\circ \text{ F.} \end{array} \right) - \left(\begin{array}{c} \text{Heat of liquid at} \\ \text{low press. above} \\ \text{ } 32^\circ \text{ F.} \end{array} \right)} & (b); \quad = \frac{2545}{E(H_1 - q_2)} & (c) \end{aligned} \right\} (829)$$

Steam generated continuously at the high pressure and issuing from a perfect nozzle to a region of the low pressure would have a velocity measured by the work done on it by its own expansion. Since its increase in kinetic energy from rest, is $\frac{1}{2}Mu^2 = \frac{w}{2g}u^2$, the work done per pound of steam w , where $w=1$, will give to the pound of steam, the velocity in feet per second of

$$u = \sqrt{2g \times W} = \sqrt{64.4 \times W} \text{ (app.)} \quad \dots \dots \dots (830)$$

It is clear, therefore, that this maximum attainable jet velocity is directly proportional to the square root of the work done in foot-pounds or of the

difference between the total heats per pound of steam at the conditions before and after adiabatic expansion. This fact prompts the paralleling of the heat scale on the total heat-entropy or Mollier diagram, Chart 45 of the Tables, by a velocity scale computed as above.

Attention has already been called to the possibility of evaluation of the work of cycles of this sort by means of the temperature-entropy and the total heat-entropy diagrams; this is the only practical method for engineering calculation, a fact that is appreciated only by solving problems first by the equation method and later by the tabular and chart methods. The Mollier diagram is constructed to read the differences ($Q_1 - Q_2$) directly so that work in

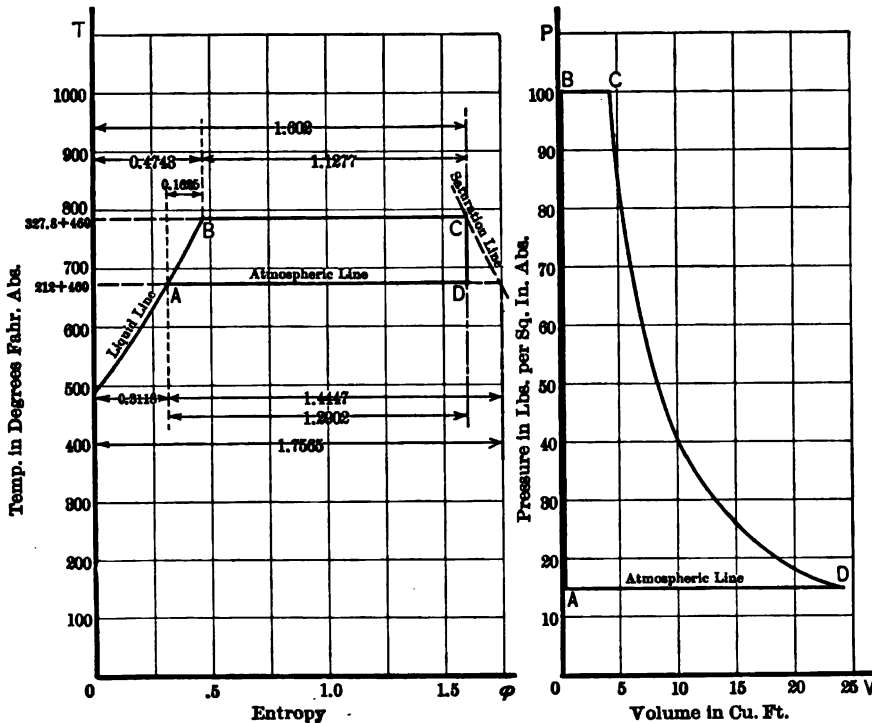


FIG. 151.—Example of the Rankine Steam Cycle.

heat units is only a question of reading the scale, and efficiency, a mere question of slide-rule division. Moreover, the operations are precisely the same for steam of any initial quality, ranging from all water to the highest superheat, and changes in quality, superheat and volume are just as easily read off as the changes in total heat.

As the Rankine cycle is the only steam cycle that reasonably approximates in representation the action of steam in cylinders and nozzles, for which work and efficiency can be read off the entropy charts, this is an additional reason why it has become the standard of reference.

To make clear the great difference in labor involved in the solution of

problems by tabular and by chart methods compared to the algebraic method, the following numerical examples are worked out. It is important to remember that the table and chart method is more accurate, as well as shorter, because it involves no assumption as to constancy of specific heats of water or steam, nor as to the accuracy of temperature relations with latent heat, all of which are necessary to secure even workable equations.

Required the work per pound of steam, the thermal efficiency, heat consumption, cyclic water rate and mean effective pressure for the Rankine cycle between 100 lbs. per square inch absolute and one atmosphere, for steam initially dry. This case is illustrated by Fig. 151.

First Method. By steam tables alone, without entropy data.

$$\text{From the tables } \left\{ \begin{array}{l} T_1 = 327.8^\circ \text{ F.} = 788^\circ \text{ F. absolute (app.).} \\ T_2 = 212^\circ \text{ F.} = 672^\circ \text{ F. absolute (app.).} \\ \text{Heat of liquid (32^\circ \text{ F. to } 327.8^\circ \text{ F.})} = 298 \text{ B.T.U.} \\ \text{Heat of liquid (32^\circ \text{ F. to } 212^\circ \text{ F.})} = 180 \text{ B.T.U.} \\ \text{Heat of liquid (212^\circ \text{ F. to } 327.8^\circ \text{ F.})} = 118 \text{ B.T.U. } A \text{ to } B. \\ \text{Latent heat at } 327.8^\circ \text{ F.} = 888 \text{ B.T.U. } B \text{ to } C. \\ \text{Latent heat at } 212^\circ \text{ F.} = 970.4 \text{ B.T.U., } A \text{ to } X. \\ \text{Specific volume at } 212^\circ \text{ F.} = 26.79 \text{ cu.ft.} \end{array} \right.$$

Hence

$$Q_1 = \text{Heat added} = 888 + 118 = 1006 \text{ B.T.U.}$$

$$Q_2 = \text{Heat abstracted} = \text{final quality} \times 970.4 \text{ B.T.U.}$$

$$\text{Final quality} = \frac{\overline{AD}}{\overline{AX}} = \frac{\phi_d - \phi_a}{\phi_x - \phi_a} = \frac{\phi_c - \phi_a}{\phi_x - \phi_a},$$

$$\phi_b - \phi_a = \log_e \frac{788}{672} = .1625, \quad \phi_c - \phi_b = \frac{888}{788} = 1.1277,$$

$$\phi_c - \phi_a = \phi_b - \phi_a = 1.1277 + .1625 = 1.2902, \quad \phi_x - \phi_a = \frac{970.4}{672} = 1.4447.$$

$$\text{Whence final quality} = \frac{1.2902}{1.4447} = .893, \quad \text{and}$$

$$Q_2 = \text{Heat abstracted} = 970.4 \times .893 = 866.57, \text{ B.T.U.}$$

$$W = J(Q_1 - Q_2) = J(1006 - 866.57) = J(139.43) = 108476.5 \text{ ft.-lbs.}$$

$$\text{Efficiency} = \frac{139.43}{1006} = 13.8\%. \quad \text{Final vol. of steam} = .893 \times 26.79 = 23.92 \text{ cu.ft.}$$

$$\text{m.e.p.} = \frac{J(139.43)}{144 \times .893 \times 26.79} = \frac{108,476.5}{144 \times 23.92} = 31.5 \text{ lbs. sq.in.}$$

$$\text{B.T.U. per hr. per I.H.P.} = \frac{2545}{E} = \frac{2545}{.138} = 18442.$$

$$\text{Water rate of the cycle} = \frac{2545}{.138 \times 1006} = 18.33 \text{ lbs.}$$

$$\text{Steam jet velocity} = \sqrt{64.4 \times 108,476.5} = 2643 \text{ ft. per second.}$$

Second Method. By ordinary entropy tables.

From 32° F. to 328° F. entropy change for liquid = .4743

From 32° F. to 212° F. entropy change for liquid = .3118

From 212° F. to 328° F. entropy change for liquid = .1625

At 328° F. entropy change for vaporization = 1.1277

From liquid at 212° F. to dry steam at 328° F. entropy change = 1.2902

From liquid at 212° F. to dry steam at 212° F. entropy change = 1.4447

Difference in entropy = .1545

Moisture after expansion = $\frac{.1545}{1.4447} = 10.7\%$,

Quality after expansion = $\frac{1.2902}{1.4447} = 89.3\%$,

Heat supplied per lb. = $(298 - 180) + 888 = 118 + 888 = 1006$ B.T.U.

Heat abstracted per lb. = $.893 \times 970.4 = 866.6$ B.T.U.

Work = 139.4 B.T.U.

Efficiency = $\frac{139.4}{1006} = 13.8\%$

The other quantities follow in the same way.

Third Method. By equal entropy table, from Peabody's Steam Tables.

For dry steam at 328° F. find (page 94) for quality = 1, (.9984), the entropy = 1.60, and B.T.U. per pound above 32° F. = 1184.5. For wet steam at the same entropy at 212° F. read directly the quality = 89.15 per cent, and B.T.U. per pound = 1044.9 above 32°. Heat of the liquid (Peabody's Tables) 180.3.

Hence, Heat supplied above 212° = $1184.5 - 180.3 = 1004.2$ B.T.U.

Heat abstracted above 212° = $1044.9 - 180.3 = 864.6$ B.T.U.

Work = $(1004.2 - 864.6) = 139.6$ B.T.U.

Efficiency = $\frac{139.6}{1004.2} = 14\%$.

The other quantities are derived by obvious means.

Fourth Method. Ordinary temperature entropy diagram with constant volume, quality and total heat lines. The final quality, initial and final heats are read off directly, work and efficiency are given by difference and ratio of these heats, which are slide-rule determinations.

Fifth Method. Total heat entropy diagram. This method eliminates one step in finding work, but is no shorter than the last for efficiencies. Work is given by the length of a vertical line joining initial and final conditions and is known without first finding the total heats.

Sixth Method. By formulas. As this is merely a question of numerical substitution in equations, which by inspection are clearly complex, it is omitted from the text, but students are advised to do the work to familiarize themselves with the application of the formulas.

To show more clearly the relations between the various initial conditions and results derived from them, for this Rankine cycle, curves have been plotted and are given in the handbook of tables, for reference. The curves are in

sets, as it is not convenient to show simultaneously all relations on a single curve. Thus, Chart No. 46, gives the thermal efficiency direct and shows how it varies with initial and back pressures and with the corresponding temperatures for steam initially dry and saturated. As heat consumption is in constant inverse proportion to the thermal efficiency, its scale is made to parallel that of thermal efficiency. Water rate being related to heat consumption through the high-pressure total heat per pound of steam above the terminal temperature, for the Rankine cycle, it is necessary to use the family of curves in the left-hand angle to pass from the vertical ordinate of heat consumption, on thermal efficiency, to the horizontal coordinate of corresponding water rate on the left. The resulting chart gives a direct solution of a problem on the relative value of two Rankine steam cycles for any pressure ranges when the steam is initially dry and, moreover, gives a numerical answer for the thermal efficiency, heat consumption and water rate.

It is desirable to compare for given pressures the relative performance of the cycle with steam in various initial conditions of wetness or superheat; in order to assist in this the curves of Chart No. 47 have been plotted according to a system similar to that used in Chart No. 46, but with initial quality as the prime variable, drawn only for steam initially at 200 lbs. per sq.in. gage pressure, which may be regarded as the high limit of present day general practice.

These last two curves show the essential relations between the efficiency, heat consumption or cyclic water rate, and the initial pressure or quality, but as capacity for work is just as important as efficiency of performance another set of diagrams has been plotted which are given in Charts No. 48 and 49, showing the work per pound of steam, and the corresponding maximum jet velocity for turbine nozzles or mean effective pressures for cylinder engines. In Chart 48, these two results, fundamental to capacity for work, are shown as functions of initial pressures for steam initially dry, and in the second case as functions of initial quality for 200 lbs. per square inch gage initial pressure, in both cases, for various back pressures. The very low mean effective pressure for low initial pressures, such for example as in the region of one atmosphere, even with back pressures of only a fraction of a pound, is the very best demonstration of the value of modern low-pressure steam turbines against their only competitor, a low-pressure cylinder. Cylinder sizes must be inversely proportional to mean effective pressures, but there is no similar relation for turbines, the size of which depends on bucket speed and number of stages for a given output, so that the turbine is a logical substitute for cylinders in low-pressure ranges even when its mechanism losses exceed those of cylinders, which is often the case.

Referring to the cyclic efficiency curves, Chart No. 46 a fairly rapid rise, with increase of initial pressure is found at first, which becomes less as the pressure rises, but a most striking increase in efficiency with back-pressure reduction is demonstrated. Thus the gain from $\frac{1}{2}$ lb. to $\frac{1}{4}$ lb. absolute is almost as great as from 15 to 10 lbs., proving the superior value of low over high back pressure from the efficiency standpoint. Initial quality has

most influence when the steam is very wet, more wet than is ever used, as appears from Chart 47, but the initial quality is of greater importance for low than for high back pressures shown by the slopes of the .25 lb. and 25.00 lb. curves near saturation especially. Initial superheat has a beneficial effect in all cases, but never large at any time.

Example 1. *Calculation and use of diagram Chart No. 46, giving Rankine cycle efficiency, water rate and heat consumption as functions of initial pressure.* The method used in plotting is illustrated by the location of point A, for assumed conditions of initially dry saturated steam at 100 lbs. per square inch gage pressure and 10 lbs. per square inch absolute back pressure. The efficiency is given directly by Eq. (815, b) as reference to the general $T\phi$ diagram, Fig. 150, shows these data to fall under Case c. Therefore

$$E_a = 1 - \left(\frac{T_a L_1 + T_a \log_e \frac{T_b}{T_a}}{T_b - T_a + L_1} \right) = 1 - \left(\frac{653.2 \times 880 + 653.2 \log_e \frac{797.9}{653.2}}{797.9 - 653.2 + 880} \right) = 16.94 \text{ per cent,}$$

and the point A is located opposite this per cent on the efficiency scale. Passing to the left-hand angle of the diagram the point A' is located by Eq. (829, c),

$$\text{Water rate} = \frac{2545}{E(H_1 - q_2)} = \frac{2545}{.169(1188.7 - 161.1)} = 14.62 \text{ lbs. per I.H.P. hour,}$$

which fixes the abscissa of point A', the ordinate being 16.94 per cent efficiency.

To illustrate its use: Find the Rankine cycle efficiency, heat and steam consumption for an initial pressure of 150 lbs. per square inch gage and dry saturated steam with a back pressure of 10 lbs. per square inch absolute. Starting at the initial pressure point B, project up to the 10-lb. back pressure curve point C, and then across to the efficiency scale point D, reading there a thermal efficiency of 19.3 per cent and a heat consumption of 13,200 B.T.U. per hour per I.H.P. Continuing across horizontally to the back pressure curve of 10 lbs. in the left-hand angle to point E and thence downward to the water-rate scale point F, the value 12.6 lbs. steam per hour per I.H.P. is read off directly.

Example 2. *Calculation and use of the curves in Chart No. 47, Rankine cycle performance as a function of initial quality.* The determination of the position of the curve ABCD will illustrate the whole series. This represents the case of 2 lbs. per square inch absolute back pressure and 200 lbs. per square inch gage initial pressure. Point A represents an initial condition of all water; B, 50 per cent steam; C, dry saturated steam; D, 200 degrees superheat. Reference is first made to the general $T\phi$ diagram to find under which case each set of data belongs so as to permit the selection of the proper formula. The conditions represented by point A are those of Case b, and Eq. (813, b) applies. Therefore,

$$E_a = 1 - \left(\frac{T_a \log_e \frac{T_b}{T_a}}{T_b - T_a} \right) = 1 - \frac{586.1 \log_e \frac{847.9}{586.1}}{847.9 - 586.1} = 17.34\%,$$

which is the ordinate of point A, the abscissa of which is zero. Similarly a condition of 50 per cent quality falls under Case a, and Eq. (810, b) applies. Therefore,

$$E_b = 1 - \left(\frac{\frac{T_a}{T_b} x_c L_1 + T_a \log_e \frac{T}{T_a}}{T_b - T_a + x_c L_1} \right) = 1 - \left(\frac{\left(\frac{586.1}{847.9} \times .5 \times 838 \right) + \left(586.1 \log_e \frac{847.9}{586.1} \right)}{847.9 - 586.1 + (.5 \times 838)} \right) = 25.68\%.$$

Dry saturated steam falls under Case *c*, and Eq. (815, *b*) applies, so that

$$E_c = 1 - \left(\frac{\frac{T_a}{T_b} L_1 + T_a \log_e \frac{T_b}{T_a}}{T_b - T_a + L_1} \right) = 1 - \left(\frac{\left(\frac{586.1}{847.9} \times 838 \right) + 586.1 \log_e \frac{847.9}{586.1}}{847.9 - 586.1 + 838} \right) = 27.66\%.$$

An initial superheat of 200° falls under Case *d*, and Eq. (817) applies, hence

$$E_d = 1 - \frac{x_a L_2}{q_1 - q_2 + L_1 + h_1} = 1 - \frac{x_a L_2}{T_b - T_a + L_1 + h_1} \text{ (approx.)}$$

Referring to the Mollier diagram, Chart 45 of the tables to find x_a , we note $x_a = .851$, which could have been calculated from Eq. (818). Substituting it,

$$E_d = 1 - \frac{.851 \times 1021}{847.9 - 586.1 + 838 + 110.5} = 28.24\%.$$

In the left-hand angle the corresponding curve to be located is $A'B'C'D'$. The ordinates of these points are the same as for *A*, *B*, *C*, and *D*, so it is only the abscissas that are to be determined as follows from Eq. (829, *c*):

$$\begin{aligned} (WR)_{a'} &= \frac{2545}{(T_b - T_a)E} = \frac{2545}{(847.9 - 586.1) \times .1734} = 56.1 \text{ lbs. I.H.P. hr.} \\ (WR)_{b'} &= \frac{2545}{(T_b - T_a + x_a L_1)E} = \frac{2545}{(847.9 - 586.1 + .5 \times 838) \times .2568} = 14.55 \text{ " " } \\ (WR)_{c'} &= \frac{2545}{(T_b - T_a + L_1)E} = \frac{2545}{(847.9 - 586.1 + 838) \times .2766} = 8.37 \text{ " " } \\ (WR)_{d'} &= \frac{2545}{(T_b - T_a + L_1 + h_1)E} = \frac{2545}{(847.9 - 586.1 + 838 + 110.5) \times .2824} = 7.45 \text{ " " } \end{aligned}$$

To illustrate the use of the chart, find the thermal efficiency, heat and steam consumption, for the Rankine cycle, when steam is 90 per cent initially dry at 200 lbs. per square inch gage pressure, and the back pressure 15 lbs. per square inch absolute. From the scale of quality at 90 per cent, point *E*, project up to point *F* on 15-lb. curve, and then horizontally to point *G* at 18.98 per cent thermal efficiency and 13,400 B.T.U. per hour per I.H.P. heat consumption. Continue across to *H* and down to *K*, reading the water rate value 14.4 lbs. of steam per hour per I.H.P. on the bottom scale.

Example 3. Calculation and use of diagram Chart No. 48, giving work, jet velocity and mean effective pressure, as functions of initial pressure for the Rankine cycle. Assume for point *A* an initial condition of dry saturated steam at 150 lbs. per square inch gage and 5 lbs. per square inch absolute back pressure. Then the cycle work will be given by Eq. (814, *b*) under Case *c*, so that

$$\begin{aligned} W_a &= J \left[(T_b - T_a) \left(1 + \frac{L_1}{T_b} \right) - T_a \log_e \frac{T_b}{T_a} \right] \\ &= 778 \left[(825.9 - 622.3) \left(1 + \frac{856.9}{825.9} \right) - 622.3 \log_e \frac{825.9}{622.3} \right] = 185,700 \text{ ft.-lbs.} \end{aligned}$$

Passing to the left the abscissa of point *A'* is to be found and this is given by Eq. (826) as

$$(\text{m.e.p.}) = \frac{W}{144 V_1} = \frac{185,700}{144 \times 60.49} = 21.32 \text{ lbs. per square inch.}$$

In the above the value of V_1 is found from the specific volume of dry saturated steam

and final quality, or better, it may be taken directly from the Mollier diagram. The velocity scale paralleling that for work is calculated directly from Eq. (830) and when

$$W = 250,000, \quad u = \sqrt{64.14 \times 250,000} = 4010 \text{ ft. per second.}$$

To illustrate the use of this chart (No. 48), find the jet velocity, work per pound of steam, and mean effective pressure for the Rankine cycle for steam at 75 lbs. initial pressure gage, dry and saturated expanding to 10 lbs. absolute. Project up from point *B* to point *C* and across to point *F* where there is read, work done = 115,000 ft.-lbs. per pound of steam. Continuing across to *D* and down to *E*, (m.e.p.) = 23.5 lbs. per square inch, or continuing *CD* across to *G* the jet velocity is 2790 ft. per second.

Example 4. Calculation and use of Chart No. 49, giving for the Rankine cycle, work, jet velocity and mean effective pressure as functions of initial quality. Curve *ABCD* is fixed by the four points, *A* for all water initially; *B* for quality 75 per cent; *C* for 100 per cent, and *D* for 250 degrees superheat, so each point belongs to a different case of the Rankine cycle and is given by a different though corresponding equation.

From Eq. (812, *b*), Case *b*.

$$W_a = J \left(T_b - T_a - T_a \log_e \frac{T_b}{T_a} \right) = 778 \left(847.9 - 662.3 - 622.3 \log_e \frac{847.9}{622.3} \right) = 25,750 \text{ ft.-lbs.}$$

$$\begin{aligned} \text{From Eq. (808), Case } a. \quad W_b &= J \left[(T_b - T_a) \left(1 + \frac{x_c L_1}{T_b} \right) - T_a \log_e \frac{T_b}{T_a} \right] \\ &= 778 \left[(847.9 - 622.3) \left(1 + \frac{.75 \times 838}{847.9} \right) - 622.3 \log_e \frac{847.9}{622.3} \right] = 156,000 \text{ ft.-lbs.} \end{aligned}$$

$$\begin{aligned} \text{From Eq. (814 } b), \text{ Case } c. \quad W_c &= J \left[(T_b - T_a) \left(1 + \frac{L_1}{T_b} \right) - T_a \log_e \frac{T_b}{T_a} \right] \\ &= 778 \left[(847.9 - 622.3) \left(1 + \frac{838}{847.9} \right) - 622.3 \log_e \frac{847.9}{622.3} \right] = 200,000 \text{ ft.-lbs.} \end{aligned}$$

$$\begin{aligned} \text{From Eq. (816), Case } d. \quad W_d &= J(T_b - T_a + L_1 - x_d L_2 + h_1) \\ &= 778 [847.9 - 622.3 + 838 - (.899 \times 1000.3) + 135.2] = 231,200 \text{ ft.-lbs.} \end{aligned}$$

To illustrate the use of the diagram, find work, jet velocity, and mean effective pressure, for the Rankine cycle when initial pressure is 200 lbs. per square inch gage, 50° superheat and back pressure 1 lb. per square inch absolute. Projecting up from point *E* to *F* and across to *G*, read, work = 272,000 ft.-lbs., velocity = 4190 ft. per second, and stopping on the 1-lb. curve at point *H* the mean pressure 7.4 lbs. per square inch is read directly below at *K*.

Prob. 1. A locomotive engine with steam at 250 lbs. per square inch absolute initial pressure and 90 per cent quality, exhausting to atmosphere, uses 24 lbs. of steam per hour per I.H.P. What fraction of the Rankine cycle is realized?

Prob. 2. A low-pressure steam turbine supplied with atmospheric steam 75 per cent quality, exhausting to a 25-in. vacuum at 3000 ft. elevation, is developing 70 per cent of the Rankine cycle heat. What is its heat consumption per hour, its water rate, thermal efficiency, and work per pound of steam?

Prob. 3. A combination low-pressure turbine and high-pressure piston engine operates on 60 per cent of the Rankine cycle efficiency in the former and 70 per cent in the latter. The initial pressure is 190 lbs. per square inch absolute with 100° superheat; receiver quality 80 per cent, pressure 30 lbs., and back pressure $\frac{1}{2}$ lb. per square inch absolute. What is the combined thermal efficiency and water rate?

Prob. 4. What would be the jet velocity of the turbine of Problem 2 if single stage, and all losses took place in the nozzles?

Prob. 5. If a five-pressure stage turbine suffered no heat losses whatever and each stage were to develop the same energy and jet velocity from 190 lbs. and 100° superheat to 1 lb. absolute, (a) what would be the pressure in each stage, (b) the jet velocity, (c) the initial quality? (d) Plot a curve through these quality points.

Prob. 6. Compare the thermal efficiencies of two turbines working between the pressures of Problem 5, one with 100 per cent initial quality and the other with 200° superheat, each developing 65 per cent of the Rankine cycle. What will be the differences in the respective heat consumptions and water rates.

Prob. 7. What mean effective pressure would be developed by a Rankine cycle for the data of Problem 6 and how much cooling water between 45° F. and 70° F. would be required per hour by a condenser per cyclic horse-power.

Prob. 8. The value of a reheating receiver is to be judged by comparing the Rankine cycle efficiency with and without it for the following data. Initial pressure 200 lbs. per square inch absolute, 50° superheat, receiver pressure 40 lbs., and back pressure 2 lbs. per square inch absolute. Find the thermal efficiency for (a) one-stage adiabatic expansion; (b) two-stage adiabatic expansion with receiver reheat to original temperature; express the value of the reheat in any proper terms.

Prob. 9. Which case of the Rankine cycle corresponds to the following data:

Initial press. lbs. sq.in. abs.	Initial quality.	Back press. lbs. sq.in. abs.	Initial press. lbs. sq.in. abs.	Initial quality.	Back press. lbs. sq.in. abs.
(a) 200	200° superh.	20.0	(g) 140	80 per cent	2.0
(b) 190	150	10.0	(h) 130	70	4.0
(c) 180	100	5.0	(i) 120	80	5.0
(d) 170	50	4.0	(j) 110	90	10.0
(e) 160	dry saturated	2.0	(k) 100	dry saturated	20.0
(f) 150	90 per cent	1.0			

Prob. 10. Find for any one of the cases of Problem 9, the heat added and abstracted, work per pound, mean effective pressure, thermal efficiency, heat consumption, and cyclic water rate by each of the methods of the text.

100. The Carnot Steam Cycle and Derivatives. Work, Mean Effective Pressure, Water Rate, Heat Consumption and Efficiency of Steam Cycle II, Adiabatic Expansion and Compression, Constant Pressure Heat Addition and Abstraction. To this cycle illustrated in Fig. 152 the same general Eqs. (800) and (801) of Sec. 99 apply, and need not be reproduced. It is in the evaluation of the heat supplied and abstracted that all differences between this and the Rankine Cycle arise. In this case there is no heating of the liquid, so that the heat supplied to or abstracted from the cycle will have a different relation to the total heat per pound of steam. Steam in condition C has received from the beginning of the cycle some part of the latent heat at the high pressure, so that the heat supplied is given by the product of quality into latent heat and this is equal to the total heat of the wet steam above 32°, less the heat of the liquid. Hence if H_1 is the total heat of the steam in the initial condition whether wet or dry,

$$\begin{aligned}
 Q_1 = H_1 - q_1 &= xL_1 \text{ for steam initially wet;} & (a) \\
 &= L_1 \text{ for steam initially dry saturated;} & (b) \\
 &= L_1 + h_1 \text{ for steam initially superheated.} & (c)
 \end{aligned}
 \quad . \quad (831)$$

The heat abstracted is, however, differently related to the total heat per pound, because the steam is never fully condensed at the low temperature in this cycle, but is reduced to water by steam compression at the end, therefore only part of its low-pressure latent heat is ever abstracted. Calling the low-pressure quality at which abstraction begins, x_2 , and that at which it ends, x_2' , then the heat per pound of steam not abstracted is $x_2'L_2 + q_2$, and the amount abstracted is the total heat above 32° for the condition at which abstraction begins less the above quantity, symbolically by Eq. (832).

$$\left. \begin{aligned} Q_2 &= H_2 - x_2'L_2 - q_2 = (x_2 - x_2')L_2 \text{ for wet steam} & (a) \\ &= (1 - x_2')L_2 \text{ for dry saturated steam} & (b) \\ &= (1 - x_2')L_2 + h_2 \text{ for superheated steam} & (c) \end{aligned} \right\} \quad (832)$$

Each of the cases arising from various positions of the adiabatic expansion line due to different initial quality will be taken up separately for evaluation.

Case a. Steam initially wet, ABCD, Fig. 152, Carnot cycle.

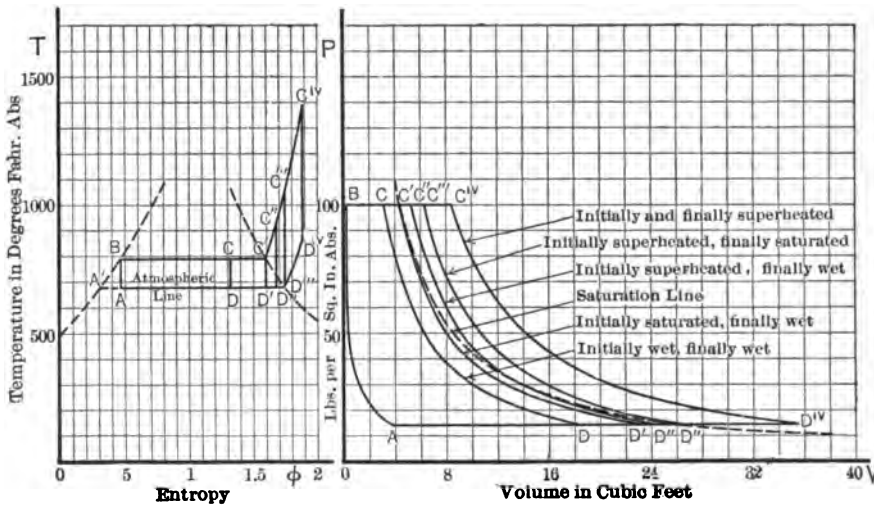


FIG. 152.—Carnot Steam Cycle and Derivatives.

$$W = J(Q_1 - Q_2) = J[x_c L_1 - (x_d - x_a)L_2]; \quad (833) \quad E = 1 - \frac{Q_2}{Q_1} = 1 - \frac{(x_d - x_a)L_2}{x_c L_1}. \quad (834)$$

In these two equations, Eq. (833) for work, and Eq. (834) for efficiency, there are two dependent variables x_d and x_a which are related high pressure qualities at C and B by conditions of equal entropy, Eq. (835) and Eq. (836):

$$(\phi_a - \phi_a') = (\phi_b - \phi_a'), \quad \text{or} \quad \frac{x_a L_2}{T_a} = \log_e \frac{T_b}{T_a}. \quad \text{Therefore} \quad x_a L_2 = T_a \log_e \frac{T_b}{T_a}. \quad (835)$$

$$\begin{aligned} \text{Also} \quad (\phi_d - \phi_a') &= (\phi_c - \phi_a') = (\phi_c - \phi_b) + (\phi_b - \phi_a'), \quad \text{and} \quad \frac{x_d L_2}{T_a} = \frac{x_c L_1}{T_b} + \log_e \frac{T_b}{T_a}, \\ \text{or} \quad x_d L_2 &= \frac{T_a}{T_b} x_c L_1 + T_a \log_e \frac{T_b}{T_a}. \quad \dots \dots \dots (836) \end{aligned}$$

Whence combining Eq. (835) with (836) to evaluate the quality difference term of Eq. (833) and Eq. (834), this becomes:

$$(x_a - x_a)L_2 = \frac{T_a}{T_b}x_cL_1 + T_a \log_e \frac{T_b}{T_a} - T_a \log_e \frac{T_b}{T_a} = \frac{T_a}{T_b}x_cL_1. \quad (837)$$

Substitution of Eq. (837) in Eq. (833) gives Eq. (838), which is the work done by the cycle: $W = J \left[x_cL_1 - \frac{T_a}{T_b}x_cL_1 \right] = Jx_cL_1 \left(1 - \frac{T_a}{T_b} \right).$ (838)

Substitution in Eq. (834) gives the efficiency,

$$E = 1 - \frac{\frac{T_a}{T_b}x_cL_1}{x_cL_1} = \left(1 - \frac{T_a}{T_b} \right) = \left(\frac{T_b - T_a}{T_b} \right). \quad (839)$$

This is a remarkable case in several ways, characteristic of the Carnot cycle: all heat is received *at* the high temperature and abstraction of all that is abstracted occurs *at* the low temperature; it will be noted, that the *efficiency depends on the temperatures only and not at all on the quality of the steam.*

It will also be observed that Eq. (839) is a simple algebraic expression for the second law of thermodynamics as previously enunciated in section 95.

Between two temperatures, maximum and minimum, it is not possible to convert more heat into work or to get a higher thermal efficiency for any substance than is possible for this Carnot cycle. It is, therefore, a standard of comparison of value, because no system, whether using gases or vapors as media in an engine mechanism, however perfect, could yield a higher efficiency than such a Carnot cycle.

Its value as a standard of comparison is, however, often overrated, because it represents an unattainable high value for efficiency, whereas other standards like that of the Rankine for steam are attainable in proportion as the losses are located and reduced to zero. The difference between the attainable maximum for steam efficiency of the Rankine and the unattainable maximum of the Carnot, is not so great as the corresponding difference for gas engine standards of reference. In the case of gas engine efficiencies there is a very great difference between the efficiency of the Carnot for the temperatures used, and the efficiency of the most closely representative attainable ideal cycle for the particular method of working, and this difference is significant.

It is not necessary to prove that the efficiency of the Carnot cycle is a *maximum for the temperature range* since this is a self-evident fact or axiom, if the impossibility of perpetual motion in mechanisms be admitted. Yet the demonstration is very simple, by the aid of the $T\Phi$ diagram $ABCD$, Fig. 152, which is a rectangle whose height is the temperature range, its base the entropy range, and the enclosed area is the work done. It is evident that between two temperatures of working the maximum work area of any cycle will be that of the rectangle embracing and enclosing all points and this is the Carnot cycle.

Case b. *Steam initially dry saturated, ABC'D', Fig. 152, Carnot cycle.* For this case x_c becomes $x_c = 1$, and x_a becomes x_a , whence

$$W = J(Q_1 - Q_2) = J[L_1 - (x_{c'} - x_a)L_2], \quad (840) \quad E = 1 - \left(\frac{x_{c'} - x_a}{L_1}\right)L_2. \quad (841) \quad (840)$$

$$\text{But } x_a L_2 = T_a \log_e \frac{T_b}{T_a}, \text{ and } x_{c'} L_2 = \frac{T_a}{T_b} L_1 + T_a \log_e \frac{T_b}{T_a}. \therefore (x_{c'} - x_a)L_2 = \frac{T_a}{T_b} L_1 \quad (842)$$

Substitution of Eq. (842) in Eqs. (840) and (841) gives work by Eq. (843), and efficiency by Eq. (844).

$$W = J L_1 \left(1 - \frac{T_a}{T_b}\right); \quad (843) \quad E = \left(1 - \frac{T_a}{T_b}\right) = \left(\frac{T_b - T_a}{T_b}\right). \quad (844) \quad (843)$$

This is another case of the Carnot cycle, that for dry steam, as the last was that for wet steam, and the cyclic work is of course greater. Comparing the work per pound in the two cases it is found to be directly proportional to the quality of the steam initially.

Case c. *Steam initially superheated and finally wet (ABC'C''D''A), Fig. 152.* To the latent heat at the high pressure is now to be added the heat of superheat, to get the heat supplied, while the heat abstracted has the same form as before, so that the work is now given by, $W = J(Q_1 - Q_2)$, therefore,

$$W = J[L_1 + h_1 - (x_{c'} - x_a)L_2] = J[L_1 + S_{c''}(T_{c''} - T_b) - (x_{c'} - x_a)L_2], \quad (845)$$

$$\text{and efficiency, } E = 1 - \frac{(x_{c'} - x_a)L_2}{L_1 + h_1} = 1 - \frac{(x_{c'} - x_a)L_2}{L_1 + S_{c''}(T_{c''} - T_b)}. \quad (846)$$

In these equations, the two dependent variables $x_{c''}$ and x_a are related to initial quality or superheat by the expansion conditions. As before

$$x_a L_2 = T_a \log_e \frac{T_b}{T_a}. \text{ But } (\phi_{c'} - \phi_a) = (\phi_{c''} - \phi_{c'}) + (\phi_{c'} - \phi_b) + (\phi_b - \phi_a), \\ \frac{x_{c'} L_2}{T_a} = S_{c''} \log_e \frac{T_{c''}}{T_b} + \frac{L_1}{T_b} + \log_e \frac{T_b}{T_a}, \text{ or } x_{c'} L_2 = S_{c''} T_a \log_e \frac{T_{c''}}{T_b} + \frac{T_a}{T_b} L_1 + T_a \log_e \frac{T_b}{T_a}. \\ \therefore (x_{c'} - x_a)L_2 = S_{c''} T_a \log_e \frac{T_{c''}}{T_b} + \frac{T_a}{T_b} L_1. \quad (847)$$

Substitution of Eq. (847) in Eqs. (845) and (846) gives work,

$$W = J \left[L_1 + h_1 - S_{c''} T_a \log_e \frac{T_{c''}}{T_b} + \frac{T_a}{T_b} L_1 \right], \quad (848)$$

$$\text{and efficiency, } E = 1 - \frac{S_{c''} T_a \log_e \frac{T_{c''}}{T_b} + \frac{T_a}{T_b} L_1}{L_1 + h_1} \quad (849)$$

These expressions do not reduce to the simple form found for the Carnot cycle, from which Case c is a departure. This case is less efficient, indicating that superheat heat is of less value than latent heat in this cycle when judged by the *temperature limits*, which are now $T_{c''}$ and T_a . However, for the *same pressures*, initial and final, this cycle is more efficient than the Carnot, since

in practical machines it is pressure limits that control. The superior value of superheat in ordinary working over saturated steam was shown to be true also with the Rankine cycle.

Case d. Steam initially superheated and finally dry saturated, $ABC^I C^{III} D^{III} A$, Fig. 152. By substituting in the last case $x_a'' = x_a''' = 1$, the work and efficiency Eqs. (850) and (851) for this case follow at once, but h_1 , the heat of initial superheat, has a particular value, that which yields after expansion dry saturated steam. Hence

$$W = J \left[L_1 + h_1 + S_{c'''} T_a \log_e \frac{T_{c'''}}{T_b} + \frac{T_a L_1}{T_b} \right], \quad \dots \quad (850)$$

$$E = 1 - \left[\frac{S_{c'''} T_a \log_e \frac{T_{c'''}}{T_b} + \frac{T_a L_1}{T_b} }{L_1 + h_1} \right] \dots \dots \dots (851)$$

Case e. Steam continually superheated, $ABC^I C^{IV} D^{IV} D''' A$, Fig. 152. The work and efficiency equations in their first form can be set down by inspection,

$$W = J[L_1 + h_1 - (1 - x_a)L_2 - h_2]; \quad (852) \quad E = 1 - \frac{(1 - x_a)L_2 + h_2}{L_1 + h_1}. \quad (853) \quad (852)$$

In these equations the heats of superheat are given by $h_1 = S_{c'''}(T_{c'''} - T_b)$ and, $h_2 = S_{c'''}(T_{c'''} - T_a)$, and the amounts of superheat are related by the necessity for equal entropy. Therefore $(\phi_{c'''} - \phi_a) = (\phi_{c'''} - \phi_b)$, which is

$$\text{equivalent to } S_{c'''} \log_e \frac{T_{c'''}}{T_a} + \frac{L_2}{T_a} = S_{c'''} \log_e \frac{T_{c'''}}{T_b} + \frac{L_1}{T_b} + \log_e \frac{T_b}{T_a}. \quad \dots \quad (854)$$

This relation between superheats, together with $x_a L_2 = T_a \log_e \frac{T_b}{T_a}$, will, when substituted in Eqs. (852) and (853) give expressions for work and efficiency but these are somewhat too complex to be very useful.

Unlike the Rankine cycle, this one cannot be solved by the temperature entropy and Mollier diagram charts, though the tables and diagrams are of value in finding individual quantities. For example, on the $T\Phi$ diagram containing the various characteristic lines, the final quality or superheat can be read off directly, thus avoiding the equations; likewise heats of superheat for any temperature can be read off by passing along a constant-pressure line from any high temperature and total heat, to the lower temperature of saturation for the same pressure and its total heat. The difference between these total heats is the heat of superheat. There are many other short-cut ways of picking off various quantities from the diagram that cannot be detailed here but may be learned by experience. The really important fact to emphasize is that while the $T\Phi$ and Mollier diagrams do help in finding separate quantities they do not yield the answer directly.

Dividing any of these work expressions by the volume range will give the mean effective pressure as in all other cases, but it is to be noted that the volume range is exactly the same as for the Rankine Cycle I, so that the mean

effective pressure will be given by Eq. (855), using in it the value of low-pressure volume, Eq. (856):

$$(m.e.p.) = \left(\frac{W}{144 \times \text{vol. per lb. at low pressure}} \right) \text{lbs. sq.in.} \quad (855)$$

$$\left\{ \begin{array}{l} \text{Vol. per} \\ \text{lb. at low} \\ \text{pressure} \end{array} \right\} = V_2 x_2, \text{ if wet finally} \quad (a)$$

$$= V_2 \left(\frac{\text{Temp. of superheated steam}}{\text{Temp. of saturated steam}} \right), \left\{ \begin{array}{l} \text{(approx.) if} \\ \text{superheated} \\ \text{finally} \end{array} \right\} (b) \quad (856)$$

$$\left\{ \begin{array}{l} \text{B.T.U.} \\ \text{per hr.} \\ \text{per I.H.P.} \end{array} \right\} = \frac{2545}{E}, \text{ in general} \quad (a)$$

$$= \frac{T_b(2545)}{T_b - T_a}, \text{ for saturated steam wet or dry} \quad (b) \quad (857)$$

$$\left\{ \begin{array}{l} \text{Cyclic wa-} \\ \text{ter rate} \\ = WR \end{array} \right\} = \frac{2545}{E[(\text{latent heat}) + (\text{heat of superheat})]} \left\{ \begin{array}{l} \text{for super-} \\ \text{heated} \\ \text{steam} \end{array} \right\} (a)$$

$$= \frac{2545}{E[(\text{latent heat}) \times (\text{quality})]}, \text{ for wet steam} \quad (b) \quad (858)$$

It might seem as though steam-jet velocity could be computed for this cycle as for the Rankine, but as a matter of fact such would be at present a purely academic proceeding, because it is difficult to conceive of continuous steam generation necessary to nozzle operation with a cycle that requires adiabatic compression of wet low-pressure steam. This would be regarded to-day as an impracticable procedure, though it is within the range of possibility.

There is given in the handbook of tables a set of four series of curves similar to those derived for the Rankine cycle, two Charts, Nos. 50 and 51 indicate thermal efficiency and the quantities derived from it, and two others, Charts Nos. 52 and 53 indicate the work per pound of steam, and its derivatives. One curve of each set gives the result as a function of pressures for initially dry steam and the other as a function of quality for a given initial pressure. Comparison of this cycle with the Rankine is possible by taking off points from these curves for any given data and solving similarly for the same data by the Rankine curves.

Inspection of the efficiency as a function of initial pressure, Chart No. 50, shows the variation of this cycle to be of the same general character as for the Rankine cycle curves, Chart No. 46, but usually higher numerically. Thus, dry saturated steam expanding from 100 lbs. per square inch gage to 1 lb. per square inch absolute yields 29.8 per cent for the Carnot, as against 26.8 per cent for the Rankine, and the values for 200 lbs. to 1 lb. are 33.6 per cent for the Carnot, and 30.1 per cent for the Rankine cycles. However, efficiency plotted as a function of initial quality or superheat, brings out a striking difference, comparing this cycle, Chart No. 51, with the Rankine

cycle, Chart No. 47. For this case the efficiency is independent of quality up to saturation, whereas for the Rankine it increases with quality, so that the Rankine is somewhat less efficient for dry steam and very much less for very wet steam. In the superheat region the character of the curves for the two cycles is the same, both showing a small gain by superheat, and at about the same rates.

Example 1. Calculation and use of Chart No. 50, giving for the Carnot cycle, the efficiency, heat and steam consumption, as functions of initial pressure. Two points *A* on the right and *A'* on the left will be located to show the method used in plotting. Assume initially dry saturated steam at 100 lbs. per square inch gage and 10 lbs. per square inch absolute back pressure. Then by Eq. (844) as these conditions fall under Case *b*, $E_a = 1 - \frac{T_a}{T_b} = 1 - \frac{653.2}{797.9} = 18.13$ per cent.

Passing across to *A'* its abscissa is given by Eq. (858, *b*),

$$(WR)_{a'} = \frac{2545}{EL_1} = \frac{2545}{.1813 \times 880} = 15.95 \text{ lbs. per hour per I.H.P.}$$

To illustrate the use of the Chart No. 50, solve the problem: For the Carnot cycle with dry saturated steam between 150 lbs. per square inch gage and 10 lbs. absolute find the thermal efficiency, heat, and steam consumption. From point *B* pass up to *C* and across to *D*, reading efficiency = 21.1 per cent, and heat consumption 12,060 B.T.U. per hour per I.H.P. Passing horizontally to *E* and down to *F'* the water rate of 13.9 lbs. per hour per I.H.P. may be read off directly.

Example 2. Calculation and use of the Chart No. 51, giving for the Carnot cycle and its derivatives, the efficiency, heat and steam consumption, as functions of initial quality. The location of the curve *ABCD* will illustrate the calculation of the series, point *A*, representing the condition of all water initially; *B*, 50 per cent steam; *C*, dry saturated steam; and *D*, 200° superheat all for 200 lbs. per sq.in. gage initial pressure and 2 lbs. per sq.in. absolute back pressure. Each falls under a special case of the cycle, all being Carnot cycles, except the last, where superheat is present. The first point *A* apparently has the same thermal efficiency as *B* and *C*, but really it is an impossible condition, because without some steam the cycle cannot exist.

$$\text{From Eq. (839), } E_a = E_b = E_c = 1 - \frac{T_a}{T_b} = 1 - \frac{586.1}{847.9} = 30.8 \text{ per cent.}$$

$$\text{From Eq. (846), } E_d = 1 - \frac{(x_{a'} - x_a)L_2}{L_1 + h_1} = 1 - \frac{(.851 - .2159) \times 1021}{838 + 110.5} = 31.62 \text{ per cent,}$$

in which $x_{a'}$ and x_a , the two qualities, are determined by the steam tables or from the Mollier diagram. Passing to the left-hand angle the water rates for points *A'*, *B'*, *C'*, and *D'* are given by the formulas, Eq. (858): $(WR)_{a'} = \frac{2545}{Ex_c L_1} = \frac{2545}{0}$ (indeterminate).

$$(WR)_{b'} = \frac{2545}{.3088 \times .5 \times 838} = 19.67 \text{ lbs. per hour per I.H.P.}$$

$$(WR)_{c'} = \frac{2545}{.3088 \times 838} = 9.838 \quad \text{“} \quad \text{“} \quad \text{“}$$

$$(WR)_{d'} = \frac{2545}{E(L_1 + h_1)} = \frac{2545}{31.62(838 + 110.5)} = 8.51 \quad \text{“} \quad \text{“} \quad \text{“}$$

Lines of constant dryness are drawn across this set, and to show how these are found the point *M* on the 60 per cent quality line will be located for 200 lbs. initial gage and 5 lbs. absolute back pressure. From Eq. (844) the ordinate or efficiency is found, and from Eq. (858, *b*) the abscissa or water rate, whence

$$E_m = 1 - \frac{T_a}{T_b} = 1 - \frac{622.3}{847.9} = 26.61 \text{ per cent.}$$

$$(WR)_m = \frac{2545}{Ex_c L_1} = \frac{2545}{.2661 \times .6 \times 838} = 19.02 \text{ lbs. per hour per I.H.P.}$$

The use of the diagram requires no further explanation after what has been given for the Rankine cycle, nor is it necessary to explain the other diagrams completing this set to correspond to the set of four for the Rankine cycle.

Prob. 1. For the data of Problem 1, Section 99, what fraction of the Carnot cycle efficiency is being developed?

Prob. 2. For the same back pressure as in Problem 2, Section 99, what initial pressure for the Carnot cycle would yield the same efficiency?

Prob. 3. Compare the Rankine and Carnot derivative cycle efficiencies for 190 lbs. initial, $\frac{1}{2}$ lb. back pressures per square inch absolute with 100° superheat.

Prob. 4. With the operating data of Problem 3, a steam engine shows by test a thermal efficiency of 18 per cent, what per cent of the Rankine and of the Carnot derivative cycles is being developed.

Prob. 5. Solve for the data of Problem 8, Section 99, the corresponding case with the Carnot derivative as the standard of reference.

Prob. 6. Which case of the Carnot cycle or its derivatives corresponds to each set of data of Problem 9, Section 99.

Prob. 7. Find for the data referred to in the last problem the performance of the corresponding Carnot cycle or its derivative.

CHAPTER XV

THERMAL CYCLES FOR GAS.

101. Gas Cycles Representative of Ideal Processes and Standards of Reference for Gas Engines. While comparatively few thermal processes may be considered important as representatives of what may happen in transforming heat into work by steam as a medium, it is not so when perfect or reasonably perfect gases become the medium. Gases may receive their heat at constant pressure, constant volume or constant temperature, whereas steam must receive nearly all of its heat at constant pressure and temperature, as heats of liquid and superheat, when also involved, are both small parts of the total. Gases may, moreover, be expanded isothermally with the product of pressure and volume constant, or adiabatically with the product of pressure and the gamma power of the volume constant; moreover, compression by either of these laws may or may not precede heat addition in various ways peculiar to it. It is possible to construct for gases a very great number of cycles, much greater than for vapors within the range of reasonable possibility of execution, but when the criterion of easy execution by mechanism that shall be very simple is imposed, the number drops at once. In selecting those worth examination there are three conditions to be satisfied. First, the cycles selected shall most closely represent processes now being executed in practical machines so that the cycles may serve as standards of reference for comparison with actual performance, which comparison shall show how perfectly the mechanism is carrying out the imposed process. Second, the selected cycles should include some that might reasonably be carried out, especially if they promise high returns in efficiency, controllability, mean effective pressures or any other desired end not yet attained otherwise. Third, they should include those that yield general guiding principles, however academic the cycles may be or however impracticable. As a matter of fact comparatively few cycles, but more than it was found necessary to study when steam was the medium, will serve these ends, the same cycle falling at times in two or all three of the above groups.

Consideration of gas cycles involves differences in the bases of comparison from those that were satisfactory with steam by reason of the differences in limiting conditions. With the steam it is the pressures that constitute the fundamental limits to results attainable, boiler pressures as high and condenser pressures as low as are commercially economical are used, and the temperatures that correspond are mere accidental incidents. The one exception is introduced by superheated steam with a rise of temperature at the same

high initial pressure, but this is of little importance as a departure, because the temperatures used seldom exceed 200° superheat and the heat of superheat, 10 per cent of the total. Thus, with steam the amount of work that will be done per pound, depending as it does on the amount of heat per pound, is fixed within fairly narrow limits by the physical properties of steam itself, far more than by the various phase combinations that may form different steam cycles, because these latter are so few.

Gas cycles are not only more numerous, even when the limit of practicability is imposed, than vapor cycles, but the amount of work per pound of gas, depending on the efficiency of the cycle and the amount of heat added to the gas per pound, is not limited by the physical properties of gases. Gases may be caused to take up as much heat as is convenient to give them, but steam may not carry more than a definite amount fixed by the safe and economical high pressure. *There is, however, a practical limit to the heat per pound of gas, at least at present, which curiously enough is of about the same order of magnitude as for steam.* Gases may receive their heat from an external source, in which case the engines embodying the cycle are termed external combustion engines and in this case the amount of heat a pound of gas may receive is limited by the temperature to which it is safe to heat metals—somewhere about 1100° F. as a high limit—and the temperature of cooling water as a low limit, these limits corresponding roughly to about 200 B.T.U. per pound. Thus, external combustion gas cycles may receive only about one-fifth the amount of heat per pound of medium as may steam, which is roundly 1000 B.T.U. for ordinary conditions. This low limit is partly responsible for the abandonment of external combustion gas engines in favor of *internal combustion*, in which explosive mixtures are made with fuel and air, or in which air is caused to support non-explosive combustion of fuel in closed pressure chambers. With internal combustion as a mode of heating, the heat per pound of working fluid will depend on the thermo-chemical reaction equations, which fix both the weight of air needed to burn a pound of fuel, and the heat of combustion per pound of fuel, which together fix the heat of combustion per pound of working gases. This for air-fuel mixtures is in round numbers about 1000 B.T.U. per pound or about the same as for steam, but for oxygen-fuel mixtures it would be about five times as much.

Gas cycles may receive, therefore, a definite amount of heat per pound, limited at present to about 1000 B.T.U. per pound of gases maximum, so that the problem is, to decide how the greatest part may be transformed into work, or which of all the various cycles will give the most work when receiving this amount of heat. This is not the only question, however, because one cycle will require larger *volumes* of gas to do the same work than another, demanding that a piston engine embodying this larger volume cycle be larger. Another cycle will require the gas to rise to a very high pressure to do the given amount of work, and its engine will have to be very strong and heavy to resist these high pressures. Therefore, gas cycle efficiency must be studied, not alone, but in conjunction with corresponding pressure and volume ranges which are infinitely

more varied than for steam. The basis of comparison should in all gas cycles be the amount of heat received per pound of working gases, rather than the pressures which impose the corresponding limit on steam cycles, together with corresponding temperatures of boiling and condensation.

One of the first methods employed for operating a gas engine was used by Brown, who burnt the gas in a large flame at the suction port, filling the cylinder with hot gases of combustion at atmospheric pressure during the out stroke. He then injected water, which more or less suddenly chilled the gases, causing a pressure drop at constant volume, and with this vacuum the return stroke began. This, by reason of the considerable residual volume of the cooled gases, was necessarily a compression stroke, and by reason of the slowness of operation, large weights of metal and of injected water, was approximately an isothermal compression, the non-gaseous bodies taking up the heats of compression of the gases as fast as liberated. Thus, the cycle of the Brown engine closely approximates, (a) heating at constant pressure; (b) cooling at constant volume to original temperature; (c) heat abstraction at constant temperature to original volume and pressure. There were many modifications of this, constituting other so-called *atmospheric* cycles, but as all were essentially inefficient and required large displacement volumes, they were abandoned.

The next practical proposal embodied in an engine was that of Lenoir, about fifty years ago, from which modern gas engines may be said to date, so that steam engines of the practical sort are about one hundred years older than gas engines. Lenoir drew into his cylinder an explosive gaseous mixture and at about one-third stroke exploded it by an electric spark. This caused a rapid rise of pressure, ideally a constant-volume heating, which was followed by adiabatic expansion for the rest of the stroke. If too much mixture had not been taken in, this expansion might be complete and end at atmosphere at the end of the stroke, otherwise there would be a terminal drop at release. Neglecting the charging stroke that ideally neither does nor uses up work, the cycle would be, (a) constant-volume heating, beginning at atmosphere; (b) adiabatic expansion to atmosphere; (c) constant-pressure cooling to original volume, this last being the thermal equivalent of exhaust as for steam.

This same series of operations was embodied in the Otto and Langen free-piston engines that succeeded the Lenoir, but with one modification. The piston engaged the shaft by a clutch on the down stroke only, charging, explosion and expansion occupying the up stroke, the cylinder being vertical, single acting and open upward. This construction allowed the piston to rise as a free projectile till stopped by the vacuum created in the cylinder behind it by overexpansion. Returning slowly by its own weight, the low-pressure gases in the water-jacketed cylinder were compressed nearly isothermally to atmosphere. Hence the Lenoir cycle is here extended below atmosphere adiabatically with an isothermal return. Both these cycles and others of the same general sort are termed non-compression cycles because the gases are

not compressed before heating; they are inefficient compared to others that have preliminary compression, as will be shown, which is the reason they were abandoned. They did, however, demonstrate the superior advantages of internal combustion over external, about the same time that external combustion engines had demonstrated the superiority of compression over non-compression cycles, though they were not purely such, as some of their heat was exchanged during compression.

These external combustion cycles, involving precompression, are best illustrated by the Stirling and Ericsson engines. The Stirling used an enclosed mass of air so arranged that it could be brought in contact with a hot part of the chamber and a cold part alternately, and likewise caused to pass through a *regenerator* or large mass of porous or divided solid matter like wire gauze packed together. In passing through this regenerator the air gave up heat if hotter itself, which was the case when it flowed from the hot to the cool chamber in one direction, and took up heat when it was cooler on the return. The various parts of the mechanism were so arranged that the flow through the regenerator took place at constant gas volume, which could be accomplished by two pistons moving synchronously in opposite directions, the regenerator being between the cylinders. Expansion is accomplished by one piston moving out faster than the other; compression inversely. During compression the gas is in the cold cylinder, which keeps its temperature about constant—ideally it would be so; after isothermal compression the air transfer takes place through the regenerator, which adds heat to the gas at constant volume. Expansion now takes place in the hot cylinder, the air absorbing heat from the fire, ideally at constant temperature, and this is followed by a constant-volume return flow through the regenerator, during which the gas is cooled to its original temperature and the regenerator is heated. The cycle is, therefore, (a) constant-temperature compression; (b) heating at constant volume; (c) expansion at constant temperature; (d) cooling at constant volume. The regenerator heat is, however, algebraically equal to zero at the limit and the only heat derived from the fire is that for isothermal expansion.

Ericsson arranged his pistons differently, so far as their relative motions are concerned, and secured two constant-pressure phases between the two isothermals, by making one piston move out faster than the other during heating; in one form of engine there was substituted a transfer piston that did no work at all, leaving but one piston acted on by gas pressures. The Ericsson cycle was ideally, (a) constant-temperature compression; (b) constant-pressure heating; (c) constant-temperature expansion; (d) constant-pressure cooling.

Embodying the advantages of internal combustion, which are, (a) more heat per pound of gases, and (b) faster working because of the removal of conduction limitations to heat absorption by the gas, and in addition introducing the fundamentally necessary precompression before heating and after heating, utilizing the strong transforming power of adiabatic expansion, Otto employed in his engine a new cycle, previously proposed by Beau de Rochas, which

has come to be the standard of nearly all modern engines. This Otto cycle is, (a) adiabatic compression; (b) explosion or heating at constant volume; (c) adiabatic expansion to the original volume; (d) exhaust at constant volume which is equivalent thermally to constant-volume cooling to original pressure. This was modified by Atkinson, who expanded to more than the original volume, and various attempts at compounding were made subsequently, to carry the expansion to atmospheric pressure instead of limiting it to the original volume before compression as in the Otto engine.

Contemporaneously with Otto in Germany, Brayton developed an engine in America, using non-explosive but internal combustion operating on another cycle, which bore to that of Otto the same relation that the Ericsson did to the Stirling. Brayton substituted constant pressure for Otto's constant-volume phases between the two adiabatics. His cycle, also known by other names such as Joule, consists of, (a) adiabatic compression; (b) constant-pressure heating by internal combustion; (c) adiabatic expansion to atmosphere if the cylinder is big enough for the amount of heating employed; (d) constant-pressure cooling as the thermal equivalent of atmospheric exhaust. Later Diesel utilized the same cycle in a single cylinder, Brayton having used two, which structural change necessitated a modification of cycle, as the expansion in one cylinder can proceed only to the original volume; the Diesel cycle thus becomes, (a) adiabatic compression; (b) constant-pressure heating by internal combustion; (c) adiabatic expansion to original volume; (d) constant-volume cooling as the equivalent of exhaust, to atmospheric pressure.

Following the success of the steam turbine, many proposals have come forward for the operation of gas turbines involving, (a) compression of air adiabatically and delivery into and through a combustion chamber for, (b) heating at constant-pressure followed by, (c) adiabatic expansion in turbine nozzles instead of in cylinders, to atmospheric pressure; this must be followed to complete the thermal cycle by, (d) constant-pressure cooling at atmosphere, which is the Brayton cycle as used in cylinders.

This review, while decidedly incomplete, serves to indicate six distinct primary types of gas cycles, with two additional modifications, all well worth further investigation.

In the diagram, Fig. 153, the cycles selected for consideration are illustrated to PV and $T\Phi$ coordinates. The PV diagram is of greater value than the $T\Phi$ in this analysis of gas cycles, each point of which must be calculated before the heats, work and efficiency, and the volume and pressure range with quantities derived therefrom, may be determined. In order that there shall be some proportionality between these cyclic diagrams, so necessary for a fair visual comparison, they are all plotted to the same scale on the following assumptions of initial condition: one atmosphere pressure, absolute temperature 492° F. $\text{app.} = 32^{\circ}$ F., volume 12.38 cu.ft. per pound, that for air. To the pound of gas in each cycle is added 1000 B.T.U. after a compression of 7 atmospheres absolute for compression cycles, except for the two external combustion cases, which are allowed 200 B.T.U. and 3 atmos-

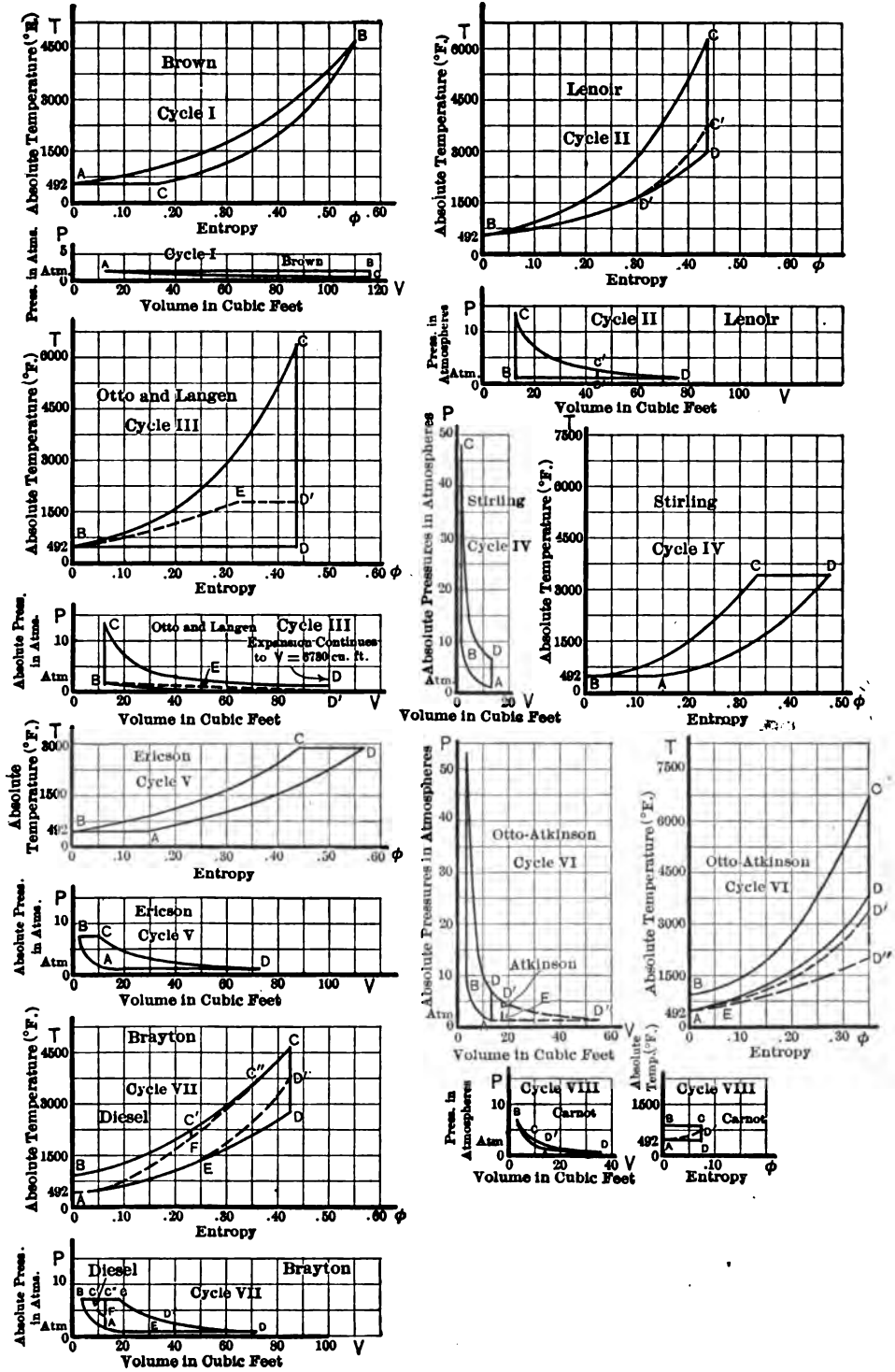


FIG. 153—Typical Gas Cycles.

pheres compression absolute. Another modification or exception necessary is that of Cycle VIII, which has isothermal heating followed by adiabatic expansion. With 1000 B.T.U. added isothermally at 10 atmospheres pressure initially, the pressure would fall below atmosphere before adiabatic expansion could begin, and the final volume would become several thousand cubic feet. This cycle has accordingly been modified by allowing only 200 B.T.U. per pound.

In all cases the specific heat of the gases is taken as constant and the working fluid is assumed to be air; any other procedure leads to unmanageable formulas. Errors involved in the practice can be allowed for at the same time as errors from other sources, in one or two inclusive correction factors when it is necessary to pass from these ideal to real engine conditions.

The cycles illustrated are described as follows, referring to Fig. 153:

BROWN ATMOSPHERIC. CYCLE I. First phase from *A* to *B*. Heating at constant atmospheric pressure. Second phase from *B* to *C*. Cooling at constant volume. Third phase from *C* to *A*. Isothermal compression.

LENOIR. CYCLE II. First phase from *B* to *C*. Heating at constant volume. Second phase from *C* to *D*. Adiabatic expansion. Third phase from *D* to *B*. Cooling at constant pressure.

OTTO AND LANGEN. CYCLE III. First phase from *B* to *C*. Heating at constant volume. Second phase from *C* to *D*. Adiabatic expansion. Third phase from *D* to *B*. Isothermal compression.

STIRLING. CYCLE IV. First phase from *A* to *B*. Isothermal compression. Second phase from *B* to *C*. Heating at constant volume. Third phase from *C* to *D*. Isothermal expansion. Fourth phase from *D* to *A*. Cooling at constant volume.

ERICSSON. CYCLE V. First phase from *A* to *B*. Isothermal compression. Second phase from *B* to *C*. Heating at constant pressure. Third phase from *C* to *D*. Isothermal expansion. Fourth phase from *D* to *A*. Cooling at constant pressure.

OTTO. CYCLE VI. First phase from *A* to *B*. Adiabatic compression. Second phase from *B* to *C*. Heating at constant volume. Third phase from *C* to *D*. Adiabatic expansion. Fourth phase from *D* to *A*. Cooling at constant volume.

BRAYTON. CYCLE VII. First phase from *A* to *B*. Adiabatic compression. Second phase from *B* to *C*. Heating at constant pressure. Third phase from *C* to *D*. Adiabatic expansion. Fourth phase from *D* to *A*. Cooling at constant pressure.

CARNOT. CYCLE VIII. First phase from *A* to *B*. Adiabatic compression. Second phase from *B* to *C*. Isothermal expansion. Third phase from *C* to *D*. Adiabatic expansion. Fourth phase from *D* to *A*. Isothermal compression.

These typical gas cycles exemplify the following:

- (1) An atmospheric cycle which has a low mean effective pressure and is essentially inefficient.

- (2) Non-compression cycles, likewise inefficient when compared with compression cycles.
- (3) The external combustion compression cycles, which demonstrate the value of precompression for gas cycles, and illustrate the action of a regenerator; they also afford a basis of comparison of isothermal with adiabatic compression cycles.
- (4) Constant-volume heating, adiabatic precompression cycles, typical of most modern engines, supplying a standard of reference for them.
- (5) Constant-pressure heating, adiabatic precompression cycles, typical of the rest of the modern engines and of the suggested gas turbine, supplying a standard of reference for them.
- (6) Constant-temperature heating, adiabatic precompression cycles, which complete the list of possibilities and demonstrate a most valuable general law, viz., that for all cycles of four phases in which the compression and expansion lines follow the same law and are included between a pair of similar heating and cooling lines the efficiency is the same, and equal to that of the general Carnot cycle, the standard for both gas and vapor cycles.

Prob. 1. Modify the Brown atmospheric Cycle I, by substituting adiabatic for isothermal compression and plot between the same temperature limits.

Prob. 2. Plot the modification of the Lenoir Cycle II, introduced by assuming constant-pressure heating to follow constant volume, each equally sharing the whole heat added.

Prob. 3. Modify the Otto and Langen Cycle III by substituting adiabatic compression and isothermal expansion.

Prob. 4. Plot to scale a modification of the Stirling Cycle IV, due to substitution of adiabatic compression and complete isothermal expansion to original pressure.

Prob. 5. Modify the Ericsson Cycle V by substituting adiabatic compression to maximum temperature.

Prob. 6. If the Otto-Atkinson Cycle VI were executed in two cylinders, one carrying out the Otto part and the other the rest of the extended expansion, plot their indicator diagrams to the same stroke, base, and correct pressure scale.

Prob. 7. Modify the Otto cycle for two-stage adiabatic compression with perfect intercooling and both with and without final perfect cooling.

Prob. 8. Assume the Brayton Cycle VII to be carried out in four cylinders, two belonging to a two-stage adiabatic compressor with perfect intercooling and two to a compound steam type of engine with perfect reheating, each without clearance, and plot the cycle.

Prob. 9. For all cylinders having 5 per cent clearance each, plot to the same stroke base the indicator cards for Problem 8, using correct pressure scales.

Prob. 10. Plot a combination of Diesel and Carnot cycles in which after adiabatic compression half the total heat added is received at constant pressure and the other half at constant temperature.

102. Non-compression Gas Cycles, Brown, Lenoir, Otto and Langen. Work, Mean Effective Pressure, Efficiency, Heat and Gas Consumption. Space does not permit an analytical study of each of the types enumerated

in the preceding section, although such an investigation would be very interesting. It is sufficient to state that the principal types of gas cycles applicable to present day practice will be studied by thermal analysis, and that a comparative study of the older cycles will be made by reference to charts and diagrams given in the Handbook of Tables.

The curves for the non-compression gas cycles, shown in Charts Nos. 54 and 55, give efficiencies and mean effective pressures, and indicate the relation between the important quantities for the complete cycle in each case. They prove the inferiority of the Brown atmospheric cycle compared to the other two by reason, not only of its low efficiency, but also its small mean effective pressure, which requires large cylinders. For efficiency the Otto and Langen complete expansion cycle is superior to the Lenoir, but its mean effective pressure is very much too low to compete. It should be noted that while the mean effective pressure of such cycles as the Lenoir is the work divided by the volume range, the mean effective pressure to be expected from the execution of even the perfect cycle in cylinders must be the work divided by the whole or maximum volume, because that part of the displacement up to explosion is used in getting the charge into the cylinder, so that the cylinder mean effective pressures will be smaller than those for the cycles in the ratio of volume range to maximum volume. Comparison of the performance of all these non-compression cycles will show them to be beneath consideration in comparison with compression cycles, in both efficiency and mean effective pressure. All the non-compression cycles show an increase of efficiency with heat supplied, so that for small heats supplied, which would correspond to weak explosive mixtures or much excess air in any case of internal combustion, the efficiency would be less than for no excess air. This also is in striking contrast to the compression cycles, the efficiency of which is not only better but independent of the amount of heat supplied.

Referring to the efficiencies, Chart 54, all three cycles show rising efficiency with increase of heat added, the Brown being lowest with the value of 20.8 per cent for 1000 B.T.U. per pound of gases, with Lenoir at 38.4 per cent and Otto and Langen at 78.4 per cent for the same heat supplied. For 200 B.T.U. per pound these three cyclic efficiencies fall off considerably, having the values 10.8 per cent, 18 per cent, and 48.4 per cent. These are all, however, high values and stimulate interest in what is attainable in cycles with compression, which, it was pointed out, promise still higher results. It must be remembered, however, that in executing these gas cycles in engines realization has fallen far short of cyclic promise, due to excessive heat losses in the mechanism itself.

To illustrate the use of the curves, Chart 54, find for a Lenoir cycle receiving 800 B.T.U. per pound of working gases, the thermal efficiency, heat consumption, and cubic feet of 300 B.T.U. per cubic foot fuel gas, per hour per I.H.P. From the 800 point *E* pass vertically to point *F* on the Lenoir curve and thence horizontally to *G* on the efficiency scale, reading 35.2 per cent, and heat consumption, 7250 B.T.U. per

hour per I.H.P. Passing across to the 300 B.T.U. calorific power curve at *H* and down to *K*, the gas consumption is found to be 24 cu.ft. per hour per I.H.P.

Illustrating the use of Chart No. 55, the solution of the following problem is required: Find the work per pound of working gases and the mean effective pressure for an Otto and Langen cycle receiving 500 B.T.U. per pound of gases. Starting at the 500 B.T.U. point *G*, pass up to the cycle curve at *H* and then across to the point *K* on the work scale, reading 280,000 ft.-lbs. Passing horizontally across to the point *L* and thence downward to point *M* the mean effective pressure is found to be 1.18 lbs. per square inch.

103. Stirling and Ericsson Cycles. Work, Efficiency and Derived Quantities for Isothermal Compression Gas Cycles, with and without Regenerators. STIRLING CYCLE IV. There are two cases to be considered, Fig. 154, first, that dealing with the heats involved as a whole, and second, that dealing

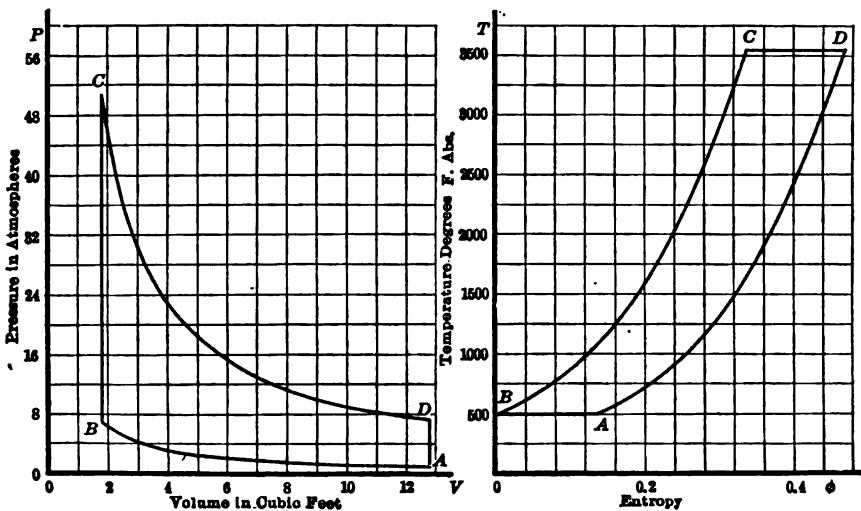


FIG. 154.—Stirling Gas Cycle. Cycle IV.

with the heat derived from the source of supply, separately considered from the regenerator heat which is alternately added and abstracted, and by hypothesis without loss. So far as the diagram points are concerned, there is no difference between the two; this is also the case for work and mean effective pressure.

Let Q_1 = the whole heat supplied from *B* to *C* to *D*;
 Q_1' = the heat supplied from the fire from *C* to *D*;
 Q_1'' = the heat supplied from regenerator from *B* to *C*;
 Q_2 = the whole heat abstracted from *D* to *A* to *B*;
 Q_2' = the heat abstracted by water jacket from *A* to *B*;
 Q_2'' = heat abstracted by regenerator from *D* to *A*;

$$C = \text{compression in atmospheres} = \frac{P_b}{P_a};$$

C_p and C_v = specific heats of gas at const. pr. and vol., assumed const.

r_v and r_p = volume and pressure ranges respectively = (max.), - (min.);
 H = B.T.U. per cubic foot gas as supplied;
 E = real thermal efficiency of whole cycle;
 E' = thermal efficiency of cycle referred to heat from source of supply (C to D).
 R = gas constant such that $PV = RT$;
 $Z = 1 + \frac{\text{heat added from regenerator}}{\text{temp. at beginning of addition} \times \text{specific heat at con. vol.}}$

Then, work, $W = J(Q_1 - Q_2) = R(T_d - T_a) \log_e C$ (859)

$$E = \frac{Q_1 - Q_2}{Q_1} = \frac{\frac{1}{J} R(T_d - T_a) \log_e C}{\frac{1}{J} R T_a \log_e C + C_v(T_d - T_a)} \quad \text{. (860)}$$

$$E' = \frac{Q_1 - Q_2}{Q_1'} = \frac{\frac{1}{J} R(T_d - T_a) \log_e C}{\frac{1}{J} R T_a \log_e C} = \frac{T_d - T_a}{T_a} = 1 - \frac{T_a}{T_d} \quad \text{. . . (861)}$$

This efficiency is therefore equal to the whole temperature range divided by the maximum temperature.

$$(\text{m.e.p.}) = \frac{W}{144 r_v} = \frac{R(T_d - T_a) \log_e C}{144 V_a \left(1 - \frac{1}{C}\right)} \quad \text{. (862)}$$

$$\text{B.T.U. per hr. per I.H.P.} = \frac{2545}{E'} = \frac{2545 T_d}{T_d - T_a} \quad \text{. (863)}$$

Evaluation of these quantities requires first the finding of the pressures, volumes and temperatures at each point in terms of the initial conditions, as imposed.

Point B: $T_b = T_a$ (a); $P_b = P_a C$ (b); $V_b = V_a \frac{P_a}{P_b} = \frac{V_a}{C}$ (c). (864)

Point C: $V_c = V_b = \frac{V_a}{C}$ (a); $T_c = T_b + \frac{Q_1''}{C_v} = T_a \left(1 + \frac{Q_1''}{C_v T_a}\right) = T_a Z$ (b) $\left. \begin{array}{l} P_c = P_b \frac{T_c}{T_b} = P_b Z = P_a C Z \text{ (c); where } Z = 1 + \frac{Q_1''}{C_v T_a} \end{array} \right\} \text{. . . (865)}$

Point D: $T_d = T_c = T_a Z$ (a); $V_d = V_a$ (b); $P_d = P_a \frac{T_d}{T_a} = P_a Z$ (c). (866)

$$Q_2 = Q_2' + Q_2'' = \frac{1}{J} P_a V_a \log_e \frac{V_a}{V_b} + C_v(T_d - T_a) = \frac{R}{J} T_a \log_e C + C_v(T_d - T_a) \quad \text{. . . (867)}$$

$$Q_1 = Q_1' + Q_1'' = \frac{1}{J} P_c V_c \log_e \frac{V_d}{V_c} + C_v(T_d - T_b) = \frac{R}{J} T_d \log_e C + C_v(T_d - T_a) \quad \text{. . . (868)}$$

As this cycle is executed on enclosed air and not by internal combustion of gas mixtures, the gas consumption has not the same significance as in the other cycles. If fuel is burned externally, the consumption per hour per H.P.

will be measured only partly by the above value of heat consumption, which is simply the heat that reaches the enclosed air, the rest of the generation escaping from the furnace, so that the other factor in fuel consumption will be a furnace efficiency which may be quite low, defined as the ratio of the heat appearing in the cycle along CD , Fig. 154, to the amount generated in the fire. While these engines had, as indicated by Eq. (861), a high cyclic efficiency, external combustion characteristics gave a higher fuel consumption than ought to correspond. This condition also applies to the following Ericsson cycle.

ERICSSON CYCLE V. This cycle when executed with regenerators alternately raising the temperature from B to C , Fig. 155, and lowering it from D to A , may, like the Stirling, have two efficiencies, one for the cyclic regard-

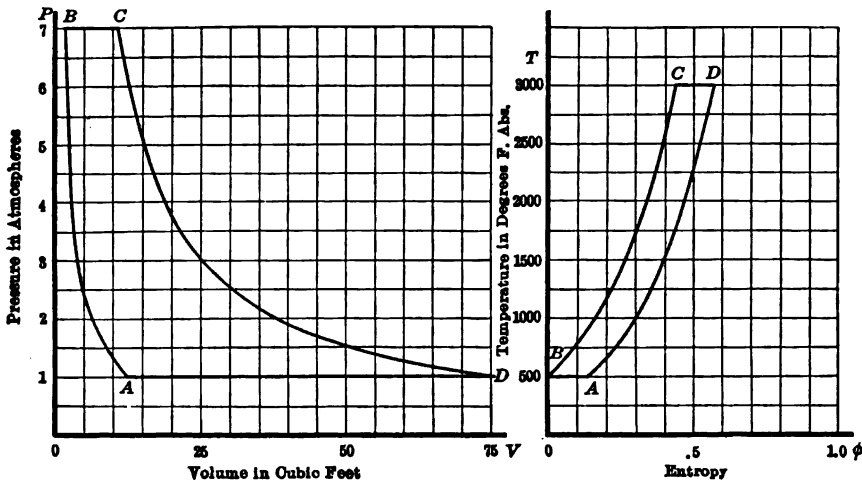


FIG. 155.—Ericsson Gas Cycle. Cycle V.

less of the source of the heat, and the other considering only the heat received from the fire source.

The position of point B is the same as for the Stirling, and its coordinates therefore need not be set down.

$$\left. \begin{aligned} \text{Point } C: P_c = P_b = P_a C \text{ (a)}; \quad T_c = T_b + \frac{Q_1''}{C_p} = T_a \left(1 + \frac{Q_1''}{C_p T_a} \right) = T_a Z' \text{ (b)} \\ V_c = V_b \frac{T_c}{T_b} = \frac{V_a}{C} Z' \text{ (c)}; \quad \text{where } Z' = 1 + \frac{Q_1''}{C_p T_a} \end{aligned} \right\} \quad (869)$$

$$\text{Point } D: P_d = P_a \text{ (a)}; \quad T_d = T_c = T_a Z' \text{ (b)}; \quad V_d = V_a \frac{T_d}{T_a} = V_a Z' \text{ (c)}. \quad (870)$$

$$Q_2 = Q_2' + Q_2'' = \frac{1}{J} P_a V_a \log_e \frac{V_a}{V_b} + C_p (T_d - T_a) = \frac{R}{J} T_a \log_e C + C_p (T_d - T_a). \quad (871)$$

$$Q_1 = Q_1' + Q_1'' = \frac{R}{J} T_a \log_e C + C_p (T_d - T_a) \quad (872)$$

$$W = J(Q_1 - Q_2) = R(T_d - T_a) \log_e C \quad (873)$$

$$E = \frac{Q_1 - Q_2}{Q_1} = \frac{\frac{R}{J}(T_d - T_a) \log_e C}{\frac{1}{J}RT_d \log_e C + C_p(T_d - T_a)} \quad (a); \quad E' = \frac{Q_1 - Q_2}{Q_1'} = \frac{T_d - T_a}{T_d} - 1 - \frac{T_a}{T_d} \quad (b) \quad (874)$$

$$r_v = V_d - V_b = V_a \left(Z - \frac{1}{C} \right) \quad (a); \quad r_p = P_b - P_a = P_a(C - 1) \quad (b). \quad (875)$$

$$(\text{m.e.p.}) = \frac{W}{144r_v} = \frac{R(T_d - T_a) \log_e C}{144V_a \left(Z - \frac{1}{C} \right)} \quad (876)$$

$$\text{B.T.U. per hr. per I.H.P.} = \frac{2545}{E'} = \frac{2545T_d}{T_d - T_a} \quad (877)$$

To show the important relations in these cycles between the work done, or the efficiency of transformation, and the two prime variables—quantity of heat supplied per pound of working gases and the amount of compression—requires a series of curves; these have been plotted in the form of charts which are given in the Handbook of Tables. Four pairs would be necessary but some of these have been omitted to save space, since the practical value of the cycles is now small, though their scientific value is great. The first pair Charts Nos. 56 and 57, for the Stirling and Ericsson respectively, show efficiencies and corresponding heat consumption in the working gas, and as supplied in the fire, for various furnace efficiencies, plotted to B.T.U. supplied per pound of working gases as the prime variable. Three compressions, 5, 10 and 30 atmospheres are each represented by one curve. As heat supplied to the working gases may be derived from the fire alone, or from fire and regenerator together, a double heat-supplied scale is necessary and each curve must be referred to its appropriate scale. These two sets show a very high potential efficiency when the fire source of heat is considered alone, efficiencies rising to over 90 per cent, whereas half of this value approximately corresponds to charging all the heat supplied to gases. A regenerator of less than 100 per cent efficiency as here considered would yield efficiencies somewhere between these two limits. No difference in the efficiency of the two cycles is found when regenerator heat is not charged, but with it charged the Stirling has a higher efficiency than the Ericsson, the figures being 44 per cent and 36 per cent for 1000 B.T.U. added with 10 atmospheres compression.

The second pair of curves, Charts Nos. 58 and 59, show efficiencies as functions of compression for various arbitrarily chosen quantities of heat added, as derived from fire alone, or from both fire and regenerator together.

As actually used in engines these cycles were operated with very low compression, about three atmospheres, and with very small quantities of heat supplied, such as correspond to maximum temperatures not over 800° F., and as the curves show, rather lower but still fair efficiencies are obtainable, though mean effective pressures are hopelessly low compared to modern engines. These curves have been extended beyond the working ranges used in engines that have been built because they show possibilities that are remarkable

though not yet realizable. It would require a somewhat severe stretch of imagination to picture a steam cycle offering any such efficiencies as are here shown—over 90 per cent—and while these prospects have as yet not been fruitful they certainly point a way for further investigation.

To illustrate the use of the diagram, Chart 56, find the efficiency, cyclic and fuel heat consumption for a Stirling cycle, for 300 B.T.U. supplied from fire per pound of working gases, 30 atm. compression, and a furnace efficiency of 40 per cent. Starting at point *E* at the value 300 on the upper scale, pass vertically up to point *F* on the efficiency curve referred to fire heat, and horizontally to *G*, reading thermal efficiency of 62.8 per cent, and cyclic heat supplied 4050 B.T.U. per hour per I.H.P. Continuing across to point *H* on the 40 per cent furnace efficiency curve and down to fire heat scale at *K*, the fire heat supplied is found to be 10,200 B.T.U. per hour per I.H.P.

A similar procedure applies to the curves, Chart 57, for the Ericsson cycle, using the appropriate formulas for efficiency.

Prob. 1. Derive a formula for the efficiency of the Stirling cycle by the methods of this chapter, for any regenerator efficiency.

Prob. 2. Derive a similar formula for the Ericsson cycle.

Prob. 3. An Ericsson hot-air engine with two atmospheres compression and a volume increase on constant-pressure heating of 20% burns 100 cu.ft. of 600 B.T.U. per cu.ft. fuel gas per hour per I.H.P. If the furnace efficiency is 30% and regenerator efficiency 100%, what part of the cyclic efficiency is realized?

Prob. 4. If in Problem 3 the regenerator were only 70% efficient, what would be the cyclic efficiency and per cent realization?

Prob. 5. Compare the Stirling and Ericsson cycle efficiencies for equal compressions and equal temperature limits.

Prob. 6. What are the numerical values of the efficiencies of Problem 5?

Prob. 7. For the data of Problem 3, find the amount of regenerator surface necessary for the execution of 50 cycles per minute, if the rate of heat transfer were 3 B.T.U. per hour per square foot per degree difference. Use arithmetical mean differences.

Prob. 8. Find the water jacket and furnace surface for any assumed reasonable values of water and fire temperature, corresponding to Problem 7, and for the same coefficient of heat transfer.

104. Otto, Complete Expansion Otto, Atkinson, Brayton, Diesel and Carnot Cycles. Work, Efficiency and Derived Quantities for Adiabatic Compression Gas Cycles.

OTTO CYCLE. VI. As this and succeeding cycles include as a first phase, adiabatic compression to any arbitrarily fixed degree, the amount of this compression expressed by either the temperature, pressure or volume change which may characterize it, will constitute an additional independent variable or initial condition.

It is most convenient to express this in terms of pressures; accordingly, let C = compression in atmospheres = $\frac{P_b}{P_a}$. Then the volume and pressure at point *B*, Fig. 156, representing the Otto cycle, are given by the ordinary adiabatic relation.

Point B: $P_b = P_a C$, (a);

$$\left. \begin{aligned} V_b &= V_a \left(\frac{P_a}{P_b} \right)^{\frac{1}{\gamma}} = \frac{V_a}{C^{\frac{1}{\gamma}}}, \quad (b); \quad T_b = T_a \left(\frac{V_a}{V_b} \right)^{\gamma-1} = T_a C^{\frac{\gamma-1}{\gamma}} \quad (c) \end{aligned} \right\} \cdot (878)$$

Point C: $P_c = P_b \frac{T_c}{T_b} = P_b X = P_a C X$ (a);

$$\left. \begin{aligned} V_c &= V_b = \frac{V_a}{C^{\frac{1}{\gamma}}} \quad (b); \quad T_c = T_b + \frac{Q_1}{C_s} = T_b \left(1 + \frac{Q_1}{C_s T_b} \right) = T_b X = T_a C^{\frac{\gamma-1}{\gamma}} X \quad (c) \end{aligned} \right\} \cdot (879)$$

In which $X = 1 + \frac{\text{heat added}}{\text{temperature at beginning of addition} \times \text{specific heat at con. vol.}}$

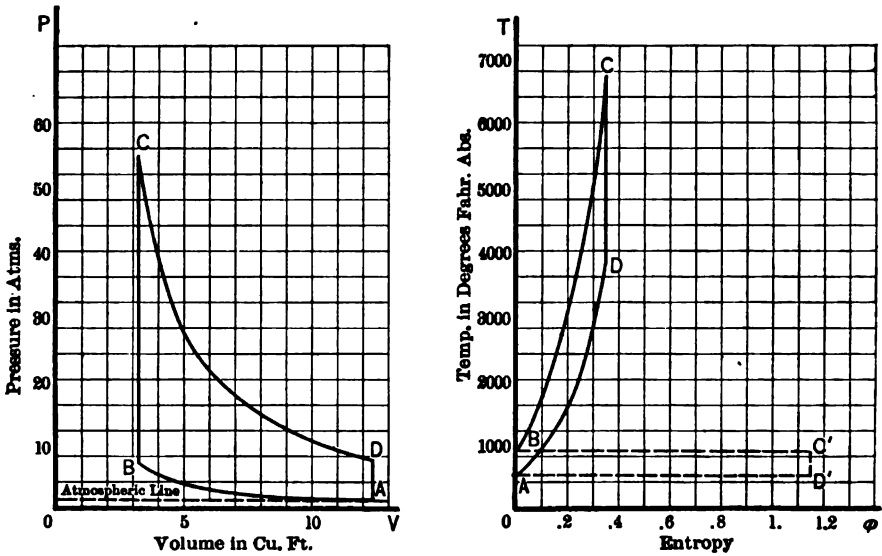


FIG. 156.—Otto Gas Cycle. Cycle VI.

Point D: $V_d = V_a$ (a);

$$\left. \begin{aligned} P_d &= P_c \left(\frac{V_c}{V_d} \right)^{\gamma} = P_a C X \left(\frac{1}{C^{\frac{1}{\gamma}}} \right)^{\gamma} = P_a X \quad (b); \quad T_d = T_a \frac{T_c}{T_b} = T_a X \quad (c) \end{aligned} \right\} \cdot (880)$$

$$Q_2 = C_s (T_d - T_a) = C_s T_a (X - 1) = C_s T_a \left(\frac{Q_1}{C_s T_b} \right) = \frac{T_a}{T_b} Q_1 \quad (881)$$

$$W = J(Q_1 - Q_2) = JQ_1 \left(1 - \frac{T_a}{T_b} \right) \quad (a);$$

$$\left. \begin{aligned} &= JQ_1 \left[1 - \left(\frac{P_a}{P_b} \right)^{\frac{\gamma-1}{\gamma}} \right] \quad (b); \quad = JQ_1 \left[1 - \left(\frac{V_b}{V_a} \right)^{\gamma-1} \right] \quad (c) \end{aligned} \right\} \cdot (882)$$

$$E = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{T_a}{T_b} \quad (a); \quad = 1 - \left(\frac{P_a}{P_b} \right)^{\frac{\gamma-1}{\gamma}} \quad (b); \quad = 1 - \left(\frac{V_b}{V_a} \right)^{\gamma-1} \quad (c). \quad (883)$$

$$r_v = V_a - V_b = V_a \left(1 - \frac{1}{C^\gamma}\right) = V_a \left[1 - \left(\frac{P_a}{P_b}\right)^{\frac{1}{\gamma}}\right] \quad (a); \quad r_p = P_c - P_a = P_a(CX - 1) \quad (b) \quad (884)$$

$$(\text{m.e.p.}) = \frac{W}{144r_v} = \frac{JQ_1 \left[1 - \left(\frac{P_a}{P_b}\right)^{\frac{\gamma-1}{\gamma}}\right]}{144V_a \left[1 - \left(\frac{P_a}{P_b}\right)^{\frac{1}{\gamma}}\right]}, \quad \dots \quad (885)$$

$$\begin{aligned} \text{B.T.U. per hr.} &= 2545 \left(\frac{T_b}{T_b - T_a}\right) \quad (a); & \text{Cu.ft gas per} &= \frac{2545}{H} \left(\frac{T_b}{T_b - T_a}\right) \quad (b) \quad (886) \\ \text{per I.H.P.} & & \text{hr. per I.H.P.} & \end{aligned}$$

Once more the efficiency of a cycle, this time that of the Otto, is given by an expression similar to that of the Carnot cycle, but in this the temperature range is not that for the whole cycle, but only that for the adiabatic compression. Interpreted thus, the Otto cycle has the same efficiency as a Carnot, for the same amount of adiabatic compression. This is indicated graphically in Fig. 156 by the dotted lines $ABC'D'A$, representing the equal Carnot rectangle to same height AB , that of the temperature rise in adiabatic compression.

Extended expansion, beyond the original volume as practiced by Atkinson, reaches its limit when the pressure falls to atmosphere if all expansion below atmosphere be excluded as impracticable. This cycle evidently will be more efficient than the Otto because work area is added without any additional heat supply, but the mean effective pressure will be much lessened since the volumes increase very rapidly in the lower ranges of pressures. Stopping the expansion at some pressure and volume greater than the initial, will produce a cycle having characteristics midway between the two limiting cases. This is really the Atkinson cycle. The complete expansion case will be called the Complete Expansion Otto and is of interest because of efforts to produce a perfect compound Otto gas engine for which it is a reference standard. These two modifications of the Otto will be considered separately as special cases of the Otto Cycle.

ATKINSON CYCLE, EXTENDED EXPANSION OTTO CYCLE. VI. All points up to and including point C are given by the same expressions as for the Otto, but points D and E , Fig. 157, are located at some volume greater than the volume at A , and as this is arbitrary, it adds one more independent variable.

Accordingly, let $n = \frac{V_e}{V_a} = \frac{\text{Volume after expansion}}{\text{Volume before compression}}$; then for

$$\left. \begin{aligned} \text{Point } D: \quad V_d &= V_a n \quad (a); \\ P_d &= P_c \left(\frac{V_c}{V_d}\right)^\gamma = P_c CX \left(\frac{\frac{V_a}{C^\gamma}}{V_a n}\right)^\gamma = P_c X \left(\frac{1}{n}\right)^{\frac{1}{\gamma}} \quad (b); & T_d &= T_c X \left(\frac{1}{n}\right)^{\gamma-1} \quad (c) \end{aligned} \right\} \quad (887)$$

$$\text{Point } E: \quad P_e = P_a \quad (a); \quad V_e = V_a n \quad (b); \quad T_e = T_a \frac{V_e}{V_a} = T_a n \quad (c). \quad \dots \quad (888)$$

$$Q_2 = C_v(T_c - T_d) + C_p(T_c - T_d) = C_v T_a n \left[X \left(\frac{1}{n} \right)^\gamma - 1 \right] + C_p T_a (n - 1) \quad (889)$$

$$W = J(Q_1 - Q_2) = J \left\{ Q_1 - C_v T_a n \left[X \left(\frac{1}{n} \right)^\gamma - 1 \right] - C_p T_a (n - 1) \right\} \quad (890)$$

$$E = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{T_a}{Q_1} \left\{ \left[X \left(\frac{1}{n} \right)^\gamma - 1 \right] n C_v - C_p (n - 1) \right\} \quad (891)$$

$$r_v = (V_c - V_b) = V_a \left(n - \frac{1}{C^\gamma} \right) \quad (a); \quad r_p = (P_c - P_a) = P_a (CX - 1) \quad (b) \quad (892)$$

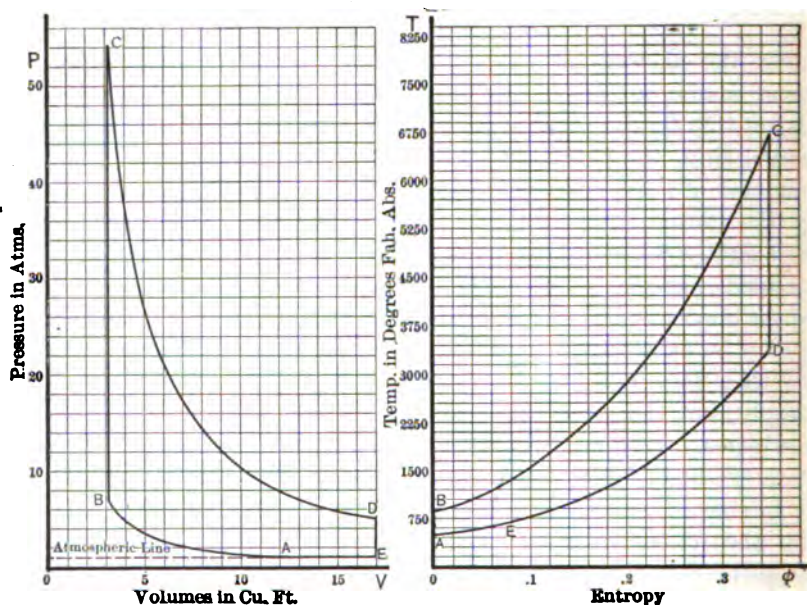


FIG. 157.—Atkinson Gas Cycle, Extended-Expansion Otto. Cycle VI.

$$(\text{m.e.p.}) = \frac{W}{144 r_v} = \frac{W}{144 V_a \left(n - \frac{1}{C^\gamma} \right)} \quad (893)$$

These expressions are not so simple as for the Otto cycle and will not be carried further; it is, however, clear that efficiency is greater, and now no longer independent of the amount of heat supplied, but on the contrary a function of it, and of the amount of extended expansion.

COMPLETE EXPANSION OTTO CYCLE. VI. The only point to be located is that at D, Fig. 158.

Point D: $P_d = P_a$ (a);

$$\left. \begin{aligned} V_d &= V_a \left(\frac{P_c}{P_d} \right)^{\frac{1}{\gamma}} = \frac{V_a}{C^\gamma} (CX)^{\frac{1}{\gamma}} = V_a X^{\frac{1}{\gamma}} \quad (b); \quad T_d = T_a \frac{V_a}{V_d} = T_a X^{\frac{1}{\gamma}} \quad (c). \end{aligned} \right\} \quad (894)$$

$$Q_2 = C_p(T_d - T_a) = C_p T_a (X^{\frac{1}{\gamma}} - 1). \quad (895)$$

$$W = J(Q_1 - Q_2) = J[Q_1 - C_p T_a (X^{\frac{1}{\gamma}} - 1)]. \quad (896)$$

$$E = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{C_p T_a}{Q_1} (X^{\frac{1}{\gamma}} - 1). \quad (897)$$

$$r_v = (V_d - V_b) = V_a \left(X^{\frac{1}{\gamma}} - \frac{1}{C^{\frac{1}{\gamma}}} \right) \quad (a); \quad r_p = (P_c - P_d) - P_a (CX - 1) \quad (b). \quad (898)$$

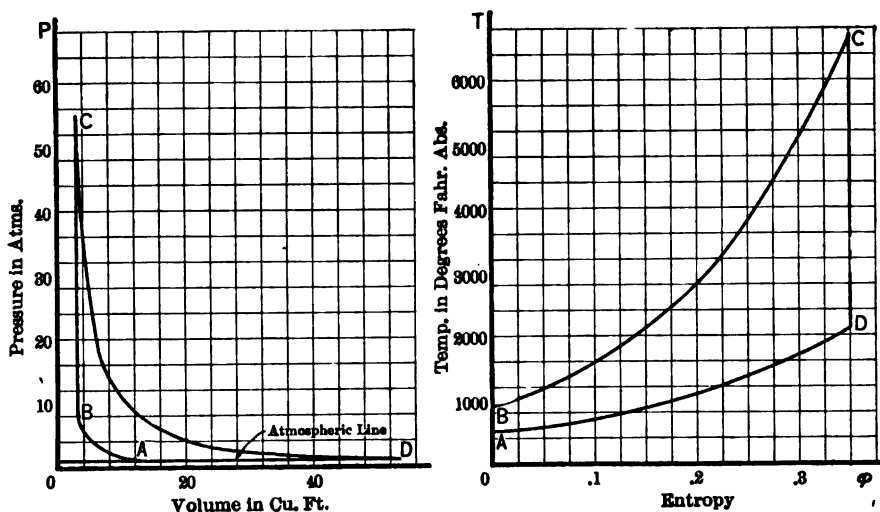


FIG. 158.—Complete Expansion Otto Cycle. Special Case of Atkinson Cycle VI.

$$(\text{m.e.p.}) = \frac{W}{144 r_v} = \frac{W}{144 V_a \left(X^{\frac{1}{\gamma}} - \frac{1}{C^{\frac{1}{\gamma}}} \right)}. \quad (899)$$

BRAYTON CYCLE. VII. The first data to be determined are for point C, Fig. 159. In these equations and following,

$$Y = 1 + \frac{\text{heat added}}{\text{temperature at beginning of addition} \times \text{specific heat at constant pressure}}.$$

$$\left. \begin{aligned} \text{Point C: } P_c &= P_b = P_a C \quad (a); \\ T_c &= T_b \left(1 + \frac{Q_1}{C_p T_b} \right) = T_a C^{\frac{\gamma-1}{\gamma}} Y \quad (b); \quad V_c = V_b \frac{T_c}{T_b} = V_b Y = V_a Y \left(\frac{1}{C} \right)^{\frac{1}{\gamma}} \quad (c) \end{aligned} \right\} \quad (900)$$

$$\text{Point D: } P_d = P_a \quad (a); \quad T_d = T_a \frac{T_c}{T_b} = T_a Y \quad (b); \quad V_d = V_a Y \quad (c). \quad (901)$$

$$Q_2 = C_p(T_d - T_a) = C_p T_a (Y - 1) = C_p T_a \frac{Q_1}{C_p T_b} = \frac{T_a}{T_b} Q_1. \quad (902)$$

$$r_v = (V_d - V_b) = V_a \left[Y - \left(\frac{C}{1} \right)^{\frac{1}{\gamma}} \right] \quad (a); \quad r_p = (P_b - P_a) = P_a (C - 1) \quad (b). \quad (903)$$

$$(\text{m.e.p.}) = \frac{W}{144r_v} = \frac{JQ_1 \left[1 - \left(\frac{P_a}{P_b} \right)^{\frac{\gamma-1}{\gamma}} \right]}{144V_a \left[Y - \left(\frac{1}{C} \right)^{\frac{1}{\gamma}} \right]} \quad \dots \quad (904)$$

The work, efficiency, heat, and gas consumption are the same as for the Otto cycle, and may be evaluated from Eqs. (882), (883), and (886). The efficiency is independent of everything but the amount of compression, being the same as for the Carnot cycle with the same compression. This Brayton cycle has the smallest pressure range for this efficiency, as the Otto has the smallest volume range, and the Carnot the smallest temperature range, the efficiency in each case being measured by the amount of compression. To

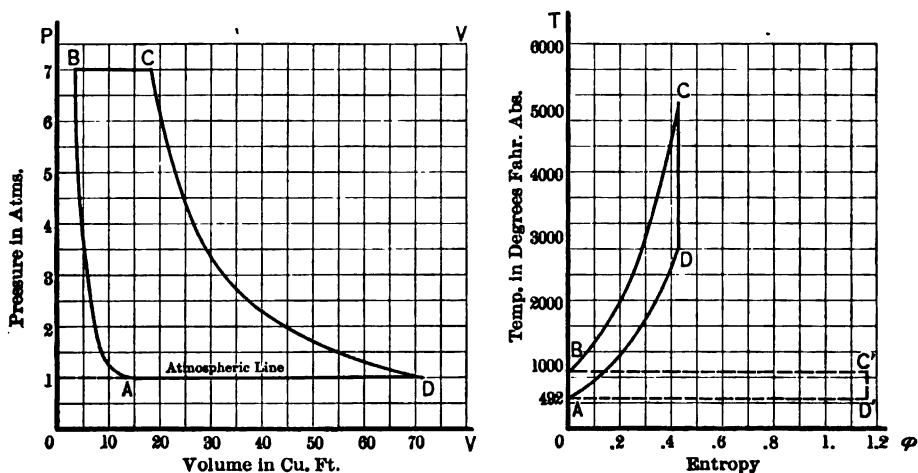


FIG. 159.—Brayton Gas Cycle. Cycle VII.

make the comparison more clear the Carnot cycle rectangle for the same temperature rise in compression, that is, for the same height, is plotted in dotted lines to $T\phi$ coordinates $ABC'D'A$.

DIESEL OR INCOMPLETE EXPANSION BRAYTON CYCLE. VII. A special case of the Brayton cycle is that of the Diesel which, executing all the processes in a single cylinder, can sweep through no more displacement volume on the working stroke than on the compression stroke; it therefore cuts off the expansion at the initial volume. This is obviously a less efficient cycle, as much expansion work is lost. It is represented in Fig. 160, $ABCD A$.

Point D: $V_d = V_a$ (a);

$$\left. \begin{aligned} P_d &= P_c \left(\frac{V_c}{V_d} \right)^\gamma = P_c C \left[\frac{V_a Y \left(\frac{1}{C} \right)^{\frac{1}{\gamma}}}{V_a} \right]^\gamma = P_a Y^\gamma \quad (b); & T_d &= T_a \left(\frac{P_d}{P_a} \right) = T_a Y^\gamma \quad (c) \end{aligned} \right\} \quad (905)$$

$$Q_2 = C_v(T_d - T_a) = C_v T_a(Y^\gamma - 1). \quad \dots \quad (906)$$

$$W = J(Q_1 - Q_2) = J[Q_1 - C_v T_a (Y^\gamma - 1)]. \quad (907)$$

$$E = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{C_v T_a (Y^\gamma - 1)}{Q_1}. \quad (908)$$

$$P = (P_b - P_a) = P_a(C - 1); \quad r_v = (V_a - V_b) = V_a \left[1 - \left(\frac{1}{C} \right)^{\frac{1}{\gamma}} \right] = V_a \left[1 - \left(\frac{P_a}{P_b} \right)^{\frac{1}{\gamma}} \right] \quad (909)$$

$$(\text{m.e.p.}) = \frac{W}{144 r_v} = \frac{J[Q_1 - C_v T_a (Y^\gamma - 1)]}{144 V_a \left[1 - \left(\frac{P_a}{P_b} \right)^{\frac{1}{\gamma}} \right]} \quad (910)$$

$$\text{B.T.U. per hr.} = \frac{2545}{E} \quad (a); \quad \text{Cu.ft. gas per hr.} = \frac{2545}{HE} \quad (b), \quad (911)$$

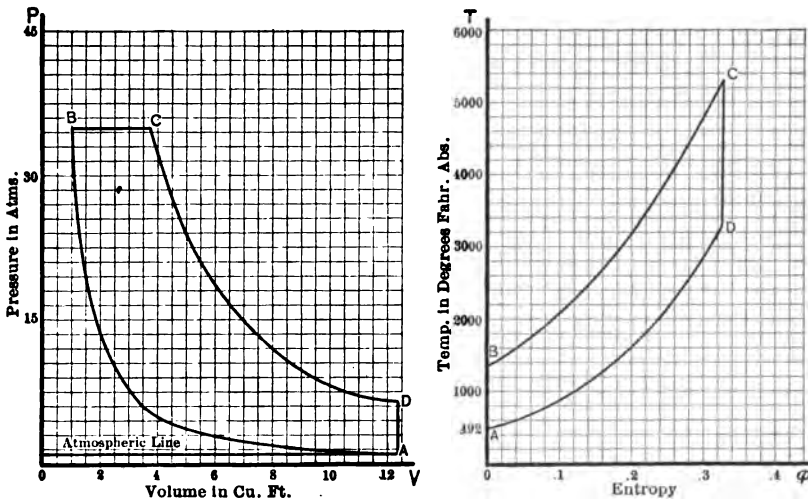


FIG. 160.—Diesel Gas Cycle. Special Case of Brayton Cycle. Cycle VII.

CARNOT CYCLE FOR GASES. VIII. After adiabatic compression from *A* to *B*, Fig. 161, the heat addition takes place isothermally, thus locating the point *C* by the amount of this addition. When the pressure at *B* is small and the heat addition large, the point *C* may easily fall below atmosphere. This is of no importance to the cycle as a whole, but if expansion were to cease at the initial volume, then adiabatic expansion may not only be impossible, but only a small amount of heat could be added to the gas isothermally and only a little work done. Moreover, the efficiency would be very low, likewise the mean effective pressure would also be low, because adiabatics lie very close to isothermals, making the work area between them small for very considerable displacements. The original proposal for Diesel engines involved a graduated introduction of the oil fuel into highly compressed air to keep the heating line nearly of this isothermal form, but the small mean effective pressures obtained made the engines too big and there was substituted the modified Brayton cycle, just discussed, which follows

by a more rapid oil injection and combustion, rapid enough to keep the pressure from falling till the fuel supply is cut off.

It is of no importance, therefore, to investigate this early Diesel proposal, but the Carnot gas cycle will be analyzed not with any idea of its practical importance, but rather to show the extremes of low mean effective pressures and large volumes that it involves in attaining the same thermal efficiency for equal compressions as the Otto and Brayton cycles, which give the same work with so little volume range and such small pressure ranges respectively. As

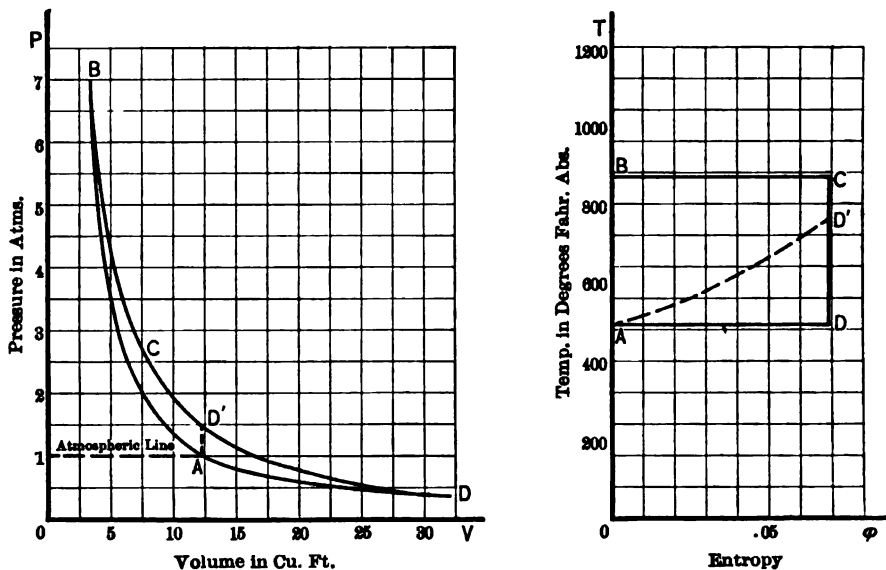


FIG. 161.—Carnot Gas Cycle. Cycle VIII.

there is to be no practical application of this cycle, the desired quantities will be set down without locating the several points.

$$Q_1 = \frac{1}{J} \left(P_b V_b \log_e \frac{V_c}{V_b} \right) = \frac{R}{J} T_b \log_e \frac{V_c}{V_b} \quad (a); \quad Q_2 = \frac{R}{J} T_a \log_e \frac{V_d}{V_a} = \frac{R}{J} T_a \log_e \frac{V_c}{V_b} \quad (b) \quad (912)$$

$$E = \frac{T_b - T_a}{T_b} = 1 - \frac{T_a}{T_b}. \quad \text{But } E = \frac{W}{JQ_1}. \quad \dots \dots \dots (913)$$

Therefore
$$W = JQ_1 \left(\frac{T_b - T_a}{T_b} \right) \text{ ft.-lbs.} \quad \dots \dots \dots (914)$$

Also
$$\frac{V_d}{V_a} = \frac{V_c}{V_b} = \log_e^{-1} \left(\frac{JQ_1}{RT_b} \right) \therefore V_d = V_a \log_e^{-1} \left(\frac{JQ_1}{RT_b} \right). \quad \dots \dots \dots (915)$$

$$P_d = P_a \frac{V_a}{V_d} = \frac{P_a}{\log_e^{-1} \left(\frac{JQ_1}{RT_b} \right)} \quad (a); \quad r_v = V_d - V_b = V_a \left[\log_e^{-1} \frac{JQ_1}{RT_b} - \left(\frac{1}{C} \right)^{\frac{1}{\gamma}} \right] \quad (b) \quad (916)$$

$$(\text{m.e.p.}) = \frac{W}{144r_v} = \frac{JQ_1 \frac{T_b - T_a}{T_b}}{144V_a \left[\log_e^{-1} \left(\frac{JQ_1}{RT_b} \right) - \left(\frac{1}{C} \right)^{\frac{1}{\gamma}} \right]} \quad \dots \dots \dots (917)$$

These volumes are very large, and pressures, both final and mean, very low, so that a Carnot cycle for gases, even if the apparatus for executing it were available, would not be practicable, whereas for steam or other vapors it is not very far from the attainable or actually used processes.

In the Charts Nos. 60 to 65, laid down in the book of tables, some of the important relations between the quantities involved in these cycles are more clearly indicated than by the equations they represent.

Referring to Chart No. 60, which gives thermal efficiency as a function of heat supplied per pound of working gases, a wide variation is found for the different cycles. One, the Diesel, shows a decreasing efficiency with increase of heat supply, three, the Otto, Brayton, and Carnot, a constant efficiency, while the complete expansion Otto or perfect Atkinson increases in efficiency. This crossing of efficiency lines makes the relative efficiencies change places with the position chosen on the heat supply scale. Furthermore compression has the same effect, three different compressions, 5, 10 and 30 atmospheres being represented for each cycle, so any question of superiority of one cycle over another will in general depend on conditions. There are, however, certain clear relations, for example, the Diesel is always less, and the perfect Atkinson always more efficient for the same compression than the constant value for the three, Otto, Brayton, and Carnot, as shown by the three curves starting from the origin for a given compression: one rising (perfect Atkinson), one horizontal (Otto, Brayton, Carnot), and one falling (Diesel), the whole set rising with rise of compression. The Diesel, it is interesting to note, is quite impossible beyond a given heat supply where adiabatic expansion ceases, due to constant-pressure heat supply for whole stroke. For 1000 B.T.U. supply per lb. of working gases and 30 atmospheres compression the efficiencies are 71.8 per cent for the complete expansion Otto, 62.2 per cent for Otto, Brayton, and Carnot, and 45.7 per cent for the Diesel. Equal heat supply and 10 atmospheres compression lowers these efficiencies to 63.4 per cent, 48.2 per cent for the first two, and is an impossible condition for the Diesel. Reduction of heat supply lowers the first, leaves the second unchanged, and raises the last; for example, with 30 atmospheres compression and 200 B.T.U. added per lb. of gases the efficiencies become 66 per cent, 62.2 per cent, and 58 per cent.

The precise effect of compression alone is shown more clearly in Chart 61, where all cycles are shown rising in efficiency, the curves of all being nearly parallel, each exhibiting at first a rapid rise, which later becomes more gradual. The relative positions for a given heat supply are the same as indicated by the last set of curves.

Mean effective pressure and work per pound of working gases as functions of the two prime variables are shown in Chart 62, with B.T.U. per pound of gases, and in Chart 63, with compression. The most striking feature indicated is the great superiority of the Otto and the hopelessly small Carnot values. As the Otto and Diesel, the two gas cycles of practical importance, are frequent subjects of computation, the curves of Chart 64 for the Otto and Chart 65 for the Diesel, are given to separate large scales from which

the mean effective pressure may be accurately read off, for 200, 500, and 1000 B.T.U. added. Engineers engaged in this work are advised to reproduce these charts on a still larger scale, adding more heat supply curves to meet working conditions, as they are great labor savers and may be used in estimating actual horse-power after the determination by tests of performance, or diagram factors.

Comparison with the curves of the non-compression cycles shows how very much less effective the latter are, and why no engine embodying these non-compression cycles is ever likely to successfully compete with engines executing compression cycles, and also why in the practical construction of gas engines designers are ever on the alert to discover means by which the allowable working compression may be raised. In fact, it is quite possible that one engine mechanism that permits of higher compression than another, even when the former is working on a less efficient cycle than the latter, may give a better actual efficiency. This has for some years been demonstrated by the less efficient Diesel cycle compared with the more efficient Otto, the engines using the Diesel compressing only pure air to about 30 atmospheres without difficulty, while the Otto when compressing explosive mixtures, are limited to compressions between four and twelve atmospheres by the temperature of ignition of the mixtures, the low limit for pure kerosene-air mixtures and the high for blast furnace gas. These compressions have resulted in better actual efficiencies in general for Diesel than for Otto engines, thus reversing the order of cyclic efficiencies.

Another point that must not be forgotten is, that in the execution of all these cycles, both steam and gas, realization always falls short of promise, but not to the same degree in the different cases; thus, in round numbers the efficiency actually realizable in steam engines may easily exceed 70 per cent of the most closely representative Rankine cycle for the same pressures, while few Otto cycle gas engines have so far been able to realize more than 60 per cent of the Otto cycle efficiency.

Example 1. *Calculation and use of Charts 60 and 61, giving for the adiabatic compression cycles, Otto, Brayton, complete expansion Otto, and Diesel the efficiency, heat, and fuel gas consumption for any compression and heat supplied per pound of working gases.* The efficiency and its derived quantities are given in Chart 60 as functions of heat supplied and the calculation of these curves will be illustrated by the location of one point on each, *A* for the Diesel; *B* for Brayton, Otto, and Carnot, all having the same efficiency for the same compression, and *C* for the complete expansion Otto. These points are all for 800 B.T.U. supplied per pound of working gases, after 10 atm. adiabatic compression.

Point *A* is located by Eq. (908) for efficiency of the Diesel cycle.

$$\begin{aligned}
 E_a &= 1 - \frac{C_p T_a}{Q_1} \left[\left(\frac{Q_1}{C_p T_a C^{\frac{\gamma-1}{\gamma}}} + 1 \right)^{\gamma} - 1 \right] \\
 &= 1 - \frac{.17 \times 492}{800} \left[\left(\frac{800}{.239 \times 492 \times (10)^{.286}} + 1 \right)^{1.4} - 1 \right] = 23.68\%.
 \end{aligned}$$

The limiting case for the efficiency of this cycle is that receiving enough heat to make $V_c = V_a$. Whence

Q_1 (limit) $= C_p T_a C^{\frac{\gamma-1}{\gamma}} \left(\frac{1}{C^{\frac{\gamma-1}{\gamma}} - 1} \right) = .239 \times 492 \times 10^{.286} (10^{.713} - 1) = 947.3$ B.T.U., which is indicated on the curve carrying the point A by a crossing line and appropriate lettering.

The point B is located from Eq. (883, b) giving

$$E_b = 1 - \left(\frac{P_a}{P_b} \right)^{\frac{\gamma-1}{\gamma}} = 1 - \frac{1}{C^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{(10)^{.286}} = 48.2\%.$$

By means of Eq. (897) the efficiency represented by point C is fixed, giving,

$$E_c = 1 - \frac{.239 \times 492}{800} \left[\left(\frac{800}{.17 \times 492 \times 10^{.286} + 1} \right)^{.713} - 1 \right] = 62\%.$$

The location of the lines in the left-hand angle needs no explanation in view of the examples for preceding gas cycles.

Illustrating the use of the curve the solution of the following problem is traced graphically on the diagram. Required the thermal efficiency, cyclic heat, and fuel consumption for the Diesel cycle, supplied with an oil yielding 1500 B.T.U. per cubic foot in its vapor, the cycle receiving 600 B.T.U. per pound of working gases after 10 atm. compression. From the 600 point E on the heat-supplied scale pass up to the 10 atm. compression Diesel curve F , and horizontally across to the efficiency scale G , reading 28.6 per cent and 8900 B.T.U. per hour per I.H.P. Continuing across to the fuel calorific power curve of 1500 B.T.U. per cubic foot H , and thence down to K , the fuel consumption is found to be 6 cu.ft.

The second set of efficiency curves, Chart 61, is obtained and used in exactly the same way as the first, the only difference between the two being the scales, so this series requires no explanation.

Example 2. *Calculation and use of Chart No. 62 in the tables, giving for the adiabatic compression cycles the work per pound of working gases and mean effective pressures.*

From Eq. (882, b), $W = JQ_1 \left(1 - \frac{1}{C^{\frac{\gamma-1}{\gamma}}} \right)$. For 5 atm. comp. and 1000 B.T.U. supplied,

$$W_a = 778 \times 1000 \times \left(1 - \frac{1}{5^{.286}} \right) = 287,160 \text{ ft.-lbs.}$$

From Eq. (907), for 30 atm. compression and 1000 B.T.U. supplied,

$$W_b = 778 \left\{ 1000 - .17 \times 492 \left[\left(\frac{1000}{.239 \times 492 \times (30)^{.286} + 1} \right)^{1.4} - 1 \right] \right\} = 355,000 \text{ ft.-lbs.}$$

From Eq. (896), for 5 atm. compression and 1000 B.T.U. supplied,

$$W_c = 778 \left\{ 1000 - .239 \times 492 \left[\left(\frac{1000}{.17 \times 492 \times 5^{.286} + 1} \right)^{.713} - 1 \right] \right\} = 447,300 \text{ ft.-lbs.}$$

Mean effective pressures are derived from these cyclic work values, for the location of points and curves of the left-hand angle, by the following equations for one point of each curve A' , A'' , A''' , B' , and C' .

From Eq. (917) for the Carnot cycle,

$$(m.e.p.)_c = \frac{W}{144r_v} = \frac{287,160}{144 \times 12.39 \left[\log_e^{-1} \left(\frac{778 \times 1000}{53.3 \times 492 \times 5^{.286}} \right) - \left(\frac{1}{5} \right)^{.713} \right]} = .0000012 \text{ lb.-sq.in.}$$

From Eq. (904) for the Brayton cycle,

$$(m.e.p.)_{br} = \frac{W}{144r_v} = \frac{287,160}{144 \times 12.39 \left[\frac{1000}{.239 \times 492 \times 5^{.286}} + 1 - \left(\frac{1}{5} \right)^{.713} \right]} = 26.61 \text{ lbs. per sq.in.}$$

From Eq. (885) for the Otto cycle,

$$(m.e.p.)_{ot} = \frac{W}{144r_v} = \frac{W}{144V_a \left[1 - \left(\frac{1}{C} \right)^{\frac{1}{\gamma}} \right]} = \frac{287,160}{144 \times 12.39 \left[1 - \left(\frac{1}{5} \right)^{.713} \right]} = 235.81 \text{ lbs. per sq.in.}$$

From Eq. (910) for the Diesel cycle,

$$(m.e.p.)_d = \frac{W}{144r_v} = \frac{W}{144V_a \left[1 - \left(\frac{1}{C} \right)^{\frac{1}{\gamma}} \right]} = \frac{355,000}{144 \times 12.39 \left[1 - \left(\frac{1}{30} \right)^{.713} \right]} = 218.79 \text{ lbs. per sq.in.}$$

It may be noted for equal works and compressions by the Otto and Diesel cycles the mean effective pressures are equal because the volume range is the same for both.

From Eq. (907) for the complete expansion Otto cycle,

$$(m.e.p.)_{ce} = \frac{447,300}{144 \times 12.39 \left[\left(\frac{1000}{.17 \times 492 \times 5^{.286}} + 1 \right)^{.713} - \left(\frac{1}{5} \right)^{.713} \right]} = 58.3 \text{ lbs. per square inch.}$$

No explanation of the use of the diagram is needed nor is it necessary to give the detailed steps for, or illustrate the use of, the other work and mean effective pressure diagrams, Charts Nos. 63, 64 and 65, for all are similarly derived and all serve to solve graphically, work and mean effective pressure problems for these cycles without calculation and with sufficient accuracy for general use.

Prob. 1. An Otto cycle engine is supplied with producer gas having 135 B.T.U. per cubic foot, the compression is 12 atmospheres, heat per pound of working gases 700 B.T.U. Find the cyclic efficiency, cyclic heat consumption, gas consumption, and mean effective pressure.

Prob. 2. For the same data as in Problem 1, solve for the Brayton cycle.

Prob. 3. For the same data as in Problem 1, solve for the complete expansion Otto, and from the answers state the value of perfect compounding of Otto gas engines.

Prob. 4. A Diesel engine is operated with 25 atm. compression, on oil having 19,000 B.T.U. per pound; the cut-off is such as to add 500 B.T.U. per pound of working gases. Find its thermal efficiency, weight of oil per hour per I.H.P., and mean effective pressure.

Prob. 5. If 50 per cent of the cyclic efficiency were realized in the Otto cycle engine of Problem 1, all losses taking place during explosion, what would be the new answers for the same data?

Prob. 6. If the Diesel engine of Problem 4 realizes 60 per cent of its cyclic efficiency and all losses occur during combustion, find the corresponding thermal efficiency, mean effective pressure, and oil consumption.

Prob. 7. Otto and Diesel cycle engines are in commercial competition and the best of each have nearly equal thermal efficiency. If this be taken as 35 per cent actual and if each realizes 50 per cent of the cyclic efficiency, all losses taking place during combustion, and finally, if the Otto cycle receives 800 B.T.U. per pound of working gases, what are the characteristics of a Diesel engine cycle of equal performance to an Otto with 15 atm. compression?

Prob. 8. Solve Problem 7 for an Otto engine with 20 atm. compression.

Prob. 9. Find what cut-off volume corresponds to various quantities of heat per pound of working gases for the Diesel cycle at various compressions from 15 to 30 atm. Plot a curve of relations.

Prob. 10. Compare the terminal pressures and temperatures for all adiabatic compression cycles receiving 750 B.T.U. per pound of working gases after 15 atm. compression.

CHAPTER XVI

EFFICIENCY OF TRANSFORMATION OF HEAT INTO WORK BY STEAM AND GAS, CYCLIC AND ACTUAL.

105. Comparison of Steam and Gas Cycles taking the Rankine as Standard for Steam, and the Otto and Diesel as Standard for Gas. Relations of Otto and Diesel to Rankine Cycles. Conditions for Equal Efficiency. Transformation of heat into work is a practical and economic possibility by the use of either a vapor or a gas as the working medium, as well as mixtures of vapors and gases. Such transformation is effected by a series of thermal processes, the efficiency of which is given by certain equations derived from analysis of the corresponding thermal cycles, which also give the work of the cycle, mean pressure, and other important quantities. Comparison of these equations for the different cycles establishes principles of practice that are invaluable. For example, the relative value of gases and vapors as the working media fixes corresponding relations between the steam and gas power systems as such, independent of interferences and thermal losses imposed by the mechanisms of the actual machinery. This is the most valuable contribution of such thermal analysis of cycles, because such conclusions could not have been obtained in any other way. For steam alone the analysis shows how fast efficiency increases with the rise of the initial pressure and temperature and with the lowering of the back pressure; it shows the gains to be expected by high boiler pressures or high superheats over low, also by most perfect condensing equipment over a poor equipment, and these gains are to be balanced against the additional cost of equipment involved in arriving at a good engineering judgment of how far it is worth while to go in any given direction.

The situation is the same with gas power efficiency, for the rate of increase of efficiency is rapid at first with increase of compression, and slower as compression rises. Higher cylinder compression means more costly construction of engine, so that a means is available for comparing the increased cost with the gain to be derived. However, in this case there is another limit to the compression and that is the ignition temperature of the explosive mixture when such is the substance being compressed. Similarly, there is another variable in the whole gas power problem and that is the cycle itself, for it has been shown that for the same compressions the maximum pressure which controls the weight and strength to be given to the working parts is not fundamentally related to the conditions for high efficiency, but depends rather on the cycle itself, being highest for cycles of the Otto group and lowest for those of the Brayton group.

The most striking general result of the cyclic analysis is the fact that in the abstract, neither steam nor gas as a working medium is always superior to the other as a transformer; both may yield high efficiencies if the proper and special conditions prescribed by the cycle are fulfilled. When, however, the possibilities of their easy and cheap fulfillment examined, then there is a noticeable superiority of gas cycles over steam so far as efficiency is concerned. On the other hand, judging by mean effective pressure standards the differences change again. Although in general the conditions for most efficient working are coincident with low mean effective pressures, which means that large cylinders are generally needed to fulfill the conditions of high efficiency, this handicap has been removed from the steam system by the substitution of the turbine for the cylinder machine, but it still remains with the gas system. However, the mean effective pressure for gas systems is not so much a function of the efficiency conditions as it is of the cycle. Nothing could be more convincing in this respect than the comparison of the Otto and Carnot cycle mean effective pressures, those for the Otto extremely high—many times more than is possible with efficient steam cycles—while those for the Carnot are extremely small. It is sometimes believed that the engine size or mean effective pressure handicap is to be removed in the gas power system by the turbine method which has succeeded so well with steam. While this may some day work out, the present difficulties are very great, but more important still, the handicap is nowhere nearly so great as it was for steam and, therefore, the necessity for working it out is less, especially so if the best gas cycle be chosen. This is beyond all question the Otto cycle, because of the high mean pressures and the corresponding high efficiency; from this cycle no departure is permissible without proof of its ineffectiveness in some practical respect, with equally strong proof that a substitute is available, which will give a fair return for what is lost by comparison with the Otto. One such is the Diesel, in which the limitation of compression by ignition temperatures is removed by compressing pure air alone to which fuel is subsequently added, the air compression being carried far enough to itself ignite the fuel on admission and burn it at constant pressure. Though a comparatively inefficient cycle for equal compressions, this may yield with its 30 atmospheres compression better over-all results in efficiency, in the ratio of about 6 to 5, than the Otto for the lower compressions of about 12 atmospheres, to which the latter has been confined by self-ignition limits even with the least sensitive fuel. This is not, however, the case with the corresponding maximum and mean effective pressures, because Diesel cycle engines are approximately twice as costly as Otto cycle engines, due to very high maximum, without correspondingly high mean effective pressure. Furthermore, Diesel cycle engines are confined to oil fuel to-day, as it is still regarded as impracticable to so highly and separately compress gas fuel for injection into the air at the end of the compression; gas fuel-air mixture compression is, however, retained in Otto cycle engines notwithstanding the ignition temperature limitation the practice imposes.

Another illustration of reasonable departure from the standard Otto would be the substitution of a cycle represented by a mechanism that would give speed regulation and reversibility, equal in perfection to that obtainable with steam, which is impossible with Otto cycle engines; or to cite another case, the substitution of a cycle and mechanism that would permit safe and direct internal combustion of solid fuel, also now considered impossible. Either of these would offer advantages sufficient to offset cyclic deficiency measured by lower thermal efficiency or smaller mean effective pressures as compared with the Otto if not carried too far.

It appears then that while there are innumerable reasons for not taking the teachings of the cyclic analyses too seriously, this is not to be regarded as proof of a lack of value. As a matter of fact cyclic analysis, whether worked out graphically or algebraically, is quite invaluable, but its teachings must be used with judgment.

To close the comparison, it is desirable that the efficiency and work of the cycles, together with the quantities derived from them for the standard steam and gas cycles, be brought together so as to show more clearly the conditions to be met in each case for the attainment of equally good results, especially with respect to efficiency, as there are practical ways of meeting the low mean effective pressure handicap, but none for overcoming inherently low efficiencies. The standard cycle for steam is the Rankine, and those for gas the Otto and Diesel, so that the comparison need be made only for these; any other steam cycle should, and may easily, be compared with the Rankine, and the same is true for the gas cycle with respect to the Otto and Diesel. Thus, a given gas cycle is best judged by saying its efficiency is more or less than that of the Otto if it uses constant volume heating, or its mean effective pressure is as great, for either the same amount of heat supplied or for equal compressions, or perhaps for equal maximum pressures. If the gas cycle uses constant-pressure heating it may be compared in performance with the Diesel. This permits judging all gas cycles by a gas cycle standard, and all steam cycles by a steam cycle standard, the comparison of a steam cycle with a gas cycle being made through the relative position of the Rankine with the Otto and Diesel standards. This may at first glance look like a cumbersome and round about method, but in reality it is not.

The comparison may be conveniently made by a diagram, Fig. 162, on which efficiency is plotted vertically, the curves for Otto and Diesel gas cycles to the left, and the Rankine steam cycle to the right, thus making a comparison of conditions quite simple through the common efficiency scale. Compressions for gas cycles are carried to 40 atmospheres so as to exceed somewhat the working maximum of the Diesel engine, but steam pressures are carried to 500 lbs., though 250 lbs. may be regarded as the corresponding working limit, because the efficiencies of the steam cycles would not otherwise rise to the same values as those for gas. For a similar reason wet steam is not considered, back pressures are kept at the lowest practicable limit of half a pound absolute corresponding to 1.03 ins. Hg and a temperature of 80° F.; superheats are

carried in each case to an amount that brings the maximum temperature of the steam up to 600° F., beyond which dissociation, pipe oxidation, and expansion are excessive, though 100° less is a better practicable limit. The diagrams show that it requires some 450 lbs. per square inch initial pressure of steam to reach 35 per cent cyclic steam efficiency, a pressure beyond working practice, whereas less than five atmospheres compression, about the least ever used, gives an equal efficiency to the Otto cycle gas engine standard. For the same 35 per cent efficiency it appears that the Diesel requires $7\frac{1}{2}$, $11\frac{1}{2}$, and $18\frac{1}{2}$ atmospheres compression when there is added to the cycle 200, 500 and 1000

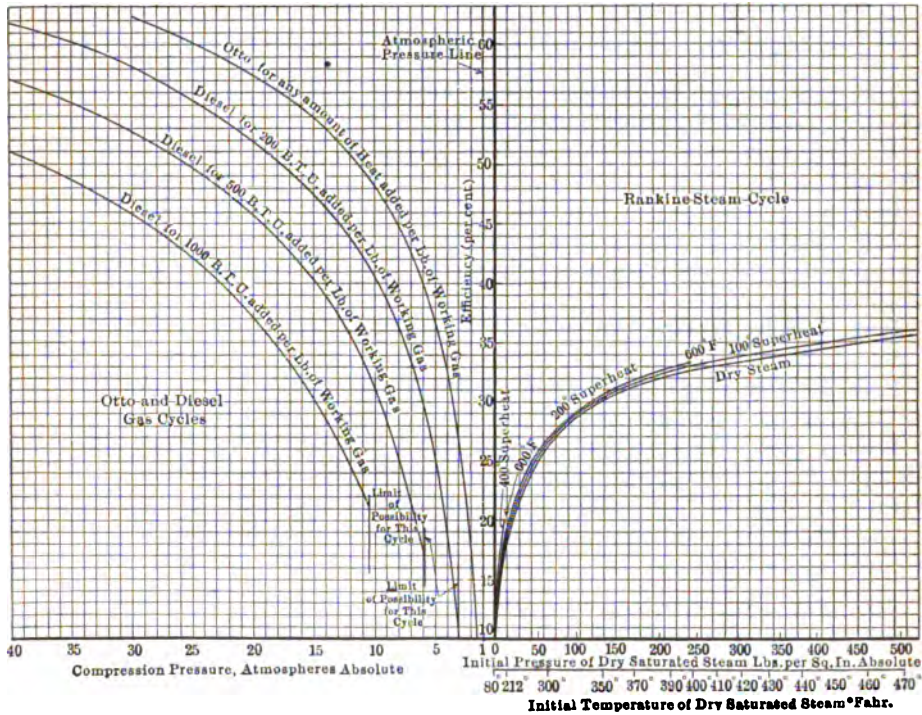


FIG. 162.—Comparison of Otto and Diesel Gas Cycles, with Rankine Steam Cycle, for 0.505 pound per square inch Absolute Back Pressure and Various Initial Pressures and Steam Temperatures Up to 600° F.

B.T.U. respectively per pound of working gases. The most noticeable fact brought out by the comparison is the sustained rapid rise of efficiency for all gas cycles with compression, while the steam cyclic efficiency rises very slowly beyond 200 lbs. initial pressure and by almost a straight line, superheat helping some but not much in proportion.

This chart shows clearly the inherently higher efficiency possibility for gas systems over steam and, as realization has not been in the same proportion, it indicates that the gas engine mechanism, as the means of executing the gas cycles, must receive very great improvement before it can equal the effectiveness

of steam mechanism. It is therefore of great practical importance that gas engine mechanism losses be most carefully studied and reduced, for, once brought to an equality with steam, the cyclic promise may be realized and the fuel consumption of the world for power purposes will be greatly diminished.

Example. Under what operating conditions may a steam engine realizing 60 per cent of the Rankine cycle efficiency be worked, to equal in thermal efficiency an Otto cycle gas engine realizing 40 per cent of the cyclic efficiency on 10 atm. compression. Reference to the diagram, Fig. 162, in the left-hand angle shows that the Otto cycle efficiency is 48.6 per cent. If 40 per cent of this is realized the actual efficiency is 19.44 per cent, and if this is equal to that of the steam engine its cyclic efficiency must be $\frac{19.44}{.60} = 32.4$ per cent. On the right-hand part of the diagram this would be realized for $\frac{1}{2}$ lb. back pressure and absolute initial pressures from 190 to 220 lbs. per square inch approximate with superheats from 200° to zero.

More exact determinations may be made by separate reference to the charts dealing with these cycles.

Prob. 1. A steam engine operating on 200 lbs. initial pressure and 1 lb. back pressure realizes 55 per cent of the Rankine cycle. Under what operating conditions may an Otto cycle engine realizing 50 per cent of its cyclic efficiency equal the performance?

Prob. 2. Solve Problem 1 for a Diesel cycle realizing the same fraction of its cyclic performance.

Prob. 3. What would be the mean effective pressures for the three cases, the steam, the Otto, and the Diesel gas cycles?

Prob. 4. Plot to $T\Phi$ coordinates the pure cycles and estimate the probable diagram, accounting for the losses given, and check by areas the efficiencies reported.

Prob. 5. Plot to PV coordinates the pure cycles and estimate the position of the probable diagram, accounting for the losses given, and check by areas the mean effective pressures reported.

106. Gas Cycle Performance as Affected by Variability of the Specific Heats of Gases, Applied to the Otto Cycle. All results so far obtained for gas cycles have been derived on the assumption that the specific heats of gases are constant, but it is known that at high temperatures such as characterize the Otto, for example, specific heat varies, so it is necessary to examine the effect of this variability on cyclic performance.

When specific heats vary, it has been assumed, in accordance with thermodynamic prediction, that they are functions of the temperature only, and this is true if the gases are *perfect*. Actual gases depart from this law and the most notable example is superheated steam, for which the specific heat is fairly well established and now known to be a function of both pressure and temperature. It is, therefore, quite likely that practically all gases are similar in this respect. The usual procedure is to assume a linear relation between specific heats and scale temperature above 32° F., expressing the specific heat *at*, or the mean specific heat *to* a given temperature, as equal to the value at 32°, with a constant multiple of the temperature excess over 32°, added. While this is too simple a relation probably, to correctly state the

facts, it is yet too complicated to enable the derivation of cyclic performance equations that shall be simple enough to be useful in engineering practice.

Careful studies of the indicator cards of Otto cycle gas engines by Clerk, Burstall and others have led to some conclusions as to specific heats that may be used for the purpose in hand, though always with doubt as to accuracy. Without in any way implying inferior merit in the work of others the data of Clerk will be adopted here, and these are:

$$\left. \begin{aligned} C_v &= .194 + .000051t \\ C_p &= .265 + .000051t \end{aligned} \right\} \text{ in terms of centigrade scale temperatures;}$$

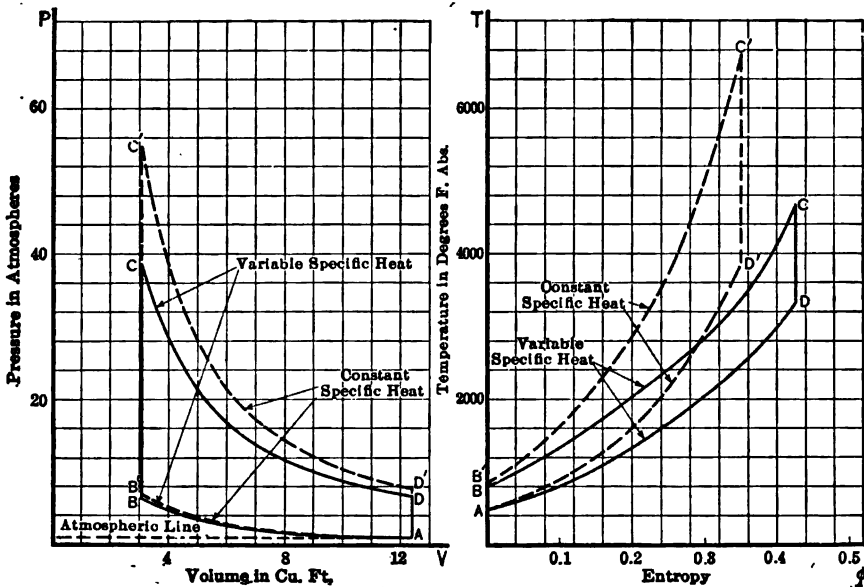


FIG. 163.—Comparisons of Otto Cycles for Constant and for Variable Specific Heat.

$$\left. \begin{aligned} C_v &= .194 + .0000283 \{t - 32^\circ\} \\ C_p &= .265 + .0000283 \{t - 32^\circ\} \end{aligned} \right\} \text{ in terms of Fahrenheit scale temperatures;}$$

$$\left. \begin{aligned} C_v &= .1801 + .0000283T \\ C_p &= .2511 + .0000283T \end{aligned} \right\} \text{ in terms of Fahrenheit absolute temperatures.}$$

In general, then,

$$C_v = \alpha + \beta T; \quad C_p = \alpha' + \beta T; \quad R = J(C_p - C_v) = J(\alpha' - \alpha); \quad \gamma' = \frac{\alpha' + \beta T}{\alpha + \beta T}.$$

Referring now to the cycle, Fig. 163, the first phase of which is adiabatic compression, it will appear that the first effect of the above variability of specific heat relations is to cause the adiabatic compression line AB to lie below the corresponding curve for constant specific heats AB' , which latter is shown dotted. This is proved algebraically by starting with the general

equation for a small increment of heat which for adiabatic changes is put equal to zero.

$$dH = C_d dT + \frac{1}{J} P dV = \frac{C_v}{JR} (P dV + V dP) + \frac{1}{J} P dV = 0,$$

by substituting the new values for C_v and R and integrating this becomes

$$\frac{P_b}{P_a} = \left(\frac{V_a}{V_b} \right)^\gamma e^{\frac{\beta}{\alpha} (T_a - T_b)} \quad (918)$$

This is the new adiabatic relation between pressures and volumes, when specific heats are variable, but it also involves the temperature. By the general gas law, the other relations can be found as follows for T and V :

$$\frac{T_b}{T_a} = \frac{P_b V_b}{P_a V_a} = \frac{V_b V_a^\gamma e^{\frac{\beta}{\alpha} T_a}}{V_b^\gamma V_a^\gamma e^{\frac{\beta}{\alpha} T_b}} = \left(\frac{V_a}{V_b} \right)^{\gamma-1} e^{\frac{\beta}{\alpha} (T_a - T_b)} \quad (919)$$

$$\text{Similarly for } T \text{ and } P, \quad \frac{T_b}{T_a} = \left(\frac{P_b}{P_a} \right)^{\frac{\gamma-1}{\gamma}} e^{\frac{\beta}{\alpha \gamma} (T_a - T_b)} \quad (920)$$

$$\text{Finally for } P \text{ and } V, \quad P_a V_a^\gamma e^{\frac{\beta}{R\alpha} P_a V_a} = K, \quad \text{or} \quad \frac{P_b}{P_a} = \left(\frac{V_a}{V_b} \right)^\gamma e^{\frac{\beta}{R\alpha} (P_a V_a - P_b V_b)} \quad (921)$$

By means of these three new adiabatic equations the compression line is completely determined and can be plotted, though the solution is not simple. The next phase is that of heat addition from B or B' , to C or C' , and as the specific heat increases with temperature rise, the increase in temperature will be less for the variable than for the constant value of the specific heat. The relation between temperature rise $(T_c - T_b)$ and the quantity of heat added is given by Eq. (922):

$$Q_1 = \int_{T_b}^{T_c} (\alpha + \beta T) dT = \alpha (T_c - T_b) + \frac{\beta}{2} (T_c^2 - T_b^2) = (T_c - T_b) \left[\alpha + \frac{\beta}{2} (T_c + T_b) \right] \quad (922)$$

To express the final temperature T_c in terms of the heat added, Q_1 , requires the solution of the second degree equation above; this solution is given by,

$$T_c = \sqrt{\frac{2}{\beta} \left[Q_1 + \left(\alpha + \frac{\beta}{2} T_b \right) T_b \right] + \left(\frac{\alpha}{\beta} \right)^2} - \frac{\alpha}{\beta}. \quad \text{As before, } P_c = P_b \frac{T_c}{T_b}. \quad (923)$$

By means of these equations the point C of the cycle is located and there remains only the fixing of D with reference to C and A . The same adiabatic relations as applied to the compression line AB also apply to the expansion line CD and in addition, $V_c = V_b$ and $V_d = V_a$.

$$\frac{P_c}{P_d} = \left(\frac{V_a}{V_b} \right)^\gamma e^{\frac{\beta}{\alpha} (T_d - T_c)} (a); \quad \frac{T_c}{T_d} = \left(\frac{V_a}{V_b} \right)^\gamma e^{\frac{\beta}{\alpha} (T_d - T_c)} (b); \quad \frac{T_c}{T_d} = \left(\frac{P_c}{P_d} \right)^{\frac{\gamma-1}{\gamma}} e^{\frac{\beta}{\alpha \gamma} (T_d - T_c)} (c) \quad (924)$$

The efficiency of the cycle is given by

$$E = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{(T_d - T_a) \left[\alpha + \frac{\beta}{2} (T_d + T_a) \right]}{(T_c - T_b) \left[\alpha + \frac{\beta}{2} (T_c + T_b) \right]} \quad (925)$$

When the specific heat is constant, $\beta=0$, and this expression reduces to that previously found for the constant specific heat, Otto cycle; but it must be remembered that for equal compressions and heat additions, the temperatures for the variable specific heat are not the same as those for the constant value.

Substitution of the various temperatures in terms of prime variables in the equation for efficiency gives an expression that cannot be solved. However Wimperis secured a solution by a series of algebraic approximations that assumed equal temperatures for the points B , C , and D , for the two cases of constant and variable specific heat which, in reality, cannot be for equal compressions and heats added. Calling the efficiency for the former case E' , he found,

$$E = E' \left\{ 1 - \frac{\beta}{2\alpha} [(1 - E')T_c + T_a] \right\} \dots \dots \dots (926)$$

Clerk's calculations of the efficiencies are given in Table XXXII compared with those for constant specific heat for the *same temperatures*. These relations are clearly of limited value since equal temperatures cannot result from equal compressions and heat additions, nor can any other temperatures for the cycle be equal if the maxima are.

Graphic methods are more satisfactory, the plotting of the cycle shows the relation at a glance, and the degree of accuracy in evaluating work and heat area to obtain mean effective pressure and thermal efficiency is entirely a question of cross-section paper and scale used.

TABLE XXXII

OTTO CYCLE THERMAL EFFICIENCIES FOR CONSTANT AND VARIABLE SPECIFIC HEATS. EQUAL LIMITING TEMPERATURES

Compression. (Volume before Volume after)	Thermal Efficiency.		
	Variable Specific Heat.		Constant Specific Heat.
	Max. temp. = 2912° F.	Max. temp. = 1832° F.	
2	.197	.200	.242
3	.286	.293	.356
4	.354	.356	.426
5	.384	.394	.473
7	.439	.433	.541

Prob. 1. A gas engine operates on an Otto cycle, and the ratios of volumes before and after compression is 5. What is the cyclic efficiency, assuming constant specific heats? What according to Wimperis' approximation?

Prob. 2. If in a gas engine the temperature of compression is 500° F. absolute and 1200 B.T.U. are added per pound of gas, what will be the temperature rise if the specific heat is considered constant, and what if it follows the values as given by Clerk? What will be the difference in pressure rise due to the two values of specific heat?

107. Actual Performance of Otto and Diesel Gas Engines, and its Relation to the Cyclic. Diagram Factors for Mean Effective Pressure and Thermal Efficiency. Effect of Load on Efficiency. Heat Balance of Gas Engines: Alone and with Gas Producers. All modern gas engines are operated on the internal-combustion method of adding heat to the mass of working gas both explosively and non-explosively, a fact that is responsible for one group of limitations in the results obtainable. There are two direct consequences of importance, first, the charge must be renewed each cycle, fresh air and fuel coming into the working chamber and hot products of combustion leaving alternately, with the other phases between; and second the obtainable temperature rise with its corresponding pressure increase in Otto cycle engines, or volume increase in Diesel cycle engines, is fixed by fuel-air mixture properties. Furthermore, as all these operations take place in cylinders having pistons that require lubrication and must, therefore, be kept cool enough to prevent destruction of the lubricant, a heat exchange takes place between gases and walls alternately in each direction that causes corresponding pressure, volume, and temperature changes in the gas, not contemplated by the cycle nor subject to computation or formulation. Besides these influences there are others chargeable to mechanical construction or adjustments that are responsible for further departure in the pressure, volume, and temperature changes from the ideal cyclic ones, as for example, too early an exhaust curtails expansion; too late a closure of admission valve similarly delays the beginning of compression and reduces the total amount. In Otto cycle engines where mixtures are exploded the combustion line may not be vertical and heating fail to be truly at constant volume, while in Diesel cycle engines where the fuel burns as fast as it enters, inaccurately graduated injection may cause the combustion to depart from that of true constant pressure—too rapid a feed causing pressure to rise; too slow a feed permitting a fall of pressure.

It is not surprising, therefore, to find that the mean effective pressure, thermal efficiency, or other characteristic performances differ in real gas engines from their cyclic equivalents, but this does not in any way reduce the value of cyclic analysis, in fact it emphasizes the necessity for it. There are two sorts of contributions of practical value in the cyclic analysis taken in conjunction with the performance of real engines. The first is of the fundamental scientific order, giving numerical value to the extent of the possible improvement of real engines of any one class and, pointing out just where the losses occur, with the amount of each, indicates where any improvement must be made. The second is of more direct practical value to builders and users of engines, as it is concerned with the prediction of results, for a given cylinder and fuel—just what horse-power and fuel consumption may be expected, or what cylinder size is required for a given output. In these latter cases the cyclic analysis is responsible for the *form* of equations for mean effective pressure and thermal efficiency, actual tests furnishing numerical values for the constants of proportionality.

As cyclic performance is to be the basis of all computations on the approach

to perfection of performance in real engines and of their probable power and efficiency, it is necessary to select the standard cyclic equations as a first step. It has been shown that comparatively simple performance equations are derivable for the Otto and Diesel cycles if the specific heats of gases are assumed to be constant, whereas it is known that they are not constant. But with any proposed law of variation in specific heats it has also been shown that the equations for cyclic performance are very difficult of any solution and impossible to solve exactly. It is therefore not easy to decide on a course of procedure for practical computations in the first group of comparisons that are concerned with the approach to perfection of real engines. Strictly speaking, they should be compared with cyclic performance as computed on the basis of variability of specific heats, but in view first, of the uncertainty of the law of variation, and second, of the complexity of the cyclic equations, this is not yet feasible as a matter of regular engineering routine, however desirable it may be in exceptional cases. It is, therefore, generally agreed that the cyclic results based on constancy of specific heats shall be used as a basis of comparison of real engine performance; in the simple everyday predictions of probable power and efficiency this is quite as good as any other, because a single constant factor can include all cyclic departures as well as the losses in the engine itself.

The next step in establishing the cyclic standard is to fix the physical properties of the working gases, for it must be remembered that these working gases include some fuel, some air, and some products of combustion left in the cylinder from a previous explosion. Every change in fuel, or in proportions of fuel, to air, and burnt gases, involves a different specific heat for the working mixture and even for a constant mixture, the expansion stroke being made only with burnt or burning gases, will have different specific heats than the compression stroke where the mixture is yet unburnt. Attempts have been made to evaluate the specific heats for these various mixtures, but this practice is defensible only in those exceptional cases where approach to perfection is under study, and here variability of specific heat with temperature and pressure must be taken into consideration as well as variability due to chemical composition, a procedure so far impossible. This has led to the assumption that for the cycle the working gases have the physical properties of *air*, or are air, have specific heats known to apply to air at 32° F., and that these do not vary.

On this basis the thermal efficiency of real engines is compared with a so-called *air-card standard* which is the cycle most nearly corresponding, but with air as the gas. Compared to this air-card standard, actual performance is found to be surprisingly constant, regardless of the fuel, the compression, or the size of engine; the ratio or *diagram factor*, ranging from .40 to .60 for Otto cycle engines. In some cases the reason for an increase or decrease in this number can be found, but in general this is not possible. For example, it is generally higher in large than in small engines, and higher when the cylinder and piston heat-absorbing surface per cubic foot of working gases

is small; but other influences may entirely counteract these and the precise effect of each separate influence cannot be evaluated.

According to the above, Otto cycle engines yield efficiencies referred to indicated horse-power that are somewhere about half the corresponding air-card efficiencies, so that if absolutely perfect they apparently would be only twice as good as they now are, or would consume only half the fuel. Wimperis has made some comparisons by means of his formula based on Clerk's linear variation of specific heat with temperature, and finds that when the air card with constant specific heat yields efficiencies of 54 per cent for a compression from 7 to 1 volumes, the air card with variable specific heats yields 44 per cent and 46 per cent, for a maximum temperature of $1600^{\circ}\text{C.}=2912^{\circ}\text{F.}$ and $1000^{\circ}\text{C.}=1832^{\circ}\text{F.}$, respectively. If for these cases the diagram factor based on constant specific heats were .50, then based on variable specific heats, it would be .615 and .59 respectively, showing the performance to be really nearer to perfection than at first appeared.

A similar computation made by Meyer on test data of a Deutz engine, 8.67×13 ins., with a volumetric compression ratio of 8.9, delivering about 10 H.P., showed air-card efficiencies of 39.75 per cent and 40.30 per cent, based on constant specific heat, and 29.69 per cent and 31.78 per cent, when based on his values of variable specific heat; the corresponding diagram factors are .665 and .655 for the former, and .89 and .83 for the latter. From the standpoint of constant specific heat the performance could be half as good again, but if the variability values are right the improvement possible is only about one-ninth better than was obtained, to be quite perfect.

For these Otto cycle engines some little assistance in estimating the value of the diagram factor is afforded by the determinations and the specific instances cited in the Tables (No. LXXX, Handbook of Tables). Calculating from Meyer's test for a four-cycle engine 7.8×11.8 ins., 250 R.P.M., with four different compressions from 40 to 80 lbs. per square inch above atmosphere, Clerk finds the diagram factor practically constant at .58 even when the air-card efficiencies range from 33 per cent to 44 per cent. He also finds from Burstall's tests for the English Engineering Society Gas Engine Research Committee, on a four-cycle engine, 6×12 ins., 200 R.P.M., with compressions ranging from 30 to 105 lbs. per square inch above atmosphere, values of diagram factors from .38 to .59, while air-card efficiencies range between 33 per cent and 47 per cent, though not regularly. He points out, however, a generally higher diagram factor for low maximum temperatures, the values ranging from .50 to .59, when the maximum temperature is about $1100^{\circ}\text{C.}=2012^{\circ}\text{F.}$, whereas the factor lies between .40 and .50 for maximum temperatures of about $1750^{\circ}\text{C.}=3182^{\circ}\text{F.}$ Clerk also reports values of the diagram factors of .6 and .66 respectively for a 10-in. National and a 26-in. Crossley engine, and while the difference favors the larger engine it is not enough to warrant any conclusion.

The load that a constant-speed engine carries will change its performance because the governing mechanism when in operation always introduces

losses greater at one time than at another, and the hand-control mechanism does the same for variable-speed engines. Such variations are considered here as incidental rather than fundamental data, and a test series by Hopkinson on a four-cylinder Daimler automobile engine, 3.56×5.11 ins., will serve to illustrate the point. At speeds of 400, 600, 1000, 1100, and 1225 R.P.M., the actual thermal efficiencies were 19.5 per cent, 21.5 per cent, 24.2 per cent, 24.6 per cent, and 22.3 per cent, referred to I.H.P. Since the compression ratio by volumes was 3.85, the cyclic efficiency is, $1 - \left(\frac{1}{3.85}\right)^{\frac{1}{4}} = 41.8$

per cent, so that at the successively higher speeds the diagram factors apparently were .48, .53, .58, .59, and .54. These are not the real diagram factors, because at low speeds the engine was throttled and the cycle changed by the lowered suction line.

Examination of a very large number of tests show that for various fuels used in typical Otto engines, the diagram factors lie somewhere within the following limits, for natural gas, coal gas, carburetted water, and producer gas, .45 and .55; for blast furnace gas, gasolene and kerosene vapors previously made and taken in during suction, from .40 to .50, but when injected during compression, the mixture non-homogeneous, the factors may fall quite low or be very high, depending on circumstances, difficult, if not impossible, to predict. These figures are for ordinary commercial engines, not for special engines operated under finest test conditions where the factor slightly exceeds .70.

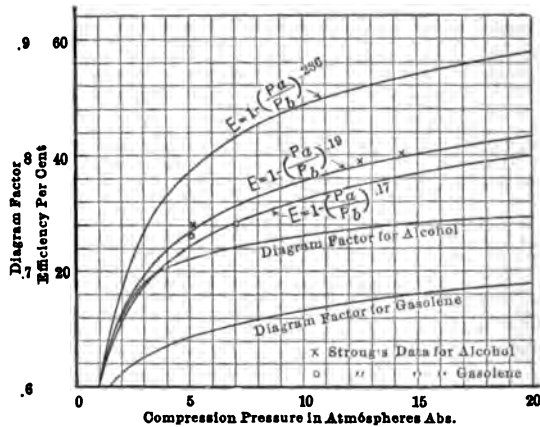


FIG. 164.—Variation of Otto Cycle Gas Engine Diagram Factors with Compression, in Small Gasolene and Alcohol Engines.

One most interesting and valuable series of tests for diagram factor is that conducted by Strong for the U.S.G. Survey on a 10-H.P. Nash and a 15-H.P. Otto engine, using gasolene and alcohol, the results of which are shown in Fig. 164. In this series the compression was varied from 85 to 215 lbs. per square inch above atmosphere and both fuels were tested in each engine with unusual care. At 6 atmospheres absolute compression, the diagram factors were .65 for gasolene and .716 for alcohol; at 8 atmospheres, .66 and .724, respectively, this being the maximum compression for gasolene; but at compressions of 11 and 15 atm. absolute for alcohol, the factors were .732 and .74. Strong finds that the efficiency is given by the equations noted in the diagram. The efficiencies obtained in the respective tests, divided by the expression for maximum attainable efficiency, shown by the upper curve,

will give the corresponding diagram factor. Diesel cycle engines have diagram factors, at about their rated load corresponding to fuel admission for 10 per cent of the stroke, of very closely, .50, though it must be admitted that there are insufficient data available to make this quite as well established as the factors for Otto engines.

Prediction of thermal efficiency for a given engine is, through the use of established diagram factors, reduced to a simple operation of finding the cyclic or air-card efficiency for the allowable compression pressure, and multiplying it by the factor. From this the heat consumption per hour and gas consumption follow directly.

Let E = thermal efficiency of air card;

E_I = thermal efficiency of engine referred to I.H.P.;

E_B = thermal efficiency of engine referred to B.H.P.;

E_M = mechanical efficiency = $\frac{\text{B.H.P.}}{\text{I.H.P.}}$;

F = diagram factor;

H = B.T.U. per cu. ft. gas, standard conditions, 29.92 ins. Hg and 32° F.

Then $E_I = EF$ (a); $E_B = E_I E_M = EFE_M$ (b). (927)

B.T.U. per hr. = $\frac{2545}{E_I} = \frac{2545}{EF}$ (a); B.T.U. per hr. = $\frac{2545}{E_B} = \frac{2545}{E_I E_M}$ (b). (928)

Cu.ft. gas per hr. = $\frac{2545}{E_I H} = \frac{2545}{EFH}$ (a); Cu.ft. gas per hr. = $\frac{2545}{E_B H} = \frac{2545}{E_I E_M H}$ (b). (929)

That these equations may be numerically evaluated requires some data which are given in the following Tables XXXIII and XXXIV; the former showing mechanical efficiency as a function of type of engine structure, and the latter allowable compressions as functions of the fuel class, from 2 atm. to 35 atm. The allowable compressions are always limited by the temperature of ignition of the mixture, when mixtures are under compression, and by the effectiveness of the internal wall cooling. When only air is compressed, the compression may be anything, but is generally limited by the consideration of cost. Mechanical efficiencies include all losses, mechanical friction, fluid friction, the work of charging and exhausting, and in addition the precompression or pump work in two-cycle engines.

Predictions of gas-engine horse-power are best made by establishing values for the mean effective pressure of the whole cycle referred to one stroke, with which the *horse-power of any gas engine, two cycle or four, and of any number of cylinders may be found, by multiplying this mean effective pressure by the piston area in square inches, the stroke in feet, the total number of cycles per minute in all working chambers, and dividing the product by 33,000.* The prediction of mean effective pressure of gas-engine cycles is, therefore, a matter of considerable practical importance not only as directly fixing the horse-power, but as an indication of good working of the whole mechanism. Any loss in weight of charge by suction resistances, or heating on entrance, as well as

any failure to compress as much as the fuel will stand, or to realize the full temperature, pressure or volume increase on combustion, or to completely expand and expel burnt gases with least resistance, all these effects will be shown in reduced mean effective pressures.

TABLE XXXIII

MECHANICAL EFFICIENCIES OF GAS ENGINES $= E_M = \frac{\text{B.H.P.}}{\text{I.H.P. (motor cylinders)}}$

Type of Engine.	Mechanical Efficiency.	
	Four Cycle.	Two Cycle.
Small high-speed automobile multi-cylinder, single acting.75	
“ single-cylinder boat engine, single acting.85	.68
“ or medium single-cylinder stationary, single acting.87	.7
“ “ “ two-cylinder stationary, single acting.84	
“ “ “ three-cylinder stationary, single acting.82	
“ “ “ four-cylinder stationary, single acting.80	
Large single-cylinder stationary, single acting.90	.70
“ two-cylinder stationary, single acting.86	to
“ four-cylinder stationary, single acting.84	.80
Double-acting single cylinder.83	.75
“ “ tandem, two cylinder.81	.73
“ “ “ twin four cylinder.77	.69

TABLE XXXIV

ALLOWABLE COMPRESSION OF GAS ENGINES IN STANDARD ATMOSPHERES ABSOLUTE

Fuel.	Compression in Standard Atmospheres (Abs.) for Water-cooled Engines.
Gasoline in suction mixture.	6 to 8
Kerosene in suction mixture.	7 with water injection
Alcohol in suction mixture.	3 with hot mixture without water
Oil injection in hot bulb before compression.	9 to 13
Oil injection after compression.	4
Carburetted water gas.	18 in Franchetti-Otto cycle engine
Coal gas.	35 in Diesel
Natural gas.	6 to 8
Producer gas.	6 to 9
Blast-furnace gas.	8 to 11
Coke-oven gas.	8 to 11
	10 to 13
	8 to 10

Referring to the ideal diagram of the cycle, Fig. 165, without any losses, except the heat suppression during combustion,

Let $(V_a - V_b)$ = displacement = volume of fresh charge per cycle;

V_b = clearance volume = volume of neutral gases added per cycle;

Let W = foot-pounds work per cycle;
 E = air-card thermal efficiency;
 E_I = thermal efficiency referred to I.H.P.;
 Q_1 = heat supplied per cycle;
 $(\text{m.e.p.})'$ = mean effective pressure, pounds per square inch for compression and expansion strokes referred to one stroke;
 H = B.T.U. per cubic foot gas (std.), low value;
 F = diagram factor.

$$\text{Then } W = 144(\text{m.e.p.})'(V_a - V_b) = 778EFQ_1 = 778 \left[1 - \left(\frac{P_a}{P_b} \right)^{\frac{\gamma-1}{\gamma}} \right] FQ_1 \quad (930)$$

$$\text{whence } (\text{m.e.p.})' = 5.4 \left(\frac{FQ_1}{V_a - V_b} \right) \left[1 - \left(\frac{P_a}{P_b} \right)^{\frac{\gamma-1}{\gamma}} \right] \quad (931)$$

but $\frac{Q_1}{V_a - V_b} = \frac{\text{Heat liberated per cycle}}{\text{Cu.ft fresh mixture taken in per cycle}} = \frac{\text{Heat per cycle}}{\text{Displacement per cycle}}$
 therefore, if the working charge during suction be always at 32° F. and one standard atmosphere pressure, then

$$\frac{Q_1}{V_a - V_b} = \text{B.T.U. per cu.ft. fresh mixture (std.)} = \frac{H}{a+1} \quad (932)$$

where a = cubic feet of air per cubic foot of gas for best mixture.

Substituting Eq. (932) in Eq. (931), the mean effective pressure becomes

$$(\text{m.e.p.})' = 5.4F \frac{H}{a+1} \left[1 - \left(\frac{P_a}{P_b} \right)^{\frac{\gamma-1}{\gamma}} \right] (a); \quad = 5.4F \frac{H}{a+1} \left[1 - \left(\frac{P_a}{P_b} \right)^{.29} \right] (b) \quad (933)$$

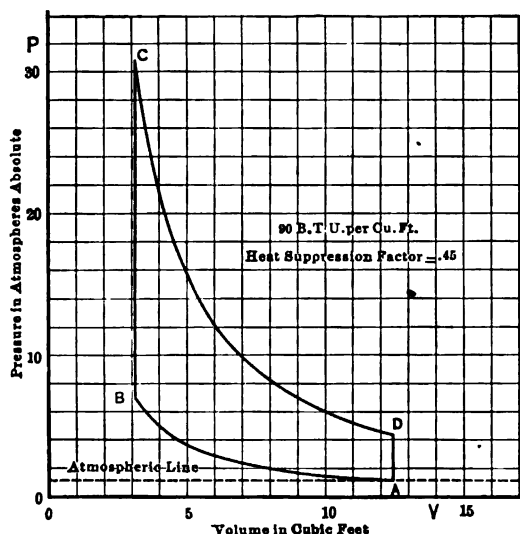


FIG. 165.—Otto Cycle Standard Reference Diagram without Suction or Exhaust Lines.

reduce the mean effective pressure. The

It appears from this equation that the same diagram factor F applies to mean effective pressure as to thermal efficiency. For convenience in calculation the values of the bracket are set down in Table LXXXII in the book of tables and these are to be multiplied by $\frac{H}{a+1}$, and by the diagram factor (F) applied to efficiencies previously discussed.

Values of H for various typical gases will also be found in the tables (Table LXXVII).

The actual indicator card is subject to losses in suction and exhausting which tend to exhaust stroke ends at a pres-

sure slightly greater than atmosphere, so that no new charge at atmospheric pressure can enter till an expansion, actually or in effect, has lowered the pressure at this point; some volume of atmospheric fresh mixture begins to enter in effect, and this continues to the end of the suction stroke where compression begins at a pressure less than atmosphere. This resisted suction is characterized by a negative mean suction pressure to be subtracted from the positive mean pressure of the upper loop. There is, however, another suction effect and that is a volume increase of the original fresh cool charge, due to mixture with hot burnt gases in the clearance and to its passage over hot metal surfaces, reducing the weight of the charge, and the real volumetric efficiency. Various tests have shown that the mixture suffers a rise of temperature during suction, ranging from 125° F. to 200° F., and mean suction resistances depending on speed and valve openings from less than 1 lb. square inch to over 5 lbs. These data together lead Güldner to assign values of real volumetric efficiency according to the following Table XXXV with values of terminal suction pressure, p_a , from which the mean suction resistance is estimated at two-thirds of the terminal.

A similar situation exists for the exhaust stroke, where a mean resistance is encountered, ranging from half a pound to three or four pounds per square inch, depending on the design and valve setting, and this is added to the suction resistance. The sum is to be subtracted from the mean pressure of the compressing and expanding area $ABCD$, Fig. 165, to obtain the mean effective pressure of the whole, referred to one stroke, neglecting the small inconsequential area at the beginning of exhaust,

TABLE XXXV

GÜLDNER'S VALUES OF OTTO ENGINE REAL VOLUMETRIC EFFICIENCY
WITH ESTIMATED MEAN SUCTION RESISTANCES

Type of Engine.	True Volumetric Efficiency EV .	Terminal Suction Pressure, Lbs. per Sq. In. Abs.	Estimated Mean Suction Pressure, Lbs. per Sq. In.
Slow-speed mechanical valve.....	.88 to .93	12.8 to 13.7	1.2 to .7
Slow-speed automatic valve.....	.80 to .87	12.5 to 13.3	4.5 to .9
High-speed mechanical valve.....	.78 to .85	11.8 to 12.5	1.9 to 1.5
High-speed automatic valve.....	.65 to .75	11.5 to 12.2	2.1 to 1.3
Very-high-speed air-cooled automatic valve	.50 to .65	8.8 to 11.0	3.9 to 2.5

If the gas and air mixture exist at a temperature external to the cylinder, other than 32° F. and at a pressure other than one standard atmosphere, then the heat per cubic foot of mixture measured external to the cylinder, will not have the value $\frac{H}{a+1}$ previously assigned, but a correction may be applied.

Let T_m = absolute temperature F of the mixture external to the cylinder;

h_m = absolute pressure of the mixture external to the cylinder, inches Hg.

Then from Eq. (588),

B.T.U. per cu.ft. of fresh mixture externally measured

$$= H_m = \left(\frac{H}{a+1} \right) \div \left(\frac{29.92}{h_m} \right) \left(\frac{T_m}{492} \right) (a); = \left(\frac{H}{a+1} \right) \frac{h_m}{.06 T_m} (b). \quad (934)$$

It has been shown that the mean pressure of the compression and expansion strokes referred to one stroke is given by an expression containing the term

$$\frac{Q_1}{V_a - V_b} = \frac{\text{Heat per cycle}}{\text{Displacement per cycle}}$$

(Displacement per cycle) $\times E_v$ = cu.ft. externally measured mixture, per cycle

$$\text{or } \frac{Q_1}{V_a - V_b} = E_v \times \left(\frac{\text{Heat per cycle}}{\text{Cu.ft. of externally measured mixture per cycle}} \right)$$

$$= E_v \times H_m = \frac{E_v}{.06} \left(\frac{H}{a+1} \right) \frac{h_m}{T_m} \quad (935)$$

Substituting Eq. (935) in Eq. (931), giving the mean effective pressure for a compression and expansion stroke, and subtracting the mean resistance on suction p_s and on exhaust p_e there results for Otto cycle gas engines

$$(\text{m.e.p.}) = 5.4 \times \frac{F E_v}{.06} \left(\frac{H}{a+1} \right) \frac{h_m}{T_m} \left[1 - \left(\frac{P_a}{P_b} \right)^{.29} \right] - p_s - p_e \quad (936)$$

$$= 90 F E_v \left(\frac{H}{a+1} \right) \frac{h_m}{T_m} \left[1 - \left(\frac{P_a}{P_b} \right)^{.29} \right] - p_s - p_e \quad (937)$$

For Diesel cycle engines the following Eq. (938) will serve:

$$(\text{m.e.p.}) = F E_v \times (\text{m.e.p. of the cycle}) - p_s - p_e \quad (938)$$

As already shown for steam engines, the efficiency and fuel consumption of gas engines is not the same at all loads, even when expressed in terms of indicated horse-power, the variations being due to the kind of control used. There is, however, an interesting relation that should be noted, based on the fact that the thermal efficiency referred to I.H.P. would be constant if, (a) there were no suction and exhaust resistances; (b) the ignition were always adjusted to give vertical explosion lines; (c) all the fuel used were burned at the right time. There are three typical methods of control, each yielding a characteristic fuel or heat-consumption load curve: the quality control mainly used in oil engines; the hit and miss in cheap engines that need not regulate closely; and the suction throttle, with sometimes a fourth, suction cut-off. The first two have always the same compression characteristics and valve resistances; while with the throttle, resistance varies all the time with load so it would be expected that two characteristic curves would result. For the former, fuel per hour or B.T.U. supplied per hour, gives a straight line when plotted to load, except where, at overload, excess and unburnt fuel causes a change of curvature. This is equivalent in steam engines and turbines to the straight Willans line; the curve which characterizes the

throttle type throughout is expressed as a second degree equation similar to that found for cut-off-governed steam engines. Several efficiency curves of

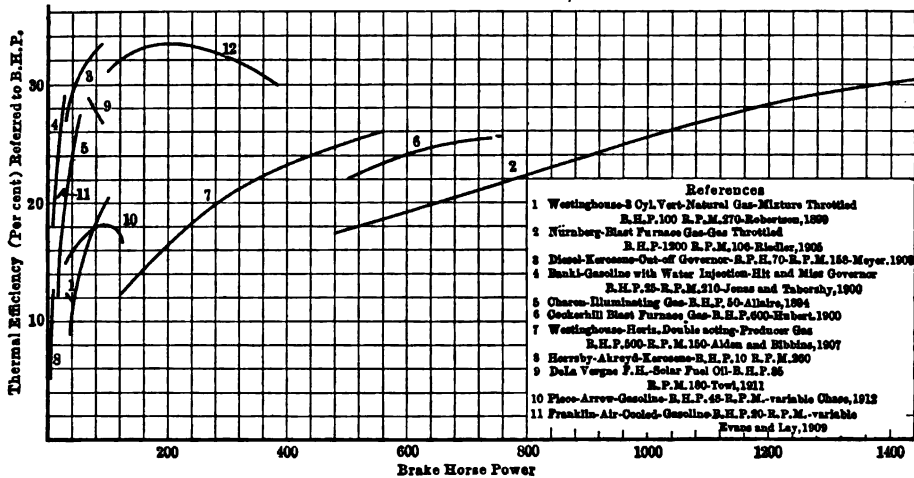


FIG. 166.—Variation of Gas Engine Thermal Efficiency with Actual Load.

each engine type are shown in Fig. 166 on actual, and in Fig. 167 on a fractional load basis to bring all sizes in accord.

Any examination of the question of gas-engine efficiency would be incomplete without at least a brief review of the ultimate disposition of the heat

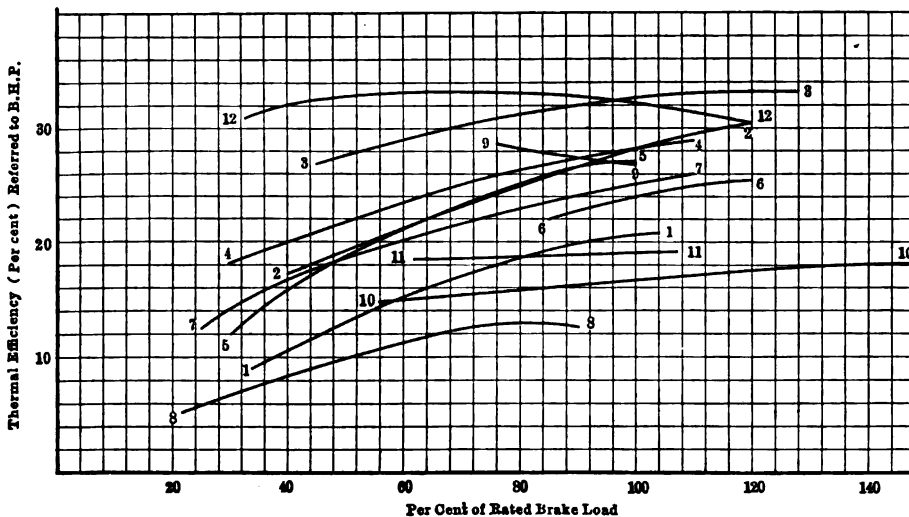


FIG. 167.—Variation of Gas Engine Thermal Efficiency with Fractional Load.

supplied, that does not become converted into work. The *heat balance* of gas engines and complete plants is the term applied to a tabular or graphic statement of the disposition of the heat supplied in fuel, in the form of gas or oil

for engines, or in the form of coal or coke for gas producer and engine plants together. These balances are, of course, different for different sizes and styles of engines, for different fuels and engine loads, but more widely different for complete plants of many units operating on a fluctuating load. In a recent paper by Andrus and Porter some data were presented showing, for average plant working conditions in England, that the overall efficiency of complete gas plants averages about 14 per cent from useful power to heat of coal, over periods ranging from one month to a year. The minimum reported was 12.4 per cent and the maximum 15.05 per cent. They also compared similar data for steam plants which showed an average of 6.83 per cent overall efficiency for periods of one to twelve months, but here the lowest was only 3.70 per cent and the highest 9.30 per cent, showing in round numbers about twice as good a return for gas systems as for steam. Comparing engines alone it will be found that the proportion is different, charging heat in steam or in gas as heat supplied, the performances are closer on the average, though the best gas or oil engine performance has about twice the efficiency of the best steam engine or turbine.

One interesting overall estimate of heat balance by Coster for marine plants is as follows:

TABLE XXXVI

COMPARATIVE HEAT BALANCES OF PRODUCER AND ENGINE PLANTS

	Gas Producer Plant.	Steam Plants.	
		Turbine.	Reciprocating Engines.
B.H.P.	25.0	13.0	10.8
Friction loss.	1.4	1.0	1.2
Exhaust loss.	30.4	56.0	58.0
Gas-engine jacket loss.	20.0		
Gas-engine radiation, etc.	3.2		
Producer or boiler loss.	20.0	30.0	30.0
	100.00	100.0	100.0

The numbers give the per cent of the heat of coal converted into work, or otherwise disposed.

Table LXXXI in the book of tables gives the heat distribution for several different engines alone to show the range of variations.

There is no doubt that one of the prime factors in the failure of the performance of real gas engines to agree with that of the corresponding cycle, is the combined effect of leakage and heat exchanges between walls and working gases; it is certain that all jacket loss is derivable from heat transmitted from gases to walls. These exchanges have been much studied, but it must be confessed with as yet little conclusive result. Such investigations are quite useless if any leakage is taking place, and if the physical constants of the gases are not established. Just as some investigators have been misled

by air-compressor lines into believing leakage to be evidence of cylinder-wall cooling, so have others been finding heat losses from pressure-volume measurements, which are nothing more than leaks. One additional cause of difficulty with the gas engine, which make the T equivalent of its PV product of doubtful value, is the *non-homogeneity* of the charge.

If, however, all these precautions be observed, then the gains and losses of heat by the gas may be found by studying the PV lines. Accordingly, if on any part of the PV diagram or indicator card the pressure P , and volume V , be known, and the change of pressure dP to the change of volume dV measured a short distance on each side of the point, the heat change for that particular volume change may be determined from Eq. (948) in which dH is given in ft.-lbs.

$$\frac{dH}{dV} = \frac{C_v}{R} \left(P + V \frac{dP}{dV} \right) + P, = \frac{1}{\gamma - 1} \left(P + V \frac{dP}{dV} \right) + P, = \frac{1}{\gamma - 1} \left(\gamma P + V \frac{dP}{dV} \right). \quad (939)$$

It must be noted, however, that this is true for constant specific heats only, but even so, it is of some value and changes in sign indicate whether heat is being received (+), or lost (-), by the gas at that time.

Prob. 1. An engine operates on the Otto cycle and the pressures after and before compression are 85 lbs. and 14 lbs. per square inch gage. The diagram factor being taken as .5 and the mechanical efficiency as .8, what is likely to be the hourly consumption per B.H.P., of gas having a heating value of 600 B.T.U.?

Prob. 2. For a large 4-cycle single-cylinder stationary engine operating on natural gas, find the probable consumption of gas per B.H.P. per hour, assuming 900 B.T.U. as the heating value of the gas. Use Table XXXIII for efficiency and consult book of tables for other necessary information.

Prob. 3. An Otto cycle engine works with a ratio of pressures of 5 and uses producer gas No. 6, Table LXXI, Handbook of Tables. What will be the horse-power if this engine is a single-acting 2-cycle one, running at 150 R.P.M. and having a cylinder 20×30 ins.?

Prob. 4. Estimate the horse-power and fuel consumption for a gas engine of any type and size, on any fuel.

108. Actual Performance of Piston Steam Engines and Steam Turbines at Their Best Load and its Relation to the Cyclic. Effect on Efficiency of Initial Pressure, Vacuum, Superheat, Jacketing, and Reheating. Heat Balances of Steam Power Plants. Steam plants including boilers, engines and auxiliaries, are very much more complex both structurally and with reference to thermal changes than gas plants, so it is not surprising that analysis of thermal losses and heat distribution is more difficult. There are vast numbers of these plants in existence bearing little or no similarity in detail in the stationary class, though the representatives of the locomotive and marine groups do fairly well agree with their respective type forms, though less so to-day than a few years ago. It would not be very profitable to study analytically all the various test data of steam plants and of the separate component units, so the treatment of the subject will be mainly confined to methods of determining the influence of controlling factors.

The overall heat balance will be most variable, as shown in the comparison of the balance of a large street railway steam central station, and a locomotive, together with Gebhardt's estimates of a small non-condensing and a simple condensing stationary plant, as given in Table XXXVII.

Those losses that occur in boilers have already been examined and plant auxiliary requirements are items that need no separate detailed thermodynamic analysis, but the performance of the engine or turbine does. It is not so much with the actual efficiency of a given steam engine that this analysis is concerned but rather with the best obtainable efficiency under whatever conditions prevail,

TABLE XXXVII
STEAM PLANT HEAT BALANCES

		Stationary Plant.						Locomotive, Goss.	
		Simple, Non- Condensing, Gebhardt.		Refined Condensing, Gebhardt.		Large Central Station Con- densing, Stott.			
Supplied in coal.....		1.00	1.00	1.00	1.00	
Boiler	{ Sensible heat of stack...50	{	.18022719
	{ Unburned fuel loss.....05002417
	{ Radiation, etc.....07008007
Piping and auxil- iaries	{ Used for boiler feed.....010014		
	{ Used by condenser pumps024016		
	{ Piping and auxiliary021023		
	{ losses.021023		
	{ Returned by feed heater162099			
Engine	{ Discharged by engine624601		
	{ exhaust.....183114054
	{ Converted into I.H.P.037008		
	{ Engine friction.....146106		
	{ Converted into B.H.P..022					
Totals (I.H.P. excluded).....		1.00	1.162	1.162	1.099	1.099	1.00	

with a view to making clear a policy, or the value of imposing one or another condition of service. It has already been shown that the steam consumption per H.P. hour varies with load according to a characteristic curve always having a minimum point, and since efficiency is directly related, so must every steam engine have a best efficiency at some load. The method to be used in determining this efficiency is that of comparison with the corresponding cyclic efficiency, using for this purpose the Rankine cycle. This comparison will show, first, the approach to perfection of the actual machine, but also it will demonstrate: secondly, the specific value of high initial pressures, low terminal pressures, superheat or initial wetness. Exactly similar methods of comparison will demonstrate the value of reheat in compound engines and

throw some light on the desirability or undesirability of using jackets on steam cylinders of piston engines, and on the limiting conditions of staging and vane speed of turbines.

The first step in this analysis is to ascertain from accurate tests of different steam engines, the best load thermal efficiency, with the initial and final conditions of the steam, and by comparison with the Rankine cycle establish *cyclic efficiency* factors. A number of these, for both piston and turbine engines is given in Table LXXXV in the Handbook of Tables.

Examination of this table brings out clearly two important characteristics of steam engines and turbines, first, that the efficiency factors for both classes are substantially the same and approach 80 per cent as a high limit, which is better by about 15 per cent than that found for gas engines; second, that the factor may be very low, less than 50 per cent. This is due to the fact that piston engines leak and suffer heat exchanges, and that expansion is limited by the valve adjustment and cylinder ratios. In turbines, whereas expansion is always complete, the vane and steam jet speed relations may be improper or friction losses occur in the flow path. Where such turbine losses exist they do not differ greatly in range from the different losses in piston engines even at best load. It is most interesting to note that so far as capabilities of conversion of steam heat into work are concerned, as measured by this factor there is little to choose between these two classes of machines, though piston engines appear to be able to make a little better use of the steam than turbines, when constructed to do so, and allowed to in service.

In all cases where cylinder leakage, heat exchange, and expansion losses, or turbine velocity and flow friction losses, are not intentionally suffered to meet load or first-cost conditions, the factor for steam engines and turbines is consistently and materially higher than in gas engines. This indicates that there is much more room for improvement of efficiency in gas than in steam engines, though it is not yet clear whether the present modes of operation forbid its realization or not. The doubt is, however, all on the side of the gas engine, as the nature of the losses in steam machines is fairly well understood and the value of the various factors that control them have been experimentally determined many times.

It takes no elaborate thermal study to see that the higher the initial steam pressure and the lower the terminal pressure the better should be the efficiency. As experimental data to illustrate this point the test results of Goss on locomotives will be used. Several series of runs are available, some with superheated steam and some with saturated, each with boiler pressures from 100 to 250 lbs. per square inch gage, some of which are shown graphically in Fig. 168, on which lines are located to indicate the Rankine cycle efficiency and water rate at the left. These results are replotted in the center, to a scale of efficiency factors, as functions of boiler pressure, but it must be remembered that in no case is expansion complete in a locomotive and the degree must be less for the higher pressures. The corresponding $T\Phi$ diagrams are also given at the right. Examination of these curves indicates clearly that no

great improvement in efficiency or water rate follows the pressure rise, and what changes occur tend first to decrease the steam consumption and later to increase it, the minimum in one case being at 200 lbs., per sq. inch gage and in the other a little less, with the exception of the saturated steam, which consistently improves in performance. The Rankine cycle regularly improves, so it appears that as pressures rise the performance departs more and more from that of the Rankine cycle, the factor being highest at 140 lbs., 64 per cent for superheated, and at 100 lbs., 55 per cent for saturated steam, showing an increase in losses due to leakage, initial condensation, and curtailed expansion. To actually prove the value of high initial pressure would require test data of several engines each designed for a successively higher pressure

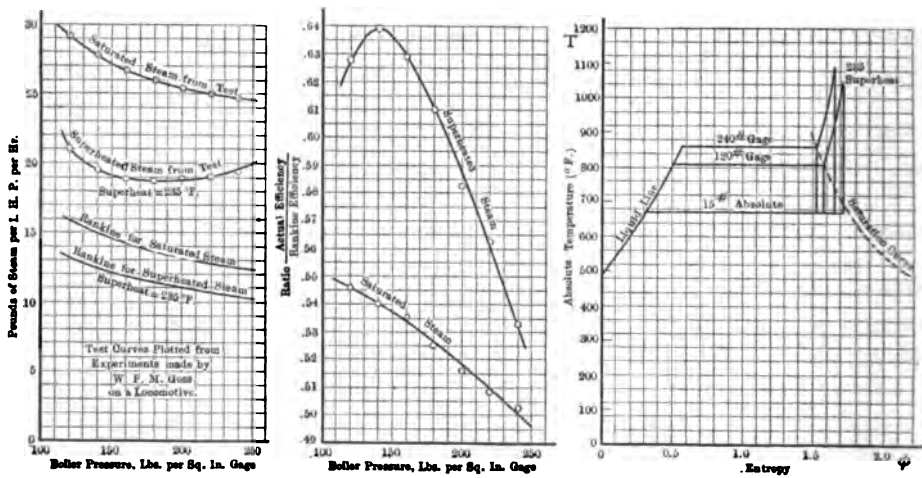


FIG. 168.—Three Diagrams showing the Value of High Initial Pressures with Superheated and Saturated Steam in Locomotive Engines.

and operated under its own best conditions, but no such data are available. The real limit to high pressures is not thermal, but one of expense, gains due to excess pressures over 175 lbs. for stationary plants and 200 lbs. per sq. in. gage for locomotives are obtained at too great cost generally to warrant it.

Lowering the back pressure should have the same effect as raising the initial pressure, in improving efficiency, though to a very much greater degree as appears from both the pressure-volume and temperature-entropy diagrams, Fig. 169. These are drawn for adiabatic expansion from 175 lbs. per square inch initial pressure absolute, with dry saturated steam, to one atmosphere, $ABCD$; for a 5 lb. per square inch rise of initial pressure there is added the area $BCEF$, which is small compared to that obtained from a 5 lb. per square inch reduction of back pressure, given by area $ADGH$. Whether a real engine is capable of taking advantage of this or not depends on its structure and adjustment. For example, if the cylinder volume is represented by AD' on the PV diagram, the gain due to reduced back pressure will be $AD'G'H$ instead of $ADGH$, since expansion must stop at D' , so that piston engines

that ordinarily do not completely expand the steam, can hardly be expected to utilize all the available gain. The situation is better for turbines where the expansion is always complete, but even here the full advantage is not realizable, unless the vane speeds and steam-jet velocities are correctly related. If so related for the higher back pressure, a reduction of it will result in higher jet speeds and, unless vane speeds and R.P.M. correspondingly increase, the

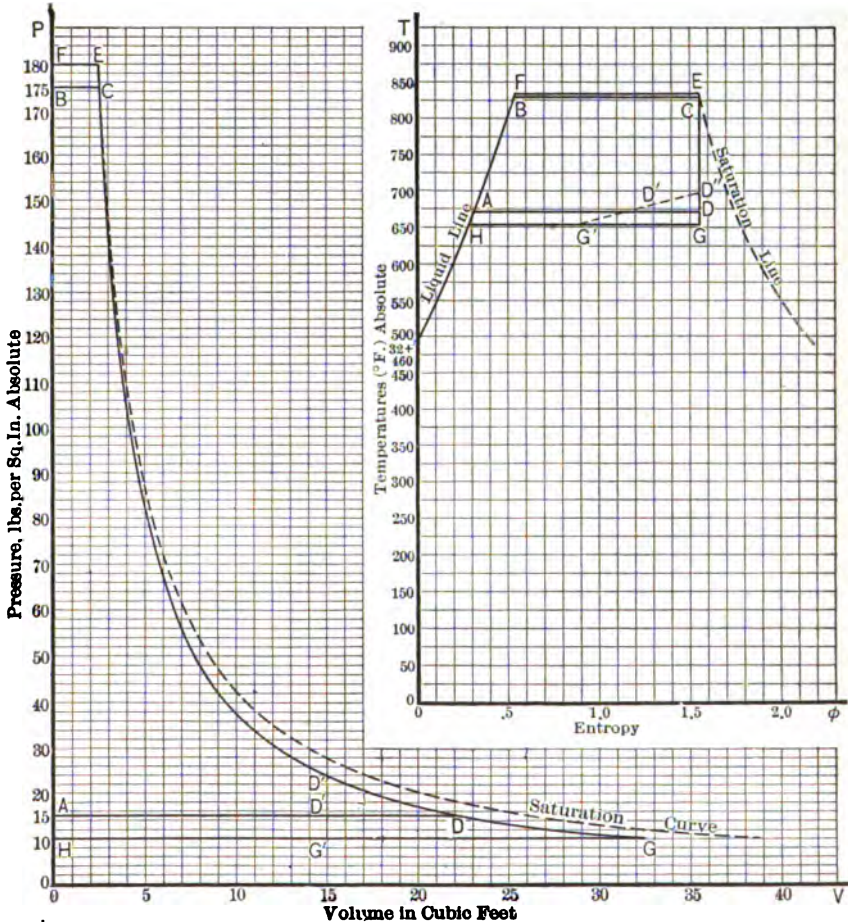


FIG. 169.—Effect on Steam Engine Performance of Raising the Initial Steam Pressure 5 lbs. and Lowering the Back Pressure by 5 lbs. per sq.in.

steam will leave the vanes with some residual velocity and some kinetic energy not utilized.

All experimental data prove that both piston engines and turbines do gain in efficiency with lowered back pressure and very much more than with corresponding increase of initial pressure, the amounts, of course, depending on the pressures themselves; it is also experimentally established that turbines are benefited more than piston engines. To illustrate the effect of back

pressure on a turbine, the test data of a 1250 K.W. Westinghouse turbine is given in Table XXXVIII, with the corresponding Rankine cycle efficiencies and cyclic efficiency ratio calculated and added.

TABLE XXXVIII
STEAM TURBINE EFFICIENCY WITH VARYING BACK PRESSURE AND STEAM
APPROXIMATELY AT CONSTANT INITIAL PRESSURE, DRY SATURATED

Initial pressure pounds per square inch absolute.....	161	162	161	161.6
Back pressure inches Hg absolute.....	5	4	3	2
Pounds of steam per B.H.P. hour (best).....	14.39	14.10	13.59	13.49
Pounds of steam per I.H.P. hour for 94% mech. efficiency.....	13.53	13.25	12.78	12.68
Actual thermal efficiency referred to I.H.P. per cent.....	17.22	17.42	17.92	17.84
Rankine cycle efficiency per cent.....	25.30	26.18	27.08	28.64
Ratio $\left(\frac{\text{Actual efficiency}}{\text{Rankine cycle efficiency}} \right)$	68.05	66.56	66.16	62.29

The efficiency factor varies from 62 per cent for 2 ins. Hg to 68 per cent for 5 ins. Hg, indicating that not so much use is made of the high velocities of the lower back pressure as of the lesser jet velocities of the higher back pressure.

There are no similar tests available for reciprocating engines but the performance of a large 28×54×48 ins., non-condensing engine, running on a 26-inch vacuum, indicates its ability to utilize as much of the available gain as the turbine, in this particular case, more. This engine required 19.4 and 13.65 lbs. of steam per hour per I.H.P. for non-condensing and condensing operation, with a back pressure of one atmosphere and 4 ins. Hg absolute, respectively, and an initial pressure of 165 lbs. per square inch absolute. These water rates for initially dry steam correspond to actual thermal efficiencies of 12.86 per cent, and 16.83 per cent, for which the equivalent Rankine cycle efficiencies are 17.3 per cent, and 26.29 per cent, so that the ratio $\left(\frac{\text{Actual thermal efficiency}}{\text{Rankine cycle efficiency}} \right)$, or the efficiency factor is equal to .743 for one atmosphere back pressure, and .644 for 4 ins. Hg absolute back pressure.

The turbine is not only able to utilize a fairly large fraction of the available gain due to reduction of back pressure, but is generally much cheaper to build than piston engines, especially for very low back pressures which require in piston engines abnormally large low-pressure cylinders. Thus the practice has been established of using turbines for the low ranges of pressure in preference to piston engines, and of combining reciprocating engines and turbines to work together—the turbine taking the exhaust steam from the engine at whatever release pressure it is discharged. This practice has been found especially valuable in adding capacity to existing overloaded reciprocating engines. The most notable example of this is the installation at the 59th Street power station of the Interborough Ry. in New York City. Here the original installation consisted of 15,000 K.W. twin horizontal-vertical compound engines

42×86×60 ins.; need for more power led to an investigation of the relative advantages to be obtained by installing new high-pressure turbines, or of adding low-pressure turbines to existing reciprocating engines; the latter were adopted on the basis of a guarantee of 8 per cent better efficiency than the former, and a 7500 K.W. low-pressure turbine was added to one piston engine unit for trial. Careful tests were conducted which proved beyond question the value of the combination, an average thermal efficiency between loads of 6500 K.W. and 15,500 K.W., of 20.6 per cent being obtained. The engine alone showed a best thermal efficiency of 10.3 per cent on 177.7 lbs. per square inch initial pressure gage, with 8.9° F. superheat, and a back pressure of a little less than 2 ins. Hg absolute, for which the Rankine cycle efficiency is 15.6 per cent, so that the engine realized at best 66.2 per cent of the Rankine cycle. When operated together, however, the rise of back pressure on the engine enabled it to realize a larger per cent of the Rankine cycle, the figure rising to 79 per cent, while the best that was obtained for the low-pressure turbine was about 64 per cent, though not at the same load. Operated together, the overall performance was the best on record, the thermal efficiency being 21.8 per cent, which corresponds to about 78 per cent of the Rankine possibilities, a remarkable performance probably not exceeded in service by any single unit of whatever type. At this time the initial pressure was 199.1 lbs. per square inch absolute for the engine, and 10.35 lbs. per square inch absolute for the turbine with a final back pressure of .46 lb. per square inch absolute, the initial steam being about 1 per cent wet.

Experiments by Carpenter and Sawdon on a Shuman 24×24 ins. simple

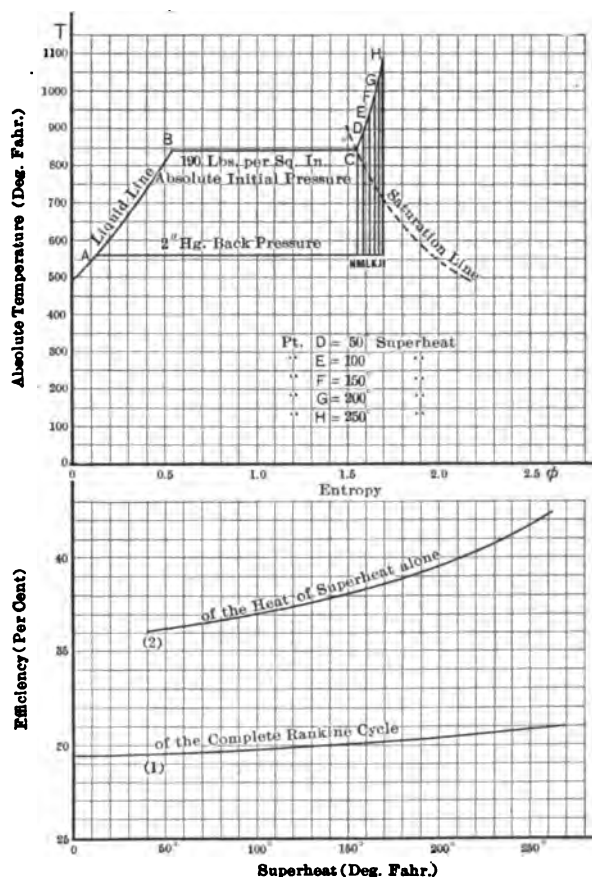


FIG. 170.—Effect of Superheat on Efficiency of Transforming Heat of Steam into Work.

engine with low pressures varying from 8 to 16 pounds absolute indicate that the efficiency ratio is essentially constant at about 50 per cent, which is considerably lower than is easily obtainable in low-pressure turbines.

Superheat is another important factor in steam-engine efficiency, as examination of the $T\phi$ diagram will prove. In Fig. 170 is shown a series of Rankine cycles for 190 lbs. per square inch absolute initial, and 2 ins. Hg absolute back pressure, for every 50° of superheat up to 250° . The curve (1) shows the efficiency of the whole cycle $ABCNA$ to $ABCHKA$, and indicates a slow and regular improvement with increase of superheat from $29\frac{1}{2}$ per cent efficiency for zero superheat to 31 per cent efficiency for 270° superheat. The other curve (2) gives the efficiency of the superheat part of the cycle alone as represented by the areas $CDMNC$ to $CHINC$, and this also increases with the amount of superheat and faster than the whole Rankine cycle of which it is a part. According to this, a gain in thermal efficiency with increase of superheat is to be expected very nearly proportional to the superheat as line (1) is nearly straight, but as maintenance costs increase rapidly beyond a moderate superheat practice has fixed on values about 100° , occasionally going as high as 200° , as the economic limit.

Piston engines as exemplified in locomotive tests showed an improvement in efficiency by superheating, depending on the boiler pressure, of

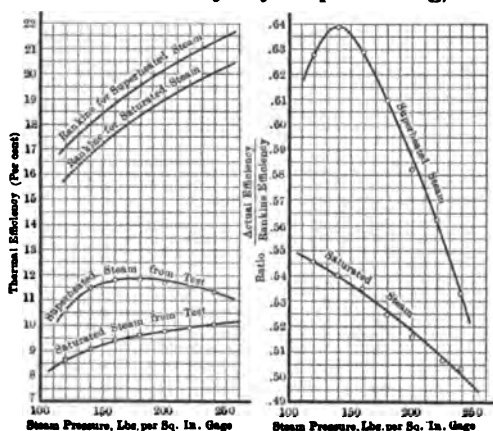


FIG. 171.—Effect of Superheat on Locomotive Engine Efficiency.

2 per cent for 120 lbs. gage pressure, 2.3 per cent for 180 lbs. gage, and only 1.3 per cent for 240 lbs. as would be expected, though another factor of variability of amount of superheat is introduced by the peculiarities of the locomotive structure. All gains are based on the performance with saturated steam as a standard of comparison. The superheat varies in these machines with the boiler pressure and rate of boiler firing and in the tests ran over and under 150° . The net results are given in Fig. 171, compared with

the Rankine cycle. This comparison shows a greater gain by superheat than corresponds to the Rankine cycle expectations as the efficiency factors with superheat are much higher; it is also shown by the greater distance between the actual and the Rankine curves for saturated and superheated conditions. This result is most interesting, usually explained on the theory of lessened leakage with superheated steam, and less wall heat absorption due to the insulating action of a layer of gaseous steam which is always more effective than a film of water, as already shown.

Somewhat better data are available for showing the improvement in

efficiency due to superheat with turbines, as tests are available at constant pressures with superheat as the only variable. Hodgkinson's results for a 1250 K.W. Westinghouse turbine well illustrate this. With an initial pressure of 162 lbs. per square inch absolute and a back pressure of 2 ins. Hg absolute the water rate per I.H.P. hour on an assumed mechanical efficiency of 94 per cent was for dry saturated steam, 12.7 lbs., 12.4 for 75° superheat, and for 100° superheat 12.05 lbs. These data correspond to actual thermal efficiencies of 17.80 per cent, 17.55 per cent, and 17.86 per cent and the corresponding Rankine cycle efficiencies are 28.68 per cent, 28.90 per cent, and 29.01 per cent, so that the cyclic efficiency ratios are 0.6206, 0.6073 and 0.6156.

Tests on the use of superheated steam in pumping engines has led to the estimates by Foster of a saving over saturated steam, depending on the efficiency of the unit previously, as follows: 6, 10, 20, 40 per cent, when the duty is 150, 100, 50, and 10 million foot-pounds useful pump work per million heat units supplied. These figures are higher than those of the Jacobus tests of a New York pumping unit with and without superheated steam, which showed a gain of 18.5 per cent with 207° superheat and 80 lbs. gage pressure when the duty was about 28 million, based on heat in the steam.

The fact that changes in initial pressure, back pressure and superheat in any one type of engine will produce a definite and regularly varying change in efficiency, has led builders to adopt curves of correction for their machines. Thus Emmet states with respect to vacuum, that near 27 ins. Hg the change in efficiency per inch is 6.6 per cent, near 28 ins., 7.8 per cent; near 29 ins., 9.5 per cent; and according to Parsons each inch between 23 and 28 ins. affects efficiency 3 per cent in a 100-K.W., 4 per cent in a 500-K.W., and 5 per cent in a 1500-K.W. turbine. If the turbine is correctly designed for best economy the gain per inch of vacuum will be almost exactly that for the Rankine cycle, but if not so designed, no prediction can be made. It is fairly well agreed that superheat corrections are properly fixed for prevailing pressures at about 10 per cent per 100° F. superheat, up to about 100° F. superheat as a maximum, but it does fall off in the higher ranges, being about 8 per cent per 100° between 100° and 150°. However, as superheat affects steam jet velocity, just as high initial or low back pressure does, it is clear that the realization must depend on design proportions.

The cylinder jacket of piston engines is a factor in steam engine efficiency, the analysis of which cannot be based on the Rankine cycle. It is expected that the jacket will impart heat to the working steam during expansion by condensing some live steam outside the cylinder wall. Exact analysis thermally, of this sort of expansion is quite impossible because the law of heat gain, as expansion proceeds in the working steam, depends on laws of transfer that are not established. Sometimes it is assumed that heat will be received at a rate to keep the working steam at constant quality and this will serve to show the *sort of effect* that the process of heat addition during expansion may be expected to produce. Such a case, for initially dry saturated steam at 160 lbs. per square inch absolute pressure, expanding to 2 lbs. absolute, com-

pared with the corresponding Rankine cycle shows that the saturated expansion cycle has an efficiency of 24.11 per cent, which is less than the 26.07 per cent for adiabatic expansion, so from purely thermal grounds a loss rather than a gain is to be expected from jackets on this assumption of the cycle. However, this does not prove that in real engines jackets may not be efficiency aids, since in the case of superheat more gain may be realized than expected, but on the other hand with pressure range increases, less was realized, so test data are necessary once more. A compound engine $9 \times 16 \times 14$ ins. at 265 R.P.M. tested by Carpenter at 112 lbs. per square inch gage, at 22-in. vacuum showed a minimum steam consumption per I.H.P. hour of 18.6 lbs. with both cylinders jacketed, and 19.1 lbs. without jackets, on initially dry steam, which is a reduction of $\frac{.5}{19.1} = 2.6$ per cent; but at some loads

there was an actual loss by the use of jackets. This is true for practically all cases that have been subject to test, and the conclusion is, that a gain of about 2 per cent may be expected from jackets when the engine works always at the load at which the gain is realizable, which is possible for pumping engines, for example, but when engines have to do work at variable loads the net effect is either a loss or so small as not to warrant the expense of obtaining it.

Reheating the steam between the high- and low-pressure cylinders as a means of improving economy, is in much the same class as jacketing cylinders,

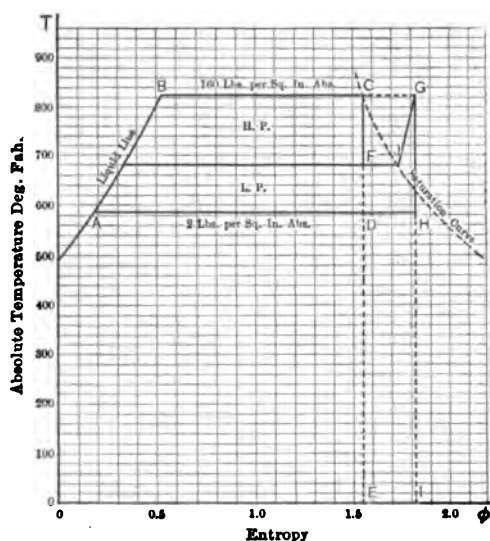


FIG. 172.—Steam Cycle with Perfect Receiver Reheating in Compound Engine Represented by Two Rankine Cycles.

efficiency without reheat to be 26.07 per cent, which is a slight loss. Independent laboratory tests made some years ago on a triple expansion Corliss engine

in fact it is really a process of jacketing receivers of multiple-expansion engines. What may be expected thermodynamically from perfect reheating, that is, up to the initial temperature, of steam initially dry saturated at 160 lbs. per square inch absolute pressure expanding adiabatically to 2 lbs. absolute, is shown in Fig. 172 for the case of equal division of work without reheating. The work diagram without reheat is bounded by *ABCD*, the heat of reheat which first dries and then superheats the steam is given by the area *EFJGIE*, the work derived from the heat of reheat by *FJGHDF* and the complete work diagram with reheat is *ABCFJGHA*. Evaluation of these areas shows the thermal

per cent, with reheat 25.03 per cent, laboratory tests made some years ago on a triple expansion Corliss engine of 200 H.P. and on a compound

Corliss of 150 H.P. respectively, with and without reheat in the receivers, showed a net gain of about 1.5 per cent by reheating at certain loads, and a loss or no measurable difference at other loads, which agrees with practical experience in the use of reheating receivers. These receivers are always troublesome to keep tight and well drained, therefore the tendency is to use simple well-lagged receivers and apply heat in the form of superheat initially, which does yield a gain in efficiency at all times.

One interesting test on a large McIntosh & Seymour engine, $29 \times 60 \times 56$ ins. in the power station of the Boston Edison Co., by Cooke, shows the combined effect of both reheat and jackets, data being available with heaters both in and out of service. At full load, with an initial pressure of 160 lbs. gage and 95° of superheat, 26-in. vacuum, the indicated water rate was, for two runs, 10.85 and 11.18, or 11.01 lbs. mean, with both jackets and reheaters in use, and without them 11.55 lbs. Therefore, the gain by the use

of both was $\frac{.54}{11.55} = 4.7$ per cent at full load. At half load the water rates were

10.33 with reheaters and jackets, and 10.61 without them, or a gain of $\frac{.28}{10.61} = 2.64$ per cent.

In nearly all cases it has been found that expected gains in efficiency due to increases of initial, decrease of back pressure, increase of superheat, use of jackets and receiver reheating, do not agree with that realized and the reason must be sought in interferences due to leakage and heat exchanges between walls and working steam. Analysis of these influences has failed to give results worthy of reproduction, so these factors must as yet be classed with the unknown so far as any prediction is possible, just as in the case of wall heat exchange and leakages in the cylinders of gas engines. One of the most interesting methods of attack yet offered is that recently used by Clayton, as it deals directly with the detection of missing water and leakage, and their separate evaluation, though it has not yet been applied to enough cases to warrant any general conclusions on these troublesome quantities. By making very careful tests of piston engines for the determination of steam consumption and studying the correct indicator cards when the clearance was accurately known, Clayton secured data for plotting the pressure volume expansion line using for the purpose logarithmic cross-section paper on which any equation of the form of $PV^s = \text{Constant}$, becomes a straight line if s is constant. As might be expected from the purely thermal investigation of the values of s for steam, it was found to be substantially constant over the length of one expansion line if not too long, but a variable with respect to the initial wetness or quality of the steam at cut-off. Knowing the quality of the steam supplied to the cylinder, any direct knowledge of the volume or from it the quality of the steam in the cylinder at cut-off, gives by differences the initial condensation if leakage is absent; this difference is itself the missing water per charge of steam-water mixture supplied. The slope of the expansion lines gives the value of s , Fig. 7, and if s is a function

of initial wetness only, it follows that the slope of the expansion line is a measure of missing water. Preliminary tests showed s to vary from .70 to 1.34, depending on engine type, size, speed, pressures, and ratio of expansion, but it always increased in any one engine with increase of cut-off and was always above 1.0 and as high as 1.34 for superheated steam, while for wet steam it was usually less than 1.0 and as low as .7. For a 12×24 in. Corliss engine, no value of s below 1.00 was found for values of cut-off quality above 80 per cent, and no value above 1.10 for cut-off quality less than 72 per cent; in the region of $s=.90$ to $s=1.00$, cut-off quality ranged from 50 to 70 per cent, the several values corresponding equally to saturated steam with long cut-offs and superheated steam with short cut-offs, indicating an independence of cut-off quality with respect to cut-off and a primary dependence of s on cut-off quality. The average of all results is given by Eq. (940) from which the maximum departure was 4.6 per cent.

$$\text{Cut-off quality} = 1.258s - .614. \quad (940)$$

Further tests showed these constants to vary with the pressures and speeds, but no general equation was derived from which it could be evaluated. The conclusion is, therefore, that in any one engine or perhaps in all engines of one type, such an equation as the above holds and can be experimentally found, so that once established a single indicator card may serve to determine the missing water as well as the indicated water, provided leakage is absent or its quantity evaluated. *Incidentally, the failure in real engines of the old assumption of logarithmic expansion is well established and renders the use of the exponential equations for horse-power and indicated water rate, more necessary than they have been regarded in the past.*

Prob. 1. Steam is being used in an engine at 100 lbs. per square inch initial pressure and 20 lbs. per square inch absolute back pressure. Show by PV and $T\Phi$ diagrams the efficiency gain, if initial pressure be doubled while back pressure is held constant, and the gain if the initial be held constant and the back pressure lowered to 5 lbs. per square inch absolute. The engine cylinder volume may be taken as 75 per cent of the volume of 1 lb. of steam at 20 lbs.

Prob. 2. By means of a $T\Phi$ diagram show the increase in efficiency due to superheat of 100°, 200°, 300°, 400°, 500°, in a turbine using steam at an initial pressure of 150 lbs. per square inch absolute and running on a 3 in. vacuum.

Prob. 3. An engine is running with an unjacketed cylinder on an initial pressure of 125 lbs. per square inch absolute and a back pressure of 5 lbs. Show by the $T\Phi$ diagram the change in efficiency if there was a jacket imparting sufficient heat to cause the expansion line to be saturated.

Prob. 4. The engine of Problem 3 is run under the same conditions except that there is initially 200° of superheat. Without the jacket adiabatic expansion occurs and with it the steam is constantly superheated 200°. What will be the difference in efficiencies with and without the jacket in this case?

Prob. 5. A compound engine runs on an initial pressure of 185 lbs. gage and a back pressure of 5 lbs. absolute. The cut-off is so adjusted as to give a receiver pressure of 30 lbs. absolute. Show by $T\Phi$ diagram the efficiency with and without complete reheat

for (a) no initial superheat; (b) 200° superheat and reheat to this temperature; and (c) 200° superheat and reheat to saturation temperature of initial pressure.

Prob. 6. Find the efficiency for the engine of the last problem, considering the low-pressure cylinder to be jacketed and low-pressure expansion to follow the saturation law.

Prob. 7. Predict the thermal efficiency, heat consumption, and water rate for any piston steam engine or turbine under any working conditions, at best load.

CHAPTER XVII

FLOW OF EXPANSIVE FLUIDS THROUGH ORIFICES, NOZZLES, DUCTS, FLUES, CHIMNEYS AND PIPES

109. Flow of Hot Water, Steam, and Gases through Orifices and Nozzles. Velocity, Weight per Second, Kinetic Energy, and Force of Reaction of Jets. Nozzle Friction and Reheating. Relative Proportions of Series Nozzles for Turbines for Proper Division of Work of Expansion. Any expansive fluid such as steam, either superheated, dry saturated, wet, or even a liquid at the boiling-point, as well as all the gases, should on suffering a loss of pressure in passing through a nozzle or orifice of any kind, expand adiabatically and transform into work some of its heat content—the amount depending on the quality of the fluid. What actually occurs is an approximation to this, as close as interferences permit, and these interferences are of the friction and impact order. In the nozzle itself friction will have the effect of lowering velocity developed previously, and so converting kinetic energy back into heat, and as this is a continuous process, the net effect is the same as if heat were added during expansion, the expansion line will therefore take a position above as shown in Fig. 173 on the entropy curve instead of being vertical, as it would be for an adiabatic change. The usual way of defining this line and the energy change responsible for it is to say that x per cent of the energy developed by a partial expansion, assumed for a small drop in pressure to be adiabatic is, after the expansion is completed, added to the fluid as heat at constant pressure. Thus in Fig. 173 let the initial condition be represented by B , C , D , or E , for the cases of water, wet, dry saturated, and superheated steam respectively. If expansion proceeds adiabatically from D for example, the work developed in the form of kinetic energy is represented by the area $BDHA$. Adding at constant pressure an amount of heat equivalent to $\left(\frac{x}{100}\right) \times (\text{area } BDHA)$, the final condition will be as represented by the position of point H' , and a line joining a series of such points, is the representation of expansion with reheat constantly going on. If all the work developed be converted back into heat the expansion line would follow the law of constant energy, and then the kinetic energy of the jet and the velocity would both be zero. Of course, this limiting case is never reached in a nozzle, but is reached when gases and vapors escape through so-called porous plugs or diaphragms or when a jet, once formed, loses velocity by impact; for such cases the final condition of the fluid may be found by these methods. For gases that are perfect, the final temperature would be the same as the initial, since the internal energy is a function of temperature only, but real gases

will always suffer a small temperature change usually to be neglected in engineering computations. Any jet undergoing friction reheat less than 100 per cent, and this includes those through all real nozzles and orifices, will have a velocity to be calculated according to an expansion line lying *between*

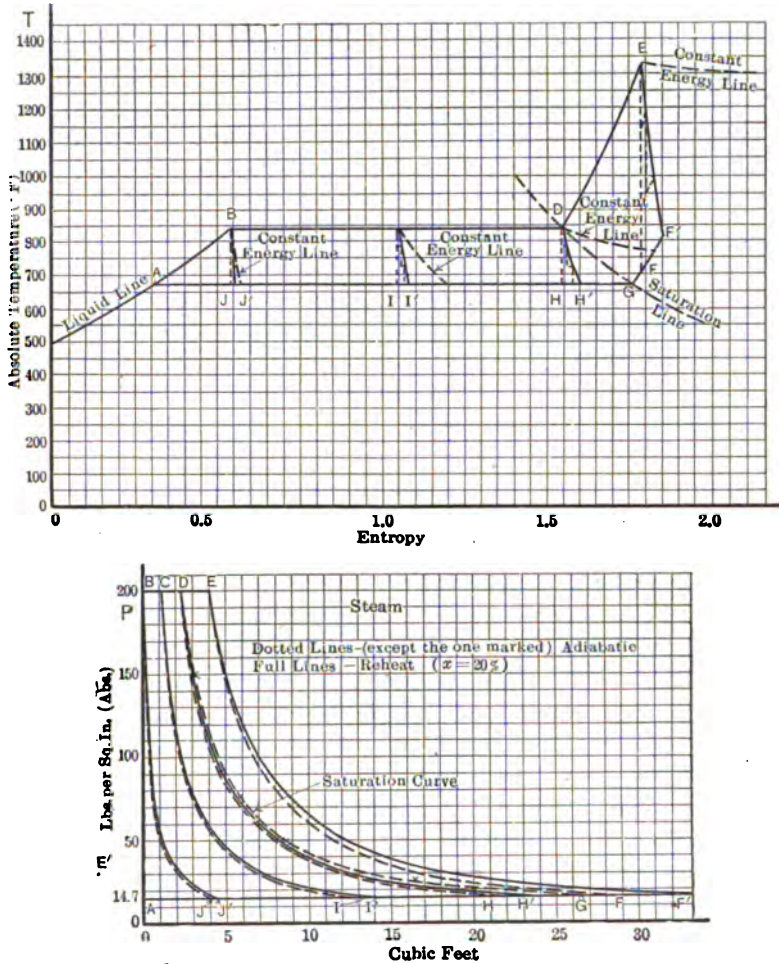


FIG. 173.—Effect of Nozzle Reheat Due to Friction in Nozzle Expansion on Final Condition and Available Gain in Kinetic Energy.

the adiabatic and the constant energy line, just where, depends on the amount of reheat.

After the jet leaves the nozzle with the above velocity and corresponding kinetic energy, it may for a time retain all of it, lose all of it or lose part, depending on subsequent friction and impact. In the throttling steam calorimeter the jet is brought to rest by impact and all kinetic energy converted back into heat, so the final condition of the steam will be that corresponding to a heat content equal to what it had originally, but at the lower pressure, and like-

wise the same as if it had escaped through a porous plug emerging with no velocity, continuously expanding along the constant total heat line. Relations between initial and final qualities can be read off on $T\Phi$ or Mollier diagrams by following constant total heat lines. About the same thing happens when steam or air passes through reducing valves—some velocity is developed and subsequently all, or nearly all is lost by impact on walls or on the mass of fluid on the low-pressure side; experience shows that the loss is practically complete so that perfect gases will not suffer any temperature change ultimately, and steam or hot liquids will have a final quality due to equal total heats. Hot liquids thus will always be partly converted into vapor as in boiler blow-offs and refrigerating expansion valves, while steam may be dried wholly, or in part, or more superheated than it was originally.

In steam turbines the jet once formed is brought to moving vanes and there has its direction of motion changed so that it leaves the vanes with a velocity with respect to them, but ideally with no velocity with respect to the casing or nozzle. When this process is perfectly executed the whole kinetic energy of the jet is imparted to the moving vane wheel, but actually there is developed some vane friction which acts as additional reheat, giving the steam a new quality higher than it had before it reached the vane, which is to be measured by an added amount of energy equivalent to the *vane reheat* expressed as y per cent of the energy of expansion. This is very important in those turbines that have a series of successively expanding nozzles, as work to be developed in the second depends in part on the nozzle and vane reheats of the first. Such reheat is, therefore, not as much a loss in such multi-stage turbines as in those of one stage, but always constitutes one of the main energy losses in these machines, the other losses being leakage and windage friction of the rotor in the steam atmosphere, which latter adds more reheat. The whole reheat is commonly taken as 40 per cent, on the average.

To calculate the velocity of a jet two factors are necessary: first, the work that would be done by adiabatic expansion, and second, the reheat factor. This reheat factor is for turbine nozzles about 10 per cent, but is not established for other forms, like valve orifices. The determination of velocity without nozzle friction has already been explained; the PV formula of Chapter I is to be used for gases, but for steam the Mollier diagram is the most expeditious, as both the work done and velocity can be read off directly. Determinations of final quality of the fluid jet can also be made for the case of no reheat from the same diagram directly, and by a simple additional step the quality can be found graphically as illustrated in Fig. 174. This is a section of the Mollier diagram for steam, where AB represents adiabatic expansion, during or after which, 20 per cent of reheat occurs. To find the final quality lay off $\overline{BC} = .2 \times \overline{AB}$, so that \overline{AC} represents the effective work and \overline{BC} the heat of reheat. A horizontal through C cutting the constant-pressure line through B locates D , the condition of final quality. It is interesting to note that if this heat is constantly added, that is, if the reheat is a constant fraction in the nozzle, the quality at any pressure can be found by a similar construction

used by Roe. With B as a center, an arc CE is drawn, and a tangent to it drawn through A . Similar tangent arcs drawn from other centers give the means of finding the quality at the pressure indicated by that center, by the intersection of horizontals tangent to the tops of the arcs, with constant-pressure lines through the centers. The line AD then represents the variation of condition of the steam in the nozzle. If it receives vane or other reheat beyond the nozzle the jet will have a sudden change in quality at the end of this nozzle condition line.

The weight of flow of expansive fluids in pounds per second per square inch of orifice, has a critical value at a given pressure ratio, which divides a range of pressures over which the flow does not change with pressure change, from the other region where it does. In the first Chapter it was shown by PV analysis that this critical pressure was a function of the exponent γ given by Eq. (26) or by Eq. (941).

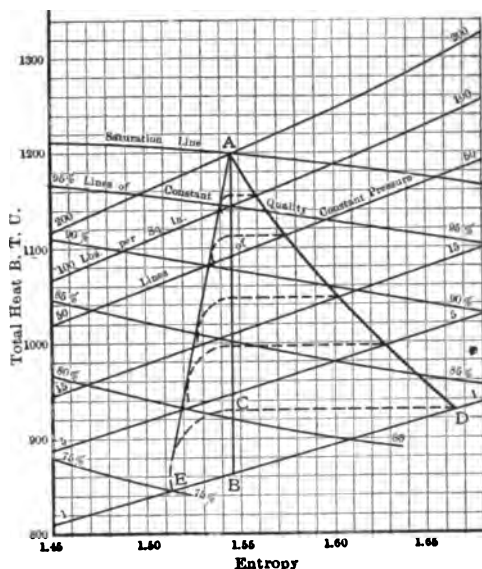


FIG. 174.—Graphic Method of Locating Nozzle Steam Expansion Line and Final Quality with Given Per Cent Friction Reheat.

$$\text{At the critical point, } \left(\frac{\text{Final pressure}}{\text{Initial pressure}} \right) = \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}} \dots \dots \dots (941)$$

This is easily fixed for gases not undergoing wide temperature changes because for them γ is constant, but for vapors the variability of γ makes it more difficult. However, for the conditions in turbine nozzles the ratio is usually taken at the round number of .58, though it depends on the quality, the pressures and temperatures of the steam. Careful investigation of this critical phenomenon seems to indicate that at some point in every orifice the pressure falls to this critical value of itself, and that the weight of flow is fixed by the pressure at that point regardless of how much lower the pressure may fall. Furthermore, it appears that at this critical point of the orifice or throat, each fluid acquires a fixed velocity peculiar to itself, about the same, according to Emden, as the velocity of sound in the medium, and for steam this is between 1400 and 1500 ft. per second for such conditions as exist in turbine nozzles. Beyond this critical point, further expansion may take place, increasing the velocity above the critical value but not the weight, *provided the orifice or nozzle is so shaped as to keep the flow axial and prevent sidewise dissipation, in which case the nozzle is an expanding nozzle.*

Weight of flow for steam and for air, have been calculated from Eq. (25),

using $\gamma=1.4$ for air, and by the Mollier diagram for steam. These are given in graphical form, charts 66 and 67 in the book of tables, to which are added some curves of experimental flow laws stated symbolically below:

$$\left\{ \begin{array}{l} \text{Lbs. dry saturated steam} \\ \text{per sq.in. area per hour} \end{array} \right\} = \left\{ \begin{array}{ll} 60p^{.97} \text{ (Grashof)} & (a) \\ 51.43p \text{ (Napier)} & (b) \\ 49.6p \text{ (Harter)} & (c) \\ 3.6p (16.3 - 96 \log p) \text{ (Rateau)} & (d) \end{array} \right\} \quad (942)$$

$$\left\{ \begin{array}{l} \text{Lbs. superheated steam per} \\ \text{sq.in. area per hr.} \end{array} \right\} = \left\{ \begin{array}{ll} 60p^{.97} [1 + .00065 \times (\text{degrees} \\ \text{superheat})] \text{ (Moyer)} & (a) \\ 49.6p \text{ to } 45.0p \text{ for superheat} & \\ 0^\circ \text{ to } 185^\circ \text{ F. at 160 lbs. sq.} & \\ \text{in. (Harter)} & (b) \end{array} \right\} \quad (943)$$

$$\left\{ \begin{array}{l} \text{Lbs. air per} \\ \text{sq.in. area} \\ \text{per hour} \end{array} \right\} = \left\{ \begin{array}{ll} 3816 \sqrt{\frac{p_2(p_1 - p_2)}{T_1}} \text{ for less than 2 atm. to atm.} & (a) \\ \text{(Fliegenger)} & \\ 1900 \frac{p_1}{\sqrt{T_1}} \text{ for more than 2 atm. to atm.} & (b) \\ \text{(Fliegenger)} & \end{array} \right\} \quad (944)$$

The superheated steam Napier coefficients and corresponding experimental steam weights are also given in the Handbook of Tables, Chart No. 68, reproduced from Harter's report.

For steam, the best and most used relation is that of Grashof, which for wet steam, such as always found entering low-pressure turbines, becomes

$$(\text{Lbs. wet steam per square inch area per hr.}) = \frac{60 p^{.97}}{\sqrt{\text{dryness fraction}}} \quad (945)$$

By means of these equations the size of nozzle *throat* or narrowest point actually or, in effect, to pass a given weight of steam or air can be calculated, or inversely, the weight that will pass a given area. The form of the nozzle, however, affects the result by making the actual minimum area differ from that which is effective, because of stream contraction. The effective area may be only 60 per cent of the actual throat area if sharp corners exist. The product of weight per hour and foot-pounds per pound of steam gives the kinetic energy of the jet in foot-pounds per hour, which has, of course, a horse-power equivalent. This jet horse-power will not be realized, however, unless the nozzle beyond the throat has a proper shape and a proper ratio of maximum terminal nozzle area, to that of the throat, to enable the full expansion work to be realized beyond the nozzle throat. The ratio of the largest area of the nozzle—that at the mouth, to the least—that at the throat, is the expansion ratio of the nozzle for which two usable values not in accord, are given by Eq. (946);

$$\left\{ \begin{array}{l} \left(\frac{\text{Mouth area}}{\text{of nozzle}} \right) \\ \left(\frac{\text{Throat area}}{\text{of nozzle}} \right) \end{array} \right\} = \left\{ \begin{array}{ll} \frac{.1550}{\sqrt{\left(\frac{p_2}{p_1}\right)^{1.762} - \left(\frac{p_2}{p_1}\right)^{18.81}}} \text{ (Zeuner)} & (a) \\ \left[.172 \left(\frac{p_1}{p_2}\right) + .7 \right] \text{ when } \frac{p_1}{p_2} < 25 \text{ (Moyer)} & (b) \\ \left[.175 \left(\frac{p_1}{p_2}\right)^{.94} + .7 \right] \text{ when } \frac{p_1}{p_2} > 25 \text{ (Moyer)} & (c) \end{array} \right\} \quad (946)$$

where p_1 and p_2 are the initial and final working pressures for the nozzle. Between throat and mouth various curves and angles are used, with, however, not so much difference in results as might be expected; about the commonest practice is to use an angle of 20° and a straight-line element. Too sudden an expansion due to too wide an angle will cause the steam to rebound from side to side and set up waves that result in increased frictional reheat. Too long a nozzle also brings about increased friction; too large a mouth results in overexpansion, with subsequent compression and more waves, and too small a mouth, incomplete expansion.

The reaction force on the nozzle due to the exit of the jet is

$$(\text{Reaction in lbs.}) = \frac{1}{32.2} [(\text{lbs. steam per sec.}) \times (\text{jet velocity ft. per sec.})] \quad (947)$$

A series of steam turbine nozzles of different forms were tested for efficiency of conversion of heat into kinetic energy by means of measured weights and reaction forces, by Selby and Kemble, and the best results were obtained with a straight taper of $14^\circ 31'$, and rounded entrance. With initial absolute pressures of 100 to 145 lbs. per sq. in., back pressures from 1.12 to 1.63 lbs. per sq. in. absolute, the flow varied from 389 to 558 lbs. per hour and the ratios of the work equivalent to the kinetic energy computed from flow weights and reactions, to the Rankine cycle were, 98.3, 97.9, 97.5, and 97.1 per cent respectively, showing how nearly perfect such nozzles may be made.

When several nozzles are in series and the same steam weights pass through all, their absolute and relative sizes can be fixed by the previous relations if the pressures in the intermediate chambers be fixed, because each may be treated independently of the other except for initial quality of approach to each, which must be determined. Thus, assume expansion to take place in four pressure stages or four nozzles or sets of nozzles, each with x per cent uniform frictional reheat, and with y per cent vane reheat between nozzles, then will the whole expansion process be according to the line $ABCDEFGH$ of Fig. 175 to both $T\Phi$ and $H\Phi$ coordinates for equal division of pressures from 200 lbs. to 1 lb. per square inch absolute, and with originally dry saturated steam. This line is determined by drawing the four constant pressure zones, each 50 lbs. per square inch, across the two diagrams, dropping a vertical AM on the $H\Phi$ chart, giving the Rankine cycle work for the first

stage pressure range, laying off $\overline{B'M} = \left(\frac{x}{100}\right) \times \overline{AM}$, the nozzle reheat loss, and

$\overline{B'B''} = \left(\frac{y}{100}\right) \times \overline{AM}$, the vane reheat loss, referred to the same Rankine cycle

basis, so that point B locates the condition of the steam at exit from first nozzle, and C its condition at the entrance to the next. The work of the first

stage is, $\left[1 - \left(\frac{x+y}{100}\right) \times (\text{Rankine cycle work})\right]$ for the two pressures, and the

first-stage efficiency is, $\left[1 - \left(\frac{x+y}{100}\right)\right]$. A similar construction applies to the

other stages, and where graphically carried out will show unequal work and stage efficiencies even with equal per cents of reheat, when the stages are equally divided as to pressure range. It is more usual to divide the work per stage equally; if the stage efficiencies were 100 per cent this could be easily carried out by dividing the vertical AN representing the whole expansion work on the $H\Phi$ diagram into four equal parts, through each of which the constant pressure lines could be drawn. With any nozzle and vane reheat losses, however, this will not give equal stage work, but is a good starting point for a graphical trial and error method of finding the pressure

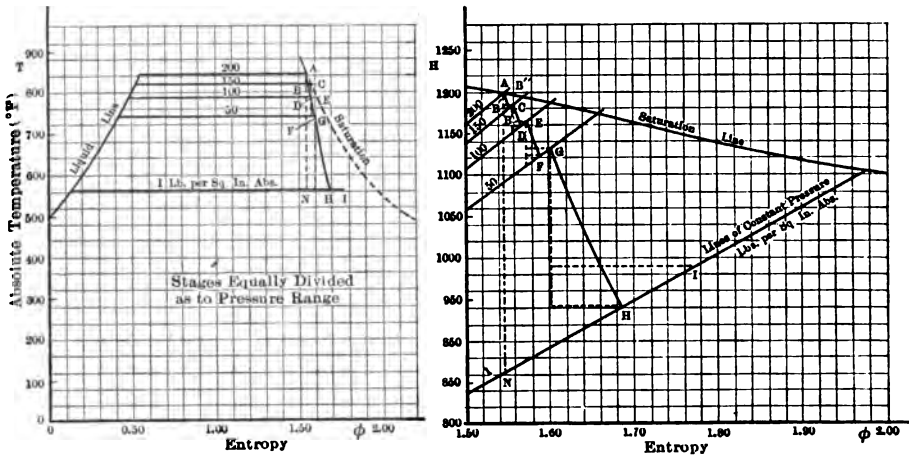


FIG. 175.—Multi-stage Series Nozzle Expansion of Steam, with Nozzle Reheat, $x=20$ Per Cent, and Vane Reheat, $y=20$ Per Cent. Stages Equally Divided as to Pressure Range. Work of Stages Unequal.

division that will give it, after which the nozzle sizes can be found to maintain that pressure relation.

Prob. 1. Steam at 100 lbs. per square inch absolute containing 5 per cent moisture expands in a nozzle to 10 lbs. per square inch absolute with 20 per cent nozzle reheat. What is the final quality, velocity, and horse-power per square inch of nozzle and what fraction is each of that which would result without nozzle friction?

Prob. 2. How would the results of the previous problem differ by changing final pressure to 1 lb., initial pressure to 150 and 200 lbs., and initial quality to 50 per cent, 100 per cent or 100° superheat, in any combinations?

Prob. 3. Air at 150 lbs. per square inch absolute and 1000° F. expands into a standard atmosphere, where its temperature in the jet is 400° F. If the reheating had been uniform, plot the $T\Phi$ expansion curve, find s and the reheat factor.

Prob. 4. What would be the nozzle reaction per square inch of orifice for Problems 1 and 3 and what dimensions should expanding nozzles have?

Prob. 5. Water from a boiler with 150-lb. gage pressure is blown off. What per cent will evaporate? How much will the evaporation change with initial pressure? Plot a curve.

Prob. 6. Air flows from a tank where the pressure is 150 lbs. per square inch absolute, to a standard atmosphere through a $\frac{3}{4}$ -in. diameter throat nozzle. How many cubic

feet of free air must be supplied per hour to maintain the tank pressure and what compressor horse-power will be required if two stage?

Prob. 7. A steam turbine has four $\frac{1}{2}$ -in. diameter nozzles in parallel, the back pressure is 2 lbs. per square inch absolute. How much steam per hour and what jet velocities result for, (a) 150 lbs. per square inch absolute and 100° superheat initially, (b) same pressure and dry saturated steam, (c) 100 lbs. per square inch absolute and dry saturated steam, (d) same pressure as in (c) and 75 per cent quality. Use Mollier diagram.

Prob. 8. Compare results of Problem 7 with those computed by the formulas of Grashof, Napier, Rateau, Harter, Moyer.

Prob. 9. For same initial pressure and temperature and same final pressures compare the preceding results on steam flow with air flow, using rational formula and Fliegner's empiric form.

Prob. 10. If four nozzles of a steam turbine are in series, and the first $\frac{1}{2}$ -in. diameter is supplied with dry saturated steam at 180 lbs. per square inch absolute, what must be the sizes of the other three

- | | | | | |
|-----|--|-----|---------------------------|---|
| (a) | to equally divide the pressure range with no nozzle and no vane reheat | | | |
| (b) | " " " " " | 20% | vane reheat | |
| (c) | " " " " " | 20% | nozzle and no vane reheat | |
| (d) | " " " " " | 20% | " 20% | " |
| (e) | to equally divide the temperature range with no | " | no | " |
| (f) | " " " " " | no | " 20% | " |
| (g) | " " " " " | 20% | " no | " |
| (h) | " " " " " | 20% | " 20% | " |

The back pressure is 2 lbs. per square inch absolute.

110. Flow of Expansive Fluids under Small Pressure Drops through Orifices, Valves, and Venturi Tubes. Relation between Loss of Pressure and Flow. Velocity Heads and Quantity of Flow by Pitot Tubes. When gases and vapors flow with a small difference between initial and final pressures, the flow characteristics have a special engineering importance great enough to warrant more precise determinations of equations and constants than those which apply to flow in general or with large pressure drop. The loss of pressure through the valves of air compressors, steam and air engines and gas engines, and its relation to velocity and weight of flow per square inch of opening, are examples of one class of application. Others of a similar nature include the flow of gas through burner nozzles, air from ventilating duct openings and through fan passages; also various meter orifices including the Venturi and Pitot tubes involve in their use, similar relations.

As a starting-point for the analysis of all these cases, the flow through an orifice, assumed to be adiabatic, must be adopted. Accordingly, calling P_1 the initial and P_2 the final pressure in pounds per square foot, Eq. (25), of Chapter I, gives the flow when the difference is not too great.

$$(\text{Lbs. per sec. per sq.ft.}) = \frac{w}{A} = \sqrt{2g} \frac{\gamma}{\gamma-1} \frac{P_1}{V_1} \left[\left(\frac{P_2}{P_1} \right)^{\frac{2}{\gamma}} - \left(\frac{P_2}{P_1} \right)^{\frac{\gamma+1}{\gamma}} \right] \dots \quad (948)$$

Applying this to air passing through a round orifice of d ins. diameter, this becomes for air at 60° F.

$$\left\{ \begin{array}{l} \text{Lbs. air per sec. through} \\ \text{circle of } d \text{ ins. dia. at } 60^\circ \text{ F.} \end{array} \right\} = .0816d^2 \sqrt{\frac{P_1}{V_1} \left[\frac{1}{R_P^{1.428}} - \frac{1}{R_P^{1.712}} \right]} \quad \dots (949)$$

Substituting $V_1 = \frac{RT_1}{P_1}$, and noting that $p = \text{pr. lbs. per sq. in.}$ this becomes

$$\left\{ \begin{array}{l} \text{Lbs. air per sec. through} \\ \text{circle of } d \text{ ins. dia. at } 60^\circ \text{ F.} \end{array} \right\} = .00049d^2 P_1 \sqrt{\frac{1}{R_P^{1.428}} - \frac{1}{R_P^{1.712}}} \quad (a) \\ = .0707d^2 p_1 \sqrt{\frac{1}{R_P^{1.428}} - \frac{1}{R_P^{1.712}}} \quad (b) \quad \dots (950)$$

With orifice friction and poor orifice forms the actual area may not be effective, so there must be introduced coefficients representing these influences as experimentally established. It is found, however, that the actual flow values agree fairly well with a simpler and more practical formula for very small pressure drops. Such a formula can be developed on the theory that when the drop in pressure is small the work derived by expansion may be neglected, and the fluid may be considered as non-expansive, and of a density corresponding to the mean pressure on the two sides and to the original temperature, supposed to remain constant. For this case

Let h_w = the difference in pressure in inches of water;

H_A = the head in feet of air corresponding to h_w ;

P = mean pressure, lbs. sq.ft. = $\frac{P_2 + P_1}{2}$;

V_A = cu.ft. per lb. air = $53.34 \frac{T}{P}$;

A = sq.ft. area of orifice;

u = velocity in feet per second;

w = pounds air per second.

Then since 5.2 pounds per square foot is equal to one inch of water column,

$H_A = V_A(P_2 - P_1) = 5.2V_A h_w$, and $\frac{wV_A}{A} = u$, so that

$$\left\{ \begin{array}{l} \text{Pounds air per sec.} \\ \text{per sq.ft. of orifice} \end{array} \right\} = \frac{w}{A} = \frac{u}{V_A} = \frac{\sqrt{2gH_A}}{V_A} \quad (a); = \frac{\sqrt{2g \times 5.2h_w V_A}}{V_A} \quad (b) \\ = \sqrt{\frac{64.4 \times 5.2h_w P}{53.34T}} \quad (c); = 2.506 \sqrt{\frac{h_w P}{T}} \quad (d) \quad \dots (951)$$

For a round orifice of d ins. diameter this becomes

$$\left\{ \begin{array}{l} \text{Lbs. air per sec.} \\ \text{through circle} \\ \text{of } d \text{ ins. diam.} \end{array} \right\} = \frac{.7854d^2}{144} \times 2.506 \sqrt{\frac{h_w P}{T}} \quad (a); = .01366d^2 \sqrt{\frac{h_w P}{T}} \quad (b). \quad (952)$$

If the discharge is from a low-pressure chamber into a standard atmosphere, this becomes very nearly

$$\left\{ \begin{array}{l} \text{Lbs. air per sec. to atmos. through} \\ \text{circle of } d \text{ ins. diam.} \end{array} \right\} = .6283d^2 \sqrt{\frac{h_w}{T}} \quad \dots (953)$$

The best determinations of the constants for either the approximate formula, Eq. (952), or the exact one, Eq. (950), are those made by Durley, whose exper-

imental coefficients for various orifices are given in Table XXXIX. His results show that the coefficient for small orifices increases with the head but more and more slowly and is constant for a 2-in. diameter orifice, after which it decreases. Increase of diameter involves small coefficients, but it was shown that temperature had no effect between 40° F. and 100° F.; this is probably so for even wider ranges. The orifice must be not more than $\frac{1}{8}$ the area of the approach chamber for these coefficients to apply. They are all for a plate .057 in. thick and for sharp edges; changing the thickness or rounding the edges will

TABLE XXXIX
COEFFICIENT OF DISCHARGE C FOR AIR UNDER VARIOUS WATER
HEADS AND DIAMETERS OF ORIFICE (DURLEY)

Diameter of Orifice. Inches.	1-inch Water.	2-inch Water.	3-inch Water.	4-inch Water.	5-inch Water.
$\frac{1}{8}$.603	.606	.610	.613	.616
$\frac{1}{4}$.602	.605	.608	.610	.613
1	.601	.603	.605	.606	.607
$1\frac{1}{2}$.601	.601	.602	.603	.603
2	.600	.600	.600	.600	.600
$2\frac{1}{2}$.599	.599	.599	.598	.598
3	.599	.598	.597	.596	.596
$3\frac{1}{2}$.599	.597	.596	.595	.594
4	.598	.597	.595	.594	.593
$4\frac{1}{2}$.598	.596	.596	.593	.592

Weight of air discharged per second = $.6283Cd^2 \sqrt{\frac{hw}{T}}$ lbs. (Barometer at 30 inches.)

change the coefficient. Moss reports some tests with rounded approach orifices in which the coefficient was as high as $C=0.942$.

When the thickness of the plate is great in proportion to the diameter of the orifice, the latter becomes a *short tube* for which differences enter. It appears that the old results of Weisbach on air for both small orifices and short tubes are fairly good and these are reproduced in the Tables.

His general conclusions on the coefficients for these small orifices give them values between .97 and .99 for orifices with rounded entrance, .56 to .79 for sharp edge in thin plates, .81 to .84 for short straight tubes, .92 to .93 for short tubes with rounded entrance and .90 to .99 for converging orifices.

When the fluid is not air, the flow is to be determined by the general law of proportionality as a function of density. The velocities due to pressure drop for different gases and vapors in these cases are to each other inversely as the square root of the density; thus, for any gas of specific volume, V_g , or density, $\frac{1}{V_g}$, its velocity is given in terms of that of air by,

$$u_g = u_A \frac{\sqrt{\frac{1}{V_A}}}{\sqrt{\frac{1}{V_g}}} = u_A \sqrt{\frac{V_g}{V_A}} \quad \dots \dots \dots (954)$$

Putting this in terms of weights by the relation, $\frac{wV}{A} = u$,

$$w_G = w_A \frac{V_A}{V_G} \sqrt{\frac{V_G}{V_A}} = w_A \sqrt{\frac{V_A}{V_G}} \quad \dots \dots \dots (955)$$

so that flow weights are to each other directly as the square roots of the densities.

One very common and important case of orifice flow with small pressure drop is that through the valves of gas engines and compressors, valves of the poppet type with both flat and conical seats, the flow being in both directions, that is, toward the cylinder and from it, while the valve always seats away from the cylinder in the direction of the action of the superior pressure. Experimental determinations of the coefficients for such flows were made in the laboratories of Columbia University on both mechanically operated and spring-closed valves, all, however, of small size. One valve had a flat seat 1.58 in. diameter, and the other two, conical 45° seats 1.5 and 2.0 ins. diameter respectively. These were supplied with measured air flowing in either direction first with the valves fixed at known lifts, and later rising and falling with the piston known amounts and at measured rates. From the data obtained the relation between flow and pressure drop up to 30 ins. of water was obtained and compared with the flow-pressure drop relation for no losses for which, $u = \sqrt{2gH_A}$. The ratio of the measured velocity or volume to that computed for no loss, as above, gave the coefficient of efflux for the given pressure drop. The corresponding coefficients are not easily obtained for intermittent flow, the more direct method being to compare momentary velocities actual and computed, at each point of the rise and fall of the valve for the measured pressure drop, and plot a curve of pressure drop to velocity, as shown in Fig. 176 (a). To give a direct comparison between the steady and the intermittent flow, the corresponding curves for steady flow are plotted in Fig. 176 (b). It is impossible to give any single generalization of these results, as the coefficients varied from 0.2 to 1.0, depending on type of valve, its lift and the rate or flow as well as direction of flow, and the only thing that appears to be sure is, that such flow is very complex indeed, and may involve very serious losses, but on the other hand may be very highly efficient.

Venturi tubes are usually made by a converging pipe of straight cone form and small angle, ending in a short small diameter cylinder, which discharges into a diverging cone of smaller angle, ending in the original diameter; while designed originally for measuring the flow of water they have been found of great practical value in measuring gases. The larger sizes are especially useful, since gas meters of the volumetric displacement type of equal capacity, are prohibitively costly. In such a tube the same weight of fluid is passing each point, and neglecting frictional losses and static head differences which apply to tubes set vertically, but not to horizontal ones, the sum of the velocity head and pressure head is constant. Therefore calling P_1 and P_2 the upstream and throat pressures lbs. per square foot, and A_1 and A_2 the corresponding areas in

square feet, the equation of condition is, $P_1 + \frac{u_1^2}{2g} = P_2 + \frac{u_2^2}{2g}$ (956)

If the fluid be of constant density as it passes from the large area to the small one, the velocities must vary inversely as areas, since equal volumes pass each point in this case, $\frac{u_1}{u_2} = \frac{A_2}{A_1}$. Therefore the throat velocity u_2 will be given by Eq. (957), for constant density

$$u_2 = \frac{A_1}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g(P_1 - P_2)} \quad (957)$$

When gases and vapors undergo very small pressure drops Eq. (957) may be applied to their flow in Venturi tubes, but when the difference between upstream and throat pressures is so small as to warrant the use of this relation, it is too small for the accurate readings needed to determine the quantity flowing. Therefore, for such cases it is necessary to return to the fundamental equation for expansive fluids, equating the increase in kinetic energy to the work of the cycle as for turbine nozzles:

$$\frac{u_2^2 - u_1^2}{2g} =$$

$$\frac{\gamma}{\gamma - 1} \frac{P_1}{\delta_1} \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} \right] \quad (958)$$

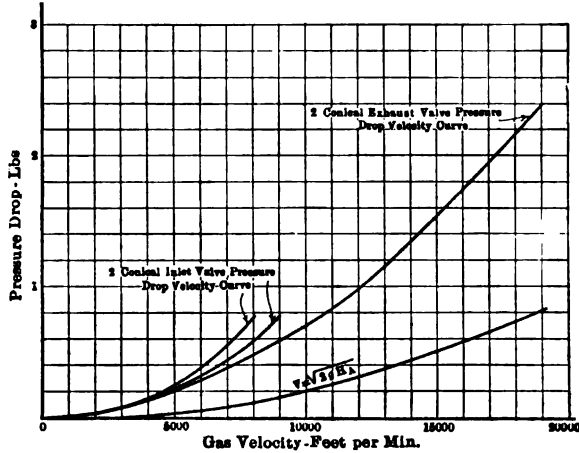
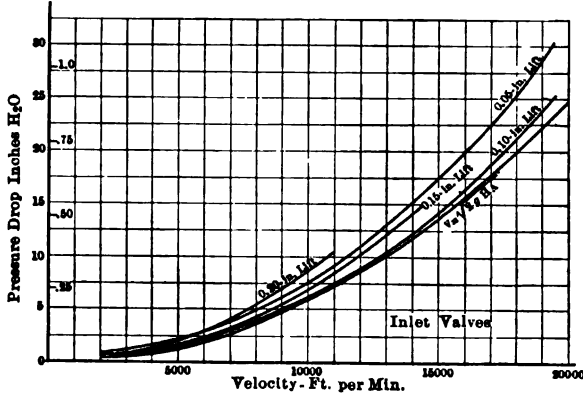
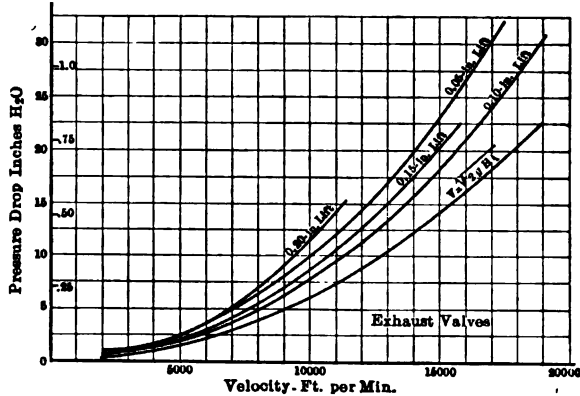


FIG. 176 (a).—Intermittent Flow through Poppet Valves.



Outer Scale Inches of Water. Inner Scale Coefficient.

FIG. 176 (b).—Steady Flow through Poppet Valves. Relation between Pressure Drop and Velocity.

In Eq. (958), δ_1 is the density of the fluid on the high pressure or upstream side in pounds per cubic foot. Since equal weights must pass both the upstream and throat sections, the velocities, densities and areas must be related thus:

$$\left. \begin{aligned} w = w_1 = w_2 = A_1 u_1 \delta_1 = A_2 u_2 \delta_2 = A_2 u_2 \delta_1 \left(\frac{P_2}{P_1} \right)^{\frac{1}{\gamma}}, (a) \\ \text{therefore} \quad u_1 = u_2 \frac{A_2}{A_1} \frac{\delta_2}{\delta_1} = u_2 \frac{A_2}{A_1} \left(\frac{P_2}{P_1} \right)^{\frac{1}{\gamma}} (b) \end{aligned} \right\} \quad (959)$$

Substituting this value of u_1 in Eq. (958), solving for u_2 and introducing the resulting value of u_2 in the expression for w , Eq. (959), the weight of flow is found as in Eq. (960):

$$\left. \begin{aligned} \text{Lbs. of gas or vapor} \\ \text{per second} \end{aligned} \right\} = w = A_2 \left(\frac{P_2}{P_1} \right)^{\frac{1}{\gamma}} \sqrt{2g \frac{\gamma}{\gamma-1} P_1 \delta_1 \left[\frac{1 - \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}}{1 - \left(\frac{A_2}{A_1} \right)^2 \left(\frac{P_2}{P_1} \right)^{\frac{2}{\gamma}}} \right]} \quad (960)$$

In the actual use of Venturi tubes the value of $\left(\frac{P_2}{P_1} \right)$ should be between .95 and .995, which values correspond nearly to throat velocities from 300 to 100 ft. per second for low-pressure gases near one atmosphere in pressure, but for higher gas pressures very much higher values may be used. For steam the Mollier diagram may be advantageously used in Eq. (961):

$$\frac{\gamma}{\gamma-1} \frac{P_1}{\delta_1} \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \right] = 778 \times \left\{ \begin{array}{l} \text{Work of Rankine cycle between} \\ P_1 \text{ and } P_2, \text{ in heat units by} \\ \text{Mollier diagram} \end{array} \right\} \quad (961)$$

At the works of the Lackawanna Steel Co., Coleman made a check test on one of these meters, using throat velocities of 572 and 208 ft. per second with steam $\frac{1}{2}$ per cent wet at 110 lbs. per square inch pressure, and weighing the feed water, found errors of only +.056 per cent and -.84 per cent. Checking another meter on air in a 6-in. line, by measuring velocity heads across both upstream and throat diameters by Pitot tubes, the weights found by the former were .88289 lb. per second and by the latter .88986, giving .9922 as the ratio of Venturi to Pitot indications. Other tests with the Pitot tube along one diameter only, gave results that averaged .9963 in a 10-in. Venturi tube and .9976 in a 6-in. tube. The Pitot tube gives the velocity head of the gas in inches of water directly at the point of measurement, the center of a small area a , and the average gas velocity over the whole area is, $\frac{\sum au}{\sum a} = (u)(\text{average})$. In the same plant Dr. Lucke checked a 16-in. Venturi tube against the displacement of the gas pumps of a 1000 H.P. Korting two-cycle gas engine, against a regular proportional gas meter in the blast-furnace gas pipe, and the Venturi results agreed with the pump dis-

placement within 1 per cent, whereas the proportional meter disagreed with all other data.

The Pitot tube carries a double orifice at the end of a double tube that can be inserted in a pipe through stuffing boxes to allow the orifice to be located anywhere along the pipe diameter. One orifice points upstream and measures the velocity head by reducing to zero the velocity of the small portion of the stream that strikes it, to which is added the static gas pressure; while the other measures the gas pressure proper by an orifice tangent to the flow. The difference in pressure between the two orifices is the velocity head only. When the fluid is of constant density with respect to pressure, the velocity is given by $u = \sqrt{2gH}$ where H is the head in feet of fluid corresponding to the destroyed velocity. Usually water or mercury is used in the Pitot tube giving velocity head in inches of water or mercury. These heads are related to gas heads according to Eq. (962), where H_w , and H_m , are heads in feet of water and mercury:

$$H = H_w \left(\frac{\text{lbs. per cu.ft. water}}{\text{lbs. per cu.ft. gas}} \right) (a); \quad = H_m \left(\frac{\text{lbs. per cu.ft. mercury}}{\text{lbs. per cu.ft. gas}} \right) (b) \quad (962)$$

When air is the fluid its density is to be determined from its pressure, temperature, and gas constant, $R = 53.34$, if it is dry, but when moist by the Smithsonian Eq. (963), where p_v is the partial pressure of the water vapor in the air:

$$\left\{ \begin{array}{l} \text{Lbs. per} \\ \text{cu.ft. of} \\ \text{moist air} \end{array} \right\} = \left[\frac{.080723}{1 + .0020389(t - 32^\circ)} \right] \left(\frac{\text{barometer in ins. Hg} - .378p_v}{29.921} \right) \quad (963)$$

These relations are proper for such very small velocities as correspond to no appreciable change of density of the gas for the pressures used; to obtain measurable velocity heads, the density variation cannot as a rule be neglected, and in such cases the standard nozzle relation must be used as a starting-point.

$$\frac{u_1^2 - u_2^2}{2g} = \frac{\gamma}{\gamma - 1} \frac{P_2}{\delta_2} \left[1 - \left(\frac{P_1}{P_2} \right)^{\frac{\gamma - 1}{\gamma}} \right] \quad (964)$$

If P_2 is the pressure measured at the impact orifice, $u_2 = 0$, and, $u_1 = u$.

$$\frac{P_2}{\delta_2} = RT_2 = RT_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} \quad (965)$$

Substituting Eq. (965) in Eq. (964) there is obtained:

$$\frac{u^2}{2g} = \frac{\gamma}{\gamma - 1} RT_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] \quad (966)$$

This is developed into a series by Taylor as follows:

$$u = \sqrt{2gK \frac{P_1}{\delta_1} \left\{ 1 - \frac{K}{2\gamma} + \left(\frac{1 + \gamma}{6\gamma^2} \right) K^2 - \left[\frac{(1 + \gamma)(1 + 2\gamma)}{24\gamma^3} \right] K^3 \right\}} \quad (967)$$

Introducing the readings of P_2 and P_1 , in inches of mercury h_{M2} and h_{M1} , and multiplying by 60, the velocity in feet per minute for air, is given by:

$$\left\{ \begin{array}{l} \text{Velocity of} \\ \text{air in feet} \\ \text{per minute} \end{array} \right\} = 4046.16 \sqrt{\left(\frac{h_{M2} - h_{M1}}{\delta_1} \right) (1 - .355K + .202K^2 - .137K^3)} \quad (968)$$

Calculation of velocity by this Eq. (968) and by the equation neglecting density changes, gives comparative results which have been plotted. These are presented in the book of tables Chart No. 69, reproduced from Kneeland.

Prob. 1. In a test on air flow sharp-edged orifices of sheet iron were used, to which Durley's coefficients apply. For the following data what was the volume and weight of air flowing in each case? Air temperature = 70° F.

Orifice dia. in.		Head in inches of water.			
1 1.4,	1.95,	2.8,	4.1,	and 4.8
2 1.25,	1.7,	2.25,	"	4.8
3 1.60,	1.95,	2.65,	"	5.0

Prob. 2. What size of orifice would pass 200 cu.ft. of air with 5-inch water pressure drop, if (a) the coefficient were 100 per cent, (b) if Durley's coefficient were used?

Prob. 3. Air flows from a tank into the atmosphere at the rate of 75 cu.ft. per minute. The pressure in the tank is 25 lbs. per square inch absolute, and temperature 60° F. According to Weisbach how big should the hole be?

Prob. 4. A gas burner is supplied at 2-inch water pressure and passes 5 cu.ft. of gas per hour, the density being 60 per cent that of air. What area of burner is needed? Check this by comparing with a standard gas tip.

Prob. 5. A Venturi in an air pipe showed an upstream pressure of 17 lbs. per square inch absolute and a throat pressure 2½ ins. Hg less. The throat diameter is 6 ins., pipe diameter 20 ins. At 60° F., what quantity was flowing?

Prob. 6. If the pipe diameter of Problem 5 had been 12 ins., what would be the difference in pressure between throat and upstream in inches of water when the upstream velocity is 6000 ft. per minute?

Prob. 7. Gas is being measured by the Pitot tube. The pressure is 50 lbs. per square inch gage and the tube reading is 2½ ins. Hg. If the gas density is .8 that of air, pipe 4 ins. diameter, temperature 40° F., what is the flow rate, (a) by exact and, (b) by approximate formula?

Prob. 8. A steam Venturi tube gave the following readings:

	Dia.	Pressure, lbs. sq.in.	Gage quality.
Throat.....	2 ins.	90	95%
Upstream.....	4 ins.,	100	100%

What weight of steam was flowing and what was the upstream velocity?

111. Flow of Gases and Vapors in Pipes, Flues, Ducts, and Chimneys.
Relation between Quantity of Flow and Loss of Pressure. Friction Resistances. Draught and Capacity of Chimneys. Pipe or conduit flow of gases and vapors is characterized by comparatively small losses in pressure compared to the mean pressure, that is, by low values of the ratio of initial to final pressure. It is characterized also by complete conversion into heat of all the kinetic energy lost by friction, and by appreciable gains or losses of

heat by wall conduction and surface radiation—gains when the fluid temperature is lower than the surroundings as it is with refrigerating media, and losses otherwise, as for steam. All relations between the rates of flow and the change of fluid conditions corresponding to it can be numerically evaluated only when friction energy and heat conduction can be evaluated, and these quantities depend so much on structural details as to defy algebraic analysis. Accordingly, while some thermal relations for pipe flow may be set down, the really useful equations for engineering work are empiric.

Reduction of pressure with no conduction losses in a pipe line would result in the doing of work and the development of kinetic energy equal to that of the Rankine cycle for steam and its equivalent for gases. Since, however, the velocity is constant in a uniform pipe, neglecting the change of density corresponding to the pressure drop, the above gain in kinetic energy is the frictional energy, which being converted into heat characterizes the process as one of constant total heat if in a non-conducting container. The velocity obtained should, therefore, be that due to attaining the work of a cycle bounded by a constant total heat line between two constant pressure lines, corrected for actual thermal loss by conduction. The desired relation between pressures lost in a pipe of given dimensions and the corresponding energy was first established on somewhat questionable empiric grounds by Unwin, who, reasoning from experimentally established laws for water, put the loss of pressure

in terms of head of fluid as proportional to, (a) the velocity head, $H = \left(\frac{u^2}{2g}\right)$, (b) the length (L) directly; and (c) inversely as the mean hydraulic radius, $m = \left(\frac{\text{area}}{\text{perimeter}}\right)$. (Loss of fluid head due to friction) $= \zeta \left(\frac{u^2}{2g}\right) \frac{L}{m} = \zeta H \frac{L}{m}$. (969)

The name, coefficient of friction, is given to the constant of proportionality, usually designated by ζ .

With this as a basis and on certain reasonably fair assumptions, the relation between flow velocity and pressure drop can be derived. These assumptions are, that the fluid behaves like a perfect gas according to $PV = wRT$, that the work of friction is the product of friction head in feet of fluid and weight of fluid flowing, that negative work is done by the fall of pressure equal to the change of pressure into the volume, and finally, that the change in kinetic energy in passing from high to low pressure is equal to the sum of the pressure change work, and friction work, neglecting all other influences.

$$\left\{ \begin{array}{l} \text{Change in kinetic energy in} \\ \text{passing from high to low} \\ \text{pressure regions in pipes} \end{array} \right\} = \left\{ \begin{array}{l} \text{Work done by} \\ \text{change of} \\ \text{pressure} \end{array} \right\} - \left\{ \begin{array}{l} \text{Work of} \\ \text{friction} \end{array} \right\} \quad (970)$$

These quantities are evaluated algebraically as follows:

Let u = velocity, mean, in feet per second;

P_1 and P_2 = initial and final pressures, pounds per square foot;

p_1 and p_2 = initial and final pressures, pounds per square inch;

V = cubic feet per pound fluid at mean pressure;

Let u_m = velocity in feet per minute;
 w = pounds fluid per second;
 A = area pipe in square feet;
 m = mean hydraulic depth;
 L = length of pipe in feet;
 D and d = diameter of pipe in feet and inches respectively.

Then for an infinitely small pipe length dL and a correspondingly small velocity and pressure change du and dP

$$\left\{ \begin{array}{l} \text{Change in kinetic energy} \\ \text{at ends of pipe length} \end{array} \right\} = \frac{w}{2g}[(u+du)^2 - u^2] = \frac{w}{g}u du \quad (a); \quad = wdH \quad (b) \quad (971)$$

because $H = \frac{u^2}{2g}$, and $dH = \frac{u du}{g}$, and the square of a differential may be neglected in comparison with the differential itself.

$$\text{Also} \quad (\text{Work done by change of pressure}) = -wV dP = -\frac{wRT}{P} dP \quad (972)$$

$$\text{and} \quad (\text{Work of friction}) = (\text{Loss of head in feet}) \times (\text{Lbs. fluid}) = \zeta H \frac{dL}{m} w. \quad (973)$$

Substituting these values in Eq. (970), developing and integrating between the required limits of head, pressure, and length, there is obtained

$$u = \frac{\sqrt{gRT_1(P_1^2 - P_2^2)}}{\sqrt{P_1^2 \left(\zeta \frac{L}{m} + \log_e \frac{P_1}{P_2} \right)}} \quad (974)$$

in which u is the velocity at the beginning of the L foot section.

For pipe lengths of usual size and the small allowable pressure drops, or small $\left(\frac{P_1}{P_2}\right)$, it is usually fair to drop $\left(\log_e \frac{P_1}{P_2}\right)$, in comparison with the quantity to which it is added, hence the velocity becomes:

$$u = \sqrt{\frac{gmRT_1(P_1^2 - P_2^2)}{\zeta LP_1^2}} = \sqrt{\frac{gm(P_1^2 - P_2^2)}{\zeta L \delta P_1}} \text{ feet per second} \quad (975)$$

When the pipe is of circular section $m = \frac{\text{area}}{\text{perimeter}} = \frac{\pi D^2}{4 \times \pi D} = \frac{D}{4} = \frac{d}{12 \times 4} = \frac{d}{48}$,

$p = \frac{P}{144}$, and u_m (feet per minute) = $60u$ (feet per second);

$$u_m = (\text{Const.}) \times \sqrt{\frac{dRT(p_1^2 - p_2^2)}{\zeta L p_1^2}} = (\text{Const.}) \times \sqrt{\frac{d(p_1^2 - p_2^2)}{\zeta L \delta p_1}} \text{ ft. per minute.} \quad (976)$$

The corresponding weight is given by, $w_m = A u_m \delta = \frac{\pi D^2}{4} u_m \delta = .00545 d^2 u_m \delta$.

$$\left. \begin{aligned} (\text{Pounds of fluid per minute}) &= (\text{Const.}) \times \sqrt{\frac{d^5 \delta^2 RT(p_1^2 - p_2^2)}{\zeta L p_1^2}} \quad (a) \\ &= (\text{Const.}) \times \sqrt{\frac{d^5 \delta (p_1^2 - p_2^2)}{\zeta L p_1}} \quad (b) \end{aligned} \right\} \quad (977)$$

In all ordinary cases of pipe flow the ratio of the absolute pressure at the end of a length to that at the beginning is sensibly equal to 1, at least to greater limits of accuracy than the coefficient of friction can be established, or the premises of the above formula can be accepted as sound. When this is so the ratio, $\frac{p_1^2 - p_2^2}{p_1}$, becomes

$$\frac{(p_1 - p_2)(p_1 + p_2)}{p_1} = (p_1 - p_2) \left(1 + \frac{p_2}{p_1} \right) = 2(p_1 - p_2), \text{ (approx.)}$$

which on substitution gives the second form of Eq. (977) the following value

$$w_m = (\text{Constant}) \times \sqrt{\frac{d^5 \delta (p_1 - p_2)}{\zeta L}} \dots \dots \dots (978)$$

The first determination of the value of the coefficient of friction based on experiments of some accuracy is that made by Unwin on data obtained from tests of the compressed-air mains of Paris, all less than one foot in diameter.

His value is given by, $\zeta = .003 \left(1 + \frac{3.6}{d} \right) \dots \dots \dots (979)$

This indicates that the velocities vary as the square root of diameter divided by the above fraction of diameter, whereas for liquids, velocities by some formulas are given as functions of the square root of diameter, and by others generally believed to be better, as increasing with diameter faster than the square root. No better values appearing for many years, engineers have adopted and used the above results with practical success, though it must be confessed without scientific accuracy; experience has led to various values of the constants, the most used of which is the Babcock value in Eq. (980). Careful

$$w = 87 \sqrt{\frac{d^5 \delta (p_1 - p_2)}{L \left(1 + \frac{3.6}{d} \right)}} \dots \dots \dots (980)$$

analysis of existing information by Martin in 1903 led to the adoption by him of a different constant, $\sqrt{7000}$, giving the following series of formulas, all equivalent, but put in convenient form for solution of any desired quantity.

$$w = \sqrt{\frac{7000 d^5 \delta (p_1 - p_2)}{L \left(1 + \frac{3.6}{d} \right)}} = V \delta = \frac{u \delta d^2}{3.056} \text{ (a)}; \quad V = \frac{w}{\delta} = \sqrt{\frac{7000 d^5 (p_1 - p_2)}{L \delta \left(1 + \frac{3.6}{d} \right)}} = \frac{u d^2}{3.056} \text{ (b)} \quad (981)$$

$$u = \frac{3.056 w}{\delta d^2} = \frac{3.056 V}{d^2} = 256 \sqrt{\frac{(p_1 - p_2) d}{\delta L \left(1 + \frac{3.6}{d} \right)}} = \sqrt{\frac{454 d H_G}{L \left(1 + \frac{3.6}{d} \right)}}; \dots \dots (982)$$

$$(p_1 - p_2) = \frac{w^2 L \left(1 + \frac{3.6}{d} \right)}{7000 \delta d^5} = \frac{V^2 \delta L \left(1 + \frac{3.6}{d} \right)}{7000 d^5} = \frac{u^2 \delta L \left(1 + \frac{3.6}{d} \right)}{65360 d}; \dots \dots \dots (983)$$

$$H_G = \frac{w^2 L \left(1 + \frac{3.6}{d}\right)}{49 \delta^2 d^5} = \frac{V^2 L \left(1 + \frac{3.6}{d}\right)}{49 d^5} = \frac{u^2 L \left(1 + \frac{3.6}{d}\right)}{454 d} = \frac{144(p_1 - p_2)}{\delta}; \quad (984)$$

$$d = \sqrt[5]{\frac{.0003 w^2 L}{(p_1 - p_2) \delta}} = \sqrt[5]{\frac{.0003 V^2 \delta L}{(p_1 - p_2)}} \text{ (approx.)}; \quad (985)$$

where $(p_1 - p_2)$ = loss of pressure in pounds per square inch;

H_G = loss of pressure in feet of fluid;

V = cubic feet of fluid per minute;

u = velocity in feet per second;

δ = density of fluid in pounds per cubic foot;

w = weight of fluid flow pounds per minute;

d = diameter of pipe in inches;

L = length of pipe in feet.

Also $27.7(p_1 - p_2)$ = loss of pressure in inches of water;

$2.04(p_1 - p_2)$ = loss of pressure in inches mercury.

These formulas are applicable to all fluids, steam, both saturated and superheated, compressed air, ammonia, carbon dioxide and to the combustible gases, but not with equal degrees of accuracy over the whole range of kinds of fluid, rates of flow and density. Engineers should determine for themselves the correct numerical constant to be used in the general equations to meet the conditions of service in any given case.

One general investigation for steam recently conducted by Eberle under the auspices of the German Society of Engineers has led to an important and useful conclusion, inasmuch as it established the coefficient of friction as being independent of pipe diameter, steam pressures, quality and velocity for very considerable ranges. The velocities varied from 2000 to 15,000 ft. per minute, pressures from three to ten atmospheres, with steam saturated as well as superheated to 180° F., in pipes from 3 ins. to 6 ins. diameter approximately, while the coefficients varied some 20 per cent. For low pressures such as are found in steam-heating systems and steam exhaust pipes it is probable that the coefficient is too high. The simplified formula is given by Eq. (986),

$$(p_1 - p_2) = 2.667 \times 10^{-5} \times \frac{u^2 L}{d} \delta \text{ (Eberle)} \quad (986)$$

A number of pipe flow formulas have been analyzed and reduced to common terms by Gebhardt. These are given in the book of tables for reference.

Actual pipe lines have bends, branches and valves, together with enlargements and reductions of cross-section, all of which introduce resistances, as well as the skin friction of the pipe, which are very much more difficult to evaluate. The general method of procedure is to give the resistance in terms of the equivalent length of straight pipe but this is of little practical value, as the proportions of elbows differ in every make of fitting. There is need for accurate tables of data in commercial work relating to the various lines of fittings used in practice.

When low-pressure gases like air or furnace products flow in ducts, flues,

and chimneys, there arises a special case of flow conditions characterized by a possibility and often a probability that the whole skin frictional resistance due to straight pipe may be entirely overbalanced by the aggregate of the special resistances due to bends, branches and changes of cross-sectional areas. There are two typical cases of such low-pressure gas flow: first, that in which the flow is all horizontal, or if vertical the whole system must be at constant temperature—this is a typical duct-flow condition; second, that in which there is a vertical flow not necessarily straight upward in which the gases are hotter than other gases communicating with them—this is a typical chimney-flow condition. Each case must be treated by itself in order that workable equations and constants may be developed; the former will be taken up first.

Usually direct flow takes place from, or to, the free atmosphere, or a place where at least the pressure is that of the atmosphere whether the temperature is the same or not. The problem always takes the form of fixing the excess pressure over atmosphere that will deliver a required quantity of air or gases through ducts of a given size, or inversely the size for a given capacity, when the whole resistance is that due to the system of passages. The whole resistance is, therefore, to be evaluated and this consists of various frictional resistances due to straight duct walls, to changes of direction of flow, to changes of cross-section of duct, to inlet and to outlet orifice contractions. To these resistances in terms of feet of head of gas, or pounds per square foot, ounces per square foot, or inches of water, there is to be added the velocity head to obtain the total equivalent head or pressure that must be applied to dead air, to establish flow at the required velocity against the imposed resistances.

Let $H_O = \frac{u^2}{2g}$ = head of gas equivalent to velocity of flow;

$P_V = \frac{u^2 \delta}{2g}$ = pressure pounds per square foot equivalent to velocity head;

δ = density of gas in pounds per cubic foot;

P_F = pounds per square foot static pressure lost in wall friction.

P_R = pounds per square foot static pressure lost in changes of flow direction, cross-section, inlets and outlets.

P = whole static pressure equivalent to velocity and all resistances;

F_F = friction factor, which, applied to velocity head gives the frictional loss of pressure;

F_R = resistance factor, which applied to velocity head gives the loss of pressure due to resistances;

ζ = coefficient of friction for straight walls;

C = circumference or perimeter of duct in feet;

A = area of duct in square feet;

L = length of duct in feet.

Then

$$P = \left\{ \begin{array}{l} \text{Velocity pressure } P_V \\ + \text{Frictional pressure } P_F \\ + \text{Resistance pressure } P_R \end{array} \right\} \text{ in pounds per square foot. (987)}$$

$$P = \frac{u^2 \delta}{2g} (1 + F_F + F_R) \quad \text{but} \quad F_F = \zeta L \frac{C}{A} \quad \dots \quad (988)$$

Hence

$$P = \frac{u^2 \delta}{2g} \left(1 + \zeta L \frac{C}{A} + F_R \right) = \left\{ \begin{array}{l} \text{Velocity pressure} \quad \frac{u^2 \delta}{2g} \\ + \text{Maintained resistance} \quad \frac{u^2 \delta}{2g} \left(\zeta L \frac{C}{A} + F_R \right) \end{array} \right\} \quad (989)$$

This can easily be evaluated numerically, when there are available experimental values of the coefficient of friction for straight ducts, and of the flow change resistance F_R , but as it seldom if ever happens that a duct is uniform, and as these values depend on the dimensions, there must be a summation of the pressures lost at each point of change in each straight section. Hence,

$$\text{Maintained resistance} = \frac{\delta}{2g} \left[\sum \left(u^2 \zeta \frac{C}{A} \right) + \sum u^2 F_R \right] \quad \dots \quad (990)$$

Values of the coefficient of friction are given graphically in Chart No. 70, in the Handbook of Tables. For straight ducts of brick or iron for velocities up to 50 ft. per second, and for iron ducts different values are given for perimeters or circumferences from 8 to 100 ins. These values as well as the flow-change factors, also given in the tables, are those recommended by Ohmes from Rietschel for ventilating ducts, and are intended for the usual velocities therein attained, 6 to 24 ft. per second, when served by fans, and 3 to 8 ft. per second when the flow is due to natural or convection draft.

These pressures that are lost, due to the resistances, are convertible from pounds per square foot to inches of water by the usual factor, but in this branch of engineering practice they are usually given in ounces per square foot; this value follows directly from the formulas, if the density is used not in terms of pounds per cubic foot but in ounces per cubic foot. Of course, there may be other resistances in the flow path such as filter screens or pipe coils serving to heat or cool the air, and any such resistance is to be separately evaluated and added.

When the flow of gases in the system depends on convection that follows the heating of a rising column, which is replaced by a falling cold column, the flow characteristics are dependent on the temperatures and vertical heights involved; the most important practical case of such flow is that in chimneys, though in some ventilating systems a corresponding case is found.

Let H = vertical distance in feet equal to the difference in level between inlet and outlet of the heated column of gases;

P = difference in pressure at base of hot column between it and the atmosphere, to be called the draft, here given in lbs. per sq.ft.

h_w = draft in inches of water;

H_g = equivalent head of hot gas;

T_H = temperature of hot column abs. assumed constant throughout;

T_C = temperature of outside air abs.;

.084 = density of products of combustion at 32° F. and 29.92 ins. Hg;

$$\delta_H = \text{density of hot gases, pounds per cubic foot} = \frac{492 \times .084}{T_H} = \frac{41.43}{T_H}$$

for products of combustion, or $\frac{398}{T_C}$ for outside air;

.0807 = density of pure dry air at 32° F. and 29.92 ins. Hg;

δ_C = density of pure cold air at 32° F. and 29.92 ins. Hg =

$$\frac{492 \times .0807}{T_C} = \frac{39.8}{T_C};$$

C = circumference or perimeter of stack in feet;

A = area of stack in square feet;

w = weight of gases per second.

$$\text{Then } \left\{ \begin{array}{l} \text{Weight per cubic foot of} \\ \text{hot or cold gases} \end{array} \right\} \times H = \left\{ \begin{array}{l} \text{Pressure at base of height } H \\ \text{in pounds per square foot} \end{array} \right\} \quad (991)$$

Hence for pure air in the chimney the difference in pressures at the base is given by

$$P = H(\delta_C - \delta_H) = 39.8H \left(\frac{1}{T_C} - \frac{1}{T_H} \right) \quad (992)$$

For products of combustion of assumed density in the chimney the corresponding pressure difference is $P = H \left(\frac{39.8}{T_C} - \frac{41.43}{T_H} \right)$ (993)

The equivalent head of hot gases is given by,

$$H_G = \frac{P}{\delta_H} = \frac{P}{\frac{41.43}{T_H}} = H \left[\frac{\frac{39.8}{T_C} - \frac{41.43}{T_H}}{\frac{41.43}{T_H}} = H \right] \left(.961 \frac{T_H}{T_C} - 1 \right) \quad (994)$$

This is equivalent to draft in inches of water, the usual unit of measurement,

$$h_w = \frac{P}{5.21} = H \left(\frac{7.64}{T_C} - \frac{7.95}{T_H} \right) \text{ for chimney gases (a); } = 7.64H \left(\frac{1}{T_C} - \frac{1}{T_H} \right) \text{ for air (b)} \quad (995)$$

Solving for a chimney temperature, $t_H = 600^\circ \text{ F.}$, or $T_H = 1060^\circ \text{ F.}$, this gives $\frac{3}{4}$ in. of draft per 100 ft. of height. However, these are the full static pressures or their equivalents and after flow is established some pressure will be changed to velocity, and some more used up in overcoming the maintained resistances in coal beds, boiler tube banks, and economizers in furnace-chimney flow, which correspond to tempering coils and filters in heating and ventilating duct flow. Calling these special resistances F_s the general relation becomes:

$$P = \frac{\delta_H}{2g} \left[u^2 + \Sigma \left(u^2 \zeta \frac{C}{A} \right) + \Sigma (u^2 F_R) + \Sigma (u^2 F_s) \right] \quad (996)$$

All these resistances due to flue-wall friction, entrances, outlets, bends or changes of cross-section can be evaluated as in ventilating-duct work, but fuel-bed, economizer, and boiler-tube resistances cannot be so evaluated, because in them resistance conditions and also the velocity and temperature of gases

are constantly changing, due to changes of boiler load. It is, therefore, necessary in dealing with furnace chimney draft to proceed even more empirically than for air ducts; the usual method is to apply a general factor inclusive of all interferences and resistances, yielding a numerical result near enough to the truth to be of practical value. Thus, Stirling suggests that as an overall correction factor is to be applied there is nothing to be gained in assuming the gases to be different in composition from air, so that the static maintained draft after flow is established, as measured in the base of the stack, will be that for no flow, less something to account for velocity and friction resistances all together, derived somewhat as follows:

The flue friction loss in head is given by $\frac{u^2 \delta}{2g} \propto \frac{C}{A}$ for ducts, but $u \delta = \left(\frac{w}{A}\right)$, and for a given weight or volume per second flowing, the velocity is inversely proportional to the area, so that $u^2 \delta = (\text{Constant}) \times \frac{w}{A^2}$. Therefore, the draft loss would be $(\text{Constant}) \times \frac{\zeta C}{2g} \frac{w}{A^2} = f \frac{wC}{A^2}$, where f is given the empiric values of Eq. (997) and the maintained draft is given by Eq. (998),

$$f = \begin{cases} .0015 \text{ for steel stacks, gas temperature} = 600^\circ \text{ F. or } T_H = 1060 \\ .0011 \text{ for steel stacks, gas temperature} = 350^\circ \text{ F. or } T_H = 810 \\ .0020 \text{ for brick stacks, gas temperature} = 500^\circ \text{ F. or } T_H = 960 \end{cases} \quad (997)$$

$$h_w = 7.64H \left[\left(\frac{1}{T_C} - \frac{1}{T_H} \right) \right] - f w \frac{C}{A^2} \quad (998)$$

As a matter of fact the actual maintained draft, while it is a prime factor in the rate of combustion of coal, is not of such great importance in the problem of flow through chimneys as the capacity of chimneys. This capacity is primarily given in terms of weight of gases per hour, but for practical application it is common to express it in terms of fuel-burning capacity, or boiler horse-power equivalent to the weight of gases per hour, by assuming values for the conversion factors.

Let G = weight of gases per hour;
 G_m = maximum weight of gases per hour;
 K = proportionality coefficient.

$$\begin{aligned} \text{Then } G &= 3600w = 3600Au\delta_H = 3600AK\sqrt{2gH_G\delta_H} \\ &= 3600AK\sqrt{2gH\left(.961\frac{T_H}{T_C}-1\right)} \times \frac{41.43}{T_H} \\ &= 149148K \frac{A}{T_H} \sqrt{2gH\left(.961\frac{T_H}{T_C}-1\right)} \quad (999) \end{aligned}$$

It is clear that as T_H becomes greater the draft and the velocity corresponding to it increase, but the density becomes less, so that there would appear to

be some temperature at which the weight would be a maximum, and this is to be found by differentiation. Accordingly, putting the above in the form,

$$G = 149148KA\sqrt{2gH\left(.961\frac{1}{T_C T_H} - \frac{1}{T_H^2}\right)} \quad \text{and making } \frac{dG}{dT_H} = 0, \text{ gives}$$

$$-.961\frac{1}{T_H^2 T_C} + \frac{2}{T_H^3} = 0, \quad \text{or } T_H = \frac{2T_C}{.961} = 2.081T_C \quad \dots (1000)$$

This shows that the capacity will be a maximum when the absolute temperature of the hot column is, in round numbers, twice that of the cold surrounding air. Under these conditions the capacity is found by substituting this internal temperature in Eq. (999), which gives

$$G_m = 149148K\frac{A}{T_H}\sqrt{2gH(2-1)} = 1196250K\frac{A}{T_H}\sqrt{H} \quad \dots (1001)$$

When the problem is that of stack design, it is this maximum capacity that is the important quantity, but it must be remembered that unless the gas temperature is right, the stack will work at a lesser capacity and the actual or probable capacity of a stack at a temperature other than that giving maximum capacity is a determination quite frequently necessary. In stack designing the gas temperatures are never known, though they may be approximated, but instead of making approximations for each quantity separately as involved in the stack capacity, this is set down most commonly in terms of coal-burning capacity or boiler horse-power under some of the following assumptions: (a) The stack will operate at maximum capacity; (b) The air temperature is 50° F., or $T_C = 510$, and $T_H = 2.08 \times 510 = 1061$, or $t_H = 601^\circ$ F. (c) Twenty-four pounds of air will be required per pound of coal yielding 25 lbs. of gases; (d) Five pounds of coal per hour will be equivalent to one boiler horse-power.

The justice of these assumptions need not be discussed, as they are purely arbitrary, and the user of these equations may modify them at will, starting from the fundamental relations, but it is on such assumptions that most of the existing stacks have been designed and the coefficient K determined to apply to them. Introducing the assumptions it follows that:

$$\left(\begin{array}{l} \text{Max. lbs. coal per hr.} \\ \text{for the stack} \end{array}\right) = \frac{G_m}{25} = \frac{1196250}{25 \times 2.08 \times 510} KA\sqrt{H} = 45KA\sqrt{H} \quad \dots (1002)$$

$$\left(\begin{array}{l} \text{Boiler horse-power} \\ \text{for the stack} \end{array}\right) = \frac{1}{5} \times \left(\frac{G_m}{25}\right) = 9KA\sqrt{H} \quad \dots (1003)$$

The value of the coefficient K , which here may be regarded as a general correction factor, though it is often called a coefficient of discharge, is most commonly taken as, $K = .3$, but Kent has used another value combined with the assumption that next the walls there will be a dead layer of gas so that the effective area (KA) will be the actual area less the cross-section of this dead gas. This layer is arbitrarily taken as being 2 ins. thick, so that it will subtract about $\frac{2}{12}\pi D$ and $\frac{2}{12} \times 4D$ sq.ft., from the area A for round and

square stacks respectively, leaving the effective area as, $(A - .59\sqrt{A})$ for round, and $(A - .666\sqrt{A})$ for square, or approximately, $(A - .6\sqrt{A})$ for both. This substituted in Eq. (1003) yields the Kent formula Eq. (1004),

$$(\text{Boiler horse-power for the stack}) = .333(A - .6\sqrt{A})\sqrt{H} \quad (1004)$$

From this formula Kent has calculated a table of heights and capacities which is commonly employed for determining dimensions of chimneys for steam boilers. This table (No. LXXXVI) is given in the Handbook of Tables. The Stirling method of calculation for chimneys assumes the draft to be obtained from Eq. (995 b). Allowing 24 lbs. air per pound of coal the air will weigh 96 per cent of the flue gas. On this assumption a theoretical draft pressure will be obtained as given in Table LXXXVII in the book of tables. The available draft will be the tabular value, less the amount consumed by friction in the stack, which can be calculated from Eq. (998). If, however, the diameter of stack be determined from Eq. (1005), the net draft may be assumed to equal 80 per cent of the tabular values. If this value be accepted a very simple formula for height of chimney is obtained from Eq. (995b) which becomes: $h_w = 6H\left(\frac{1}{T_c} - \frac{1}{T_H}\right)$. For atmospheric pressure, and temperatures of 60° F., and 600° F., respectively, the draft, $h_w = 0.006H$; for a stack temperature of 450° F., this becomes: $h_w = 0.005H$.

It follows from this formula that by increasing the height of a stack of certain diameter it can be made to produce the same available draft as one of a larger diameter, the additional height being required to overcome the greater friction loss. Consequently, among the various stacks which could meet the requirements there must be one which can be constructed cheaper than the others. By deducing an equation connecting the cost of stacks with their height and diameter, and using it in connection with the formula for available draft, it has been found that the minimum-cost stack has a diameter depending solely upon the horse-power of the boilers it serves, and a height proportional to the available draft required. Assuming 120 lbs. of flue-gas per hour for each boiler horse-power this method gives:

$$\left. \begin{array}{l} \text{For an unlined steel stack, Dia. in inches} = 4.68 (\text{H.P.})^{1/4} \quad (a) \\ \text{For stacks lined with masonry, Dia. in inches} = 4.92 (\text{H.P.})^{1/4} \quad (b) \end{array} \right\} \quad (1005)$$

This formula gives the curve in Chart 71 in the book of tables, from which the stack diameter for any boiler horse-power can be taken. Stacks with diameters thus determined are assumed to have an available draft which bears a constant ratio to the theoretical draft of practically 80 per cent.

A great many empiric chimney formulas are available, all derivable from the same fundamental expressions here developed, but with different values for the constants; however since they are all empiric beyond the first steps they are omitted here. Chimneys intended to carry off the products of combustion of gas and oil fires must be proportioned somewhat differently from those for coal fires, in which the fuel bed causes the main resistance. The general procedure

with coal fires is to first fix the height that will give the draft needed to burn the coal at the desired rate on the grates, and then select an area sufficient to carry off the gases produced. With gas and oil fires the draft is of minor consequence so that a shorter, wider stack may be just as serviceable for a gas or oil fire as a narrow high one of equal capacity, whereas with coal fires the height is needed for and controlled by the bed resistance or rate of combustion.

For a more extended discussion of chimney draft and design reference is made to an excellent comparison in Gebhardt's *Steam Power Plant Engineering*.

Prob. 1. Simultaneous readings showed the pressure at the ends of a 6 inch-steam pipe line, 650 ft. long, to be 120 lbs. gage and 105 lbs. gage respectively. What was the flow as found by Babcock formula? By the Eberle?

Prob. 2. It is desired to transmit 1000 cu.ft. of compressed air per minute a distance of 2 miles and with pressure drop not to exceed 20 lbs. If the air is initially under a pressure of 100 lbs. per square inch gage, what size pipe will be needed?

Prob. 3. Natural gas is being transmitted at a pressure of 150 lbs. per square inch gage through a 10-in. pipe. What will be the amount which may be transmitted with a loss of 5 lbs. per sq. inch per mile. What size pipe would be required to carry 1000 cu.ft. (std.) per minute with this drop?

Prob. 4. A 4-in. pipe 500 ft. long is carrying dry saturated steam at a pressure of 100 lbs. per square inch gage initially, with a drop of 5 lbs. Check the weight flowing.

Prob. 5. For the same pipe find the pressure drop if the quantity flowing should be doubled.

Prob. 6. A furnace burning 1000 lbs. of coal per hour has a stack 4 ft. in diameter and 150 ft. high. If pounds of air per pound of coal is 25, outside temperature 80° F., and stack temperature 400° F., what will be the draft?

Prob. 7. What will be the maximum weight of coal which can be burned with this stack for the given amount of air per pound of coal?

Prob. 8. A stack in Glasgow has a height of 435 ft., a diameter of 13 ft. 6 ins. and is stated to have a capacity of 10,000 H.P. or 50,000 lbs. of coal per hour. Check these figures.

Prob. 9. The concrete stack at the University of Minnesota is 11 ft. 6 ins. in diameter, 235 ft. high. A straight horizontal breeching leading to it is 11 ft. by 10 ft. in section, 110 ft. long; the chimney serves eight 350 H. P. boilers. What overload capacity has it?

CHAPTER XVIII

EFFICIENCY OF COMPRESSED AIR-ENGINES WITH AND WITHOUT PREHEAT

112. Thermal Efficiency of Compressed Air Engines Alone and in Combination with Air Compressors. Effect of Preheating and Reheating. Compressor Suction Heating and Volumetric Efficiency. Wall Action. For certain special service conditions compressed air supplied to piston engines, is the most desirable method of developing the necessary power, and for this reason a brief review of the thermal characteristics not only of the engines but of the whole system including the compressors, is of value. It is, however, important for another reason, and that is the possibility of securing much better results than have yet been obtained with air engines, which would have the effect of bringing the system into wider use than at present. In mines, where exhaust steam or gas-engine exhaust gases would foul the air that must be breathed by the workers, or even set fire to combustible gases in coal workings, compressed-air engines are the only available sources of power except electric motors, and are now used as haulage locomotives, hoists, pumps, drills and cutters. Practically all excavation work must be conducted under conditions that are served by compressed-air power and the use of very small units like hammers, riveters, drills and clippers is almost universal in shops. Probably nothing has called wider attention to the possibilities of large scale use of compressed air, than the recent installation of a system of compressed-air deep-mine hoists in the Butte, Mont., mines. This is especially remarkable in view of the fact that the compressors are operated electrically from transmitted water power; it was found better to use electricity to operate compressors discharging into storage reservoirs from which the air passed through heaters on the way to hoisting engines, than to operate the hoists electrically. One factor in this conclusion is the special construction of the engine which as the cage descends operates as a compressor storing air by the energy of the falling hoist, and acts as a brake at the same time. Another similar development has recently been accomplished in a different direction by the perfection of the Porter compound compressed-air locomotive for use in mines, lumbering operations, or other places where fire is much feared; also in hauling cars about the yards of large manufacturing plants where cleanliness as well as avoidance of fire risk is important, and where the service is so intermittent that a steam locomotive might burn more coal to keep up pressure standing, than in hauling loads.

It seems a very simple operation at first glance to admit to a cylinder of ordinary steam engine construction, a charge of compressed air instead of steam,

but two important difficulties or objections appear, peculiar to the substitution of the air for steam. In the first place unless there is considerable expansion the economy will be very poor and the work obtained only a small fraction of that expended in compressing the air; and second, as a consequence of the expansion the air will become very cold, even a moderate amount of expansion lowering its temperature much below the freezing-point and filling cylinder and ports with ice from the moisture of the air unless it be dried previously. If, however, the air be heated either by internal combustion, as it can well support a fire itself, or by external heat application as it passes through chambers or tubes heated by steam or the hot products of coal, oil or gas fires, it will increase in volume thereby and one cubic foot of cool compressed air will become two or three cubic feet of hot compressed air, if its absolute temperature be doubled or trebled, thus making it capable of doing two or three times the work of the cool air. By this means, more power can be obtained than is required to run the compressor; in fact if compressor and engine be coupled together there will be possible a sufficient net output of power to warrant calling the combined unit a form of gas engine. Heat so added to compressed air is termed preheat and the heat of preheat is converted into work in part, whether the compressor and engine work separately at a distance or are coupled together as a single power-generating unit. It will be shown later that if high pressures are used, the efficiency of transformation of the heat of preheat into work may be very high, in fact the same as a Brayton gas cycle, which in turn is the same as the modern standard Otto gas-engine cycle for equal compressions. This preheat, moreover, removes any difficulties of freezing and in some cases only sufficient heating is used to accomplish this.

Reheating the air between the cylinders of compound compressed-air engines in the receiver, is exactly the same in kind and effect as preheating and yields efficiencies that correspond to the pressure at which it is taken up; the higher the pressure the more return will be available for the heat used, but as practiced other factors than thermal efficiency may control the operation. One very remarkable case of this sort is that of the Porter locomotive in which the air is stored at a pressure of about 800 lbs. per square inch in the tank, which gives the tank the large compressed-air weight and low pressure volume capacity, needed for a long haul. This air thoroughly dried by the high compression then passes through a reducing valve where it develops kinetic energy of expansion, immediately losing it by impact and so not suffering any temperature drop, when lowering pressure from 800 to 250 lbs. per square inch. At this latter pressure and at approximately atmospheric temperature, it is admitted to and expanded in the high-pressure cylinder, exhausting to the receiver at a temperature as much below that of the atmosphere as practicable, about 30° below zero F. This permits it to be reheated from *the atmospheric air itself*, which is blasted through the receiver tubes, the working air so warmed, subsequently expanding in the low-pressure cylinder *converts some of the heat of the atmosphere into work*. This process is therefore, peculiarly interesting, inasmuch as it is a means of utilizing for useful work some of the

heat of the atmosphere, incidentally improving economy in the use of the compressed air about 30 per cent, and avoiding at the same time cylinder freezing or interference with lubrication.

It has been shown that so far as economic performance independent of cylinder capacity is concerned, compressors give the same result with, as without clearance, therefore, in considering the efficiency of combined air engines and air compressors, it will be assumed that the compressor has no clearance which assumption materially simplifies the work. With respect

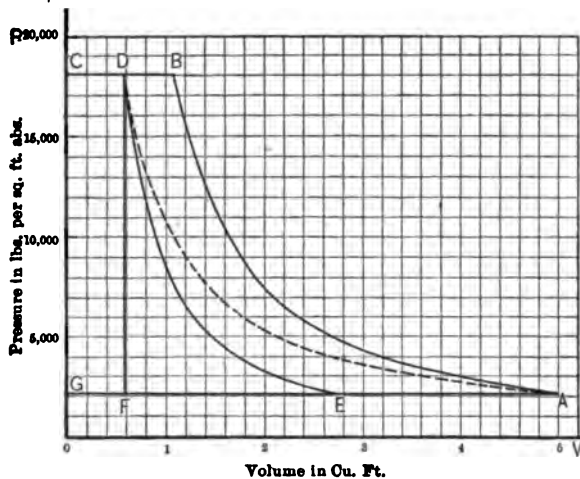


FIG. 177.—Simple Compressed-air Engine, and Single-stage Compressor, Combined Diagram.

to the engine, the same would be true if the compression were complete; since it can be made so, this assumption will also be made, and the air engine treated as if it had zero clearance. If in any case it has clearance without complete compression it will require more compressed air per unit of work done, which is an avoidable waste, most easily eliminated by making compression complete.

Consider Fig. 177, as representing a zero clearance single-stage adiabatic compressor card $ABCG$ on which the isothermal, AD . Then, if the air after compression be cooled to its original temperature in the receiver or pipe line, which usually happens, the volume of cool compressed air available for work is \overline{CD} . This may be assumed to be admitted to an air engine without clearance loss and there adiabatically expanded along DE completely, or used non-expansively along DF . These two cases represent limiting economic performances, best and worst. The efficiency of the whole process is given by the ratio of the engine work area, to the compressor work area, whence, for *single-stage adiabatic compression and simple compressed-air engines*,

$$\text{Efficiency} = \left\{ \begin{array}{ll} E_1 = \frac{\text{area } CDFG}{\text{area } CBAG} & \text{for zero engine expansion} \quad (a) \\ E_2 = \frac{\text{area } CDEG}{\text{area } CBAG} & \text{for complete engine expansion} \quad (b) \end{array} \right\}. \quad (1006)$$

These areas may be evaluated algebraically by a modification of the methods indicated in Chapter II.

Let P_b = highest pressure, pounds per square foot;

P_a = lowest pressure, pounds per square foot;

Let $\frac{P_b}{P_a} = R_P = \text{pressure ratio};$

$T_a = \text{atmospheric temperature absolute};$

$T_b = \text{maximum delivery temperature of compressor}.$

From Eq. (46) making $s = \gamma$, Area CBAG = $\frac{\gamma}{\gamma-1} P_a V_a \left(R_P^{\frac{\gamma-1}{\gamma}} - 1 \right).$

$$\text{Area CDFG} = P_a V_a \left(\frac{R_P - 1}{R_P} \right). \quad \text{Area CDEG} = \frac{\gamma}{\gamma-1} P_a V_a \left[\frac{R_P^{\frac{\gamma-1}{\gamma}} - 1}{R_P^{\frac{\gamma-1}{\gamma}}} \right].$$

Substituting these algebraic values for the areas in Eq. (1006):

$$\left. \begin{aligned} E_1 &= \frac{P_a V_a \left(\frac{R_P - 1}{R_P} \right)}{\frac{\gamma}{\gamma-1} P_a V_a \left(R_P^{\frac{\gamma-1}{\gamma}} - 1 \right)} = \frac{\gamma-1}{\gamma} \left[\frac{R_P - 1}{R_P \left(R_P^{\frac{\gamma-1}{\gamma}} - 1 \right)} \right] (a) \\ E_2 &= \frac{\frac{\gamma}{\gamma-1} P_a V_a \left[\frac{R_P^{\frac{\gamma-1}{\gamma}} - 1}{R_P^{\frac{\gamma-1}{\gamma}}} \right]}{\frac{\gamma}{\gamma-1} P_a V_a \left(R_P^{\frac{\gamma-1}{\gamma}} - 1 \right)} = \frac{1}{R_P^{\frac{\gamma-1}{\gamma}}} (b) \end{aligned} \right\} \quad (1007)$$

If the compressor were two stage and the engine simple, the diagrams would appear as in Fig. 178. The results will be different, as the denominator in the efficiency equation, which is the work of the compressor, has a different expression given by Eq. (1008).

$$(\text{Work of compressor, two-stage, no clearance}) = \frac{2\gamma}{\gamma-1} P_a V_a (R_P^{\frac{\gamma-1}{2\gamma}} - 1). \quad (1008)$$

Calling the new efficiencies E_3 and E_4 for two-stage compression compared with zero engine expansion and complete engine expansion respectively,

$$E_3 = \frac{\gamma-1}{2\gamma} \left[\frac{R_P - 1}{R_P \left(R_P^{\frac{\gamma-1}{2\gamma}} - 1 \right)} \right] (a); \quad E_4 = \frac{1}{2} \left[\frac{\frac{R_P^{\frac{\gamma-1}{\gamma}} - 1}{R_P^{\frac{\gamma-1}{\gamma}}}}{\frac{\gamma-1}{R_P^{\frac{\gamma-1}{2\gamma}} - 1}} \right] = \frac{1}{2} \left[\frac{R_P^{\frac{\gamma-1}{2\gamma}} + 1}{R_P^{\frac{\gamma-1}{\gamma}}} \right] (b) \quad (1009)$$

A complete investigation of all the possible cases of combination of compressor with engine would be too lengthy to be given here, but the methods used may be applied to other combinations, by the aid of the equations of Chapters II and III for compressors, and Chapters IV, V and VI for engines with exponential laws for the expansion.

The effect of preheating the air between compressor and engine may be shown in two different ways. According to the first method, the fraction of heat so supplied that is ultimately converted into work, can be determined by divid-

ing the net work area, between the compression and expansion lines, by the foot-pound equivalent of the heat. This will give the efficiency of the combination

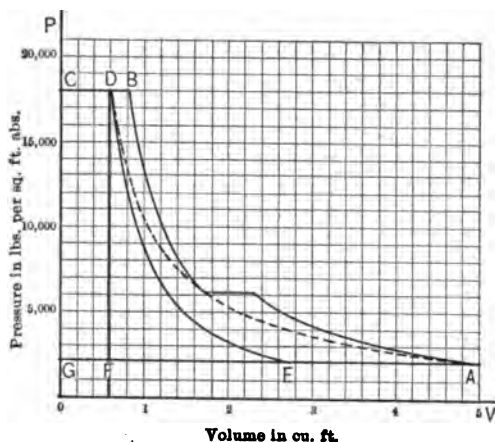


FIG. 178.—Simple Compressed-air Engine, and Two-stage Compressor, Combined Diagram.

to correspond to the thermal cycle thus created. The preheat is added from D to D' , D to B , or D to D'' , the first being small in amount, insufficient to bring the temperature and volume of the cooled air to the values corre-

as a transformer of heat of preheat into work, and is the thermal efficiency of a gas-engine cycle consisting of phases made up by the lines in question. The other way compares the whole work of the air engine taken as useful output, to the sum of the compressor work and the foot-pound equivalent of the heat supplied; this gives the efficiency of the whole combination as it is. Both results are of value, each, however, for what it shows.

In Fig. 179, the single compressor with simple air engine is reproduced with various amounts of preheat, and the $T\phi$ diagram drawn

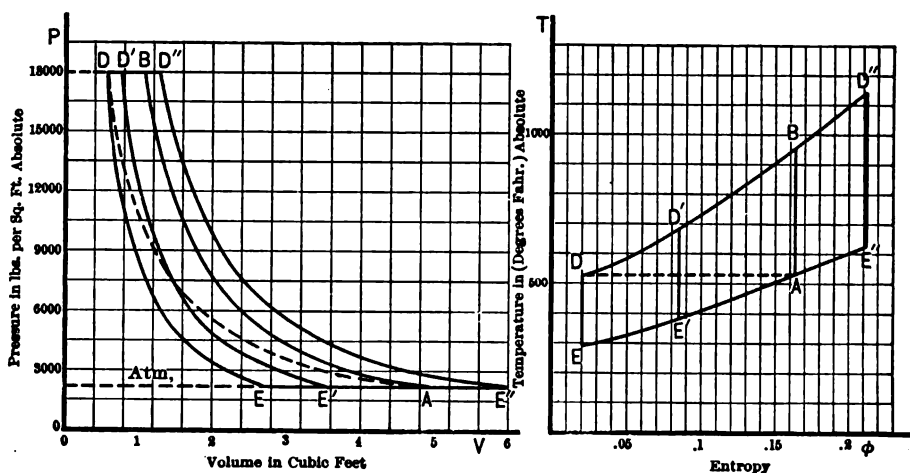


FIG. 179.—Effect of Preheat on Compressed-air Engine Performance.

sponding to compressor delivery, the second just reproduces these values, the third makes the volumes and temperatures larger.

Preheat to compressor delivery temperature will result in an engine exhaust temperature equal to that of the atmosphere or compressor suction. Less preheat than this will cause the exhaust to be cooler than the atmosphere; the minimum allowable amount should be that which will prevent

freezing or interference with lubrication. If the air be very dry, such as would be the case if compressed to 1000 lbs. per square inch or even less, the exhaust may be 20 to 30° F. below zero without making much trouble; with more moisture, such as would be carried by air at 100 to 200 lbs. per square inch pressure, the final temperature may be as low as 10 to 15° F., since the high initial temperatures will prevent ice sticking to the walls. While in many cases the preheat used is just sufficient to prevent freezing, that is, the initial temperature absolute, at the beginning of expansion, high enough for the ratio of expansion to give the above final temperatures, the real economic value of preheat is obtained only with larger amounts. There is no reason why temperatures as high as the hottest superheated steam should not be used, 500° to 600° F. in engines of exactly the same structure, or in excess of this to any degree if gas-engine structures are used as a model. These large preheats give no higher per cent thermal return than the lower ones, but they do make it possible to operate larger engines with smaller compressors, materially reducing the first cost of the equipment.

The efficiency of the heat of preheat alone is that of a cycle bounded by the maximum and minimum pressure lines and two adiabatics, the first adiabatic corresponding to the work that would be obtained without preheat, and the second that corresponding to expansion from the temperature and volume produced by the preheat. On the diagram, Fig. 179, this cycle is represented for the three different amounts of preheat by, $EDD'E'E$, $EDBAE$, and $EDD'E''E$ to both PV , and $T\Phi$ coordinates. These are, of course, all Brayton gas cycles, having established efficiencies given by:

$$(\text{Efficiency of heat of preheat}) = 1 - \frac{1}{R_P^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{T_b}{T_a} \quad \dots \quad (1010)$$

This is true no matter how much heat is added and no matter what the temperatures. It is exactly the same as an Otto gas cycle efficiency for equal compressions, and is very high, yielding not less than twice as high a return for the heat, as obtained from the heat of steam in a steam-driven compressor cylinder, and may be three or four times as high, depending entirely on the conditions of steam and air pressures. It proves the desirability of using very high air pressures for high thermal per cents of return from the heat of preheat. Thus with compressed air at 13 atmospheres or 191 lbs. per square inch absolute for a 14.7 lbs. standard atmosphere, the thermal efficiency would be, ideally,

$$E = 1 - \frac{1}{(13)^{.29}} = 52 \text{ per cent.}$$

If the diagram factors for gas engines applied to these cases the realizable amount would be about half of this, or 26 per cent, but without water jackets it is likely that fully $\frac{2}{3}$ would be realized, so the probable thermal efficiency of the heat of preheat would be approximately 40 per cent, a very high value that will bear experimental investigation.

The other standard of efficiency, that for the whole system, is given by:

$$\left\{ \begin{array}{l} \text{Efficiency of compressor and} \\ \text{engine with preheat} \end{array} \right\} = \left[\frac{\text{Work of the engine}}{(\text{Work of compressor}) + (\text{Ft. lb. equiv. to heat of preheat})} \right].$$

The foot pound equivalent of the heat of preheat is equal to, $778 \times (\text{the heat of preheat}) = 778 \times wC_p (\text{Temperature after preheat} - \text{Temperature before})$. If the temperature before preheat be taken as equal to that of the atmosphere, which corresponds to complete cooling of the compressed air in receiver and pipe line, this becomes, calling the final temperature T_H , air temperature T_A , and w the weight of air,

$$(\text{Ft. lb. equivalent to heat of preheat}) = 778wC_p(T_H - T_A). \quad (1011)$$

The work of the engine with preheat is equal to that without, plus that gained.

The work without preheat was given by, $\frac{\gamma}{\gamma-1} P_a V_a \left[\frac{R_p^{\frac{\gamma-1}{\gamma}} - 1}{R_p^{\frac{\gamma-1}{\gamma}}} \right]$, while the

(Work done by preheat) = $778 (\text{Efficiency of preheat}) \times (\text{Heat of preheat})$

$$= \left[1 - \frac{1}{R_p^{\frac{\gamma-1}{\gamma}}} \right] \times 778wC_p(T_H - T_A). \quad \text{Hence}$$

Work of engine with preheat

$$= \frac{\gamma}{\gamma-1} P_a V_a \left[\frac{R_p^{\frac{\gamma-1}{\gamma}} - 1}{R_p^{\frac{\gamma-1}{\gamma}}} \right] + \left[\frac{R_p^{\frac{\gamma-1}{\gamma}} - 1}{R_p^{\frac{\gamma-1}{\gamma}}} \right] \times 778wC_p(T_H - T_A). \quad (1012)$$

$$\text{But } P_a V_a = wRT_a = 778w(C_p - C_v)T_a, \quad \text{and} \quad \frac{\gamma}{\gamma-1} = \frac{\frac{C_p}{C_v}}{\frac{C_p}{C_v} - 1} = \frac{C_p}{C_p - C_v},$$

so that

$$\frac{1}{\gamma-1} P_a V_a = 778wC_p T_a, \quad \text{whence}$$

(Work of engine with preheat)

$$= 778wC_p \left[\frac{R_p^{\frac{\gamma-1}{\gamma}} - 1}{R_p^{\frac{\gamma-1}{\gamma}}} \right] (T_a + T_H - T_a) = 778wC_p T_H \left[\frac{R_p^{\frac{\gamma-1}{\gamma}} - 1}{R_p^{\frac{\gamma-1}{\gamma}}} \right]. \quad (1013)$$

$$\left\{ \begin{array}{l} \text{Efficiency of compressor and} \\ \text{engine with preheat} \end{array} \right\} = \frac{778wC_p T_H \left[\frac{R_p^{\frac{\gamma-1}{\gamma}} - 1}{R_p^{\frac{\gamma-1}{\gamma}}} \right]}{\frac{\gamma}{\gamma-1} P_a V_a \left[\frac{R_p^{\frac{\gamma-1}{\gamma}} - 1}{R_p^{\frac{\gamma-1}{\gamma}}} \right] + 778wC_p (T_H - T_a)}$$

$$= \frac{\frac{\gamma-1}{R_p^{\frac{\gamma-1}{\gamma}} - 1}}{R_p \left[\frac{T_a}{T_H} \left(R_p^{\frac{\gamma-1}{\gamma}} - 1 \right) + \left(1 - \frac{T_a}{T_H} \right) \right]}. \quad (1014)$$

The same methods apply, but of course, different equations result, for the more complex cases of any number of compressor stages, combined with any number of expansions with both preheat and receiver reheat. However these will not be given here owing to lack of space. Those interested will find the subject more fully treated in Dr. Lucke's larger work on Engineering Thermodynamics.

Just as in steam and gas engines there is a heat exchange between walls and gases due to the difference in temperature, sometimes in one and sometimes in the other direction, so also is there a heat exchange between air and cylinder walls in both an engine and an air compressor, which must be taken into account in any discussion of their performance, when passing from the cyclic possibilities to the probabilities for real machines. Two general methods of attack are available; the first is that developed for gas engines and is concerned with heat exchanges during expansion and compression. It was shown that the rate of heat gain or loss by the gas per unit of volume change given by, $\left(\frac{dH}{dV}\right)$, may be expressed as a function of pressures and volumes measured directly from an indicator card if the clearance be accurately known and if the cylinder and valves be tight. The only net effect of this exchange that is of any practical value is to change the value of the exponent " s " giving it some value other than the adiabatic. While for compressors and air engines this sort of analysis shows heat exchanges that are alternately positive and negative, the net effect is not serious enough to warrant the use of any exponent other than the adiabatic value, $s = \gamma = 1.4$ for air. Other values have been frequently reported, but with the exception of cases confined to small cylinders at low piston speeds, all are open to suspicion as they are in the direction that the effects of leakage would be. In cylinders the air is in about the best condition that could be found for prevention of heat transfer, so it is difficult to conceive of " s " having any materially different value than the adiabatic $s = \gamma = 1.4$.

There is, however, another period in compressors and air engines where a material heat exchange occurs, and that is during entrance, corresponding in engines to admission at the high pressure up to cut-off, and in compressors to suction at the low pressure. The effect in air engines is to cool the admitted air, so that for each cubic foot measured at conditions external to the cylinder and admitted, there will be in the cylinder less than a cubic foot before expansion begins, and at a temperature lower than that of the air supply. This is similar in kind to the initial condensation of steam engines and has the effect of increasing the consumption per hour per I.H.P. over the indicated value, but in air engines it has the additional effect of bringing the exhaust air to a lower temperature because expansion starts with air colder than the supply. In compressors the effect is to prevent the cylinder taking in as much as corresponds to the product of its apparent volumetric efficiency and displacement, measuring volumes at pressures and temperatures external to the cylinder.

Determination of the effect of both compressor suction heating and engine initial cooling have been made by measuring the volume supplied or delivered

at pressures and temperatures external to the cylinder and comparing these volumes with those indicated. The results can be expressed in terms of volumes or in terms of the temperature changes that correspond to the volume differences, and as cylinders work at such widely different temperatures and the effect is one of temperature, the latter is to be preferred. The problem may then be stated: How much will the volume change during admission to air compressor or engine cylinders, when the difference between initial and final temperatures of the air in passing through has a known value? Some general conclusions are available that will make it possible to closely approximate this effect. The result is generally to change the volume from 10 to 30 per cent, reducing it for engine admission and increasing it for compressor suction, or rather, the temperature rise is from 25 to 50 per cent of the difference between the temperatures of supply and delivery air, entering and leaving the cylinders. One series of long-run tests of an ammonia compressor using weighed anhydrous showed the rise in temperature during admission to average about 33 per cent of the difference between maximum and minimum fluid temperature. A small air engine working on an atmospheric temperature supply of compressed air showed a volumetric reduction of 10 per cent on entrance, which is equivalent to a reduction of temperature of about 30 per cent of the difference between supply and exhaust-air temperatures. One valuable contribution, that of Trinks, that the extent of the surface exposed to the suction heating has no material influence on the heating effect, was established by measuring air for an 8×8 in. compressor fitted with different clearance surfaces but with constant clearance volume. The whole range of results shows an increase of 20 to 25 per cent in the volume of air while entering. Many examples of separate data of the same sort all lead to the conclusion that the volumes will be affected as much as corresponds to a temperature rise during entrance of 25 to 50 per cent of the difference between initial and final temperature, although it is impossible to relate the figures to speed, cylinder size, construction, kind of cylinder cooling, temperature range or any other factor that might seem to affect it, but which frequently fails to produce an anticipated effect.

This being the case the volume relations may be set down as follows for air engines:

$$\left(\frac{\text{Cubic feet air indicated at cut-off}}{\text{Cubic feet air supplied}} \right) = \left(\frac{\text{Absolute temp. air at cut-off}}{\text{Absolute temp. air supplied}} \right) \quad (1015)$$

But if the temperature after entrance is equal to that at supply, less 25 to 50 per cent of the difference between discharge and supply air, then

$$(\text{Temp. at cut-off}) = (\text{Temp. supply}) - (.25 \text{ to } .50) \times \left(\frac{(\text{Temp. supply})}{-(\text{Temp. exhaust})} \right) \quad (1016)$$

$$\left(\frac{\text{Cu. ft. air indicated}}{\text{Cu. ft. air supplied}} \right) = 1 - (.25 \text{ to } .50) \times \left[1 - \left(\frac{\text{Abs. temp. exhaust}}{\text{Abs. temp. supply}} \right) \right] \quad (1017)$$

Similarly, for compressors,

$$\left(\frac{\text{Cu.ft. air indicated on suction line}}{\text{Cu.ft. air supplied}} \right) = \left(\frac{\text{Abs. temp. air after suction}}{\text{Abs. temp. air supplied}} \right). \quad (1018)$$

$$(\text{Temp. after suction}) = (\text{Temp. supply}) + (.25 \text{ to } .50) \times \left(\frac{(\text{Temp. del.})}{-(\text{Temp. sup.})} \right) \quad (1019)$$

$$\text{Hence } \left(\frac{\text{Cu.ft. air ind.}}{\text{Cu.ft. air sup.}} \right) = 1 + \left[(.25 \text{ to } .50) \left(\frac{\text{Abs. temp. air del.}}{\text{Abs. temp. air sup.}} \right) - 1 \right]. \quad (1020)$$

It is customary in compressors to use the term volumetric efficiency: E_v = apparent volumetric efficiency and E'_v = real volumetric efficiency, thus

$$E_v = \frac{\text{Cu.ft. air indicated on suction line}}{\text{Displacement}}. \quad (1021)$$

$$E'_v = \frac{\text{Cubic feet air supplied}}{\text{Displacement}} = \frac{E_v}{1 + (.25 \text{ to } .50) \left[\left(\frac{\text{Abs. temp. air del.}}{\text{Abs. temp. air sup.}} \right) - 1 \right]} \quad (1022)$$

Prob. 1. Air at 135 lbs. pressure per square inch gage and 70° F. is admitted to a cylinder in which the cut-off is $\frac{1}{4}$. What will be the temperature after expansion? What will it be for $\frac{1}{4}$ cut-off, with, and without considering initial cooling?

Prob. 2. At what temperature must air be initially, to prevent freezing of exhaust, if the expansion is from 80 lbs. per square inch gage to atmosphere?

Prob. 3. Air is compressed adiabatically in one stage from atmosphere to 10 atmospheres absolute, and then used in a single-cylinder steam engine without preheat or expansion. What will be the efficiency of the system, and to what value would it be raised by using complete expansion in the engine? The compressor may be assumed to have zero clearance and the engine to have complete compression.

Prob. 4. Should the high pressure be only 5 atmospheres instead of 10, how would the results differ?

Prob. 5. Should the compressor of Problem 3 be two-stage would the results be changed, and if so, to what extent?

Prob. 6. What would be the efficiency of the heat of preheat in a case where air was used at a pressure of 120 lbs. per square inch gage, the low pressure being zero gage, and the air preheated 100° F. above atmosphere. Would the result be different if the preheating were doubled, and if so, how much?

Prob. 7. What would the efficiency of compressor and engine when the high pressure is 140 lbs. per square inch absolute, the low pressure 14.1 lbs. per square inch absolute, and the air preheated 300° above that of the original temperature? Would the result be different if the amount of preheat were doubled? If halved? If so, by how much? What is the efficiency of the heat of preheat alone?

Prob. 8. In an air engine it was found that the temperature of the exhaust was 10° F. and that of the supply air 350° F. What would be the likely limit of the ratio of cubic feet of air indicated to cubic feet supplied, and the horse-power expected of the air engine per horse-power of compressor.

Prob. 9. Air is being adiabatically compressed from atmosphere to 100 lbs. per square inch gage. The atmospheric temperature is 50° F. and apparent volumetric efficiency is 95 per cent; what will be a fair value for the real volumetric efficiency?

CHAPTER XIX

MECHANICAL REFRIGERATION. QUANTITY OF FLUID, HORSE-POWER AND HEAT ABSTRACTED PER TON

113. Mechanical Refrigeration, General Description of Processes and Structures. Thermal Cycles and Refrigerating Fluids. Limiting Temperatures and Pressures. It is just as important that some substances be cooled to temperatures lower than the surroundings and kept cool for long periods of time, as it is to heat and keep hot other substances. Until the thermal processes here discussed were understood this could be done only to the limited degree corresponding to the use of natural ice with or without salts to lower the melting-point. The preservation of foods, the cooling of beverages, and the manufacture of ice, are all illustrations of important cooling processes, involving the abstraction of heat from a great variety of substances. The perfection of apparatus, however, to accomplish these ends has resulted in a very much wider application of that same equipment, with suitable modifications, to other equally important, though more purely industrial purposes, the refrigerating processes becoming one step in the general scheme of production of some result not itself a cooled body. For example, the drying of air is best accomplished by cooling it to as low a temperature as will reduce the moisture in saturated air to the desired low value; or just as moist air is necessary in cotton spinning, so is dry air in the manufacture of photographic films. Dried air is useful also in the drying and dry storing of wood, fabrics, crystals and powders, and in the blowing of blast furnaces, where each pound of moisture by dissociation into hydrogen and oxygen consumes heat of coke combustion. Other examples of the use of refrigeration as a process are found in the manufacture of confections, the storage of furs, the working of tobacco, the recovery of paraffine from oil, shaft sinking through soft or wet earth by freezing the whole mass, and finally the control of the rate of fermentation in various liquors and the rate of germination of seeds or growth of bacteria.

To accomplish the cooling of any given substance and to maintain it at a low temperature continuously, requires primarily that some other still colder substance to be termed the *refrigerating fluid* be brought close to it, so heat may flow by reason of the temperature difference to the colder refrigerating fluid through plates or pipes separating them. It is evident, therefore, that any sort of contact conditions that may be suitable or convenient will serve to cool anything, if only a still colder refrigerating fluid be available, so that thermally considered, the problem of refrigeration really consists in the discovery of means for making some fluid as cold as may be necessary at the

lowest cost, without danger to operators or injury to the apparatus. As the absorption of heat by any cold refrigerating fluid would ultimately raise its temperature as high as the body giving up heat to it, continuity of the refrigerating process involves as another essential element, the circulation of the refrigerating fluid, removing continuously that which has taken up some heat and replacing it with fresh quantities. As a result, there would be accumulating somewhere beyond the refrigerator large amounts of refrigerating fluid colder than the cooled substances and, of course, colder than the surrounding atmosphere and earth, so the disposition of this fluid must be provided for. Such refrigerants as have been found suitable are expensive, so they cannot be thrown away after once performing a refrigerating function, but must be restored to their original condition to be used again and again.

These fluids as the media for the execution of the refrigerating cycles must be treated in mechanical apparatus of suitable form. This apparatus, in type at least, is now fairly well standardized and falls in one of three so-called systems of mechanical refrigeration which must be understood in principle before attempting any analysis or calculations of performance. These three systems are:

A. The dense-air system, so called because the air which is the medium is never allowed to fall to atmospheric pressure, so as to reduce the size of the cylinders and pipes through which a given weight is circulating.

B. The compression system, using ammonia, carbon dioxide and sulphur dioxide, and so called to distinguish it from the third system, because a compressor is used to raise the pressure of the vapor and deliver it to the condenser after removing it from the evaporator.

C. The absorption system, using ammonia, and so called because a weak water solution removes vapor from the evaporator by absorption, the richer aqua ammonia so formed being pumped into a high-pressure chamber called a generator in communication with the condenser, where the ammonia is discharged from the liquid solution to the condenser by heating the generator, to which the solution is delivered by the pump.

No matter what the system may be, it will require circulating water to receive its heat, and the temperature of the water determines the highest temperature allowable in the system and indirectly the highest pressure, so one structural element is a cooler or condenser, or both, to which circulating water is supplied. In some cases it will be found that the same water may be used in series over two different parts when the temperature in one is higher than in the other. Another structural element common to all is the refrigerator proper, in its usual form a coil of pipe in which the air of the air system is warmed and the ammonia or carbon dioxide evaporated. This pipe may be placed directly in a room to be kept cool, in which case the arrangement is called direct expansion, but more usually the coil or an equivalent structure forms part of a brine-cooling chamber, the cold brine from which is circulated by pumps through room coils or through ice-making tanks, holding cans that contain the water to be frozen; though a great variety of com-

binations of direct cooling and brine circulation are in use. It will be assumed in what follows that in all cases a brine cooler will be used with the

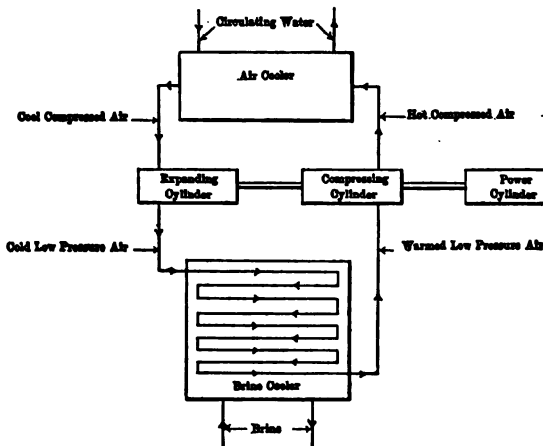


FIG. 180.—Diagrammatic Arrangement of the Air System of Refrigeration.

understanding that the cooling coils may instead, cool anything other than brine directly.

The air system is illustrated diagrammatically in Fig. 180, for a closed system, in which air previously dried of moisture is continuously circulated, showing a compressing cylinder delivering air through a water cooler to an expansion cylinder, which in turn sends the air through refrigerating coils back to the compressing

cylinder. As the work done by the expansion of the cooled compressed air is less than that needed to compress the air from the refrigerating coils through the same pressure range, there will be necessary a source of power to supply the difference, so a steam or gas-engine cylinder is added.

The compression system for ammonia, carbon dioxide or other similar condensable vapors is illustrated in Fig. 181, diagrammatically, the arrangement of equivalent parts being maintained as in the air system, Fig. 180, to bring out differences more clearly. Here the condenser replaces the air cooler, and the vapor delivered to it by the compressor is changed therein to liquid at the same pressure, which liquid escapes through a throttle, generally termed an expansion valve, to the refrigerating coils where the pressure is low and evaporation thereby induced, the discharge vapor from the coils being

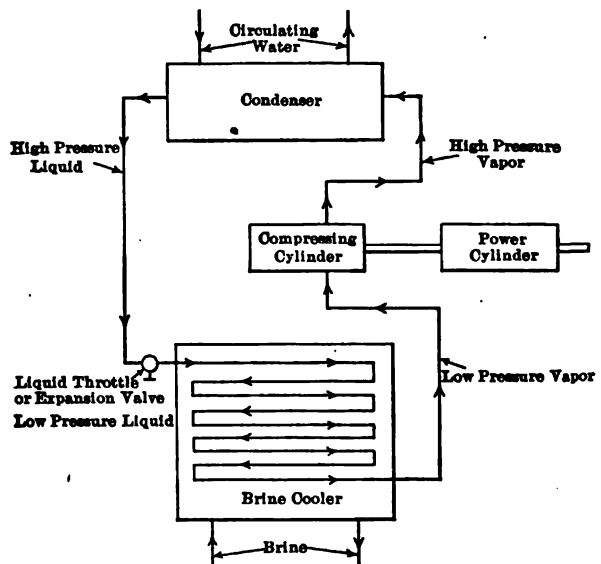


FIG. 181.—Diagrammatic Arrangement of Ammonia or Carbon Dioxide Compression System of Refrigeration.

liquid at the same pressure, which liquid escapes through a throttle, generally termed an expansion valve, to the refrigerating coils where the pressure is low and evaporation thereby induced, the discharge vapor from the coils being

regularly drawn off by the compressor suction. Of course, other elements are added in practice, for example, a bypass piping, so the condenser may be pumped out into the refrigerator for cleaning or repairs, a charging connection for taking in fresh ammonia, a forecooler at the condenser to remove superheat, which may use warmer water than the condenser itself, a liquid receiver beyond the condenser to store a quantity of fluid to meet sudden fluctuations of demand while vapor returns slowly, a purge valve at the highest point of the condenser to blow off non-condensable gases from the system, a liquid separator on the compressor suction to prevent the liquid of wet vapors entering the cylinder, oil or glycerine traps between compressor and condenser to prevent entrance into the latter of any such matter used in the cylinder and discharged from it, and other similar elements often in large numbers, none of which, however, affects the fundamental thermal process.

The absorption system of apparatus for ammonia is represented in Fig. 182, diagrammatically, which illustrates only one of several possible arrangements, but a common one that serves as a basis for analysis. Here the condenser, anhydrous ammonia liquid line, expansion valve, evaporating coils and brine cooler are the same as before, all differences being concerned with the drawing off of the vapor produced in the refrigerating coils and its delivery at

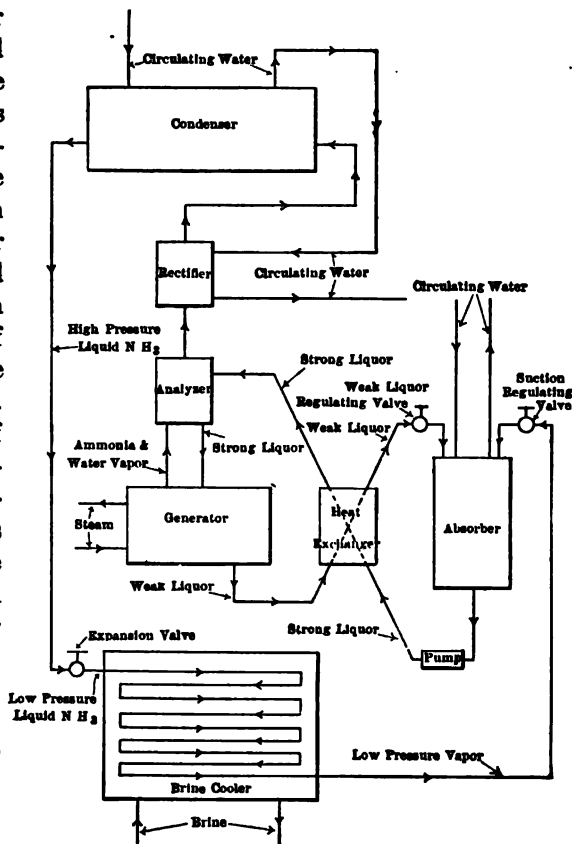


FIG. 182.—Diagrammatic Arrangement of the Ammonia Absorption System of Refrigeration.

the higher pressure to the condenser. This is accomplished by treating an aqueous solution of ammonia, alternately allowing it to absorb ammonia from the evaporating coils in an absorber at a rate determined by a regulating valve and the temperature and concentration of the solution, and later heating this richer solution in a generator by means of steam coils. Thus, referring to the diagram, the low-pressure vapor passes from the evaporating coils through the suction-regulating valve to the absorber chamber where it meets a weaker aqueous solution, being absorbed and so developing heat, which is carried off by the

circulating water coil, which latter controls the temperature of absorption. A more concentrated and cool solution of strong liquor is then pumped from this low-pressure absorption chamber to the high-pressure heating chamber or generator, passing on the way through a heat exchanger where it gets warmer by cooling weak liquor flowing in the other direction. The heating in the generator will drive off both ammonia and water vapor, but mainly the former, and this vapor mixture rises through a double cooling chamber or analyzer and rectifier. It is first cooled a little by contact with the incoming strong liquor in the analyzer, and later more cooled in the rectifier by circulating water that has been used in the condenser. At each cooling step some water vapor is thrown down and returns with such ammonia as it absorbs on the way to the generator, while the ammonia not so reabsorbed passes to the condenser, as the cooling has not been carried far enough in analyzer and rectifier to condense it. A little water vapor will pass over the condenser, making its delivery consist not of pure anhydrous ammonia, but rather of an aqueous solution, mainly NH_3 , which behaves nearly enough like anhydrous for practical purposes, unless by mismanagement it finally attains too much water, which must be removed before continuing the refrigeration effectively.

The thermal and work processes on the refrigerating fluid itself which make possible the absorption by it of heat in the low-temperature ranges, and its ultimate restoration to a condition which will permit a repetition, constitute the so-called refrigerating cycles, and as the fluid may be either a gas, or a liquid-vapor, there may be a very great variety of refrigerating cycles, at least as many as there are for transforming heat into work by both vapor and gas systems.

In general, whether the fluid be a gas or a vapor, it will pass through a cycle consisting of heat absorption in low ranges of temperature as one phase, and if the original condition is to be restored, heat absorbed must be given up to surroundings which will constitute another phase; between absorption of heat at low temperatures and its discharge to air or water later at temperatures higher than their own, the fluid itself must be raised in temperature and this can be done by doing work on it, so a third phase will be one of compression. At least one more phase is needed to close the cycle, but there may be more than one. If the fluid be a gas, the absorption of heat by it in the refrigerator as it circulates is most feasibly accomplished at constant pressure, so heating at constant pressure, A to B , would be the first phase, Fig. 183, the next, adiabatic compression, B to C , followed by heat abstraction or cooling, C to D , again at constant pressure, the final temperature always above that of the available water which is to carry it away, closing the cycle by adiabatic expansion D to A to lower the temperature to the original value.

It will be noted that this is a Brayton gas-engine cycle executed in the reverse direction, and therefore all the thermal relations established for that cycle in Section 104 of Chapter XV may be used with appropriate modifications for this reverse refrigerating cycle when its analysis is taken up later.

Should a liquid-vapor be used, absorption of heat by it at a temperature lower than in the refrigerator will evaporate liquid, and this is the first phase; compression adiabatically will raise temperature and pressure to such values as will permit condensation, since heat is given up to the circulating water; these steps constitute the second and third phases, while liquid cooling accompanied by reduction of pressure closes the cycle, though there are modifications to be noted depending on quality of the vapor at various points and the manner in which the drop in pressure after condensation takes place. If after condensation is complete at *D*, Fig. 184, the liquid adiabatically expands *D* to *A*, it will have the quality at *A* at the low temperature. If, however, it merely escapes through a valve it will follow the line of constant

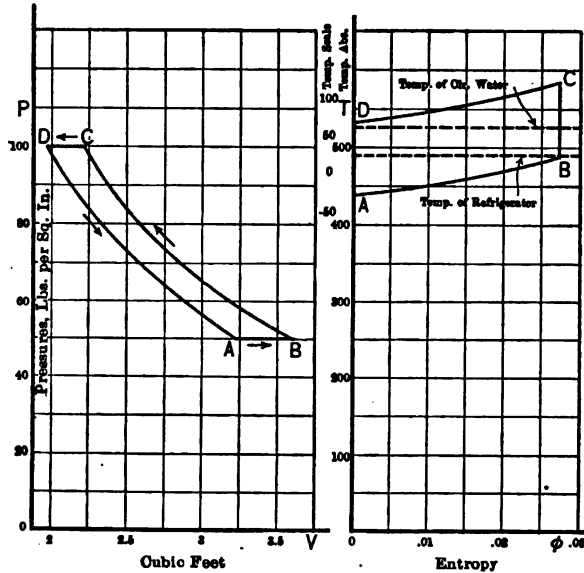


FIG. 183.—Refrigeration Cycle for a Gaseous Fluid.

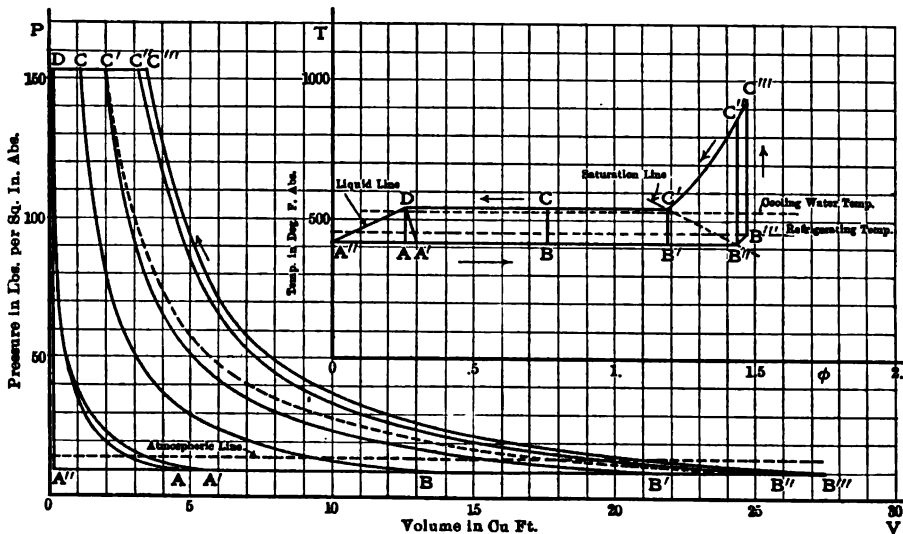


FIG. 184.—Vapor-liquid Fluid, Refrigerating Cycle.

total heat *D* to *A'* and be more wet, which is more nearly the process as practiced. From the point *A* or *A'*, evaporation proceeds at a constant tempera-

ture lower than that in the refrigerator until no more liquid remains, B'' , after which it may be superheated, $B''B'''$. At any time, that is, when the quality has any value, such for example as corresponds to points B , B' , B'' or B''' , the heat absorption may be stopped by removal of the vapor-liquid from the refrigerator to the compressor, for adiabatic compression to condenser pressure. Here, four different compression lines are shown, the first, BC , illustrating adiabatic compression from wet vapor to less wet vapor; $B'C'$, from wet to dry saturated vapor; $B''C''$, from dry saturated to superheated; $B'''C'''$, from superheated to more superheated states. When the vapor after adiabatic compression is superheated the first action in the condenser is not condensation, but a sort of precooling for the removal of superheat, $C'''C''C'$, after which true condensation proceeds C' to D , at which it is complete.

As the constant total heat line DA' for the liquid passage from condenser to refrigerating evaporator is fairly close to the liquid line DA'' for the common refrigerating fluids, it is customary to consider the resulting heats as equal in numerical value for the small temperature ranges involved, and this procedure is justified by the fact that the error is very small, smaller than that in locating the lines, and not less accurate than the knowledge of the properties of these fluids. Therefore, the refrigerating cycle involving the use of a vapor-liquid fluid may be taken as the Rankine cycle reversed, and all the methods and conclusions developed for this cycle, as a heat to work transforming process, are available with suitable modifications when analyzing it as a refrigerating cycle.

It is clear from the preceding discussion that any gas or any liquid may be used as a refrigerating fluid when subjected to these cycles of operation with only one limitation, and that is they must not pass through the critical state. This does not mean, however, that all are equally available, for they are not, and the first consideration of importance that reduces the number to a very few is the pressure necessary to cause evaporation at such low temperatures as refrigerants must assume, a high limit for which may be taken at 0°F . and a low limit at -50°F . If the pressures corresponding to the saturated vapor at such temperatures are too low, there would be needed very large pipes and compressors to handle the necessary quantity. This consideration with others to be taken up later, limits the available fluids to the following, having the boiling-points at atmospheric pressure corresponding and in order: sulphuric ether, $\text{C}_2\text{H}_5\text{O}$, $34^\circ\text{C} = 93^\circ\text{F}$.; sulphur dioxide, SO_2 , $-10^\circ\text{C} = 14^\circ\text{F}$.; methyl chloride, CH_3Cl , $-22^\circ\text{C} = -8^\circ\text{F}$.; methyl ether, $\text{C}_2\text{H}_6\text{O}$, $-24^\circ\text{C} = -11^\circ\text{F}$.; ammonia, NH_3 , $-33^\circ\text{C} = -27^\circ\text{F}$.; carbon dioxide, CO_2 , $-79^\circ\text{C} = -110^\circ\text{F}$.; nitrous acid, N_2O , $-88^\circ\text{C} = -126^\circ\text{F}$.; ethylene, C_2H_4 , $-103^\circ\text{C} = -152^\circ\text{F}$.; oxygen, O_2 , $-181^\circ\text{C} = -294^\circ\text{F}$.; air, $-192^\circ\text{C} = -314^\circ\text{F}$.; nitrogen, N_2 , $-198^\circ\text{C} = -324^\circ\text{F}$.; hydrogen, H_2 , $-243^\circ\text{C} = -416^\circ\text{F}$.

The other items in the elimination of the less desirable fluids are; (a) the condenser pressure corresponding to ordinary circulating water temperatures which, if too high, require thick and expensive pipes and unduly expensive compressors; (b) the latent heat of evaporation at the refrigerator or evaporat-

ing pressures which, if too small, require the circulation of excessive quantities of fluid to produce a given amount of refrigeration; (c) the volume of vapor per pound at the low pressure which, if too large, requires too large a compressor to circulate the required quantity, the compressor size being a function of the cubic feet of vapor per B.T.U. of latent heat taken by it from the refrigerator; (d) the cost per lb. of the fluids which must be replaced to supply leakage; (e) the corrosiveness of the fluid on the metal parts. All these influences have resulted in the placing of anhydrous ammonia in the first rank of refrigerating fluids, being used in the production of over 90 per cent of all the mechanical refrigeration now in service, with carbonic acid and sulphur dioxide next in order, and practically all others eliminated except air, which is used entirely as a gas—the only gas so used.

Referring to the air system, Fig. 180, it will be readily seen that, while any range of temperatures desired may easily be obtained with reasonable pressures, some standard is desirable for purposes of comparison, so it will be assumed that the air must leave the refrigerating coils at 0° F. and enter at -80° F., also that the cooling water available will permit an initial temperature in the expansion cylinder of 70° F. If expansion in this cylinder is complete, as it may be if the cut-off is suitably adjusted, the pressure ratio is:

$$R_P = \frac{P_H}{P_L} = \left(\frac{T_H}{T_L} \right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{530}{380} \right)^{3.46} = (1.4)^{3.46} = 3.2 \text{ (approx.)} \quad (1023)$$

If the pressure in the refrigerating coils is one atmosphere that in the water-cooling coils must be 3.2 atmospheres absolute, but if high pressures are carried in the refrigerating coils to reduce the size of equipment, four atmospheres, for example, the high pressure will be thirteen atmospheres absolute. The indicator diagram of the compressor without losses is shown for this latter case by $ABCD$ of Fig. 185, on which \overline{BC} represents the volume of hot air actually delivered by the compressor due to the clearance volume V_c . If this be cooled and admitted to an expansion cylinder having a clearance volume V_h , and working with complete compression GH and expansion EF , the admission volume \overline{HE} , is to the compressor delivery volume \overline{BC} , as the absolute temperatures after and before cooling. These two indicator cards when combined become equivalent to the PV cycle of Fig. 186, on which \overline{CB} represents the delivered volume of hot compressed air, and \overline{CE} , the admitted volume of resultant

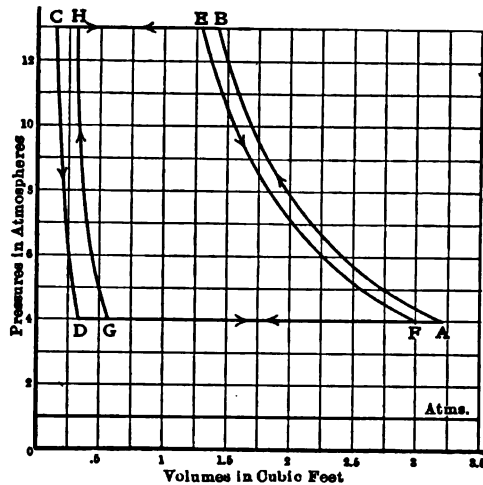


Fig. 185.—Dense Air Refrigeration, Compressor and Engine Combined Indicator Cards.

cooled air, or \overline{EB} the reduction in volume due to cooling, EF the expansion, FA the refrigerator heating of the air and AB the compression, substantially, as described previously. Compressor suction and delivery valve losses, and suction heating, as well as incomplete or overexpansion and compression in the expansion cylinder, with admission cooling of the air therein, will distort the diagram of actual machines, but the effects of such distortion are easily understood and need not be subjected to analysis, the cycle above,

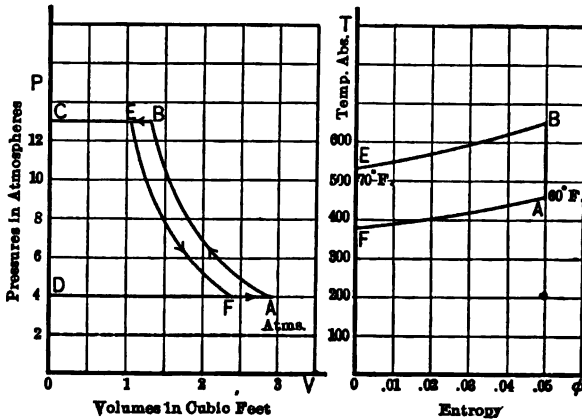


FIG. 186.—Dense Air Refrigerating Thermal Cycle.

representing the perfect conditions, will give best possible results that are to be determined analytically.

Considering the compression system for ammonia and other condensable vapors, Fig. 181, only two fluids, ammonia and carbon dioxide, will be discussed here. It will be assumed for convenience that the vapor leaving the coils in which pressure is assumed constant throughout, is dry saturated, and

that the condenser is so constructed that the pressure therein corresponds to liquid leaving at 70° F.; actually it will be higher. Under these circumstances the compressor suction pressure will be 29.7 lbs. per square inch absolute for ammonia, and 313.7 lbs. per square inch for carbon dioxide, while the corresponding condenser pressures will be 128.4 lbs. per square inch for ammonia and 851.5 lbs. per square inch for carbon dioxide, all absolute and taken directly from tables of the properties of saturated vapors.

Accordingly, the pressure ratios are, $\frac{P_H}{P_L} = 4.32$ for ammonia, and $\frac{P_H}{P_L} = 2.72$ for carbon dioxide, and neglecting wall action in the compressor as well as pipe heating or cooling, the temperature of the gas after delivery from compressor to condenser will be as given by Eq. (1024), taking $\gamma = 1.3$ for both vapors superheated.

$$T_H = T_L \left(\frac{P_H}{P_L} \right)^{\frac{\gamma-1}{\gamma}} = \left\{ \begin{array}{l} 460(4.32)^{\frac{.31}{1.3}} = 645^\circ \text{ F. abs. for ammonia} \quad (a) \\ 460(2.72)^{\frac{.31}{1.3}} = 580^\circ \text{ F. abs. for carbon dioxide} \quad (b) \end{array} \right\} \quad (1024)$$

For these pressures the two compressor indicator cards are drawn, Fig. 187, for the assumption of no losses and for equal per cents of clearance, $ABCD$ representing ammonia and $A'B'C'D'$ carbon dioxide. On each, the suction volume is \overline{DA} or $\overline{D'A'}$, and the delivery volume \overline{BC} or $\overline{B'C'}$, so they are equivalent to no clearance cards with these bottom and top lengths. As the function of the condenser is that of volume reduction at constant pressure, its action

may be equally well represented by the delivery line \overline{BC} and $\overline{B'C'}$, Fig. 188; and similarly, the coil evaporation at constant pressure is also represented by the suction line \overline{DA} and $\overline{D'A'}$, so that the whole process is represented by the PV cycles, $ABCD$ and $A'B'C'D'$, for which the corresponding $T\Phi$ cycles are shown at the right, each for one pound of substance.

Referring to the absorption system, Fig. 182, it will be assumed that the condenser discharge is pure anhydrous and in this case the coil and condenser pressures will be the same as for the compressing system. Obviously, a con-

siderable range of liquor strengths may be used and a still wider range of temperatures maintained in both absorber and generator, so for the purpose of calculation some common values must be assigned to these quantities for preliminary calculation. Accordingly, it will be assumed that the generator has a temperature of 250°F. , which corresponds to saturated steam at 29.82 lbs. per sq.in. absolute or nearly 15 lbs. gage pressure, receiving rich liquor slightly less than 32 per cent ammonia by weight, at 205°F. It is less than the 32 per cent delivered from the absorbed because at the higher temperature a small amount of the liquid has changed to gas. It will also be assumed that the weak liquor, 22 per cent, enters the absorber at 115°F. so that the exchange has reduced it 115° , while the 32 per cent liquor leaves the absorber at 100°F. and is warmed in the exchanger 10° less than the weak is cooled because of superior specific heat, so that its rise of temperature is $115^{\circ} - 10^{\circ} = 105^{\circ}$; it thus enters the generator at $100^{\circ}\text{F.} + 105^{\circ} = 205^{\circ}\text{F.}$ Finally, it will be assumed that the pressure in the absorber is 2 lbs. per sq.in. gage lower than in the evaporating coils by reason of the action of the regulating valve, which makes the absorber pressure for ammonia vapor at 0° , $29.7 - 2 = 27.7$ lbs. per sq.in. absolute. Referring to the table of properties of ammonia water solutions and the corresponding charts in the book of tables, it appears that a solution at the pressure 27.7 lbs. per sq.in. absolute in the absorber and at a temperature of 100°F. would if saturated contain 34 per cent by weight, so that the 32 per cent solution which leaves is not saturated. In the generator the condenser pressure, assumed at 128.4 lbs. per sq.in., is maintained at the temperature of the 22 per cent solution leaving it, namely, 230°F. ; if saturated this liquid would contain 24 per cent by weight instead of the 22 per cent actual.

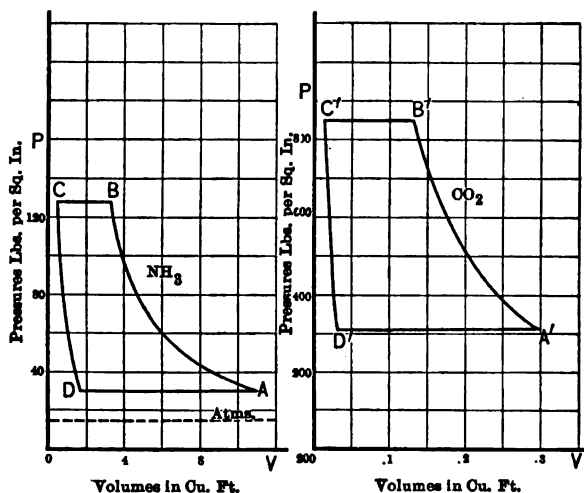


Fig. 187.—Ammonia and Carbon Dioxide Compressor Diagrams between Equal Temperature Limits.

These quantities are all subject to almost infinite variation, yet something must be assumed approximately corresponding to practice, as a basis of comparative computation, and in this case it is especially important, since no

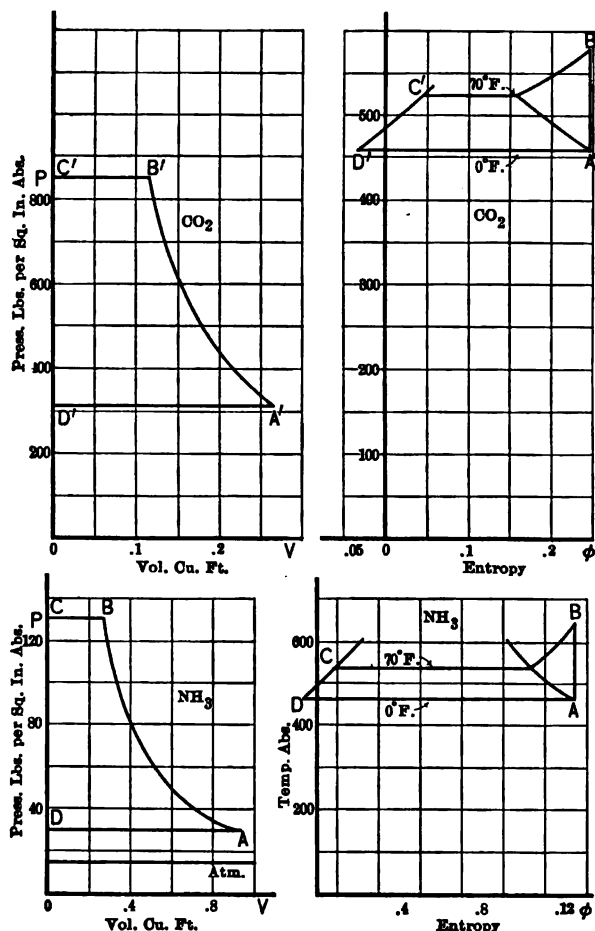


FIG. 188.—Ammonia and Carbon Dioxide Refrigerating Cycles.

thermal diagrams nor cycles can be conveniently established to represent in general terms the whole process.

This general description of the methods and the functions of the more common mechanical refrigerating systems, will serve as a basis of both absolute and comparative calculations concerning them, in which the important quantities to be determined are: the quantity of refrigerating fluid that must be circulated per minute to produce a unit of refrigerating effect; the size and displacement of cylinders necessary; the work that must be done per unit of refrigeration and the general relations between heats gained and lost by the fluid and the work that must be done on it.

114. Performance of Mechanical Refrigerating Cycles and Systems. Quantity of Fluid Circu-

lated per Minute per Ton Refrigeration, Horse-power, and Heat Supplied per Ton. Refrigeration per Unit of Work Done and its Relation to Thermal Efficiency of the System. As in the cases of the steam boiler and gas producer where capacity is to be measured in terms of a quantity of heat per minute delivered, a capacity unit has been established and given the arbitrary term "horse-power," with a different meaning in each case, so in dealing with refrigerating systems engineering practice has developed and adopted a somewhat similar term, but with a more rational basis, as the unit of capacity, and this is the *ton refrigeration*. If the latent heat of fusion of ice be taken as 144 B.T.U.,

which is nearly correct, and the ton as 2000 lbs., then one ton of ice in melting would absorb from its surroundings $2000 \times 144 = 288,000$ B.T.U., and if this were accomplished in a day of 24 hours, heat would be flowing at the rate of 288,000 B.T.U. per 24 hours, or 12,000 B.T.U. per hour, or 200 B.T.U. per minute. Whenever the refrigerating fluid absorbs heat at these rates it is said to be developing a ton refrigeration, or when brine absorbs heat at the same rate it likewise is said to be developing a ton refrigeration, so that capacity may be measured, as in the refrigerating fluid, or in the brine, and also as from the body giving up heat such as beef, or water ice. When at one point of the system, heat is being transferred at the ton refrigeration rate, it will not be necessarily at other points in the system, so it is essential to establish a standard that shall be invariable and that is in the fluid itself. Therefore, *a system has one ton refrigerating capacity when the refrigerating fluid is absorbing heat in the primary refrigerating coils at the rate of 288,000 B.T.U. per 24 hours, 12,000 B.T.U. per hour, or 200 B.T.U. per minute though the useful effect in cooling or freezing will be less if there be any gains or losses between.*

Each pound of fluid is capable of taking up in the refrigerating coils a definite amount of heat to be called *refrigeration per pound of fluid*, so that to produce the ton refrigeration a definite weight of fluid in pounds per minute must be circulated according to the general formula:

$$(\text{Lbs. of fluid per minute per ton}) = \frac{200}{(\text{Refrigeration per lb. of fluid})} \quad (1025)$$

This is a general expression for which there is a volumetric equivalent given by Eq. (1026), where the volume is that of delivery from the refrigerating coils.

$$\left\{ \begin{array}{l} \text{Cu.ft. fluid delivered from refrigerating coils per min. per ton} \end{array} \right\} = 200 \left(\frac{\text{Cu. ft. per lb. fluid at coil delivery}}{\text{Refrigeration per lb. of fluid}} \right) \quad (a) \quad (1026)$$

$$= \left(\frac{200}{\text{Refrigeration per cu.ft. fluid at coil del.}} \right) \quad (b)$$

Assuming no heating or cooling of the fluid between the points of coil delivery and compressor entrance, the same volume as left the coils will approach the compressor in compression systems, but on entering the compressor it will be heated and expanded; it will also suffer a loss of pressure, and re-expansion in the compressor will delay entrance. Defining the true volumetric efficiency E_v' , as the ratio of the volume drawn from the suction measured at conditions of pressure and temperature external to the cylinder, to the displacement volume, the compressor displacement per ton capacity may be set down as in Eq. (1027), for the air and compression systems.

$$\left\{ \begin{array}{l} \text{Compressor displacement cu.ft. per min. per ton} \end{array} \right\} = \left[\frac{200 \left(\frac{\text{Cu.ft. per lb. fluid at coil delivery}}{\text{Refrigeration per lb. of fluid}} \right)}{E_v'} \right] \quad (a) \quad (1027)$$

$$= \left[\frac{200}{E_v' (\text{Refrig. per cu.ft. fluid at coil del.})} \right] \quad (b)$$

That which corresponds in the absorption system to compressor displacement in the other systems, is the displacement of the pump, which is equal to the cu.ft. of strong liquor per minute that is capable of carrying off the quantity of ammonia circulating, if its volumetric efficiency is 100 per cent, and this it may be substantially if of suitable type and properly run.

Let w_R = lbs. rich liquor to be circulated per lb. of NH_3 taken up in absorber;

" C_R = per cent NH_3 in rich liquor;

" C_W = per cent NH_3 in weak liquor.

Then $\left(\frac{C_W}{100}\right)$, and $\left(\frac{C_R}{100}\right)$ = pounds NH_3 per pound solution weak and rich;

" $\left(\frac{100-C_W}{100}\right)$, and $\left(\frac{100-C_R}{100}\right)$ = lbs. water per lb. solution weak and rich;

" $\left(\frac{C_W}{100-C_W}\right)$, and $\left(\frac{C_R}{100-C_R}\right)$ = pounds NH_3 per pound water in weak and rich solution.

Therefore, the weight of ammonia gained per pound of water when the solution passes from the weak to the rich state is, $\left[\left(\frac{C_R}{100-C_R}\right) - \left(\frac{C_W}{100-C_W}\right)\right]$.

But 1 lb. of water is associated with $\left(\frac{100}{100-C_R}\right)$ pounds of rich solution, therefore, the pounds of NH_3 taken up per pound of rich liquor formed, is equal to the pounds NH_3 gained per pound of water, divided by pounds of rich liquor per pound of water; and the pounds rich liquor formed per pound of NH_3 taken up is the reciprocal of this, given by Eq. (1028):

$$w_R = \frac{\left(\frac{100}{100-C_R}\right)}{\left(\frac{C_R}{100-C_R}\right) - \left(\frac{C_W}{100-C_W}\right)} = \frac{100(100-C_W)}{C_R(100-C_W) - C_W(100-C_R)} = \frac{100-C_W}{C_R-C_W}. \quad (1028)$$

The displacement of the pump per ton will, therefore, be the product of the weight of rich liquor per pound of NH_3 absorbed, into the cu.ft. of liquor per pound, and the pounds of NH_3 per minute per ton.

$$\left\{ \begin{array}{l} \text{Rich liquor pump} \\ \text{displacement per} \\ \text{minute per ton} \end{array} \right\} = \left(\frac{100-C_W}{C_R-C_W}\right) \times \left(\begin{array}{l} \text{Cu.ft. rich} \\ \text{liquor per lb.} \end{array}\right) \times \left[\frac{200}{\text{Refrigeration}}\right] \text{ per lb. } \text{NH}_3. \quad (1029)$$

Solution of all these equations for weights and volumes of fluids and the corresponding displacements per minute per ton, depends on the evaluation of the term, (Refrigeration per pound of fluid). In the case of the air system this is, of course, nothing more than the product of specific heat at constant pressure into the temperature rise in the coils. For the liquid vapor system it is made up of parts depending on the final quality of vapor discharge which may be set down as in Eq. (1030):

$$\left\{ \begin{array}{l} \text{Refrig-} \\ \text{eration} \\ \text{per lb.} \\ \text{fluid} \end{array} \right\} = \left\{ \begin{array}{l} C_p \left[\begin{array}{l} (\text{Temp. air leaving coils}) - \\ (\text{Temp. air entering coils}) \end{array} \right] \left\{ \begin{array}{l} \text{For air system} \\ \end{array} \right\} \quad (a) \\ \left[\begin{array}{l} (\text{Latent heat}) \times (\text{Quality} \\ \text{of vapor leaving coils}) \\ - (\text{Heat of liquid bet.} \\ \text{supply and coil temp.}) \end{array} \right] \left\{ \begin{array}{l} \text{Final wet vapor} \\ \text{for vapor sys-} \\ \text{tems} \end{array} \right\} \quad (b) \\ \left[\begin{array}{l} (\text{Latent heat}) \\ - (\text{Heat of liquid bet.} \\ \text{supply and coil temp.}) \end{array} \right] \left\{ \begin{array}{l} \text{Final dry sat.} \\ \text{vapor for va-} \\ \text{por systems} \end{array} \right\} \quad (c) \\ \left[\begin{array}{l} (\text{Latent heat}) \\ - (\text{Heat of liquid bet.} \\ \text{supply and coil temp.}) \\ + C_p [(\text{Final vapor} \\ \text{temp.}) - (\text{Sat. vapor} \\ \text{temp. in coils})] \end{array} \right] \left\{ \begin{array}{l} \text{Final super-} \\ \text{heated vapor} \\ \text{for vapor sys-} \\ \text{tems} \end{array} \right\} \quad (d) \end{array} \right\} \quad (1030)$$

These are all tabular quantities except the heat of air and of vapor superheat, which are easily evaluated, but to facilitate determinations, Charts Nos. 72 for ammonia and 73 for carbon dioxide have been calculated and are given in the Handbook of Tables.

From the data of these charts applied to the preceding equations the displacements of compressors and pumps can be computed by the slide rule, introducing the chart quantities in the equations. When superheated vapor densities are to be evaluated either vapor may be assumed to behave as a perfect gas, volumes being directly, and density inversely proportional to absolute temperatures. The volume per pound of ammonia solutions is to be evaluated from that of water in the steam tables, dividing the water value by the specific gravity of the solution table. This may be checked by the use of Chart No. 74, in the Handbook of Tables, from which the value may be read off directly.

With a given weight and volume of fluid per minute per ton passing through the cylinders, the maximum and minimum pressures for which are determined by the temperature desired in the coils and that available by the cooling water in coolers or condensers, it follows that a definite horse-power per ton refrigeration will result for each system and each operative condition of any one. The cycle of operations to PV or $T\Phi$ coordinates offers a means of evaluating the work in foot-pounds per pound of fluid which, when multiplied by the pounds per minute per ton, gives work in foot-pounds per minute per ton, and this divided by 33,000 is the horse-power indicated, per ton refrigeration. These same cycles, but more particularly those drawn to $T\Phi$ coordinates, also offer means of directly evaluating the work per B.T.U. absorbed by the fluid in the refrigerator, the procedure being similar to that used with engine cycles in obtaining the ratio of work to heat supplied, which was the efficiency of the work developing cycle.

Compression is always actually or substantially adiabatic and vapors may be wet, dry saturated or superheated; since the value of " s " may and

usually is a variable as quality changes—which makes the PV method uncertain—the $T\Phi$ method of evaluating the work of compression for wet vapors is the only one that can be relied upon for accurate results. The Mollier diagram for NH_3 and CO_2 gives these results directly, so in Charts Nos. 75 and 76, in the Handbook of Tables are given the $T\Phi$ diagrams for NH_3 and CO_2 , and in Charts Nos. 77 and 78, the corresponding Mollier diagrams. It must be remembered that the results from these Mollier diagrams give the work in B.T.U. for the whole cycle, which is similar in form to the Rankine for steam.

The diagrams also give the B.T.U. absorbed per pound at the lower temperature, so the ratio of the work to refrigeration can be taken directly from the diagrams. This is the measure of the value of the process for refrigeration, sometimes though improperly termed efficiency, improper in that it is a different meaning for the word than that heretofore used. To facilitate calculation still further and also to show most clearly the influence of the several factors that control it, this ratio,

$$\frac{\text{B.T.U. work of refrigerating cycle}}{\text{B.T.U. absorbed at low temp.}}$$

has been plotted, and is given in Charts Nos. 79 and 80 (Handbook of Tables), for ammonia, and Charts Nos. 81 and 82, for carbon dioxide, to coordinates

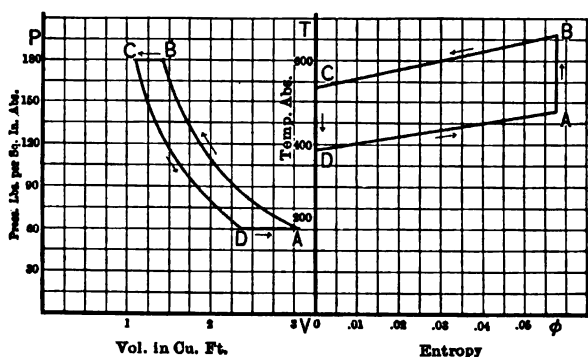


FIG. 189.—Air System. Diagram for Evaluating Work in B.T.U. per Unit of Heat Absorbed in Refrigeration.

similar to those used for the Rankine cycle efficiency, which is a similar ratio, the work there, however, being divided by the heat taken in at the high temperature. The evaluation of this performance ratio for the gas cycle is best made algebraically by methods similar to those used in the analysis of the Brayton work cycle. This cycle is shown in Fig. 189, $ABCD$, to both PV and $T\Phi$ coordinates. Here heat is taken up from D to A in the refrigerator and abstracted from B to C in the cooler, so the work done between the two adiabatics is equal to the difference on the $T\Phi$ diagram between the areas under CB and DA :

$$\left\{ \begin{array}{l} \text{Work of air refrigerat-} \\ \text{ing cycle in B.T.U.} \\ \text{per pound of air} \end{array} \right\} = \left\{ \begin{array}{l} \left(\text{Heat given up to} \right) - \left(\text{Heat taken in at} \right) \\ \left(\text{the water cooler} \right) \quad \left(\text{the refrigerator} \right) \\ = C_p(T_b - T_c) - C_p(T_a - T_d) \\ = C_p(T_b - T_c - T_a + T_d) \end{array} \right\} \quad \begin{array}{l} (a) \\ (b) \\ (c) \end{array} \quad (1031)$$

$$\begin{aligned}
 \left(\frac{\text{Work of air refrigerating cycle}}{\text{Heat given to water cooler}} \right) &= 1 - \frac{T_a - T_c}{T_b - T_c} = 1 - \frac{T_a \left(1 - \frac{T_c}{T_a} \right)}{T_b \left(1 - \frac{T_c}{T_b} \right)} \\
 &= \left\{ \begin{array}{l} 1 - \frac{T_a}{T_b} = 1 - \frac{T_c}{T_c} \\ 1 - \left(\frac{P_a}{P_c} \right)^{\frac{\gamma-1}{\gamma}} = 1 - \left(\frac{P_a}{P_b} \right)^{\frac{\gamma-1}{\gamma}} \\ 1 - \left(\frac{V_c}{V_a} \right)^{\gamma-1} = 1 - \left(\frac{V_b}{V_a} \right)^{\gamma-1} \end{array} \right\} = E_B = \left\{ \begin{array}{l} \text{Efficiency} \\ \text{of Bray-} \\ \text{ton work} \\ \text{cycle} \end{array} \right\} \left\{ \begin{array}{l} (a) \\ (b) \\ (c) \end{array} \right\} \quad (1032)
 \end{aligned}$$

$$\begin{aligned}
 \left(\frac{\text{Work of air refrigerating cycle}}{\text{Heat absorbed in refrigeration}} \right) &= \frac{T_b - T_c}{T_a - T_c} - 1 = \frac{T_b \left(1 - \frac{T_c}{T_b} \right)}{T_a \left(1 - \frac{T_c}{T_a} \right)} - 1 \\
 &= \left\{ \begin{array}{l} \frac{T_b}{T_a} - 1 = \frac{T_c}{T_c} - 1 \\ \left(\frac{P_b}{P_a} \right)^{\frac{\gamma-1}{\gamma}} - 1 = \left(\frac{P_c}{P_a} \right)^{\frac{\gamma-1}{\gamma}} - 1 \\ \left(\frac{V_a}{V_b} \right)^{\gamma-1} - 1 = \left(\frac{P_a}{P_c} \right)^{\gamma-1} - 1 \end{array} \right\} = \frac{E_B}{1 - E_B} \left\{ \begin{array}{l} (a) \\ (b) \\ (c) \end{array} \right\} \quad (1033)
 \end{aligned}$$

The importance of expressing the ratio of work done to refrigeration effect for the air cycle in terms of the Brayton work-cycle efficiency, is due purely to convenience of calculation as this efficiency is the same as that of the Otto cycle for the same compression, curves for which have been given in Section 104 that can be used to solve these refrigerating problems.

Pressure-volume determinations are equally available for the determination of the work of the compressors of the air and the compression-vapor systems; of the engine of the air system, and of the pump of the absorption system, but these methods need not be repeated here as they are completely presented in Part I, except that for pump work, which is to be evaluated as for expansive fluids with no cut-off or compression, but without any re-expansion of clearance fluid. It will be found for the vapors even if they are superheated, that the work as determined by PV methods does not check the work as determined by $T\phi$ methods; the difference being a measure of the uncertainty of the physical properties—specific heat of liquid and of superheated vapor, latent heat and densities of liquid and vapor. Of course, these data might be manipulated to give checking results for work, but this is an improper procedure; the best practice is to use both methods and assume the correct result to lie somewhere between, but nevertheless indeterminate with precision, until a redetermination of properties has been made experimentally.

From the values of the work in heat units, per unit of heat absorbed in refrigeration the horse-power per ton refrigeration follows by the simple

transformations below, which show a direct proportionality, for the compression systems.

$$\begin{aligned}
 (\text{I.H.P. per ton refrigeration}) &= \frac{778}{33000} \times (\text{Work in B.T.U. per min. per ton}) \\
 &= \frac{778 \times 200}{33000} \times \left(\frac{\text{Work in B.T.U. per minute}}{\text{B.T.U. absorbed per minute}} \right) \\
 &= 4.71 \times \left(\frac{\text{Work in B.T.U.}}{\text{B.T.U. absorbed at low temp.}} \right) \quad (1034)
 \end{aligned}$$

The heat supplied to the system per ton refrigeration is as important a determination as the work or horse-power per ton, and in the case of the absorption system more so, as the rich liquor pump work is very small while the heat supplied to the generator is large, both together representing the heat equivalent of the energy necessary to get the low-pressure vapor into the high-pressure condition. The heat supplied to develop power in steam or gas engines is given by Eq. (1035). See also Eq. (828).

$$\left(\frac{\text{B.T.U. per hour per}}{\text{I.H.P. of engine}} \right) = \frac{2545}{\text{Engine thermal efficiency}}$$

But $\left(\frac{\text{Engine thermal}}{\text{efficiency}} \right) = (\text{Efficiency of engine cycle}) \times (\text{Efficiency factor}), \therefore$

$$\left(\frac{\text{B.T.U. per hour per}}{\text{I.H.P. of engine}} \right) = \frac{2545}{(\text{Efficiency of engine cycle}) \times (\text{Efficiency factor})} \quad (1035)$$

Introducing the ratio of I.H.P. of compressor to I.H.P. of engine, which is less than unity, the heat consumption per I.H.P. of compressor follows:

$$\left\{ \begin{array}{l} \text{B.T.U. sup-} \\ \text{plied per hr.} \\ \text{to engine per} \\ \text{I.H.P. of} \\ \text{compressor} \end{array} \right\} = \left[\frac{2545}{(\text{Efficiency of engine cycle}) \times (\text{Efficiency factor})} \right] \times \left[\frac{\text{I.H.P. of engine}}{\text{I.H.P. of comp.}} \right] \quad (1036)$$

The actual I.H.P. of the compressor per ton refrigeration will be the cycle H.P. per ton divided by the diagram factor of the refrigerating cycle, which includes all losses, but which is nearly 100 per cent, certainly in any good compression not less than 95 per cent.

Therefore, (B.T.U. supplied per hour to engine, per ton refig.)

$$= \left[\frac{2545}{\text{Efficiency of engine cycle} \times \text{Efficiency factor}} \right] \times \left[\frac{\text{I.H.P. eng.}}{\text{I.H.P. comp.}} \right] \times \left[\frac{\text{Cycl.H.P. per ton}}{\text{Refrig. diag. fact.}} \right] \quad (1037)$$

These quantities have all been evaluated except the ratio of compressor to engine horse-power, which is the mechanical efficiency of the drive depending on mechanical construction, speed, and size, and must therefore be estimated.

For the absorption system the heat supplied to the pump is to be evaluated in a manner equivalent to the above; but the heat of steam supplied to the generator must be determined separately from the properties of aqueous solutions of ammonia. This heat is a very much larger amount though not necessarily at so high a temperature as that required for economical power generation. It is a little surprising to find that this heat cannot be evaluated with precision because of lack of data on the thermal properties of the solutions, but estimates may be made on various assumptions. The most common assumption made in evaluating the heat supplied to the generator per ton refrigeration or per pound of anhydrous ammonia circulating, to which it is directly related, is that nothing but NH_3 vapor will be discharged and that the heat of its liberation is equal to the heat of absorption of the same amount under inverse circumstances. For example, the equations of Section 74, give the heat of absorption per pound of ammonia absorbed, when a solution of one strength becomes of higher ammonia content. Inversely, if a rich liquor becomes weaker through the loss of the same amount absorbed as above, there will be required an equal amount of heat. It is, however, quite impossible to discharge ammonia vapor from a solution without also discharging water vapor; the difficulty involved here lies in fixing the amount of water vapor so carried off and the evaluation of heat of its evaporation, which must be added to that of the ammonia discharged. With an analyzer above the generator, condensing and returning some water vapor with its heat by incoming rich liquor as in the diagram, Fig. 182, it is necessary only to consider the water content of the NH_3 vapor beyond this point, which is less than at generator discharge, so the error of neglecting the heat of vaporization of the water vapor is appreciably less when an analyzer is present. If data were available to give the amount of water vapor in a saturated mixture beyond the analyzer, and the temperature of vapor evolution at a given pressure in the generator itself, the heat equivalent of the water-vapor discharge could be found, but neither of these are known. The law of molecular rise of boiling-point does not apply to ammonia water solutions, so the boiling-point or temperature of vapor evolution for a given liquor cannot be calculated for a given pressure. The best that can be done at present is to neglect the water vapor entirely and remember that the heat required by the generator will be something (x) more than thus found. On this basis

$$\left(\begin{array}{c} \text{B.T.U. supplied to generator} \\ \text{per lb. NH}_3 \text{ circulating} \end{array} \right) = \left(\begin{array}{c} \text{Heat of absorption per pound NH}_3 \\ \text{between } C_W \text{ and } C_R \end{array} \right) + x.$$

$$\left[\begin{array}{c} \text{B.T.U. supplied to gen-} \\ \text{erator per hour per} \\ \text{ton refrigeration} \end{array} \right] = \left[\left(\begin{array}{c} \text{Heat of absorption} \\ \text{per lb. NH}_3 \text{ be-} \\ \text{tween } C_W \text{ and } C_R \end{array} \right) + x \right] \times \left[\begin{array}{c} \text{Lbs. NH}_3 \\ \text{per hr.} \\ \text{per ton} \end{array} \right] \quad (1038)$$

The heat balance for refrigerating systems might be set down as for power-generating systems, equating energy taken in to that given out, but as such heat balances are long and complicated without the introduction of vitiating

assumptions they will be omitted. For any particular case in which they may be required the methods discussed in previous sections provide the necessary material as far as it is at present possible to do.

Example. Construction and use of Charts Nos. 72 and 73, Handbook of Tables. These diagrams are for the purpose of finding the refrigerating effect per pound of fluid, which is made up of the latent heat, or as much of it as is available, less the heat necessary to cool the liquid from its original temperature to that due to the pressure in the coils, plus the heat absorbed in superheating the vapor.

A horizontal scale of pressures is laid off in both directions for a vertical axis carrying a B.T.U. scale. In the section to the right of the center axis curves are drawn representing various temperatures of the liquid before entering the refrigerator coils. These are so drawn that the vertical scale opposite the intersection of a vertical from any pressure with any curve gives the latent heat for that pressure less the heat required to cool the liquid. This is the available heat for refrigerating if the vapor leaves the coils dry and saturated. In the section to the left of the center axis are two sets of curves, the lower, representing temperatures of the vapor leaving the coils, is so drawn that the value of the left-hand vertical scale opposite a point of intersection of a vertical from any pressure with any curve, gives the heat absorbed in superheating the vapor. The sum of this and the value found in the first section gives the total refrigerating effect for the case when the vapor leaves the coils in a superheated state. The upper curves in this section represent quality of the vapor if the liquid has not been entirely evaporated and are so drawn that the value on the vertical scale opposite the point of intersection of a vertical from any pressure with any curve, shows the heat unavailable for refrigerating, due to incomplete evaporation of the liquid, and the difference between this value and that found in the first section gives the total refrigerating effect for the case of wet vapor leaving the coils.

As an example let it be required to find the refrigerating effect per pound of ammonia when the pressure in the coils is 20 lbs. gage, the temperature of the liquid before entering the coil is 70° F. and (a) Vapor leaves dry and saturated; (b) Vapor leaves 92.5 per cent dry; (c) Vapor leaves at a temperature of 30° F. From 20 in the right-hand section project up to curve 70°. The value on the vertical scale at this point is 502 B.T.U., which is the value for case (a). From 20 in the left-hand section project to curve 92.5 per cent; the value on the left-hand vertical scale is 43, therefore, for case (b) the result is $502 - 43 = 459$ B.T.U. For case (c), project from 20 to curve 30°, the value on the vertical scale corresponding to which is 12.5, hence the result for this case is $502 + 12.5 = 514.5$.

Prob. 1. In a cold-storage room it is estimated that 350 lbs. of ice melt every minute. What would be the capacity of a machine to do an equal amount of refrigeration? What will be the capacity of a machine required to make 100 tons of ice per day from water at 60° F., the ice being at a temperature of 25° F.?

Prob. 2. Three types of machines are under consideration, (a) dense air; (b) ammonia compression; (c) carbon dioxide compression. In every case the temperature of the gas or vapor leaving the coils is 20° F. In the air system the air enters the coils at a temperature of -50° F. In the case of the ammonia system the pressure in the coils is 15 lbs. per square inch absolute, the liquid temperature before entering is 60° F.; in the carbon dioxide system the pressure is 300 lbs. per square inch gage, and the temperature before entering the coils is 80° F. What must be the compressor displacement for each case for a common true volumetric efficiency of 80 per cent?

Prob. 3. If the ammonia should leave the coils in Prob. 2 with 5 per cent unevaporated, what effect would there be on the temperature of the vapor and on the compressor size? If the carbon dioxide should leave dry and saturated what would be the change in these two quantities?

Prob. 4. An ammonia absorption system works so that the temperature of the anhydrous liquid entering the coils is 70°F . and that of the vapor leaving the coils is 30°F . with a pressure of one atmosphere, what is the displacement of the rich liquor pump, the rich liquor being 30 per cent NH_3 , and the weak liquor 20 per cent by weight?

Prob. 5. In a refrigerating system it is desired to maintain a temperature of 10°F . in the coils, and there is available for condensing purposes water having a temperature of 70°F . What will be the required cylinder displacement per minute of the compressor, and the horse-power per ton refrigeration for an ammonia-compression system, and a similar system using CO_2 ? What would be the relative worth of the two systems?

Prob. 6. Air is used for refrigeration in a system where the highest pressure which it is desirable to carry is 150 lbs. gage, the cooling water temperature is 60°F ., and the cold air is desired to have a temperature of -50° . What will be the temperature of the air leaving the refrigerator if each pound absorbs 15 B.T.U.? What will be the low pressure, work done, ratio of work to heat absorbed in refrigerator and given to cooler? Draw the PV and $T\phi$ diagrams.

Prob. 7. In an ice-making plant making 100 tons of ice per day from water at 40°F . the ice is at 20°F . when removed from the cans; the cooling water available has a temperature of 60°F . and the pressure in the coils is 10 lbs. gage. If the engine driving the compressor has a thermal efficiency of 12 per cent, the work required to overcome friction in the engine and compressor is 15 per cent of the engine horse-power, and the compressor-diagram factor is 90 per cent, how many B.T.U. must be supplied to the engine per hour, for, (a) an ammonia plant; (b) a carbon dioxide plant?

Prob. 8. An absorption system is in operation so as to produce 50 tons refrigeration. The cooling water for the condenser is at 50°F . and the liquid NH_3 is cooled to within 10° of this value. The temperature of the vapor leaving the coils is 20°F . and the pressure in them is 10 lbs. gage. The strength of the ammonia liquor varies between 15 and 30 per cent. Assuming that 25 per cent more heat must be supplied to the generator than is needed to liberate the ammonia, what will be the B.T.U. supplied to generator per hour per ton refrigeration?

INDEX

- Absolute pressure, 4
 - temperature, 286
- Absorption of gases by liquids, 333
 - process of, 281
 - system of refrigeration, 657, 659, 665
 - heat exchange in, 665
- Acceleration, velocity and work, 5
- Adiabatic changes for gases compared with isothermal, diagrams of, 521
 - expansion and cyclic analysis, 537
 - Carnot cycle, 554
 - of steam, values of s for, 617
 - processes of, 18
 - Rankine cycle, 541
 - relation, variable specific heat, 594
- Adiabatics for gases, 519
 - for steam, 523
- Adsorption, 334
- Air, amount required for combustion
 - of carbon, 457
 - of gases, 455
- Air card, standard, 597
 - coefficient of discharge through orifices (table), 629
 - composition of, 346
 - compressors, *see* Compressors.
 - compressed, engines, efficiency of, 646
 - possibilities of, 647
 - expansion of through nozzles, 624
 - flow of, Fliegenger formulas for, 624
 - gas, 418
 - characteristic diagram of, 426
 - density and calorific power of, 427
 - explosive pressures, 454
 - heats of reaction, 424
 - hypothetical, 423
 - mixture, best, calorific properties, 456
 - weight and volume proportions, 421
 - humidification of, 276
 - reheating, for air-engines, 149, 647
 - saturation of, 331
 - system of refrigeration. *See* Dense air, system of
 - vapor pressure of atmospheric, 332
- Allowable compression in gas engines (table), 601
- American mineral oil products (table), 413
- Ammonia, absorption of, 335
 - anhydrous liquid, specific heat, 295
 - P - T relations, 315
 - aqueous solution, specific heat, 297
 - heat of absorption, 335
 - of complete dilution, 336
 - of partial dilution, 337
- Ammonia, latent heat of, how obtained, 322
 - use of as refrigerant, 657
 - weight of in refrigerating system, 668
 - work in refrigeration by, 673
- Analysis, cyclic, 534
 - of boiler flue gas, 458, 460
 - of fuel, 344, 390
 - proximate, how made, 390
 - of producer gas, 433
- Anthracite coal, 395
 - rate of combustion, 470
 - sizes and ash content (table), 396
- Areas, measurement of, 34
- Ash and formation of clinker, 397
 - fusion of, 398
 - in fuel, how determined, 390
- A. S. M. E. formula for B. T. U. in coal, 399
 - rules, boiler horse-power, 473
 - commercial cut-off, 263
- Atkinson gas cycle, 566, 577
- Atomic weights, 309; example of, 344
- Avogadro, law of, 308
 - volumetric and weight relations with, 347
- Back pressure, effect of on efficiency of steam turbines (table), 612
- Balance, heat of boilers (table), 487
 - of gas engines, 605
 - of refrigerating systems, 673
 - of steam plants, 608
- Barometric pressure, 4
- Baumé specific gravity scale, oil specifications with, 413, 415
- Beau de Rochas (Otto), gas cycle, 565
- Best air-gas mixture, 456
 - load, steam engines and turbines, 607
 - receiver pressure, two-stage compressor, 74, 87
- Bituminous coal, description of, 394
 - rate of combustion, 470
 - used for coke making, 407
- Black body, 362
- Blast furnace gas, 418
 - calorific power and density of, 427
- Blau gas, 415
- Boiler furnace temperatures, 457
 - steam, efficiency of, 484, 493; table of (Donkin), 487
 - heat absorption of, 476, 482
 - heat balance of (table), 488
 - horse-power of, 473
 - test, Purdue locomotive, 477
 - U. S. Geol. Survey, 478
- Boiling- and freezing-points, 311, 316

- Boiling- and freezing-points of liquids, constant temperature process, 280
point, rise of, 316
Boudouard's equilibrium relations, CO and CO₂ (table), 420
Boyle's law, 303
and liquefaction of gases or vapors, 441
Brake horse-power, 37, 258
Brayton gas cycle, 566, 579
as reverse refrigerating cycle, 660
work of refrigeration with, 671
Breckenridge, boiler tests, 478
Brines, freezing-point for, 316
specific heat of, 294
British thermal unit (B. T. U.) and calories, value of, 290
and equivalent evaporative power, 349
formula for coals, 399, 401
Joule's equivalent in, 503
per cubic foot of gases, how determined, 354
Brown gas engine and cycle, 564
Bunte, table of detonating mixtures, 466
table of water gas characteristics, 428
- Calorie, definition of, 288, 290
Calorific power and density
of blast furnace gas, 427
of coke oven and retort coal gas, 411
of oil gas, 416
of producer gas, 447
definition of, 350
high and low, 349
of coals, 398, 401
of natural gas, 405
of oil distillates, formula for, 414
of oils, 403
of volatile of coals, curve of, 401
Calorimeter, use of, 349
Calorimetry, 288
Cannel coals, description of, 395
Capacity of single-stage compressor
no clearance, logarithmic law, 57
exponential law, 66
with clearance, logarithmic, 60, 62
exponential, 70, 71
two-stage compressor, best receiver pressure
no clearance, 75
with clearness, 88
any receiver pressure, with clearness, 81
Carbon dioxide, critical temperature, 303
density of liquid and vapor, 322
isothermals for, 304
latent and total heat of, how obtained, 322
use of as refrigerant, 657, 664
oxide ratio, 420
Carnot gas cycle, 568, 581
steam cycle, 540, 554
comparison with Otto, 589
diagram of, 555
efficiency of, 556, 559
work and efficiency, 554, 558
Carpenter, test on White automobile boiler, 478
Centigrade and Fahrenheit temperature, 284
heat units, 288
- Change of O₂ in air to CO and CO₂ (table), 421
Characteristics of explosive mixtures of
Pintsch oil gas and air (table), 455
Charcoal, production of, 407
Charles, law of, 303
Chemical reaction, combustion, process of, 281
Chimneys, boiler horse-power of, 643
capacity of, 642
draft in, 641, 644
Chimneys, height of, 642, 644
Kent's formula for, 644
pipes, flues and ducts, flow of gas in, 640
Clapeyron equation for latent heat, 320, 508
Classification of coal by composition, 392
of fuels (table), 389
of heat transfer, 367
of heating processes, 277
Clearance, amount of in piston engines, 150
and steam consumption, 139
effect of in compressed air engines, 648
in piston engines, 145
in receiver engines, 207, 218, 222, 229, 232, 247
in compressor capacity, 112
expressed as per cent of displacement, 31
method of finding, 31
Coal, anthracite sizes and ash content, 396
bituminous, used in coke making, 407
calorific power of, 398, 401
classification of by composition (table), 392
description of various kinds, 393-396
gas, properties of, 395
pulverized, in gas making, 446
rates of combustion of, 470
sizes of, 396
sulphur in, 397
Coefficient of discharge for air through
orifices (table), 629
of expansion, solids, liquids, gases and vapors, 299
of friction for gas flow, 635, 637
of heat transmission, 362, 365
in condensers, 383
variation in, 375, 380
of pressure change for gases, 299
Coke, calorific power and density of gas (table), 411
effect of temperature and time on yield of, 408
manufacture of, 407
oven and retort coal gases (table), 410
Combustible in fuel, 390
Combustion and related reactions, 341
effects, 450
exothermic, endothermic, 281, 341
explosive and non-explosive, 450
heat of, air gas, 425
producer gas, 443
heats of, 351
process of, 281
rate of, 467
temperatures due to, 453
Commercial cut-off, 257, 263
rating of boilers, 474

- Comparative heat balances of producer and engine plants (table), 606
- Comparison of coke oven and retort coal gas (table), 410
- of hypothetical water gas (tables), 430, 431
- of steam and gas power, 588
- Composition of air, 346
- of flue gas, 460
- of hypothetical air gas (table), 423
- no CO₂ and no CO (table), 424
- of oil gas, 455
- of producer gas (table), 441
- of water gas (table), 430
- Compound engine, finite receiver
- exponential law, 195, 229
- logarithmic law, 186, 222
- indicator cards from as taken and as combined, 153, 154
- infinite receiver, logarithmic law, 173, 207
- exponential law, 182, 218
- without receiver, logarithmic, 199
- exponential, 205, 237
- Compounding, effect of, 150, 159
- Compressed air engine and air compressor, efficiency of, 648
- air engines, allowable temperatures, 651
- diagram factors for, 651
- effect of preheating, 647, 650
- effect of reheating, 647
- efficiency of, 646
- value of s for, 653
- volumetric efficiency, 653, 655
- work of, 652
- Compression allowable with gas and oil engines (table), 601
- and expansion, work of, 11
- effect of on diagram factor, gas engines, 599
- exponential, 50
- in gas engines, producer gas, alcohol, gasoline, 464, 589
- isothermal, 50, *see also* under Compressors
- multi-stage, 53, *see also* under Compressors
- system of refrigeration, 657, 658
- indicator cards for, 663, 665
- work of, 11
- Compressions commonly used in gas engines (table), 465
- Compressor capacity, regulation of, 108
- characteristics, 100, 105
- limiting pressures for, 107
- maximum work of, 98
- multi-stage, maximum work, 100
- work and capacity of, 48, 50
- Compressor single-stage, exponential, in terms of dimensions, 70
- isothermal, in terms of dimensions, 61
- no clearance, exponential, 64
- logarithmic, 56
- with clearance, exponential, 68
- isothermal, 59
- three-stage, 91
- two-stage, any receiver pressure, exponential, in terms of dimensions, 81
- best receiver pressure, exponential, in terms of dimensions, 87
- Compressor single-stage, no clearance, exponential, 74
- with clearance, exponential, 78
- Compressors, economy of, 96
- efficiency of, 96, 104
- structure and processes, 48
- volumetric efficiency of air, 653, 655
- work in at partial capacity, 107
- of for refrigeration, 671
- Condensers, coefficient of heat transfer, 383
- experiments on by Orrok, 382
- Conductivity in solids, effect of joints on, 360
- of solids, liquids and gases, 358, 360
- thermal, 357
- Constant and variable temperature processes, 282
- gas, R , 295, 305
- of proportionality, 365
- pressure heating of gases, 512
- temperature heating of gases, 515
- total heat lines, method of determination, 528
- volume heating of gases, 513
- Consumption of steam engine, 259, 618
- effect of cut-off on, 266
- variation with load, 267
- rate of, 40, 266
- of simple engine, no clearance,
- logarithmic, 125; exponential, 130
- with clearance, logarithmic, 134
- exponential, 141
- of compound engine, infinite receiver
- no clearance, logarithmic, 176
- exponential, 183
- finite receiver, no clearance
- exponential, 197
- logarithmic, 189
- no receiver, no clearance
- logarithmic, 201
- exponential, 205
- infinite receiver, with clearance
- logarithmic, 210
- exponential, 219
- finite receiver, with clearance
- logarithmic, 227
- exponential, 230
- no receiver, with clearance
- logarithmic, 234
- exponential, 239
- of triple expansion engine, no clearance
- infinite receiver, logarithmic, 241
- Convection, 358
- Conversion factors in combustion of hexane (table), 346
- Critical pressure, steam, 314
- temperature, steam, 314; curve of, 323
- carbon dioxide, 303
- Cut-off, best, 259, 265
- commercial, 257, 263
- in compound engines, 178, 182, 191, 198, 209, 220
- in triple expansion engines, 242
- relation of to water rate, 266
- Cycle and work phases, 20
- gas, 562
- Atkinson, 566, 577
- Brayton, 566, 579

- Cycle, gas, Brown, 564
 Carnot, 568, 581
 Diesel, 566, 580
 Ericsson, 565, 573
 Lenoir, 564
 Otto, 565, 575, 578; Otto complete expansion, 578
 and Langen, 564
 Stirling, 565, 571
 steam, Carnot, 554
 Rankine, 541
 Cycles, gas, 562
 efficiency of with compression, 572, 575, 583
 non-compression, 569
 refrigerating, 657, 660, 666, 670
 steam, 538, 540, 554
 Cyclic analysis, 535
 results of, 589
 efficiency, 534, 559
 factors for steam, 609, 612
 relation to actual gas engines, 596
 Cylinder ratio, compound engine, finite receiver
 no clearance, logarithmic law, 193
 exponential law, 198
 no receiver, no clearance, logarithmic, 202
 exponential, 206
 infinite receiver with clearance, logarithmic, 213, 215
 sizes, graphical methods for, 272
 Dalton's atomic law, 308, 327
 Davis, total heat of steam, diagram of, 321
 formula for, 530
 Dense air system of refrigeration, 657, 658, 663
 indicator cards for, 663
 Densities of mineral oils, 413
 Density and calorific power of
 blast furnace gas (table), 427
 coke oven and retort coal gas (table), 411
 natural gas (table), 405
 oil gas (table), 416
 producer gas (table), 447
 water gas (table), 434
 of crude oils, 414
 of gases, relation of, 310
 of kerosene, correction for due to temperature, 413
 variation with temperature, 299
 Detonating mixtures, gas and air (table), 466
 Dew point, 331
 Diagram factor, 255
 gas engine, with mean effective pressures, 602
 with compressed air engine, 651
 Otto and Diesel engines, 597
 Diagram of work, 9
 PV for reference, 50
 Diesel cycle engines confined to oil fuel, 589
 engines, performance of, 596
 gas cycle, 566, 580
 Dieterici equation for specific heat of water, 530
 Dilution, complete and partial ammonia, 336
 Displacement, pump in refrigeration, 668
 Displacement, specific, definition of, 40
 Dissociation, heat of, 351
 of gases and liquids, process, 280, 281
 Distance, unit of, 1
 Distillates, oil, calorific power of, 414
 petroleum, vapor pressure of, 330
 Distillation of mineral oils, 412
 Distribution of work in multi-expansion engines, 148
 Donkin, table of boiler efficiencies, 487
 Dowson gas, 428
 Draft, effect of on combustion, 469
 on efficiency of boilers, 493
 in chimneys, 641, 644
 Dry vacuum pumps, 48
 Ducts, chimneys, pipes and flues, flow in, 634
 Dulong formula for calorific power, 399
 Durley, coefficient of discharge for air, 629
 Economical load, more than one engine, 267
 Economy of compressors, 96
 of low pressure steam turbines, 613
 Efficiencies, air card, 598
 Otto cycle with constant and variable specific heats, 595
 Efficiency, 38
 cyclic, 534, 559
 effect of high initial pressure on, 610
 of load on gas engine, 598, 605
 of low-back pressure on (steam), 610
 of reheater on, 616
 of steam jackets on, 615
 of superheated steam on, 613
 of vacuum on, 615
 factors for steam, cyclic, 609, 612
 of Carnot cycle, 556, 559
 of compressed air engines, 646
 and air compressors, 648, 652
 of compressors, 96, 104
 of gas and steam power, 588
 of gas cycles
 Atkinson, 578
 Brayton, 580
 Brown, 570
 Carnot, 582
 Diesel, 581
 Ericsson, 574
 Lenoir, 570
 Otto, 576, 595; Otto complete expansion, 579
 and Langen, 570
 Stirling, 572
 with compression, 571, 575, 583
 non-compression, 569, 584
 of gasification, air gas, 425
 of heat of preheat, 651
 of Otto cycle, thermal (table), 595
 of process, gasification, water gas, 433
 of Rankine cycle, 541, 559
 of reaction, producer gas, 443
 of refrigeration, 671
 of steam boilers, 484, 493
 table of (Donkin), 487
 of steam engines, 534
 of transformation, gas, 419
 mechanical, 258
 of gas engines (table), 601

- Efficiency, volumetric, 104
 compressed air engines and compressors, 653
 gas engine, 603
- Endothermic reaction defined, 281, 341
- Energy, intrinsic, 503
- Engine, combined diagrams of, 153
 diagram factor, 255
 multiple-expansion, work and power of, 246
 power, 252
 reference cycles, simple, 120
 compound, 157
- Engine steam, indicator cards, typical, 118
 with incorrect valve setting, 120
 steam, jacketed, 615
 water rate, 40, 259
 variation with load, 267
- Engine, single cylinder without clearance
 logarithmic law, 125
 exponential law, 130
- Engine, with clearance, logarithmic, 134
 exponential, 141
 compound, infinite receiver, no clearance
 logarithmic law, 173
 exponential law, 182
 with clearance, logarithmic, 207
 exponential, 218
 finite receiver, no clearance
 logarithmic, 186
 exponential, 195
 with clearance, logarithmic, 222
 exponential, 229
 without receiver, no clearance
 logarithmic, 199
 exponential, 205
 with clearance, logarithmic, 232
 exponential, 237
 triple expansion, infinite receiver, logarithmic, 240
- Engineering thermodynamics, 497
- Entropy, definition of, 499
 -heat diagram, use of, 626
 of the liquid, 512
 of superheated steam, 512
- Equilibrium of liquids and vapors, 329
 van 't Hoff's law of, 343
 ratio of gases, Boudouard, 419
 table of, 420
- Equivalent evaporative power B T. U., 349
- Ericsson gas cycle, 565, 573
- Evaporation, factor of, 474
 of liquids, humidification of gases, 280
 of solids, sublimation, process, 280
 steam boiler, 472
 unit of, 473; units, 349
- Evaporative power, 349
- Exothermic reaction defined, 281, 341
- Expansion, adiabatic, 18
 Carnot cycle, 554
 for gases, 519
 for steam, 523
 Rankine cycle, 541
 and compression, work of, 11
 coefficients of, 299
 exponential, 19
 extended Otto for gas cycle, 577
 infinite, of gases, 327
- Expansion, isothermal, 18
 of free gases, liquids, and solids, processes, 278, 279
 of solids, liquids, gases and vapors, coefficients for, 299
 of steam, logarithmic, 531
 multi-stage through nozzles, 626
 values of s for, 617
- Experiments on boiler evaporation, 477
 on carbon-oxide ratio, 420
 on heat transfer, 378
 on ignition temperatures, 462
 on influence of initial steam pressures (locomotive), 609
 on steam heat, résumé of, 321
 on steam turbine nozzles, 625
 on superheated steam, results of used by Marks and Davis, 297
- Explosion pressures, air and gas mixtures, 454
- Explosive and non-explosive combustion, 450
 mixtures, air gas (table), 466
- Exponent s , 17
 for air, 653
 for steam, 525, 617
 graphic method of finding, 18
- Exponential compression, 50
 expansion, 19
- Factor, diagram, 255
 heat suppression, 454
 of evaporation, 474
 Fahrenheit temperature, 285
- Falk, ignition temperatures, 462
- Fernald, U. S. G. S. tests on producers, discussion of, 437, 445
- First law of thermodynamics, 504
- Fliegner, formula for air flow, 624
- Flow of air, 624
 of expansive fluids critical value of, 623
 of gases and vapors with small pressure drops, 627, 632
 in chimneys, 640
 in ducts, 638
 through valves, 630
 through nozzles and orifices, 623
 of hot water, 620
 of steam, 620, 624
- Flue gas analysis, 458
 composition of, 460
 losses in boilers, 485, 492
- Flues, pipes, ducts and chimneys, flow of gases and vapors in, 634
- Fluid, quantity of per hour per I.H.P., 40
 weight of for refrigeration, 669
- Foot-pound, of work, 1
- Force, unit of, 1
- Fractionation of oils, 403, 413
- Freezing-points, 311, 316
 effect of pressures on, 508
 for brines, 316
- Friction, flue and chimney, 642
 horse-power, 37, 258
- Fuel analyses, 344
 proximate, 345; how made, 390
 ultimate, 345, 390
 ash in, how determined, 390
 carbon and hydrogen in, 343

- Fuel, coals, described, 393-6
 combustible in, 390
 fixed carbon in, how determined, 390
 general classification (table), 389
 how defined, 390
 lignite, 393
 mineral oil, 402
 mixtures of chemical elements, 344, 387
 moisture in, how determined, 390
 natural gas, 402
 origin of, 390
 peat, 393
 volatile in, how determined, 390
 wood, 391
- Furnace, boiler, temperatures of, 457
- Fusion, and sublimation, 311
 of ash, 398
- Gas, air, 418
 and oil engines, compressions, 589, 601
 and steam power, comparison of, 588
 and vapor mixtures, and pressures, 327
 Blau, 415
 blast furnace, 418
 constant, R , 295, 305
 cycles, 562
 Atkinson, 566, 577
 Brayton-Joule, 566, 579
 Carnot, 568, 581
 Diesel, 566, 580
 efficiency of, with compression, 572, 575, 583
 Ericsson, 565, 573
 Lenoir, 564
 non-compression, 570, 584
 Otto, 565, 575, 578; complete expansion, 578
 Otto and Langen, 564
 Stirling, 565, 571
- Gas consumption, efficiency, m. e. p., and work. *See* Separate cycles.
 density and specific volume, 307
 Dowson, 428
 efficiency of transformation, 419
 engine, compression in, 464, 589
 cycles of, 562
 efficiencies, mechanical (table), 601
 thermal (table), 595, 605
 heat balance, 605
 horse power, prediction of, 600
 performance, compared to cyclic, 596
 curves of, 605
 preignition in, 464
 suction, resistance of, 603
- Gases, absorption of by liquids, 333
 process, 281
 adiabatics for, 519
 and air, flow of in ducts, flues and chimneys, 638
 B. T. U. per cubic foot of, how determined, 354
 density of, 310
 dissociation of, process, 280
 expansion of free, process, 279
 flow of through pipes, 638
 through valves, 630
 through Venturi, and Pitot tubes, 632
- Gases, liquefaction of, 304
- Gases and vapors, weight and volumetric relations of, 347
 heat transmission by, 365
 heating of under constant pressure, 512
 constant volume, 513
 constant temperature, 515
 hot transmission of heat from, 378
 humidification of, process, 280
 mean specific heat of, 453
 perfect, specific heat of, 507
 preignition of, 464
 pressure rise in confined, process, 279
 pressure, volume and temperature relations, 302
 specific heats of, 295
 table of, 296
 variability of specific heat of, 296
- Gas flow, coefficient of friction, 635, 637
 density and calorific power of, 411
 from coke ovens and retorts, 410
 from pulverized coal, 446
 generator, 428
 oil, manufacture of, 415
 Pintsch, 415
 producer, 428
 horse-power of, 447
 plants, heat balance of, 606
 temperature variation in, 446
 various types of, 435
 yield of, 444
 standard, 339, 353
 thermometer, 285
 turbine, 566
 -water, 428
- Gasification of fixed carbon and coke, 418
- Gasolene, assumed as heptane, 456
- Gay-Lussac, law of, 303, 308
- Generator, heat supplied to in absorption refrigeration system, 673
- Gibbs phase rule, 311
- Goss, boiler tests, 477
- Goutal, formula, calorific power of coals, 399
- Graphic method of finding s , 18
 cylinder sizes, 272
 methods, work and power, 246, 272
 representation of work, 7, 9
- Guldner, table of volumetric efficiency for gas engine, 603
- Harter, formula for flow in nozzles, 624
- Heat, absorption of boilers, 476
 and gas consumption in engines, 569
 and matter, 274
 and work relations, 496
 balance of boilers (table), 488
 of gas and oil engines, 605
 producer plants (table), 606
 refrigerating systems, 673
 of steam plants (table), 608
 convection of, 358
 diagram, temperature-entropy, defined, 499
 use of, 626
 effects important in engineering, 274, 278
 effects of various processes, 278-282
 -entropy diagram, use of, 626
 equations, fundamental, for boilers, 481

- Heat, exchange in absorption refrigeration system, 665
 flow by radiation, 362
 resistance, 361
- Heat gains or losses, entropy of, 511
 internal conduction of, 357
 latent, 320, 325. *See* Latent heat.
 mechanical equivalent of, 289
 of combustion, 351
 of air gas, 425
 of water gas, 432
 of dissociation, 351
 of formation, combustion of hydrocarbons, 355
 of the liquid q , in steam, 293, 530
 of preheat in compressed air engines, 647, 650
 quantity of supplied per ton refrigeration, 672
 radiation, Stefan and Boltzmann's law, 362
- Heat resistance due to gas films, 381
 specific, 291
 variability of, 296
 effect of on conductivity of gases, 360
- suppression factor, 454
 total of vapors, 319
 transfer, classification of, 367
 coefficient for condensers, 383
 rate of, laws of, 381
 transmission of, 356
 between separate fluids, 364
 by gases, 365
 from hot gases, experiments in, 378
 variation in coefficient of, 375
- Heating processes, classification of, 277
 surface in boilers, 477
- Heats of reaction, 348
 for air gas, 424
 for water gas, 432
- Heck, formula for missing water, 264
- Height of chimneys, 642, 644
- Henry's law, 334
- Hexane, combustion of, 344
 conversion factors for, 346
- High calorific power, 349, 351
 initial pressures, effect of, 610
- Hodgkinson, test of steam turbine, 615
- Horse-power, boiler, of chimneys, 643
 brake, 37, 258
 definition of, 2
 effective, 37
 friction, 37, 258
 indicated, 36, 252
 of boilers, 473
 of gas producers, 447
 of nozzles and jets, 46
 per pound m. e. p., 36
 per ton refrigeration, 669, 672
 prediction of in gas engines, 600
- Hot water, flow of through nozzles, 620
- Humidification of air, 276
 and gases, process, 280
- Humidity, 331
- Hydrocarbons, heat of formation in combustion of, 355
 Slaby's formula for calorific value of, 355
- Ice-water-vapor, triple point, 313
- Ignition, effect of propagation on, 465
 producer gas, 465
 table of calculated, 463
 self, 464
 temperatures, 461
- Indicator, and indicator cards, 29
 cards, dense air refrigeration, 663
 combining of, from compound engines, 153
 for compression system of refrigeration, 665
 from triple expansion engines, 156
 from compound engines, 152
 relation of to pressure-volume diagrams, 29
 steam engine, with incorrect valve setting, 120
 triple expansion engines, 157
 typical for single-stage compressor, 101
 typical steam engine, 118
- Indicated consumption, 261
 horse-power, 36, 252
 of engines, various types, *see* under Efficiency.
- Indicated water rate, 261
- Initial pressure, effect of high on efficiency, 610
- Interborough Ry., tests on steam plant, 612
- Intercooling, 53, 100
- Internal conduction of heat, 357
- Intrinsic energy, defined, 503
- Isomers, 403
- Isothermal compound with adiabatics of gases, diagram of, 521
 compression, 50
 expansion, 18
 processes, 18
- Isothermals or PV products for carbon dioxide, 304
- Jacketed cylinders for steam engines, 615
- Jacobus, tests on Detroit Edison Co., boiler, 478
- Jet, calculation of velocity of, 622
 velocity of steam, 546, 559
- Joule's equivalent in foot-pounds, 503
 gas cycle (Brayton), 566
- Kerosene and distillate oils, 412
 and gasoline, calorific power of, formula for, 414
 crude oils for (table), 414
 assumed as dodecane, 456
 temperature correction for density, 413
 vapor pressure, of class, 330
- Kerosene, weight of, 413
- Kilogram-degree-centigrade, 288
- Kilogrammometer, 1
- Latent heat, 320, 325
 and entropy, 530
 Clapeyron's equation, 320
 external and internal, 325
 formula for, 530
 of ammonia, how obtained, 322
 of carbon dioxide, how obtained, 322

- Latent heat of fusion, 325
 of steam, 221
 of vaporization, 319
- Law of thermodynamics, first, 504
 second, 506, 556
- Lenoir gas engine, and cycle, 564
- Lignite, 393
 reactions of by heating, 407
- Liquefaction of gases and vapors, 304
- Liquids and vapors, equilibrium of, 329
 effect of heat on, description of processes, 278 et seq.
- Load curves for gas engines, 605
 for steam engines, 269
 division for steam engines, 272
 effect of on efficiency of gas engines, 598, 605
- Locomotive boiler test, Goss, 477
 Vaucrain, indicator cards, 152
- Locomotives, effect of initial pressures, 610
- Logarithmic expansion of steam, 531
 temperature difference, 372
- Low back pressure, steam, effect of, 610
 calorific power, 349
 pressure capacity, 56, 104
see also Compressors.
 pressure steam turbines, economy of, 613
- Mallard and Le Chatelier, experiments on
 flame propagation, 467
 ignition temperatures, 462
 mean specific heat, values of, 453
- Mariotte's law, 303
- Marks, formula, pressure-temperature of
 steam, 314
 and Davis, superheated steam, 297;
 curves, 298
- Mass flow, definition of, 381
- Maximum work of compressors, 98
- Maxwell, Clark, conductivity of gases, 360
- Mean back pressure defined, 27
 B. T. U., value of, 290
 calorie, 289
 effective pressure and mechanical efficiency,
 diagram of, 258
 compression gas cycles, 571, 575
 defined, 27
 non-compression gas cycles, 560
 of compressors by means of charts, 113
 variation of, 252
 of single-stage compressor, no clearance
 exponential law, 64
 logarithmic law, 56
 with clearance, logarithmic law,
 59, 61
 exponential law, 68, 70
 of two-stage compressor, no clearance
 exponential, 74
 with clearance, exponential, 78
 any receiver pressure, 81
 best receiver pressure, 87
 of three-stage compressors, 81
 of simple engine, no clearance
 logarithmic law, 125
 exponential law, 130
 with clearance, logarithmic, 134
- Mean effective pressure of simple engine,
 with clearance, exponential, 141
 of compound engine infinite receiver
 no clearance, logarithmic, 175
 exponential, 183
 with clearance, logarithmic,
 207
 exponential, 218
 finite receiver, logarithmic, 188
 exponential, 197, 231
 without receiver, logarithmic, 201
 with compression, 233
 exponential, 205
 of triple expansion engine, 241
 for gas cycles, Atkinson, 578
 Brayton, 580
 Brown, 570
 Carnot, 582
 Diesel, 581, 604
 Ericsson, 571
 Lenoir, 570
 Otto, 577, 604
 Otto and Langen, 570
 Stirling, 571
 steam cycles, Carnot, 559
 Rankine, 541
- Mean forward pressure, defined, 27
 variation in, 252
 specific heat of gases, 453
 temperature difference, 365, 367, 372
- Measurement of areas, 34
 of gas flow, 627, 632
 of *PV* diagrams, 34
- Mechanical efficiencies of gas engines (table),
 601
 efficiency, 38; diagram of, 258
 equivalent of heat, 288
 refrigeration. *See* Refrigeration.
- Melting of solids, 279
 points, 313
 effect of pressure on, 508
- Mineral oil distillates, calorific power of, 414
 fuel, 402
 oils, distillation of, 412
 densities of American (table), 413
- Missing water, 264; from indicator card, 618
- Mixture, air-gas, best, 456
 of gas and vapors, 327
- Molecular weight relation to specific volume and gas density, 307
- Mollier diagram, 527, 529
 use of for nozzle steam flow, 623
 in refrigeration cycles, 670
- Moyer, formula for nozzle flow, superheated
 steam, 624
 nozzle throat, 624
- Multiple-expansion engine, work and power,
 246
 engines, 148
- Multiple-stage compression, 53
 compressors, maximum work, 100
 expansion of steam through nozzles, 626
- Napier's formula for flow in nozzles, 624
- Natural gas, calorific power of, 405
 density of, 405
 fuel, 404

- Negative work, 20
- Net work, 20
- Non-reversible, and reversible processes, 282
- Nozzle friction and reheat, 621
- Nozzles and jets, horse-power of, 46
 - flow through, 620
 - for steam flow, formulas for, 624
 - best form of, 625
 - in series, 625
 - steam turbine, experiments on, 625
 - weight of flow through, 44
- Occlusion of liquids by solids, 334
- Oil, American mineral products, 413
 - distillates, calorific power of, 414
 - fuel with Diesel cycle engines, 589
 - gas, calorific power and density of, 416
 - manufacture of, 415
- Oils, calorific power of, 403
 - crude, density of, 414
 - fractionation of, 403, 413
 - mineral as fuel, 402
- Orifice, discharge of air through, 629
- Orifices, flow of hot water, steam and gases through, 620
- Orrok, experiments by on condensers, 382
- Otto and Langen gas cycle, 564
 - gas cycle, 565, 575, 578
 - affected by variability of specific heat of gases, 592
 - thermal efficiencies (table), 595
 - engines, performance of, 596
 - real volumetric efficiency, Güldner (table), 603
- Partial pressures in gas and vapor mixtures, 327
- Peat, 393
 - calorific power of, 393
 - reactions, 407
- Performance of Diesel and Otto engines, 596
 - of refrigerating cycles, 666
 - of steam engines, 607
 - and turbines, 268; together, 613
- Perry, formula for missing water, 264
 - on heat absorption, 482, 490
- Petroleum distillates, vapor pressure of, 330
- Phases and cycles of work, 20
 - thermal, primary, 510; secondary, 519
- Pintsch oil gas, explosive mixtures of, 455
 - how made, 415
- Pipes, flues, ducts and chimneys, flow in, 634
- Piston engines, work of, 115
 - See also Engines.
- Pitot tube, flow of gases, 627, 632
- Positive and negative work, 20
- Powdered coal, producer gas, 446
- Power and work, 1
 - defined, 2
 - gas and steam, comparison of, 588
 - required for refrigeration, 672
 - unit of, 2
- Preheating, effect of on air engines, 647, 650
- Preignition of gases, 464
- Pressure, absolute, 4
 - barometric rise of in confined gases, liquids, 279
- Pressure-volume change, work in terms of, 8
 - diagram, multiple expansion engine, 246
 - diagrams, relation of to indicator cards, 29
 - method uncertain for refrigeration cycle, 670
 - work in terms of, 3
- Pressures, limiting for compressors, 107
 - of gases and vapors, 327
 - partial in gas and vapor mixtures, 327
- Producer gas, 428
 - analysis of, 433
 - composition of, 441
 - density and calorific power of, 447
 - heat of combustion of, 443
 - ignition temperatures of, 463
 - Tait, 419
 - yield of, 444
 - plants, heat balance of (table), 606
- Producers, gas, 435
- Propagation, effect of on ignition, 465
 - rate of, 467
- Proximate analysis, 344
 - how made, 390
- Psychrometer, description of, 332
- Pulverized coal used in gas making, 446
- Pump displacement in refrigerating systems, 668
- PV diagrams, measurement of, 34
 - standard, 50
- PV method for horse power of nozzles and jets, 46
 - for velocity, free expansion, 42
 - for weight of flow through nozzles, 44
- PV products, isothermals, for carbon dioxide, 304
- Quantity of fluid per hour, 40
 - used in refrigeration, 667
- Quality of steam, saturated, x , 517
 - superheated, in degrees, 314
- Radiation, from black body, 362
 - heat flow, formula for, 362
- Rankine cycle, 541
 - diagram of, 542
 - example of, 547
 - relation of to Otto and Diesel, 591
 - reversed for refrigerating systems, 662, 670
 - work and efficiency, 541, 548
- Rate of combustion of gases, 467
 - of anthracite coals, 470
 - of bituminous coals, 470
 - effect of draft on, 469
 - of heat absorption in boilers, 480
 - of propagation of combustion, 467
 - of refrigeration, 667
- Rateau's formula for flow in nozzles, 624
- Reaction, heats of, 348
 - chemical, combustion, 281
 - endothermic, 281, 341
 - exothermic, 281, 341
- Receiver pressure two-stage compressor
 - best receiver pressure, 74, 78, 87
 - any receiver pressure, 81
- three-stage compressor, 91

- Receiver pressure compound engine, finite receiver,
 logarithmic law, 186, 222
 exponential, 195, 229
 triple expansion engine, infinite receiver,
 logarithmic law, 240
 variable pressures in, 162
 Receivers for steam engines, 149
 values of a and a^2 for (table), 193
 Refrigerants, how selected, 663
 list of available, 662
 Refrigerating cycles, 657, 660, 666
 performance of, 666
 fluid, 656
 substances, 275
 systems, heat balance of, 673
 Refrigeration, Brayton cycle used for, 671
 efficiency of, 671
 heat exchange in absorption system, 665
 heat supplied per ton, 672
 horse-power per ton, 669, 672
 indicator cards for dense air system, 663
 for compressor system, 665
 mechanical, 656
 per pound fluid, 669
 Rankine cycle for, 670
 systems of, 657, 659, 663, 665
 unit of, 666
 weight of ammonia used in, 668
 work of, 670
 Regenerator, use of with Stirling gas engine, 565, 571
 Regulation of compressor capacity, 108
 Reheat in nozzle friction, 621
 Reheating of air, effect of, 149
 Reheater, steam, effect of on efficiency, 616
 Resistance to heat flow, 361
 to heat transmission due to gas films, 381
 Retort coal gas comparison with coke oven, 410
 density and calorific power of, 411
 oil gas, yield of (table), 416
 Reversible and non-reversible processes, 282
 Reynolds, Osborne, rate of heat transfer, 381
 s (exponent), graphic method of finding, 18, 523
 value of, 17
 for air, 653
 for gases, 519
 steam and hot water, 524, 617
 Saturated steam, 314; vapors, 311, 314
 Saturation curve for steam, 324
 law, expansion or compression according to, 18
 Scotch marine boiler, firebox capacity of, 363
 Second law of thermodynamics, 506, 556
 Selby and Kemble experiments on steam turbine nozzles, 625
 Sherman and Kropff, calorific power of oils, 403
 Single-stage compressor, 56, 59, 61, 68, 70
 Sizes of coal, 396
 Slaby's formula, B. T. U. of hydrocarbons, 355
 Smoke, how formed, 458
 Solids, evaporation of, process, 280
 expansion of, process, 278
 Solids, melting of, process, 279
 solubility of in liquids, process, 281
 Solutions, aqueous (brines), freezing-point of, 316
 rise of boiling-points, 316
 Specific displacement, definition of, 40
 gravity, specification for oils, 413, 415
 heat, 291
 defined by equation, 507
 effect of on conductivity of gases, 360
 for perfect gases, 507
 of anhydrous ammonia, 294
 Specific heats of gases, effects of variability
 of on gas cycles, 592
 constants (table), 296
 mean of, gases, 453
 of gases and vapors, 295
 of solids, formula for, 293
 of superheated steam, 297
 of water, curve, 293
 Dieterici equation for, 530
 variability of, 296
 Specific volume, relation to molecular weight, 307
 Stack, *see* Chimney.
 Standard gas, 229, 353
 reference cycles for simple steam engines, 122
 for compound engines, 157
 PV diagrams, 50
 Steam, adiabatics for, 523
 and gas power, comparison of, 588
 boilers, *see* Boilers.
 consumption, 40, 259
 with variation of load, 267, 607, 618
 cycle, Rankine, relation of to Otto and Diesel, 591
 cycles, $T\phi$ and PV diagrams, 538
 Carnot, and derivatives, 554
 Rankine, 541
 engine, consumption of, 40, 259, 267, 618
 efficiency, at best load, 607
 perfect mechanism, 534
 referred to Rankine cycle, 609
 indicator card for reference, 118
 receivers, 149
 Steam engines, and turbines used together, 613
 effect of high initial pressure on, 610
 of low back pressure on, 611
 of superheated steam on, 613
 load curves of, 268
 performance of, actual, 607
 use of reheater with, 616
 use of jackets with, 615
 evaporation, 472
 expansion, values of s for, 617
 flow of through pipes, 634, 637
 jackets, effect of on efficiency, 615
 jet, velocity of, 546, 559, 622
 latent heat of, 221
 logarithmic expansion of, changes of quality, 531
 plant, heat balances of (table), 608
 tests on by Interborough Ry. Co., 612
 pressure, initial, effect of on efficiency, 610
 saturation curve for, 324

- Steam engines, superheated, specific heat of, 297
 curves of, 298
 total heat of, Davis formula for, 530;
 curves, 321
 Steam turbines, consumption of, 270
 efficiency of (table), 612
 effect of vacuum on, 615
 low pressure economy of, 613
 used with steam engines, 613
 quality of, 517
 value of s for, adiabatic expansion, 525
 Stefan and Boltzmann law, heat radiation, 363
 Stirling gas cycle, 565, 571
 Sublimation, curves of, 312
 process, 280
 vapor-solid relations, 312
 Suction, gas engine, resistance of (table), 603
 Sulphur dioxide, compression system of
 refrigeration, 657
 in coal, effect of, 397
 Superheated steam, definition of, 314
 effect of on efficiency, 613
 specific heat of, 297
 vapors, 311; definition of, 314
 Symbols, table of, xix
 Systems of refrigeration, 657

 Tait producer, air gas, 419
 Tar, from coal and oil retorts, 417
 Temperature absolute, 286
 critical of steam, 314, 323
 due to combustion, 453
 effect of on equilibrium of liquids, 329
 -entropy diagram, definition of, 499
 method, for refrigeration cycles, 670
 Fahrenheit and Centigrade, 285
 variation in gas producers, 446
 Temperatures allowable with compressed
 air engines, 651
 constant and variable, processes of, 282
 ignition, 461, 465
 table of for producer gas, 463
 in boiler furnace, 457
 Thermal conductivity, 357
 cycle, definition of, 498
 cycles, representing heat engine processes,
 534
 for refrigerating processes, 657, 660, 666
 efficiency, 39, 498
 of compressed air systems, 646
 Otto cycle for constant and variable
 specific heats (table), 595
 of gas engines, effect of load on, 605
 of steam engines, Rankine cycle, 548
 phases, primary, 510, 512
 secondary, 519
 Thermodynamic function (Rankine), 499
 Thermodynamics, definition of, 497
 engineering, province of, 497
 first law of, 504
 second law of, 506, 556
 Thermometry, 283
 Thermometer, dry and wet bulb, 322
 scales, 284
 Thomsen, heat of combustion of CO to CO₂,
 352
 Thomsen, heat of combustion of hydrogen
 350
 Three-stage compressor, 91
 Ton refrigeration, definition of, 666
 heat supplied per, 672
 horse-power of, 669, 672
 Total heat of steam, Davis formula for, 530
 diagram of, 321
 Transfer of heat, laws of, 380
 Transformation and transmission efficiency
 of, 37
 Transmission of heat, 356
 between separated fluids, 364
 coefficient of, 362, 365
 from hot gases experiments in, 378
 Triple-expansion engine, infinite receiver
 logarithmic, 240
 point-ice-water, vapor, 313
 Turbine, effect of low back pressure on, 611
 gas, possibility of, 566
 steam consumption of, 270
 efficiency of, 612
 Turbines and steam engines at best load, 607
 working together, 613
 Two-stage compressor, 74, 78, 81, 87

 Ultimate analysis, of fuels, 345, 390
 Unit of capacity for refrigeration, 666
 of evaporation, 473
 of distance, 1
 of force, 1
 of power, 2
 of work, 1
 Unit, British thermal, B. T. U.
 and calories, value of, 290
 and equivalent evaporative power, 349
 and mechanical equivalent of heat, 288
 Joule's equivalent in, 503
 U. S. gasoline- and kerosene-bearing crude
 oils (table), 414

 Vacuum condensers, coefficient of heat
 transmission in, 383
 Vacuum, effect of on efficiency, 615
 Value of exponent s , 17
 for adiabatic expansion of steam
 (table), 525
 for compressed air engines, 653
 for gases, 519
 for hot water, 524
 for steam tests showing, 617
 graphic method of finding, 18, 523
 Valve setting, incorrect, indicator cards
 showing, 120
 Valves, flow of gases through, 630
 van 't Hoff's law of equilibrium, 343
 Vapor and gas mixtures, 327
 pressure of atmosphere, 332
 of petroleum distillates, 330
 Vapor-solid, relations, 312
 tension, 311
 Vaporization, 311
 latent heat of, 319
 work done by, 319
 Vapors, superheated, 311, 314
 total heat of, 319
 Variation of steam consumption, 267

- Vaucrain locomotive, indicator cards, 152
 Velocity and acceleration, 5
 due to expansion, 42
 of gas, effect of on heat transfer, 380
 of steam jet, 546, 559
 determination of, 622
 of water, effect of on heat transfer, 378
 Venturi tube, flow of gases through, 627, 632
 Viscosity, change of with heat addition, 280
 Volatile of coals, calorific power of, 401
 Volume and pressure, work in terms of, 3
 or density variation with temperature, 299
 Volume-pressure change, work by, 8
 Volumetric and weight relations of gases, 347
 efficiency, 104
 compressors, *see* Efficiency.
 for dense air and compression refrigeration systems, 667
 of compressed air engines, and compressors, 653
 Otto gas engine, Güldner (table), 603
 Water consumption, 40
 gas, 428
 characteristics of Bunte (table), 428
 composition of, 430
 density and calorific power of, 434
 efficiency of process, 433
 heat of combustion, 432
 heats of reaction, 432
 volumes, weights, and heats, 429
 missing, 264
 -rate, cyclic, Carnot, 559; Rankine, 546
 of steam engines, 259, 267, 618
 specific heat of, curve, 293
 Water-vapor-ice, triple point, curve, 313
 vapor P - T relations, 314
 Weight and volumetric relation of gases, 347
 of flow through nozzles, 44
 Weisbach, coefficient of discharge for air, 629
 Willans line, 267; in steam engines, 604
 Wood, composition of, 391
 reactions of by heating, 406
 Work and power, 1
 multiple expansion engine, 246
 and capacity of compressors, 48, 50
 by pressure-volume change, 8
 compressors, partial capacity, 107
 defined, 1
 determination by mean effective pressure, 27
 diagram, 9
 distribution in multi-expansion engines, 148
 graphical representation of, 7, 9
 in terms of pressure and volume, 3
 of acceleration, 5
 of Carnot cycle and derivatives, 554
 of compressed air engine, 652
 of compressors by graphical methods, 113
 of expansion and compression, 11
 of gas cycles, Atkinson, 578
 Brayton, 580
 Work of gas cycles, Brown, 570
 Carnot, 582
 Diesel, 581
 Ericsson, 573
 Lenoir, 570
 Otto, 576; Otto complete expansion, 579
 Otto and Langen, 570
 Stirling, 572
 of piston engines, 115
 of Rankine cycle, 541
 of refrigeration, 670
 of single-stage compressor, logarithmic
 no clearance, 56; with clearance, 59
 no clearance exponential, 64
 with clearance, exponential, 68
 of single-stage compressor, in terms
 of dimensions, 61, 70
 of two-stage compressor, exponential
 no clearance, 74; with clearance, 78
 in terms of dimensions, any receiver pressure, 81
 best receiver pressure, 87
 of three-stage compressor, 91
 of simple piston engine, no clearance
 logarithmic, 125; exponential 130
 with clearance, logarithmic, 134
 exponential, 141
 of compound engine, infinite receiver
 no clearance, logarithmic, 173
 exponential, 182
 with clearance, logarithmic, 207
 exponential, 218
 of compound engine without receiver
 no clearance, logarithmic, 199
 exponential, 205
 with clearance, logarithmic, 232
 exponential, 237
 of compound engine, finite receiver
 no clearance, logarithmic, 186
 exponential, 195
 with clearance, logarithmic, 222
 exponential, 229
 of triple-expansion engine, 240
 of vaporization, 319
 phases and cycles, positive and negative
 and net work, 20
 unit of, 1
 Working fluid, must be expansive, 274
 x , quality of steam, 517
 Yield of air gas, 425
 of coke in ovens and retorts, 410
 of distillates and oil gas, 412
 of gas in coke making, 410
 chart of, 409
 of producer gas, 435, 444
 of retort oil gas (table), 416
 of water gas, 431
 Young, data of for carbon dioxide isothermals, 303

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